

On Mass Interaction Principle

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Abstract

This paper proposes mass interaction principle (MIP) as: the particles will be subjected to the random frictionless quantum Brownian motion by the collision of space time particle (STP) prevalent in spacetime. The change in the amount of action of the particles during each collision is an integer multiple of the Planck constant h . The motion of particles under the action of STP is a quantum Markov process. Under this principle, we infer that the statistical inertial mass of a particle is a statistical property that characterizes the difficulty of particle diffusion in spacetime. Starting from this principle, this article has the following six aspects of work: First, we derive the mass-diffusion coefficient uncertainty and the quantized commutation, and derive the most basic coordinate-momentum uncertainty and time-energy uncertainty of quantum mechanics, and then clearly reveal the particle-wave duality, which are properties exhibited by particles collided by STP. Second, we created the three decompositions of particles velocity. The comprehensive property of three velocities deduced the equation of motion of the particle as Schrödinger equation, and made a novel interpretation of Heisenberg's uncertainty principle and Feynman's path integral expression. And reexamine the quantum measurement problem, so that the EPR paradox can be explained in a self-consistent manner. Third, we reinterpret the physical origin of quantum spins. Each spacetime random impact not only gives the particle of matter the action of a Planck constant h , but also produces the quantum fluctuation properties of the material particles. Furthermore, the quantum spin properties of the material particles are also produced, which reveals the statistical quality of $1/2$ spin particles and the coexistence relationship of their spins. The particle spin properties ultimately reflect the spacetime properties of STP and particles, which leads to an important result that any spin $1/2$ elementary particles will be massive. Fourth, we derive the physical origin of the special relativity, and prove that the hypothesis of constant speed of light is actually the intrinsic property of spacetime. This naturally leads to the three basic inferences of the special theory of relativity, namely, "mass enhancement", "time dilation" and "length contraction" effects. Fifth, we can perfectly explain the nature of photons based on the topological and dynamic properties of STP, thus naturally obtain the complete electromagnetic theory and all important properties of charge. Sixth, we naturally derive the gravitation from the interaction of the fundamental particles of microscopic matter, namely the massive fermion and STP. Furthermore, we can judge from the overall perspective of modern physics that the inertia mass of fermions must be equal to the gravitational mass. We have obtained the equivalent principle combined with Newton's universal

gravitation, from which we can derive all the important contents of general relativity. Both inertial mass and gravitational mass are no longer the basic physical quantities. The two are indeed equivalent, which come from the statistical mass of STP collisions. This paper shows that STP can interact with matter particles in space time. The particles of matter are affected by the impact of STP and change their motional state. The mass of a particle is a statistical property exhibited by STP collision. Under the MIP framework of interaction between STP and matter particles, the relativistic quantum electromagnetic and spin properties of particles are all self-consistent. The relativistic behavior and quantum behavior of matter particles with statistical mass are all derived from the collision of STP on matter particles. We have systematically solved all the basic problems of modern physics with MIP, which is the common origin of special theory of relativity, general theory of relativity, electromagnetic theory and quantum mechanics.

Keywords: *Mass Interaction Principle , Special Relativity, Schrödinger Equation , Quantum Measurement , Entanglement , Uncertainty Relation, Path Integral , Neutrino, Electromagnetism, Photon, Spin*

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Contents

I. Introduction	7
A. Spacetime Fluctuation, STP and MIP	7
B. Inertia Mass is a Statistical Property	9
C. Realistic Interpretation of Quantum Mechanics	10
D. MIP and Statistical Properties of Spin	11
E. MIP and Electromagnetic Theory	11
F. MIP and Special Relativity	12
G. MIP and Newton's Universal Gravity	13
H. Outline	14
II. Chapter.2. Mass Interaction Principle	15
A. Proposing the MIP	15
B. the Nature of Spacetime within the framework of MIP	17
C. Energy spectrum of STP	18
III. Random Motion and Spacetime Diffusion Coefficient	20
A. Langevin Equation	20
B. Fokk-Planck Equation	23
C. From Discrete Spacetime to the Spacetime Diffusion Coefficient	25
D. From Spacetime Scattering to the Spacetime Diffusion Coefficient	26
IV. Mass-Diffusion Uncertainty relation	29
A. Classical Theory of Phase Space for Mass and Diffusion Coefficient	29
B. Mass-Diffusion Uncertainty	30
C. Energy-Time Uncertainty Relation	33
D. Position-Momentum Uncertainty Relation	35
E. Mass Measurement and Neutrino Oscillation	37
F. Neutrino Diffusion Experiment	37
V. Random Motion of Free Particle under MIP	38
A. Decompositions of the Real Velocity	38
B. From MIP to Schrödinger Equation	41

C. Physical Meanings of Potential Functions R and I	45
D. Space-time Random Motion of Charged Particles in Electromagnetic Field	46
E. Stationary Schrödinger Equation from MIP	48
F. Ground States of Hydrogen Atoms in MIP	51
1. Deriving the amount of elementary charge from MIP	53
2. Quantum number n of STP determining the orbit of hydrogen atoms	54
3. Generalisation to Hydrogen-like atoms	54
VI. Quantum Measurement in MIP	55
A. General Principle	55
B. EPR Paradox in MIP	56
VII. From MIP to Path Integral	58
A. Path Integral of Free Particle and Spacetime Interaction Coefficient	59
B. Path Integral of Particle in an External Potential and Spacetime Interaction Coefficient	61
VIII. Spin Within the Framework of MIP	62
A. MIP and Spin $1/2$	62
B. Statistical Properties of Spin	64
C. Spin Dynamics in MIP	65
1. Topological Subgroup of Lorentz Group	65
2. Topological Classifications of Non-Perturbative Fluctuations	66
3. Non-Perturbative Path and Statistical Origins of Spin	66
IX. Electromagnetism: An MIP Approach	67
A. Essential Properties of Electronic Charge In Modern Physics	67
Dynamics of STP revisited	67
B. 2+1-dim Complex Scalar Space-time field	68
C. Abrikosov-Nielsen-Olesen-Zumino Vortex	69
D. From 2+1-d to 3+1-d	71
E. The Origin of Photon from ANOZ Vortex	72
1. Dynamics on normal slice	72

	6
2. The Hodge duality	74
F. The Coulomb Force	76
G. Another Derivation of EoM of Photons	77
H. Photon and vortex tube	81
I. Conclusion of the section	82
X. MIP and Special Relativity	83
A. Equivalence between Inertial Reference Systems	83
B. STP Collision and Particle Mass	84
C. Time Dilation Effect	85
D. Relativistic Mass Effect	87
E. Length Contraction Effect	88
F. An Alternative Method of Deriving Special Relativity	90
XI. MIP and General Relativity	93
A. Electron and STP	94
B. Universal Gravity among Macroscopic Bodies	96
C. MIP and Equivalence Principle	98
XII. Summary	99
Appendix A: Brown Motion and Markov Process	100
Appendix B: Decomposition of Random Variables	101
Appendix C: From MIP to the Uncertainty Principle	102
Appendix D: Additional Physics Example with Three-speed Decomposition	104
Appendix E: Self Isomorphism on Direct Product Spin Clusters	105
Appendix F: Field Theory Calculations for Fermionic Loop Integral	106
Acknowledgement	107
References	107

I. INTRODUCTION

A. Spacetime Fluctuation, STP and MIP

We believe the energy fluctuations of spacetime are universal, which are defined as STP. In this picture, particles are classified into two groups: one is matter particles which interacting with STP, another one is massless particles which freely move in spacetime. Matter particles change their states by all the collisions with STP. The underlying property of mass is a statistical property emerging from random impacts in spacetime. Different particles have different effects of impact by STP, which can be defined as some kind of inertia property of particles. This property corresponds to mass dimension (Following we will prove it happens to be the inertial mass from Schrödinger's equation). Matter particles develop a Brownian motion due to random impacts from spacetime. We strongly suggest that all the probabilistic behaviours of quantum mechanics come from the Brownian motion, which is exactly the origin of quantum nature. In the framework of MIP, the photon represents itself as a Hopf link excitation made of the 2+1-dim gauge field and its Hodge dual partner. On the other hand, under the MIP framework, photons not only exchange electromagnetic interactions, they also exchange spin information. It just explains that the annihilation condition of positive and negative electrons is not only the opposite of charge, but also the opposite of its spin. In modern physics, the spin and charge of matter particles are independent quantum properties. However, the spin has a magnetic moment and indicates that the spin and electromagnetic interactions are related. Under the MIP framework, this apparent contradiction can be self-consistently explained.

We believe the quantum behaviours of matter particle come from spacetime fluctuation. Since the energy fluctuation of spacetime is quantised, among all the particles in spacetime which receive quantised energy from STP. The exchange of energy between particle and spacetime is not strictly random, it leads to a unique Brownian-like motion. Once the time interval of impact is fixed, the exchange of energy has to be quantised, which indeed is the quantum nature of particles. Therefore, all quantum nature of particles is a faithful representation of spacetime quantised fluctuation.

Definition 1:

Matter particles will perform random fluctuation motion in spacetime because of stochastic interactions between STP and matter particles, within which the energy exchange can not be achieved instantaneously. For free matter particles, we define the product of exchanged energy and the corresponding time interval as the change of action in the collision process.

Suppose STP begin to collide with matter particle at time t_1 and end it at at time t_2 to exchange energy E . Without the collision of STP, the action of particle at the same interval will be

$$S = \int_{t_1}^{t_2} E_0 dt \quad (1)$$

With the collision of STP, the action of particle at the same interval will be

$$S' = \int_{t_1}^{t_2} E(t) dt \quad (2)$$

Therefor the change of action in Definition 1 is

$$\delta S = S' - S = \int_{t_1}^{t_2} [E(t) - E_0] dt \equiv \int_{t_1}^{t_2} f(t) dt \quad (3)$$

By definition, integral function $f(t)$ is a monistic increasing function $f(t)$ with following property

$$f(t_1) = 0, f(t_2) = E \quad (4)$$

According to Mean value theorems for integrals, there exists one point t^* at the interval satisfying

$$\int_{t_1}^{t_2} f(t) dt = f(t^*)(t_2 - t_1) \quad (5)$$

Setting exchange of energy be $E^* = f(t^*)$ at this point, we have $0 < E^* < E$. So the exact formula of the change of action is

$$\delta S = E^* \delta t \quad (6)$$

where $\delta t \equiv t_2 - t_1$. Therefore we are sure that, it is this characteristic exchange energy $E^{(*)}$ not the energy of STP itself corresponding to the change of action. With MIP $\delta S = nh$, it's impossible to interact instantaneously, since the exchange energy E^* will blow up.

For free matter particle, we define the multiple $\delta E * \delta t$ as the variation of action(δS) of matter particle. Where the variation of energy is δE , and the time interval in this collision progress is δt .

According to the above two fundamental propositions: 1. spacetime fluctuations are universal; 2. spacetime fluctuations are quantised, we propose a MIP: Any particle with mass m will involve Brownian-like motions without frictions, due to random impacts from spacetime. Each impact changes the amount nh (n is any integer) for an action of the particle. The motion of a particle under the action of STP is a random motion of a quantum Markov process (quantum Brownian motion).

The MIP is absolutely essential to mass, spin, all quantum properties as well as relativity properties of matter particles. We will prove two important results. At first, within the framework of MIP, fundamental results of special and general relativity are natural inferences. Secondly, many important principles of modern quantum mechanics can be derived from MIP. Within this framework, MIP plays the role of the zeroth interaction, which dictates all quantum behaviours. Moreover, it will be shown that modern quantum field theory is compatible with MIP in the sense of quantum statistical partition functions.

B. Inertia Mass is a Statistical Property

Until now, our knowledge of mass, a fundamental concept of physics, mainly comes from Newton's laws of motion especially the first and second laws. The definition of mass in physics is a basic property of particles. The amount of matter contained in object is called the mass of object. The mass is related to the inertial nature of the object's original motion state.

The first law states that in an inertial reference frame, an object either remains at rest or continues to move at a constant speed, unless acted upon by a force. However according to the MIP, free particle has to do Brownian-like motions in spacetime, which is a Markov process. The mass of particle, in order to be sensed by spacetime, has to be collided randomly by STP. Mass cannot be well defined within the interval of two consecutive random collisions. In other words, mass is not a constant property belonging to the particle itself, but a discrete statistical property depending on dynamical collisions of spacetime. We will derive from MIP straightforwardly that mass must be a statistical term which has its own means and fluctuations.

Moreover, we prove the uncertainty relation asserting a fundamental limit to the precision regarding mass and diffusion coefficient. This implies that both mass and diffusion coefficient

of any particle state can not simultaneously be exactly measured. Newton's Second law states that in an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration of the object. This connects the concept of mass and inertia and in principle defines a fundamental approach to measure the mass of any particle experimentally. However, according to the MIP, forces on a particle are changed constantly by the random impact of STP. Therefore, we are no longer able to take constant mass for granted. In conclusion, we believe that mass as a statistical property is much more natural within the framework of modern science, which completely overrules Newton's concept of mass based on *Mathematical Principles of Natural Philosophy* first published in 1687.

C. **Realistic Interpretation of Quantum Mechanics**

The main idea of Copenhagen interpretation is that the wave function does not have any real existence in addition to the abstract concept. In this article we do not deny the internal consistency of Copenhagen interpretation. We admit that Copenhagen's quantum mechanics is a self-consistent theory. Einstein believed that for a complete physical theory, there must be such a requirement: a complete physical theory should include all of the physical reality, not merely its probable behaviour. From the materialistic point of view, the physical reality should be measured in principles, such as the position q and momentum p of particles. In the Copenhagen interpretation, the particle wave function $\Psi(q, t)$ or the momentum wave function $\Psi(p, t)$ is taken to be the only description of the physical system, which can not be called a complete physical theory, at most a phenomenological effective theory. Therefore, in this paper, we propose a MIP where the coordinate and momentum of particles are objective reality irrespective of observations. With the postulation of MIP, quantum behaviour will emerge from a statistical description of spacetime random impacts on the experimental scale, including Schrödinger's equation, Born rule, Heisenberg's uncertainty principle and Feynman's path integral formulation. Thus, we believe that non-relativistic quantum mechanics can be constructed under the MIP. Born rule and Heisenberg's uncertainty relation are no longer fundamental within our framework.

D. MIP and Statistical Properties of Spin

In modern quantum field theory, the spin properties of particles reflect the transform properties of particles under relativistic Lorentz transformation. The spin is a representation of the Lorentz group. The algebraic representation theory, simplifies the mathematical definition of spin, however it hides the fundamental physical properties of spin. Within the framework of MIP, particle spin is no longer an intrinsic property, but a representation of the interaction of particles and STP. Therefore, it is also a statistical property emerging from STP. In this paper, we will clarify that when a particle rotates in local spacetime, its topologically invariant path space happens to be the topological subgroup of the Lorentz group. The classification of the path spaces with these topological invariants is exactly the same as the algebraic representation of the Lorentz group. More importantly, through the Riemann function regulation, we can rigorously derive the eigenvalues of various spins. Therefore, spin is a statistical property within the framework of MIP.

E. MIP and Electromagnetic Theory

Within the framework of MIP, STP is spread over spacetime, and its energy spectrum distribution is consistent with scalar particles. It can therefore be thought of as an excitation of a scalar field. The influence of material particles on its spacetime is local, so on the 2+1-dimensional time-space slice, the influence of material particles on spacetime can be regarded as a potential energy.

In modern quantum field theory, an important point is that microscopic energy can be non-conservative, and it can fluctuate to form pairs of virtual positive and negative particles. Within the framework of MIP, the fluctuation of spacetime energy is itself STP. The number of STP particles is not conserved locally, but globally, the energy of STP is conserved. So the picture of STP as a free particle is restored on a large scale. This just shows that STP has some local symmetry, which is broken at large scale. In essence, when the domain symmetry of the authority is $U(1)$, STP is the excitation of a complex scalar field.

On the other hand, the spacetime can be regarded as 2+1-dimensional around the spacetime in which the material particles are located. On this 2+1-dimensional spatiotemporal slice, STP is the excitation of the complex scalar field, which is accompanied by the exci-

tation of the gauge field. The material particle produces a local non-perturbative potential energy in the surrounding space and time. The existence of this potential energy can cause the STP to spontaneously form a stable vortex solution. If the STP is not accompanied by a gauge field, then the vortex solution will cause the problem of energy divergence in the vortex center. The gauge field just eliminates the problem of local energy divergence.

The existence of a vortex solution also provides a possibility of duality, namely Hodge duality. The Hodge duality will extend the dynamics of the 2+1 dimensional gauge field to the 3+1 dimension. In the sense of Lagrangian, the 3+1-dimensional gauge field just describes the electromagnetic field theory. That is to say, the 3+1-dimensional equation of motion is Maxwell's equation. Therefore, we derive the classical electromagnetic theory from the vortex dynamics of STP.

In the MIP framework, the photon is essentially a topological excited state of two 2+1-dimensional gauge fields with their field strengths being Hodge's dual, and its topological configuration is a Hopf chain. Physically, photons transfer phase changes of material particles. Its equation of motion is the Maxwell equation.

On the other hand, the two topological circles of the photon, of which topological configuration Hopf link correspond to the topological subspace of the local spacetime. The Hopf links just represent the Lorentz representation of spin 1, which is a vector representation. Therefore, within the framework of MIP, the spin 1 of zero-mass photon is also self-consistently explained.

Within the framework of MIP, we have obtained five basic characteristics of electromagnetic interaction: 1. Positive and negative charge and the quantization of charge. 2. The same charges repel, and the different charges attract. 3. Forces of charge interaction satisfy the inverse square law of distance. 4. The charge is conserved. 5. The basic charge is $1.6 \times 10^{-19}C$.

F. MIP and Special Relativity

From the MIP point of view, the state change of free material particles can only be achieved by the impact of STP. We can say that when the microscopic properties of a particle of matter (such as its phase or spin eigenstate) change, the particle that propagates the information is a gauge particle. From MIP, we have obtained the classical electromagnetic

theory in section 9 of this paper. It is a theory that is invariant under the transformation of the inertial reference system.

In particular, the speed of light as a constant does not change under the transformation of inertial reference frame. Therefore, the assumption that the speed of light does not change is no longer a hypothesis, but a basic law. On the other hand, the interaction of STP on particles causes the particles to perform random fluctuations. The speed of this kind of fluctuation movement is very different from the classic speed. It is essentially a relative speed that is constant under a time reversal. This random Markov fluctuation is not related to the classical motion and is therefore invariant under the transformation of the inertial reference frame. Therefore, the equivalence between inertial reference systems is no longer an assumption, but a natural inference under the MIP framework. We can naturally derive some basic results from the special theory of relativity. Under the framework of MIP, the effects of "mass enhancement", "time dilation" and "length contraction" all have new physical meanings.

G. MIP and Newton's Universal Gravity

In the 3+1 dimension Minkowski spacetime, STP is a real scalar field. We consider the interaction between STP and matter particles. With the tree diagram approximation of quantum field theory, the interaction among matter particles induced by STP is finally embodied as the inverse square law and proportional to the product of mass of matter particles. We show that Newton's theory of gravity is an effective theory within the framework of MIP.

Furthermore, we can judge from the overall perspective of modern physics that the inertia mass of fermions must be equal to the gravitational mass. We have obtained the equivalent principle combined with Newton's universal gravitation, from which we can derive all the important contents of general relativity. Both inertial mass and gravitational mass are no longer the basic physical quantities. The two are indeed equivalent, which come from the statistical mass of STP collisions.

H. Outline

In summary, MIP provides quantum mechanics, special and general relativity a materialistic basis, where an intuitive physical picture can be constructed. In this picture, mass is a statistical property which emerges when a large number of STP collisions occur. The spin represents the statistical properties of the interaction of the particle with the STP around it. Charges interact with each other through the topological excited state of gauge field in 2+1-dim and its Hodge dual partner, i.e. photon. Within the framework of MIP, Born's probability interpretation, Heisenberg's uncertainty principle no longer are basic principles, but only the natural consequences of MIP. The main results and conclusions of this paper are as follows:

In Section 2, we propose MIP and the fundamental definition 1, and obtain the energy spectrum distribution of the STP.

In Section 3, based on MIP from the general random motion process, we derive the universal spacetime diffusion coefficients of particles.

In Section 4, within the framework of MIP, we derive the spacetime diffusion coefficient and introduce a non-relativistic statistical inertial mass. It is especially important that statistical inertial mass is a measure of how easily particles diffuse in spacetime, which is therefore a statistical property. It can be inferred that there is an uncertainty relationship between statistical inertial mass and spacetime diffusion coefficient, The most fundamental coordinate-momentum uncertainty relationship of quantum mechanics can be derived from this uncertainty relationship. Therefore, the wave-particle duality and the Heisenberg's uncertainty relationship are the characteristics of the STP colliding particles within the framework of MIP.

In Section 5, we point out that the motion of particles in spacetime is a Markov process, which will emerge as a quantum wave, which satisfies the Schrödinger equation. Within the framework of MIP, we reinterpret the Born's rule.

In Section 6, we reinterpret the Feynman path integral and construct a system of non-relativistic quantum mechanics from materialist epistemology. We demonstrate the principle of quantum mechanics and the compatibility of path integrals. Based on the path integral and MIP, we derive the path integral of the free particle, wave function in the potential field and the steady state wave function.

Section 7 of this paper provides an explanation of the quantum measurement within the framework of MIP. The effects of the measurement leading to wavepacket collapse can be well understood. EPR can also be well explained within the framework of MIP.

In Section 8, the particle will rotate around the spacetime topology subspace under the action of the STP. The statistical effect of such a rotation is reflected in the particle's spin. Within this framework, the classification and origin of spins are related to spacetime.

In Section 9, we consider the 2+1-dimensional dynamics of a complex scalar field. The existence of matter particles causes the STP to form a vortex structure solution around the material particles. The existence of this solution extends the 2+1 dimensional gauge field to the 3+1 dimension and derives Maxwell's equations. The photon appears as a Hopf chain solution of two three-dimensional selfdual gauge fields, which is a topological invariant configuration. We show that the existence of a spacetime ANOZ vortex solution can explain the origin of the charge and electromagnetic interaction.

In Section 10, we derive three important results of the special theory of relativity from MIP, which are mass enhancement, time dilation and length contraction. These effects can be self-consistently explained within the framework of MIP.

In Section 11 , we derive Newtonian gravitational forces between two matter particles from MIP.

Finally, in Section 12 we summarize and explore the future of research directions.

II. CHAPTER.2. MASS INTERACTION PRINCIPLE

A. Proposing the MIP

Particles moving in spacetime interact with STP. The generation of STP itself should be regarded as a microscopic random excitation of local spacetime energy. We can assume the following twoself-consistent ideal STP models. First, the spacetime itself is discrete, and each of the smallest spacetime units can act on the particle to change the particle's motion . However this spacetime unit acts as a random force on the particles, the motion of the particles in spacetime under the action of STP will also be random. Second, the energy distribution of STP is Gaussian, therefore, when they were scattering with matter particle, the force is random.

Furthermore, we propose in each interaction between matter particle and STP, the exchanging action should be nh , with n integer and h the Planck constant. In our MIP framework, there are no instant interactions between matter particle and STP, in other words, the interaction takes time to transfer the energy. If the scattering STP has an extremely low energy such that in Δt , the transferred action is less than h , we conclude that in Δt , the STP cannot collide the particle. We argue that such a collision is still in process, the particle as well as the STP are in a bound state, not a scattering state. This is similar to a completely inelastic collision in classic mechanics. While in such a process, the conservation of energy and momentum can not be satisfied simultaneously. Because of conservation of energy and momentum, the bound state actually is not a stable state. This observation leads to an important point: there exists a minimal energy E_{min} in Δt so that

$$E_{min}\Delta t = const. \quad (7)$$

In physics, the product of energy and time will have the dimension of action. It is natural to suggest such a constant with action dimension is the Planck constant, so we have

$$E_{min}\Delta t = nh, n \in Z. \quad (8)$$

At a certain moment, particle can be scattered by many STP with different momenta and energies. In Δt , we assume there are effectively N collisions. The state of the motion will depend on the net effect of the N times collision. This is a principle of superposition. We can use in total N vectors to superposit whole changes of the state of motion, which means if at time t the particle was at position $\vec{X}(t)$, with speed \vec{V}_0 , then at the moment $t + \Delta t$, its position will be $\vec{x}(t + \Delta t) = \vec{X}(t) + \sum_{i=1}^N \Delta X_i$, and speed $\vec{V}_0 + \sum_{i=1}^N \Delta \vec{V}_i$. This simple analysis tells us in Δt , the ultimate state of motion of the particle can be separated as N different paths. This is the effect of separation of paths. While the weights of these paths, *aka* the probability distribution of universal diffusion, highly rely on the energy distribution of STP. Collisions by STP with different energies end up with different changes of the state of motion.

B. the Nature of Spacetime within the framework of MIP

At the beginning of the 20th century, the null result of the Michaelson-Morley experiment ended the ether theory. Within the framework of MIP, the concept of spacetime looks very similar to that of ether, but it is fundamentally different. To clarify this, let us first review the concept of ether. The ether is a gas medium filled in Newton's absolute static time and space. Its definition directly introduces a reference frame of God's perspective, which is Newton's static spacetime system. The earth and this frame of reference are relatively moving, so they will feel the ether wind blowing, which is the experimental basis of the Michaelson-Morley experiment. But spacetime is not a gaseous medium filled with absolute time and space. It is the fluctuation of time and space. From a large scale, the fluctuation of spacetime does not have significant effects. Spacetime seems to be smooth and differentiable, and the differential geometry theory of general relativity can effectively describe the physical properties of large-scale spacetime. However, on the microscopic scale, the fluctuation of spacetime indicates that spacetime itself does not have continuous property. There is no absolute static spacetime reference frame in the above discussion, so the STP within the framework of MIP is not etheric.

The null result of the Michaelson-Morley experiment actually promoted Einstein's most important hypothesis of the theory of relativity, which is the constant speed of light. In the theory of relativity, the constant speed of light is the only absolute assumption, and the relativity of all other speeds remains.

Within the framework of MIP, the energy fluctuation of spacetime forms STP. If you think of spacetime as a peaceful lake, then STP is the splash of water on the surface of the lake. When it falls on the surface of the lake, it will form ripples. Therefore, the emergence of STP is always accompanied by the spread of ripple. The propagation speed of ripple is the characteristic propagation speed in spacetime. Forming a STP means that fluctuation of spacetime will spread to a certain spatial distance within a certain period of time, so the spacetime around the STP is also changed. We now know that the smallest scale of time is the Planck scale, and the smallest scale of space is the Planck length. In the Planck time STP has to spread a Planck length of space, so the propagating speed of STP is the same as light speed.

From the spacetime view of MIP, any physical observable event in spacetime will inevitably

accompany the fluctuation of spacetime energy, which will profoundly affect the spacetime after the event. Under such a view of spacetime, the current spacetime is actually the result of the joint influences of all events in the history.

C. Energy spectrum of STP

To consider the collision between STP and particle, it will be ambiguous if the energy spectrum of STP is not clear at first. In this subsection, we deal with this problem.

Let us consider a cubic with volume L^3 , which we call a system. If there are in total N systems in spacetime, we can classify the N systems by states. We label a state by j so that there are N_j systems with energy E_j . The total energy of the ensemble (collection of N systems) is denoted as \mathcal{E} , we have

$$N = \sum_j N_j \quad (9)$$

$$\mathcal{E} = \sum_j N_j E_j, \quad (10)$$

for constant \mathcal{E} and N , the possible total number of states in whole spacetime will be $\Omega = \frac{N!}{\prod_j N_j!}$. Physical reality is required by the maximum of Ω . There is a distribution $\{N_j\}$ maximizing Ω , so that

$$\ln \Omega = N \ln N - N - \sum_j N_j \ln N_j + \sum_j N_j \cdots \quad (11)$$

the question is under constraints (9,10), how to maximize $\ln \Omega$. With the method of Lagrangian multiplier,

$$\frac{\partial \ln \Omega}{\partial N_j} - \lambda_1 \frac{\partial \sum_j N_j}{\partial N_j} - \lambda_2 \frac{\partial (\sum_j N_j E_j)}{\partial N_j} = 0 \quad (12)$$

we can derive

$$\begin{aligned} -\ln N_j - \lambda_1 - \lambda_2 E_j &= 1 \Rightarrow \\ N_j &= e^{-1-\lambda_1-\lambda_2 E_j} \end{aligned} \quad (13)$$

hence the probability of being at state j

$$P_j = \frac{N_j}{N} = \frac{e^{-\lambda_1-\lambda_2 E_j}}{\sum_j e^{-\lambda_1-\lambda_2 E_j}} = \frac{e^{-\lambda_2 E_j}}{\sum_j e^{-\lambda_2 E_j}} \equiv \frac{e^{-\lambda_2 E_j}}{\mathcal{Z}} \quad (14)$$

and the average energy of the ensemble

$$E = \frac{\mathcal{E}}{N} = \sum_j E_j P_j = -\frac{\partial}{\partial \lambda_2} \ln \mathcal{Z} \quad (15)$$

In L^3 , suppose there are $n_{\vec{p}} = 0, 1, 2, \dots$ STP have momentum \vec{p} , for giving distribution $\{n_{\vec{p}}\}$, energy in L^3 is

$$E = \sum_{\{n_{\vec{p}}\}} n_{\vec{p}} E_{\vec{p}} \quad (16)$$

with $E_{\vec{p}} = c|\vec{p}| = cp$. Here STP are massless as proposed. We have

$$\begin{aligned} \mathcal{Z} &= \sum_{\{n_{\vec{p}}\}} e^{-\lambda_2 E} = \prod_{\vec{p}} (1 + e^{-c\lambda_2 p} + e^{-2c\lambda_2 p} + \dots) \\ &= \prod_{\vec{p}} \frac{1}{1 - e^{-c\lambda_2 p}} \end{aligned} \quad (17)$$

and the average energy of a system is

$$\begin{aligned} E &= -\frac{\partial}{\partial \lambda_2} \ln \mathcal{Z} = \frac{\partial}{\partial \lambda_2} \sum_{\vec{p}} \ln (1 - e^{-c\lambda_2 p}) \\ &= \sum_{\vec{p}} \frac{pe^{-c\lambda_2 p}}{1 - e^{-c\lambda_2 p}} = \sum_{\vec{p}} \frac{cp}{e^{c\lambda_2 p} - 1} \end{aligned} \quad (18)$$

when $L \rightarrow \infty$, summation becomes integration as follow

$$\sum_{\vec{p}} \frac{L^3}{8\pi^3} \int d^3\vec{p} \quad (19)$$

from which we see

$$E = \frac{L^3}{2\pi^2} \int dp \frac{p^3}{e^{c\lambda_2 p} - 1} = \frac{\pi^2 L^3}{30\lambda_2^4} \quad (20)$$

so the density of STP will be

$$\epsilon_{ST} = \frac{\pi^2}{30\lambda_2^4} \quad (21)$$

Recover c and \hbar in above equation, we obtain

$$\epsilon_{ST} = \frac{\pi^2}{30c^3 \hbar^3 \lambda_2^4}. \quad (22)$$

Now consider the physical meaning of λ_2 , which determines the constraint that representing energy distribution of STP. While the multiplier λ_1 which determines the constraint represents the number distribution of STP has no effects on the dynamics of STP. This means we

can classify STP arbitrarily, except to satisfy the total energy constraint. From dimensional analysis and MIP, we have

$$\lambda_2 = \frac{g}{E_{ST}} \quad (23)$$

where g is a dimensionless coupling constant, and E_{ST} is the characteristic energy of STP. In the limit of extreme relativity, the colliding of STP can not be seen as perturbations, but strong interactions.

III. RANDOM MOTION AND SPACETIME DIFFUSION COEFFICIENT

Let m_{ST} be the statistical mass of the particle. We will prove the spacetime interaction coefficient of a m_{ST} mass particle will be universally given as

$$\mathfrak{R} = \frac{h}{2m_{ST}}. \quad (24)$$

Within the framework of random motion[1], or Wiener process in mathematics [2], this spacetime induced random motion is equivalent to the Markov process, moreover, the spacetime interaction coefficient is nothing but the diffusion coefficient [3]. In this section, we will start our journey from probability theory of random motion[3, 4], and then give a concrete proof that for the random motion induced by MIP, the spacetime interaction coefficient is given exactly by (24). The last two subsections discussed two spacetime models in order to investigate the origin of the spacetime interaction coefficient. From both we obtained the coefficient reading as $\mathfrak{R} = \frac{w\ell}{2}$, in which w is the average speed of the particle and ℓ the mean free path.

A. Langevin Equation

The space-time background can be seen as a fluctuation environment, and the particles move in this fluctuation environment. This is a Markov process. The position of the particle \vec{q} is a random quantity. From a strict mathematical point of view, it can be decomposed into a super random part and a superimposable function

$$\vec{q}(t) = \vec{q}_0(t) + \vec{\omega}(t) \quad (25)$$

where $\vec{q}_0(t)$ is the differential part of position and $\vec{\omega}(t)$ represents random motions of particles. The whole motion of particle can be described by Langevin equation as

$$\frac{\delta q_i(t)}{\delta t} = \frac{dq_{0,i}(t)}{dt} + \frac{\delta \omega_i(t)}{\delta t} = U_i(\mathbf{q}(t)) + \nu_i(t) \quad (26)$$

In spacetime, particles are subjected to the impact of STP. But if some of the impact is relatively weak, then the change of the state of motion can only be regarded as a perturbation. Under perturbation, the velocity of the particles changes which can be seen as smoothly and continuously. The non-perturbative impacts of STP on the particles instantaneously change the motion state of the particles, leading to the completely random motion. Each impact should be treated as a sum of a differential impact and a random impact. A microscopic impact does not change the classic trajectory of the particle, but it will cause the trajectory to be superimposed on the motion of an envelope. This is precisely the “differentiable velocity function” $\mathbf{U}(\mathbf{q}(t))$ expressed by the first term in the three velocities decomposition of the Langevin’s equation. Therefore, the true velocity of the particle $\mathbf{V}(t)$ should contain three contributions, which is

$$\mathbf{V}(t) = \mathbf{v}(t) + \mathbf{u}(\mathbf{q}(t)) + \vec{\nu}(t) \quad (27)$$

Where $\mathbf{v}(t)$ is the classic statistical velocity, $\mathbf{u}(\mathbf{q}(t))$ is the quantum envelope velocity of the particle, and $\vec{\nu}(t)$ is the diffusion velocity representing random motion. $\mathbf{U}(\mathbf{q}(t))$ denotes the union of the first and the second term in eq.(27)

$$\mathbf{U}(\mathbf{q}(t)) = \mathbf{v}(t) + \mathbf{u}(\mathbf{q}(t)) \quad (28)$$

For a Markov process, the average contribution of white noise vanishes. However, because of its Gaussian nature, its variation is not zero. We have

$$\langle \nu_i \rangle_\nu = 0, \quad \langle \nu_i(t) \nu_j(t') \rangle_\nu = \Omega \delta_{i,j} \delta(t - t'), \quad t \geq t' \quad (29)$$

here the $\delta_{i,j}$ in the later equation can be obtained from the spacetime homogeneous property, while $\delta(t - t')$ determined from the Markov property. For a Markov process, only conditions at the very moment determine the dynamics of the system, and all information from future or past are irrelevant. We can write down the basic correlation function by introducing a

probability measure $[d\rho(\nu)]$, which is given as

$$[d\rho(\nu)] := \left(\sqrt{\frac{1}{2\pi\Omega\delta(t-t')}} \right)^D [d\nu] \exp \left(-\frac{1}{2\Omega} \int dt \sum_i \nu_i^2 \right) \quad (30)$$

It is easy to see that

$$\langle \nu_i(t) \rangle_\nu \equiv \int \nu_i(t) [d\rho(\nu)] = 0 \quad (31)$$

$$\langle \nu_i(t) \nu_j(t') \rangle_\nu \equiv \int \nu_i(t) \nu_j(t') [d\rho(\nu)] = \Omega \delta_{i,j} \delta(t-t') \quad (32)$$

Here Ω describes the strength of spacetime interaction on the particle. Notice $\delta(t-t')$ has the inverse dimension of time t , as

$$\int_0^\infty \delta(t-t') dt = 1.$$

However, from the definition of measure (30), we can see, ν_i have the unit of m/s , so Ω will have the unit of m^2/s . From previous analysis, each collision leads to a change of an action h . h has the unit of angular momentum, $kg \cdot m^2/s$. From this we can define a quantity with mass unit, it is

$$m_{ST} \equiv \frac{h}{\Omega}. \quad (33)$$

The mass m_{ST} has the meaning such that it is the mass collided by STP and is a statistical property. Accordingly, the collision parameter $\Omega = \frac{h}{m_{ST}}$ reflects a physical realistic viewpoint: an object in our real nature, the larger its mass means the smaller its quantum effect.

Langevin equation generates a time-dependent probability such that

$$\mathbf{P}[\mathbf{q}, t; \mathbf{q}', t'] = \left\langle \prod_{i=1}^D \delta[q_i(t) - q'_i(t')] \right\rangle_\nu, \quad t \geq t' \quad (34)$$

which means for an operator $\mathcal{O}[\mathbf{q}]$, its average value at time t will be:

$$\langle \mathcal{O}[\mathbf{q}(t)] \rangle_\nu \equiv \int \mathbf{P}[\mathbf{q}, t; \mathbf{q}', t'] \mathcal{O}[\mathbf{q}] d\mathbf{q} \quad (35)$$

Using the probability distribution (34), one can immediately verify equation (35). Actually, the distribution (34) can be seen as an evolution process, which says

$$\mathbf{P}[\mathbf{q}, t; \mathbf{q}', t'] = \iint q(t) e^{-(t-t')H(p,q)} q'(t') d^D p$$

here the evolution Hamiltonian is the famous Fokk-Planck Hamiltonian, as we will derive its formalism in next subsection.

B. Fokk-Planck Equation

Given the Langevin equation (26), we can derive the corresponding Fokk-Planck equation, as well as the Fokk-Planck Hamiltonian [3].

We consider the time segment from t to $t + \epsilon$, $\epsilon \rightarrow 0$, and have the Langevin equation as:

$$q_i(t + \epsilon) - q_i(t) = \epsilon U_i(\mathbf{q}(t)) + \int_t^{t+\epsilon} \nu_i(\tau) d\tau + O(\epsilon^2) \quad (36)$$

its related propability distribution is

$$\mathbf{P}[\mathbf{q}, t + \epsilon; \mathbf{q}', t] = \langle \delta(\mathbf{q} - \mathbf{q}(t + \epsilon)) \rangle_\nu \quad (37)$$

According MIP, everytime the STP collided with the particle, the action of particle will change nh , $n \in \mathbb{Z}$. To obtain the Fokk-Planck equation, we define following discreterization

$$\bar{\nu}_i \equiv \frac{1}{\sqrt{\epsilon}} \int_t^{t+\epsilon} \nu_i(\tau) d\tau$$

so that the discrete Langevin equation is

$$q_i(t + \epsilon) - q_i(t) = -\frac{1}{2} \epsilon f_i(\mathbf{q}(t)) + \sqrt{\epsilon} \bar{\nu}_i + O(\epsilon^2) \quad (38)$$

Notice here the time has been discreterized as

$$(t - t')/\epsilon \in \mathbb{Z}^+.$$

Now the Gaussian distribution and the property of Markov progress determins the average value of discrete white noises ν_i , and we have

$$\langle \bar{\nu}_i \rangle_\nu = 0, \quad \langle \bar{\nu}_i(t) \bar{\nu}_j(t') \rangle_\nu = \frac{\hbar}{m_{ST}} \delta_{i,j} \delta_{t,t'} \quad (39)$$

When $\epsilon \rightarrow 0$, the Fourier transformation of the probability distribution (37) is

$$\begin{aligned} \tilde{\mathbf{P}}[\mathbf{p}, t + \epsilon; \mathbf{q}', t] &= \int e^{-i\mathbf{p} \cdot \mathbf{q}} \mathbf{P}[\mathbf{q}, t + \epsilon; \mathbf{q}', t] d^D \mathbf{q} \\ &= \langle e^{-i\mathbf{p} \cdot \mathbf{q}(t+\epsilon)} \rangle_\nu \\ &= \langle e^{-i\mathbf{p} \cdot (\mathbf{q}(t) + \epsilon \frac{d\mathbf{q}(t)}{dt} + O(\epsilon^2))} \rangle_\nu \\ &= \langle \exp(-i\mathbf{p} \cdot (\mathbf{q}'(t) - \epsilon/2 \mathbf{f}(\mathbf{q}')))) \rangle_\nu \\ &\quad \times \left\langle \exp \left[-i\mathbf{p} \cdot \int_t^{t+\epsilon} \nu(\tau) d\tau \right] \right\rangle_\nu \times \langle \exp(O(\epsilon^2)) \rangle_\nu \\ &= \exp[-i\mathbf{p} \cdot (\mathbf{q}' - \epsilon f(\mathbf{q}')/2)] \\ &\quad \times \left\langle \exp \left[-i\mathbf{p} \cdot \int_t^{t+\epsilon} \nu(\tau) d\tau \right] \right\rangle_\nu \end{aligned} \quad (40)$$

Notice that the last average value can be evaluated out by Gaussian integration, which reads,

$$\begin{aligned}
 & \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d\nu] \exp \left(-\frac{m_{ST}}{2h} \int dt \sum_i^D \nu_i^2 \right) \\
 & \quad \times \exp \left[-i\mathbf{p} \cdot \int_t^{t+\epsilon} \nu(\tau) d\tau \right] \\
 = & \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d\nu] \\
 & \quad \times \exp \left(-\frac{m_{ST}}{2h} \int dt \sum_i \nu_i^2 - i\mathbf{p} \cdot \int_t^{t+\epsilon} \nu(\tau) d\tau \right) \\
 = & \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d\nu] \exp \left(-\frac{m_{ST}}{2h} \int dt \sum_i \nu_i^2 - i\sqrt{\epsilon} \mathbf{p} \cdot \bar{\nu} \right) \\
 & \times \exp \left(+\epsilon \frac{h}{2m_{ST}} \mathbf{p} \cdot \mathbf{p} - \epsilon \frac{h}{2m_{ST}} \mathbf{p} \cdot \mathbf{p} \right) \\
 = & \left(\sqrt{\frac{h}{2\pi}} \right)^D \int [d^N \left(\nu_i + \frac{ih}{2m_{ST}} \sqrt{\epsilon} p_i \right)] \\
 & \times \exp \left(-\frac{m_{ST}}{2h} \int dt \sum_{i=1}^D \left(\nu_i + \sqrt{\epsilon} \frac{ih}{2m_{ST}} p_i \right)^2 \right. \\
 & \quad \left. - \epsilon \frac{h}{2m_{ST}} \mathbf{p} \cdot \mathbf{p} \right) \\
 = & \exp(-\epsilon h \mathbf{p} \cdot \mathbf{p} / 2m_{ST})
 \end{aligned} \tag{41}$$

here we obtain the probability distribution under Fourier transformation ,

$$\tilde{\mathbf{P}}[\mathbf{p}, t + \epsilon; \mathbf{q}', t] = e^{-\epsilon h / 2m_{ST} \mathbf{p} \cdot \mathbf{p} + i\epsilon \mathbf{p} \cdot f(\mathbf{q}') / 2 - i\mathbf{p} \cdot \mathbf{q}'} \tag{42}$$

for $\epsilon \rightarrow 0$, expanding (42) will end up with

$$\tilde{\mathbf{P}}[\mathbf{p}, t + \epsilon; \mathbf{q}', t] = e^{-i\mathbf{p} \cdot \mathbf{q}'} (1 - \epsilon H_{FP}(\mathbf{p}, \mathbf{q}') + O(\epsilon^2)).$$

Here we obtained the Fokk-Planck Hamiltonian

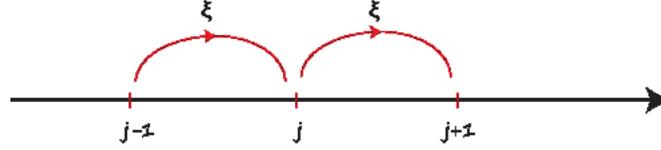
$$H_{FP}(\mathbf{p}, \mathbf{q}) = -\frac{h}{2m_{ST}} \mathbf{p} \cdot \mathbf{p} - i\mathbf{p} \cdot f(\mathbf{q}) / 2 \tag{43}$$

From which we can read off the diffusion coefficient induced by collisions between STP and the particle, is exactly $\mathfrak{K} = h/2m_{ST}$. Later we will see in deriving the Schrödinger equation of free particle in spacetime, the spacetime mass $m_{ST} = 2\pi m$ will be identified as the inertial mass, in the framework of non-relativistic quantum mechanics.

C. From Discrete Spacetime to the Spacetime Diffusion Coefficient

Beginning with MIP, we want to investigate the origin of spacetime interaction coefficient. Within the framework of discrete spacetime, spacetime diffusion coefficient $\mathfrak{R} = \frac{h}{2m_{ST}}$ should be derived in terms of parameters of discrete spacetime. Let us consider the simplest discrete model (see Fig.1), where the length union of discrete space is ℓ . $P(j, t)$ is the probability of a particle at lattice site j at time t .

Figure 1: Random jumping model on one dimensional lattice



Because of the discrete nature of the space, all jumpings can only happen between nearest pair of positions. Given the rate of jumping between the nearest neighbour ζ and the isotropy of frictionless space, the evolution of probability should be

$$\partial_t P(j, t) = \zeta \left(\frac{1}{2} P(j-1, t) + \frac{1}{2} P(j+1, t) - P(j, t) \right) \quad (44)$$

the first two terms of RHS of (44) describe the fact that jumping forward and backward from neighbors $j-1$ and $j+1$ positions respectively, have the same probability, which is $1/2$, the third term remarks the probability from j position to neighbors. Introducing the fundamental spacing of the lattice ℓ , the eq.(44) goes to

$$\partial_t P(j, t) = \frac{\zeta \ell^2}{2} \left(\frac{P(j+1, t) - P(j, t)}{\ell} - \frac{P(j, t) - P(j-1, t)}{\ell} \right) \quad (45)$$

In the continuum limit of spacetime, which says $\ell \rightarrow 0$, and $\zeta \rightarrow \infty$, but keeping the quantity $\zeta \ell^2$ unchanged, the probability $P(j, t)$ now becomes the probability density $\rho(x, t)$, and the RHS of (44) becomes the definition of second derivative. Thus we have

$$\partial_t \rho(x, t) = \frac{\zeta \ell^2}{2} \partial_x^2 \rho(x, t). \quad (46)$$

It is straightforward to generalise above equation to three dimension case, we have,

$$\partial_t \rho(\vec{r}, t) = \frac{\zeta \ell^2}{2} \nabla^2 \rho(\vec{r}, t) \quad (47)$$

Comparing with diffusion equation in Einstein's paper[6]

$$\partial_t \rho(\vec{r}, t) = \mathfrak{R} \nabla^2 \rho(\vec{r}, t) \quad (48)$$

the microscopic origin of spacetime diffusion coefficient will be

$$\mathfrak{R} = \frac{\zeta \ell^2}{2} \quad (49)$$

Furthermore, we can also discrete time with union $\tau = \frac{\ell}{w}$, where w is the average speed of particle. With $\zeta = \frac{1}{\tau}$, we obtain

$$\mathfrak{R} = \frac{w\ell}{2} \quad (50)$$

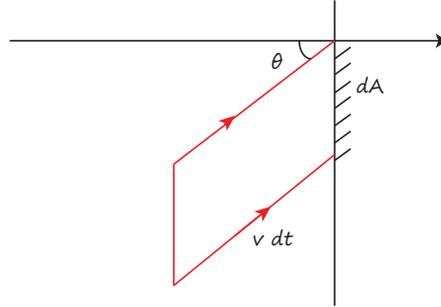
Combining the microscopic structure of discrete spacetime with the MIP, we have

$$\mathfrak{R} = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \quad (51)$$

D. From Spacetime Scattering to the Spacetime Diffusion Coefficient

Particles will be scattered randomly from the STP with the speed of light, which leads to the probability distribution of speed $f(\vec{v})$, the number of particles within $v \rightarrow v + dv$ is $f(v)d^3\vec{v}$. Therefore, all the particles cross the section area dA during time dt will be inside the cylinder (see Fig.2).

Figure 2: Probability distribution of spacetime scattering



The volume of this cylinder is

$$V = v dt \cos \theta dA \quad (52)$$

in which the number of particles is

$$N = f(\vec{v}) d^3\vec{v} v dt \cos \theta dA \quad (53)$$

Because of the isotropy of space, we have $f(\vec{v}) = f(v)$. From left to right, the number of

particle cross the unit area per unit time is

$$\begin{aligned}
 \Phi &= \int_{v_z > 0} \frac{N}{dAdt} \\
 &= \int_0^{\frac{\pi}{2}} d\theta \cos\theta \sin\theta \int_0^{2\pi} d\varphi \int_0^{+\infty} f(v)v^3 dv \\
 &= \pi \int_0^{+\infty} f(v)v^3 dv
 \end{aligned} \tag{54}$$

where $v_z > 0$ means $0 < \theta < \frac{\pi}{2}$. The average speed reads

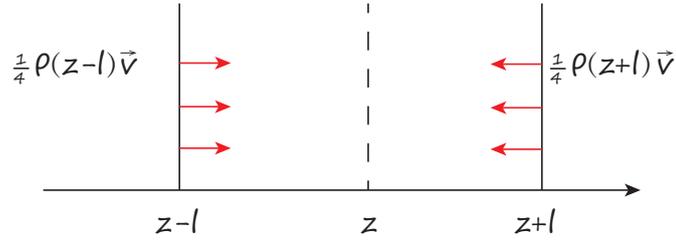
$$w = \frac{\int_0^{+\infty} f(v)v d^3v}{\int_0^{+\infty} f(v)d^3v} = \frac{4\pi}{\rho} \int_0^{+\infty} f(v)v^3 dv \tag{55}$$

where the density of particle number is $\rho = \int_0^{+\infty} f(v)d^3v$. Correspondingly, the number of particle cross the unit area per unit time will be

$$\Phi = \frac{1}{4}\rho w \tag{56}$$

Let mean free path of particles be ℓ , i.e. the average distance traveled by the particle between successive impacts from spacetime. The net flux J_z through the z plane will be (see Fig.3)

Figure 3: mean free path and scattering flux



$$J_z = \frac{1}{4}\rho(z-l)w - \frac{1}{4}\rho(z+l)w = -\frac{1}{2}\ell w \partial_z \rho \tag{57}$$

With the equation of continuity

$$\partial_t \rho + \nabla \cdot \vec{J} = 0 \tag{58}$$

and the isotropy of space, we have

$$\partial_t \rho = \frac{1}{2}\ell w \nabla^2 \rho \tag{59}$$

Combining the kinetics of spacetime scattering with quantum nature induced by STP, we obtain

$$\Re = \frac{w\ell}{2} = \frac{h}{2m_{ST}} \quad (60)$$

which is consistent with eq.(51).

- Statistical mass of fundamental particles

Let's consider the electron at first. The mass of an electron is $m_e = 9.104 \times 10^{-31} kg$. So its static energy is

$$E_e = m_e c^2 = 9.104 \times 10^{-31} \times 9 \times 10^{18} J = 8.1936 \times 10^{-12} J$$

This energy, according to MIP, comes from "effective" collisions between STP and the electron. In our MIP theory, the electron is not a point-like particle. It is finite size, statistically. Because of symmetry, its shape is a ball with a sphere boundary. The effective collisions are considered as the number of STP which coming into and going out cross the sphere. Assume every effective collision gives energy, which numerically equals to Planck constant. Hence the times of effective collisions (TEC) can be calculated as follow

$$N_e = E_e/h = 1.2347 \times 10^{20} [s^{-1}]$$

The statistical mass of electron can be written in form of TEC

$$m_e = \frac{h}{c^2} N_e \quad (61)$$

The ratio of mass and TEC is

$$k_{st} \equiv \frac{h}{c^2} = 7.37 \times 10^{-51} kg \cdot s \quad (62)$$

It has the unit of $[mass] \cdot [time]$. The fluctuation of the density of STP, around the electron, denoted as $\Delta\rho_{st}^e$, can be written as

$$\Delta\rho_{st}^e \equiv \rho^e - \rho_0 = \frac{m_e c^2}{\frac{4}{3}\pi r^3 h} \quad (63)$$

For proton, it is easy to calculate exactly the same as the electron, we have

$$N_p = \frac{m_p}{k_{st}} = 1.6726 \times 10^{-27} / 7.37 \times 10^{-51} \simeq 2.227 \times 10^{23} [s^{-1}] \quad (64)$$

The radius of proton is

$$r_p \simeq 8.735 \times 10^{-16} m \quad (65)$$

from which we obtain the mean free path of a proton in the STP sea around it.

$$l_{st} = \sqrt[3]{\frac{4}{3}\pi r_p^3 / N_p} \simeq 2.3 \times 10^{-23} m$$

IV. MASS-DIFFUSION UNCERTAINTY RELATION

We now consider the motion status of particle under impacts of STP collisions. The most important proposition of Copenhagen interpretation of quantum mechanics is the wave-particle duality. This allows one using the superposition rule of plane waves to describe the state of a particle. The kernel of the wave transformation from frequency space to time space will be the factor $\exp(ipx/\hbar)$. In fact it introduces the quantized operator formalism $\vec{p} = -i\hbar\vec{\nabla}$. Because of the duality, physical quantities of the particle can also be derived from wave, which implies some quantities can be described in phase space as eigenvalues of special operators. However, under the framework of MIP, we need to emphasize again that the wave-like property of the particle is an emergent property due to collision of STP, therefore it is not intrinsic. We can not borrow the quantization hypothesis directly. We consider the action of the particle

$$\begin{aligned} S[\phi(t, x), \partial\phi(t, x), \bar{v}(t, x)] & \quad (66) \\ & = S_0[\phi(t, x), \partial\phi(t, x)] + \sum_{I=1}^{\infty} S_I[\bar{v}(t, x)] \end{aligned}$$

where $\phi(t, x)$ describing the classical trajectory of the particle, and S_0 is the related classical action. $S_I[\bar{v}(t, x)]$ is the contribution of $I - th$ collision between STP and the particle. It does not depend on the classical trajectory at all, which only depends on the fluctuation of STP. The MIP said this term should contribute integer number of h , that is $S_I = nh$.

The partition function of the particle now is

$$Z = \int [d\phi(t, x)] \exp\left(-\frac{i}{\hbar} S[\phi(t, x), \partial\phi(t, x), \bar{v}(t, x)]\right) \quad (67)$$

hence

$$\exp\left(-\frac{i}{\hbar} S_I[\bar{v}]\right) = \exp\left(-\frac{i}{\hbar} nh\right) = e^{-i2\pi n} = 1 \quad (68)$$

from which we see the introducing of MIP does not change the classical partition function, therefore physical quantity derived from classical action will not be affected.

A. Classical Theory of Phase Space for Mass and Diffusion Coefficient

We have claimed and proven that particle mass is a statistical property describing the diffusion ability of the particle in spacetime, which shows that mass and diffusion coefficient

are indeed statistical properties, under continuous interaction of STP. However, MIP itself describes a special Markov process, which possesses the intrinsic characteristic property of being quantized.

Classically, given the position and momentum of a particle, one can describe how the particle moves in spacetime. However, if the mass of a particle is a statistical quantity, the momentum at a fixed position in spacetime will be in principle ill-defined.

The way out is to introduce the dynamic point of view. We define on each point in spacetime canonical pair (m_{ST}, \mathfrak{R}) . They satisfy the classic Poisson relation

$$\{\mathfrak{R}, m_{ST}\}_{P.B.} = 1 \tag{69}$$

in which the Poisson bracket reads

$$\{f, g\}_{P.B.} := \sum_{i=1}^D \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

while (q_i, p_i) are a pair of canonical position and momentum. It is obvious to see if we choose the canonical coordinate be \mathfrak{R} , and momentum be m_{ST} , we can immediately recover the Poisson relation (69).

B. Mass-Diffusion Uncertainty

Under the framework of MIP, for a matter particle, its mass and spacetime diffusion coefficient are not only be classical statistical conjugated quantity to each other, but also satisfy the uncertainty relation

$$\Delta m_{ST} \Delta \mathfrak{R} \geq h/2 \tag{70}$$

The existence of such an uncertainty relation is natural from the viewpoint of MIP. When we measure the mass of a particle, we can neither separate out the impacts of STP, nor perform measurement during a period of collision between STP and the particle. This rules out the instant measurement for the mass as well as the spacetime diffusion coefficient. A measurement cannot be done without collision of STP. As proposed in MIP, each collision between STP and the particle will change the action of particle at least one unit of h , which means the phase volume defined by mass and spacetime diffusion coefficient will change at least one unit of h .

The uncertainty between mass and spacetime diffusion coefficient implies one can not measure them for the same particle simultaneously. Then a realistic measurement method will be the insimultaneous measurement. However, this method will introduce the different time deviation, defined as

$$\eta := \lim_{\epsilon \rightarrow 0} m_{ST}(t + i\epsilon)\mathfrak{R}(t) - \mathfrak{R}(t + i\epsilon)m_{ST}(t) \quad (71)$$

where η describes the different time deviation. Notice here the time interval is imaginary. That is because we have integrated out the self-dynamics of STP and they only play as a background in the case of considering the dynamics of the particle. The collision progress is actually a transmittion of energy between STP and the particle, so for the particle, spacetime background served as a themal base and the collision provides an effective temperature. When $\epsilon \rightarrow 0$, the RHS of above equation can be defined as the commutation relation between mass and spacetime diffusion coefficient. We rewrite the RHS of above equation as

$$\lim_{\epsilon \rightarrow 0} \left(e^{-H\epsilon/\hbar} m_{ST}(t) e^{H\epsilon/\hbar} \mathfrak{R}(t) - e^{-H\epsilon/\hbar} \mathfrak{R}(t) e^{H\epsilon/\hbar} m_{ST}(t) \right) \quad (72)$$

Here the H is defined on the phase space (m_{ST}, \mathfrak{R}) , so one can not simply move toward the left side of $m_{ST}(t)$ or the right side of $\mathfrak{R}(t)$, from Baker-Hausdorff-Campell formula, one extracts the above equation as

$$\epsilon \frac{\partial m_{ST}(t)}{\partial t} + \mathcal{O}(\epsilon^2) = -\frac{\epsilon i}{\hbar} [H, m_{ST}] - \frac{\epsilon^2}{2\hbar^2} [H, [H, m_{ST}]] + \dots \quad (73)$$

$$\epsilon \frac{\partial \mathfrak{R}(t)}{\partial t} + \mathcal{O}(\epsilon^2) = -\frac{\epsilon i}{\hbar} [H, \mathfrak{R}] - \frac{\epsilon^2}{2\hbar^2} [H, [H, \mathfrak{R}]] + \dots \quad (74)$$

It is obvious that Heisenberg equation is satisfied under perturbative collision.

$$i\hbar \frac{\partial m_{ST}(t)}{\partial t} = [H, m_{ST}], \quad i\hbar \frac{\partial \mathfrak{R}(t)}{\partial t} = [H, \mathfrak{R}] \quad (75)$$

For the statistical inertia mass and diffusion coefficient, the evolution factor is determined by $H\epsilon$. The physical meaning is that in time interval $i\epsilon$ the energy transmitted from STP to the particle. This meets the description of MIP. So we can rewrite the action as a compact form of phase space volumn

$$S_I = \hat{\mathfrak{R}} \hat{m}_{ST}$$

The kernel relating the canonical position space and the canonical momentum space reads

$$\hat{m} \exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right) = m_{ST} \exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right), \quad (76)$$

$$\hat{\mathfrak{R}} \exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right) = \mathfrak{R} \exp\left(-\frac{i}{\hbar} S_I[\bar{\nu}]\right) \quad (77)$$

Mathematically, the unique representation of spacetime diffusion coefficient is an differential operator, as

$$\hat{\mathfrak{R}} = i\hbar \frac{\partial}{\partial m_{ST}}, \quad (78)$$

from which the equation (77) will be satisfied immediately, as

$$\begin{aligned} \hat{\mathfrak{R}} \exp\left(-\frac{i}{\hbar} \mathfrak{R} m_{ST}\right) &= i\hbar \frac{\partial}{\partial m_{ST}} \left[\exp\left(-\frac{i}{\hbar} \mathfrak{R} m_{ST}\right) \right] \\ &= \mathfrak{R} \exp\left(-\frac{i}{\hbar} \mathfrak{R} m_{ST}\right) \end{aligned} \quad (79)$$

from which we obtain the critical commutation relation

$$[\hat{\mathfrak{R}}, \hat{m}] = i\hbar \quad (80)$$

To calculate the uncertainty of $\hat{\mathfrak{R}}, \hat{m}$, denoted as $\Delta\hat{\mathfrak{R}}, \Delta\hat{m}$, we introduce an arbitrary real parameter ξ as well as an auxillary integral:

$$I(\xi) = \int |\xi \Delta\hat{\mathfrak{R}}\Psi - i\Delta\hat{m}\Psi|^2 d\tau \geq 0 \quad (81)$$

here, Ψ is a orthonormalized wavefunction. Expand the RHS as

$$\begin{aligned} I(\xi) &= \int \left(\xi^2 |\Delta\hat{\mathfrak{R}}\Psi|^2 + |\Delta\hat{m}\Psi|^2 \right) d\tau \\ &\quad - i\xi \int \Psi^* \left(\Delta\hat{\mathfrak{R}}\Delta\hat{m} - \Delta\hat{m}\Delta\hat{\mathfrak{R}} \right) d\tau \end{aligned} \quad (82)$$

Since

$$\begin{aligned} \Delta\hat{\mathfrak{R}}\Delta\hat{m} - \Delta\hat{m}\Delta\hat{\mathfrak{R}} &= [\Delta\hat{\mathfrak{R}}, \Delta\hat{m}] \\ &= \left[\left(\hat{\mathfrak{R}} - \langle \hat{\mathfrak{R}} \rangle \right), \hat{m} \right] - \left[\left(\hat{\mathfrak{R}} - \langle \hat{\mathfrak{R}} \rangle \right), \langle \hat{m} \rangle \right] \\ &= \left[\hat{\mathfrak{R}}, \hat{m} \right] = i\hbar \end{aligned} \quad (83)$$

The auxillary integral $I(\xi)$ now writes:

$$\begin{aligned} I(\xi) &= \int \left(\xi^2 |\Delta\hat{\mathfrak{R}}\Psi|^2 + \xi\hbar + |\Delta\hat{m}\Psi|^2 \right) d\tau \\ &= \xi^2 \langle \Delta\hat{\mathfrak{R}}^2 \rangle + \xi\hbar + \langle \Delta\hat{m}^2 \rangle \geq 0 \end{aligned} \quad (84)$$

$$\langle \Delta\hat{\mathfrak{R}}^2 \rangle \equiv \langle \hat{\mathfrak{R}}^2 \rangle - \langle \hat{\mathfrak{R}} \rangle^2, \quad \langle \Delta\hat{m}^2 \rangle \equiv \langle \hat{m}^2 \rangle - \langle \hat{m} \rangle^2 \quad (85)$$

where $\langle \hat{O} \rangle$ expresses the mean value under the orthogonal normalized wavefunction basis,

$$\langle \hat{O} \rangle \equiv \int \Psi^* \hat{O} \Psi d\tau \quad (86)$$

Using quadratic partition, the inequality (84) can be written as

$$\begin{aligned} I(\xi) = & \left(\xi \sqrt{\langle \Delta \hat{\mathfrak{R}}^2 \rangle} + \frac{\hbar}{2} \frac{1}{\sqrt{\langle \Delta \hat{\mathfrak{R}}^2 \rangle}} \right)^2 \\ & + \left(\langle \Delta \hat{m}^2 \rangle - \frac{\hbar^2}{4} \frac{1}{\langle \Delta \hat{\mathfrak{R}}^2 \rangle} \right) \geq 0 \end{aligned} \quad (87)$$

Hence

$$\langle \Delta \hat{\mathfrak{R}}^2 \rangle \cdot \langle \Delta \hat{m}^2 \rangle \geq \frac{\hbar^2}{4} \quad (88)$$

because of $m_{ST} = 2\pi m$, we arrive the desired result

$$\Delta m_{ST} \Delta \mathfrak{R} \geq h/2 \quad (89)$$

C. Energy-Time Uncertainty Relation

A massive particle moves in spacetime, which is represented not only by its momentum and position, but also by its energy and time. According to MIP, every impact of STP will change the action of particle by the amount of nh . As we all known, partition function is determined by energy distribution of the STP not by constraints of particle number.

We need to investigate commutation algebra of energy-time uncertainty relation based on the framework of MIP. Supposing the state function of particle is $\Psi(\vec{x}, t)$, then $Et\Psi(\vec{x}, t)$ and $tE\Psi(\vec{x}, t)$ have totally different physical meanings. From MIP, the motion of particle is a Markov process. It is impossible to define the exact state $\Psi(\vec{x}, t)$, since it is a probability function. Fundamentally, time t is a relative quantity, which depends on the zero point of time. However, the absolute zero point of time cannot be well defined. For the same reason, energy E is a relative quantity, which also depends on the zero point of energy. Due to the random impact of STP, the absolute zero point of energy cannot be well defined either. The background noise of STP is stochastic. Under MIP, time and energy both are not exactly well defined. All we can say is their probabilistic distribution. Simultaneously observing time and energy of the particle will lead to the covariance

$$\sigma(E, t) = \langle (E - \bar{E})(t - \bar{t}) \rangle \quad (90)$$

which is the average value of the change of action between particle and STP. In the interval $\Delta t = t - \bar{t}$, the change of energy is $\Delta E = E - \bar{E}$. The average value of the product is ΔS as

$$\sigma(E, t) = \langle \Delta S \rangle \quad (91)$$

For any random impact, we have $\Delta S = nh$, $n \in \mathbb{Z}$. The regularisation of ζ function leads to

$$\sigma(E, t) = \langle \Delta S \rangle = \zeta(0)h = \frac{1}{2}h \quad (92)$$

which is the very uncertainty relation of energy and time. Therefore, the energy and time are statistical quantities, which corresponds to infinite matrixes or a operator. Within this context, energy-time uncertainty relation can be interpreted as non-commutate matrixes.

However, this regularisation scheme cannot reveal the physical origin of energy-time uncertainty relation. We need a more detailed physical interpretation. When no collision between STP and matter particle happens, the particle will move with uniform velocity. After one collision, the particle will acquire an energy ΔE . This energy will feedback to the particle in two different ways. Firstly, the velocity of particle has changed, so its momentum changes correspondingly. Secondly, the mass of particle has changed, so the uncertainty of energy corresponds to the uncertainty of statistical mass. Assuming the status function of the particle depends on time and position as

$$\Psi(\vec{x}, t) \in C_\infty(\vec{x}, t) \quad (93)$$

here the $C_\infty(\vec{x}, t)$ is the set of smooth function in spacetime. As a status function, the normalizable property is very important. If the status function is a decay or increasing function, the normalizability can not be guaranteed. In other words, the particle can not be killed or created in spacetime when there are no interactions. Therefore, we can only expect the status function is an oscillation function as

$$\begin{aligned} \Psi(\vec{x}, t + \Delta t) &= \Psi(\vec{x}, t) \exp\left(-\frac{i}{\hbar} H \Delta t\right) \\ &= \Psi(\vec{x}, t) - \Psi(\vec{x}, t) \frac{iH \Delta t}{\hbar} + \dots \end{aligned} \quad (94)$$

Here H is the Hamiltonian of the particle in spacetime. Accordingly, we have

$$\Psi(\vec{x}, t + \Delta t) = \Psi(\vec{x}, t) + \Delta t \frac{\partial}{\partial t} \Psi(\vec{x}, t) + \dots \quad (95)$$

and we obtain the equivalent relation

$$\frac{\partial}{\partial t} \equiv -\frac{i}{\hbar}H \quad (96)$$

as known as

$$\frac{\partial\Psi(\vec{x}, t)}{\partial t} = \frac{-i}{\hbar}H\Psi(\vec{x}, t) . \quad (97)$$

hence we deduce that

$$\begin{aligned} \frac{\partial(t\Psi(\vec{x}, t))}{\partial t} &= \Psi(\vec{x}, t) + t\frac{\partial(\Psi(\vec{x}, t))}{\partial t} \\ &= \Psi(\vec{x}, t) - \frac{i}{\hbar}tH\Psi(\vec{x}, t) \\ &= -\frac{i}{\hbar}H(t\Psi(\vec{x}, t)) \end{aligned} \quad (98)$$

hence

$$(Ht - tH)\Psi(\vec{x}, t) = i\hbar\Psi(\vec{x}, t) \quad (99)$$

Finally, we obtain the energy-time uncertainty

$$[H, t] = i\hbar \quad (100)$$

as expected.

D. Position-Momentum Uncertainty Relation

Extending the definition of commutation relation, and recall $m = m_{ST}/2\pi$, we consider the position-momentum commutator

$$\begin{aligned} [x, p] &= \frac{1}{2\pi}x(t + i\epsilon)m_{ST}(t)\frac{\partial x(t)}{\partial t} \\ &\quad - \frac{1}{2\pi}m_{ST}(t + i\epsilon)\frac{\partial x(t + i\epsilon)}{\partial t}x(t) \\ &= i\frac{\epsilon}{2\pi} \left[m_{ST} \left(\frac{\partial x(t)}{\partial t} \right)^2 \right. \\ &\quad \left. - \frac{\partial m_{ST}(t)}{\partial t}v(t)x(t) - m_{ST}\frac{\partial^2 x(t)}{\partial t^2}x(t) \right] \end{aligned} \quad (101)$$

Define

$$a_{ST}(t) := \frac{\partial^2 x(t)}{\partial t^2} \quad (102)$$

It is the instantaneous acceleration induced by the collision between STP and the particle.

From which we can define the instantaneous "spacetime" force as

$$F_{ST}(t) = ma_{ST}(t) = m \frac{\partial^2 x(t)}{\partial t^2} \quad (103)$$

The statistical average of eq.(101) is

$$[x, p] = m \langle v(t)^2 \rangle i\epsilon - \left\langle \frac{\partial m(t)}{\partial t} v(t) x(t) \right\rangle i\epsilon - \langle F_{ST}(t) x(t) \rangle i\epsilon \quad (104)$$

The physical meaning of the third term in above equation is clear, it reflects the mean work done by STP acting on the particle. Obviously, this mean work is zero. To understand the first two terms in eq.(104), we should consider the Markov process in detail. From Langevin equation Eq. (26), the classic speed can be expressed as

$$u(t) = U(x(t)) \quad (105)$$

The fluctuation speed induced by collision between STP and the particle is

$$v(t) = \nu(t) \quad (106)$$

Therefore we could naturally deduce that

$$\begin{aligned} \langle x(t)v(t) \rangle &= \left\langle -\frac{1}{2} \int f(x(t)) dt \nu(t) \right\rangle_{\nu} \\ &= \int -\frac{1}{2} \int f(x(t)) dt \nu(t) \left(\sqrt{\frac{1}{2\pi\Omega\delta(t-t')}} \right)^D \\ &\times \exp \left(-\frac{1}{2\Omega} \int dt \sum_i \nu_i^2 \right) [d\nu] \\ &= 0 \end{aligned} \quad (107)$$

Notice in deriving the second step, we have used the property of Gaussian integral, which leads to the result that the second term of eq. (104) will also have vanishing contribution.

Under discretization of the spacetime fluctuation, the mean speed is

$$\int_t^{t+\epsilon} \nu(\tau) d\tau / \epsilon = \bar{\nu} / \sqrt{\epsilon}$$

Therefore

$$\langle \nu^2 \rangle = \langle \bar{\nu}^2 \rangle / \epsilon = \frac{h}{m_{ST}\epsilon} \quad (108)$$

Substituting this into the first term of Eq. (104),

$$[x, p] = i\epsilon m \langle \nu^2 \rangle = i\epsilon m \frac{h}{m_{ST}\epsilon} = i\hbar \quad (109)$$

we obtain the fundamental position-momentum uncertainty in quantum mechanics.

E. Mass Measurement and Neutrino Oscillation

In previous subsections, we derived the mass-diffusion uncertainty relation. We now discuss a possible important application of this .

In modern physics, neutrino oscillation is provided as a longstanding puzzle for high energy physics. The current explanation is that neutrinos have a very strange property that they can not be eigenstate of mass and flavor simultaneously. However, in the progress of nuclear reaction, neutrinos are all considered as a flavor eigenstate, which means they have definitive flavors. This leads to a strange result that we can not detect the mass of neutrinos. In this article, we will argue that the neutrino oscillation actually reflects the inertia mass is a statistical property. The mass of neutrinos is so small that the statistical deviation is comparable to the mass, so the mass can not be measured accurately.

Because of mass-diffusion uncertainty, we can calculate the effect of spacetime diffusion directly, when given the mass differences between various neutrinos. Experimentally, physicists now can measure the differences indirectly. The mass square differences are between $2.6 \times 10^{-3} eV^2$ and $7.58 \times 10^{-5} eV^2$. This gives a perfect testing arena for the mass-diffusion uncertainty. From the mass-diffusion uncertainty relation,

$$\Delta m \Delta \mathfrak{R} \geq \hbar/2 \quad (110)$$

the diffusion coefficient reads

$$\begin{aligned} \Delta \mathfrak{R} \geq \hbar/2\Delta m &= \frac{6.626 \times 10^{-34} \times 9 \times 10^{16}}{4 \times 3.1416 \times 0.05 \times 1.6 \times 10^{-19}} \\ &= 1186.4 [m^2/s] \end{aligned} \quad (111)$$

The physical meaning of this calculation is significant,. Every second the neutrino propagates with a growing diffusion cone, with the bottom of the cone, increasing its area to $1186.4m^2$. If a neutrino goes from sun to earth, its diffusion radius will be about 307 meters.

F. Neutrino Diffusion Experiment

Since sun cannot be seen as point-like source for neutrino ejection, we could design an ideal experiment in laboratory, as shown in Fig.4

Electron neutrinos came from reactor and were screened by screening matter, except those moving strictly toward x -direction. According to MIP and due to diffusion of neutrino, after

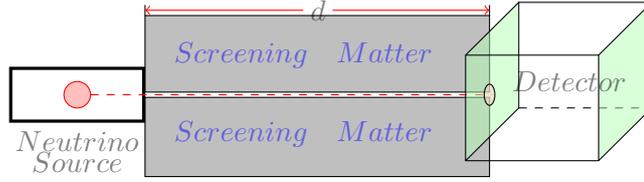


Figure 4: Ideal experiment for neutrino diffusion in lab.

propagating distance d , detectors at distance d will detect neutrinos in a disk region with equal probability. The disk area can be calculated as

$$\delta r \simeq \sqrt{\Delta R \frac{d}{c\pi}} \quad (112)$$

If $d \simeq 100km$, $\delta r \simeq 0.3548m$, the disk is macro significant detectable.

V. RANDOM MOTION OF FREE PARTICLE UNDER MIP

A. Decompositions of the Real Velocity

In modern quantum mechanics, particles do not have trajectories of motions, so their velocities are not well defined. Within the framework of MIP, the real velocity of the particles must be discussed in detail. Under the impact of STP, the velocity of the particle not only contains the classical velocity, but also the results of random mechanical interactions. It is especially important that the particles are subjected to the impact of the STP, and the change of action is quantized. Therefore, the real velocity of the particles should reflect the classical, random and quantum properties.

Within the framework of MIP, the motion of particles is a frictionless quantum Brownian motion. However, it should be noted that the impact of STP is not completely random. The exchanged action that each particle is subjected to STP is an integer multiple of the Planck constant h . Therefore, the movement of particles in spacetime cannot be a problem of random mechanics completely. It is the quantization of randomized motions. The corresponding theoretical system is a quantum Markov process. If there is no STP and other external forces, the motion of the free particles satisfies Newtonian mechanics. Its velocity is the classic velocity.

Within the framework of MIP, for the real velocity of motion of free particles $\vec{V}(\vec{x}, t)$, we can

first isolate the classical statistical velocity of the particle $\vec{v}(\vec{x}, t)$. In the context of space-time, it is a simple mean of the statistics of the impact of STP as Gaussian noise. Since the simple mean contribution of Gaussian noise is zero, the classical statistical velocity of the particle and the classical velocity under Newtonian mechanics are exactly equal. Second, after separating the classical statistical velocity $\vec{v}(\vec{x}, t)$, we will consider a random motion. This random motion is driven by the impact of STP, and we note it with the random motion velocity $\vec{W}(\vec{x}, t)$. In Appendix B of this paper, we prove that any random function can be decomposed into a random function and a superposition of differentiable functions. Random motion under the framework of MIP also follows this important principle. Therefore, in general, we can decompose the random motion velocity $\vec{W}(\vec{x}, t)$ as follow

$$\vec{W}(\vec{x}, t) = \vec{u}(\vec{x}, t) + \vec{v}(t) \quad (113)$$

Where $\vec{u}(\vec{x}, t)$ is defined as the quantum envelope velocity of the particle. For free particles, $\langle \vec{u}(\vec{x}, t) \rangle_\nu = 0$. It corresponds to the perturbation part of the random motion. It reflects the physical fact that the impact of STP is random, but it is a small perturbation to the current motion of the particle. These impacts are "differential impacts" of STP on the particles. Under the action of the perturbation of space-time, the motion of particles is not an unpredictable random motion. It allows the motion state of particles to be described by a differentiable function and describes the corresponding motion state. The equation is a non-random partial differential equation. And $\vec{v}(t)$ represents the non-microscopic impact of the particle by STP, which is a non-perturbative effect on the velocity of the particle motion. We define it as the velocity of fluctuation. Because of the existence of such random impact, the state function that we finally describe the equation of motion of the particle will not be an accurate description. It can only be a probabilistic description on the background of this fluctuation.

We will see that in the framework of MIP, quantum envelope motion reflects the wave-particle duality of particles. Considering the impact between STP and particle, the amount of exchange action is nh . For particles with a statistical mass of m_0 , the characteristic time of this collision is

$$t_c = \frac{nh}{m_0 c^2} \quad (114)$$

The so-called quantum envelope motion is essentially the differentiable part of the fluctuation motion.

The above discussion is based on the classification of particles by the impact of STP. From the above analysis we can see that there is actually another mathematical classification for the velocity of the particles, and we decompose the velocity of the particle into a differentiable part and a non-differentiable part. The differentiable part of the real motion velocity of a particle can be defined as:

$$\vec{U}(\vec{x}, t) = \vec{v}(\vec{x}, t) + \vec{u}(\vec{x}, t) \quad (115)$$

It is a superposition of classic statistical velocity $\vec{v}(\vec{x}, t)$ and quantum envelope velocity $\vec{u}(\vec{x}, t)$. We call this differentiable velocity “statistical average velocity”. Although mathematically it is a differentiable function, it is quite different from the classical velocity. Because there is a quantum envelope velocity $\vec{u}(\vec{x}, t)$, it is a representation of the Markov process formed by the impact of STP. Therefore, the decomposition of the velocity of the particles caused by the collision of STP can be written in three parts in principle[49]:

$$\vec{V}(\vec{x}, t) = \vec{u}(\vec{x}, t) + \vec{v}(\vec{x}, t) + \vec{v}(t) \quad (116)$$

Since a Markov process will still be a Markov process under time reversal, the quantum envelope velocity $\vec{u}(\vec{x}, t)$ is invariant under time reversal as

$$T : \vec{u}(\vec{x}, t) \rightarrow \tilde{\vec{u}}(\vec{x}, t) = \vec{u}(\vec{x}, t) \quad (117)$$

However, the classical statistical velocity $\vec{v}(\vec{x}, t)$ is changed by the time reversal, that is,

$$T : \vec{v}(\vec{x}, t) \rightarrow \tilde{\vec{v}}(\vec{x}, t) = -\vec{v}(\vec{x}, t) \quad (118)$$

With above properties of time reversal, we can have a well defined limit $\vec{u} = 0$ as Newtonian mechanics with

$$\vec{v} = \frac{1}{2}(\vec{U} - \tilde{\vec{U}}) \quad (119)$$

$$\vec{u} = \frac{1}{2}(\vec{U} + \tilde{\vec{U}}) \quad (120)$$

Where $\tilde{\vec{U}}$ is the time reversal of the statistical average velocity \vec{U} . In the following, the physical quantities with time reversal are marked with tilde.

The non-differentiable part is the fluctuation velocity $\vec{v}(t)$ for the random “non-differentiable impact” of the particle. It causes the particle’s velocity to deviate from the classical statistical mean, so it will be physically reflected as a random diffusion behavior of

the particle in spacetime. Based on this, we named it the “diffusion velocity” of particles in space and time.

In the following subsections, we will see that the decomposition of the above two velocities is a very important theoretical basis for deriving the equation of motion of particles, that is, the Schrödinger equation in quantum mechanics and an in-depth understanding of its physical meaning.

B. From MIP to Schrödinger Equation

Without the interaction of spacetime, the velocity of particle \vec{v} has to be the derivative $\vec{v} = \frac{d\vec{x}}{dt}$. Contrasting from usual Markov process, spacetime random motion is frictionless, otherwise the quantum effect of a particle will decay as time going, which is obviously not the case. According to the MIP, the coordinate of a free particle is a stochastic process $\vec{x}(t)$, in which the velocity \vec{V} can not be expressed in terms of $\frac{d\vec{x}}{dt}$. The velocity \vec{V} should be a statistical average corresponding to a distribution $\delta\vec{x} = \vec{x}(t + \frac{1}{\omega}) - \vec{x}(t)$, at the limit of spacetime collision frequency ω going to infinity. In Einstein’s theory on Brownian motion, $\delta\vec{x}$ is a Gaussian distribution with zero mean and variance proportional to $\frac{1}{\omega}$ [6]. However, Einstein’s theory cannot be correct at the limit of spacetime collision frequency ω going to infinity[19, 20]. Therefore, we will construct the operator D as following, which plays the same role as $\frac{d}{dt}$ in Newtonian Mechanics. For any physical function $f(\vec{x}, t)$, we have

$$\begin{aligned}
 & \omega(f(\vec{x}(t + \frac{1}{\omega}), t + \frac{1}{\omega}) - f(\vec{x}(t), t)) \\
 &= [\partial_t + \sum_i \omega(x_i(t + \frac{1}{\omega}) - x_i(t))\partial_i \\
 & \quad + \sum_{ij} \frac{\omega}{2}(x_i(t + \frac{1}{\omega}) - x_i(t))(x_j(t + \frac{1}{\omega}) - x_j(t))\partial_i\partial_j \\
 & \quad + \sum_i (x_i(t + \frac{1}{\omega}) - x_i(t))\partial_i\partial_t + \frac{1}{2\omega}\partial_t^2]f(\vec{x}(t), t)
 \end{aligned} \tag{121}$$

At the limit of spacetime collision frequency ω going to infinity, in terms of statistical average $\langle \dots \rangle$ for δx , we can define the operator D as

$$Df(x(t), t) = \lim_{\omega \rightarrow +\infty} \omega \langle f(\vec{x}(t + \frac{1}{\omega}), t + \frac{1}{\omega}) - f(\vec{x}(t), t) \rangle_\nu \tag{122}$$

$$= (\partial_t + \sum_i U_i \partial_i + \sum_{ij} \Re_{ij} \partial_i \partial_j) f(\vec{x}(t), t) \tag{123}$$

where we used

$$\vec{U} = \lim_{\omega \rightarrow +\infty} \omega \langle \delta \vec{x} \rangle_{\nu} \quad (124)$$

it relates to the discretization of Langevin equation

$$x_i(t + \epsilon) - x_i(t) = \epsilon U_i(\mathbf{x}(t)) + \sqrt{\epsilon} \bar{v}_i + O(\epsilon^2) \quad (125)$$

here

$$\epsilon = \frac{1}{\omega} \quad (126)$$

In eq.(122) , we used the following deduced result

$$\begin{aligned} \lim_{\omega \rightarrow +\infty} \frac{\omega \langle \delta x_i \delta x_j \rangle_{\nu}}{2} &= \lim_{\epsilon \rightarrow 0^+} \frac{1}{2\epsilon} \langle (x_i(t + \epsilon) - x_i(t))(x_j(t + \epsilon) - x_j(t)) \rangle_{\nu} \\ &= \lim_{\epsilon \rightarrow 0^+} \frac{1}{2\epsilon} \left[\langle \epsilon^2 U_i(\mathbf{x}(t)) U_j(\mathbf{x}(t)) \rangle_{\nu} + \epsilon \langle \bar{v}_i \bar{v}_j \rangle_{\nu} + \epsilon^{\frac{3}{2}} \langle (U_i \bar{v}_j + U_j \bar{v}_i) \rangle_{\nu} \right] \\ &= \frac{\hbar}{2m_{ST}} \delta_{i,j} \end{aligned} \quad (127)$$

Because of the isotropy of space, the MIP coefficient will be

$$\mathfrak{R}_{ij} = \frac{\hbar}{2m_{ij}} = \mathfrak{R} \delta_{ij} \quad (128)$$

which is consistent with Eq.51 and 60. The operator D and its time reversal \tilde{D} are

$$D = \partial_t + \vec{U} \cdot \nabla + \mathfrak{R} \nabla^2 \quad (129)$$

$$\tilde{D} = -\partial_t + \vec{\tilde{U}} \cdot \nabla + \mathfrak{R} \nabla^2 \quad (130)$$

Therefore, the statistical average velocity of particle \vec{V} can be written as

$$\vec{U} = D \vec{x} \quad (131)$$

$$\vec{\tilde{U}} = \tilde{D} \vec{x} \quad (132)$$

Correspondingly, its classical statistical velocity and quantum envelope velocity are

$$\vec{v} = D^- \vec{x} \quad (133)$$

$$\vec{u} = D^+ \vec{x} \quad (134)$$

with

$$D^- = \frac{1}{2}(D - \tilde{D}) \quad (135)$$

$$D^+ = \frac{1}{2}(D + \tilde{D}) \quad (136)$$

We define the statical average acceleration of particles as

$$\begin{aligned}\vec{a} &= D\vec{U} = (D^+ + D^-)(\vec{v} + \vec{u}) \\ &= D^+\vec{u} + D^-\vec{v} + D^-\vec{u} + D^+\vec{v}\end{aligned}\quad (137)$$

Under time reversal, it acts as

$$\begin{aligned}\tilde{\vec{a}} &= \tilde{D}\tilde{\vec{U}} = (D^+ - D^-)(-\vec{v} + \vec{u}) \\ &= D^+\vec{u} + D^-\vec{v} - D^-\vec{u} - D^+\vec{v}\end{aligned}\quad (138)$$

Define the classical average acceleration as

$$\vec{a}_c = \frac{1}{2}(\vec{a} + \tilde{\vec{a}}) = D^+\vec{u} + D^-\vec{v},\quad (139)$$

obviously it is invariant under time reversal. The average acceleration of a free particle must be zero, which can be written as

$$D^+\vec{v} + D^-\vec{u} = 0.\quad (140)$$

However, the average acceleration of quantum envelope motion can not simply be zero,

$$D^+\vec{u} + D^-\vec{v} \neq 0\quad (141)$$

At classical and low speed case, the average acceleration of quantum envelope motion does not relate to classical statistical velocity, therefore we can have

$$D^-\vec{v} - D^+\vec{u} = 0.\quad (142)$$

These conditions are equivalent to the coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re \nabla^2 \vec{v} - \nabla(\vec{u} \cdot \vec{v})\quad (143)$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \cdot \nabla)\vec{v} + (\vec{u} \cdot \nabla)\vec{u} + \Re \nabla^2 \vec{u}\quad (144)$$

Random motions of free particles due to the random impacts of STP satisfy the Markov property, one can make predictions for the future of the process based solely on its present state just as well as one could know the process's full history. This is the simplest situation for random motions, the free particle does not involve any external potential. Now, we have

an initial value problem, which is to solve $\vec{u}(\vec{x}, t)$ and $\vec{v}(\vec{x}, t)$ given $\vec{u}(\vec{x}, 0) = \vec{u}_0(\vec{x})$, $\vec{v}(\vec{x}, 0) = \vec{v}_0(\vec{x})$. In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$, where

$$\nabla R = \frac{1}{2\Re} \vec{u} \quad (145)$$

$$\nabla I = \frac{1}{2\Re} \vec{v} \quad (146)$$

We can obtain

$$\frac{\partial \Psi}{\partial t} = i\Re \nabla^2 \Psi \quad (147)$$

According to the MIP, the universal spacetime diffusion coefficient is the MIP coefficient $\Re = \frac{\hbar}{2m_{ST}}$. Substituting to the last equation, we will get the equation of motion of free particles as

$$i \frac{\partial \Psi}{\partial t} = -\frac{\hbar \nabla^2}{2m_{ST}} \Psi \quad (148)$$

which is the Schrödinger equation essentially. From this emergent Schrödinger equation, we can deduce a series of quantum behaviours. It's important to remark that the spacetime mass m_{ST} in the Schrödinger equation of free particles coincides with the inertial mass m of free particles. Since we only discuss non-relativistic quantum mechanics in the followings, we don't need to distinguish m_{ST} from m any more. From $|\Psi|^2 = e^{2R}$ and $\nabla R = \frac{1}{2\Re} \vec{u}$, we have

$$\vec{u} = \Re \frac{\nabla |\Psi|^2}{|\Psi|^2} \quad (149)$$

which leads to Born rule $\rho = |\Psi|^2$. $\rho(x, t)$ is the probabilistic density of particles in coordinate x at time t . The Born rule is a law of quantum mechanics which gives the probability that a measurement on a quantum system will yield a given result, which became a fundamental ingredient of Copenhagen interpretation. In this paper, we attempt to suggest an interpretation of Born rule according to the MIP, which can provide a realistic point of view for wave function. Emerging from random impacts of spacetime, it's absolutely necessary that wave function is complex. If wave function were a real sine or cosine function[25], according to $\rho = |\Psi|^2$, the probabilistic density of a free particle with definite momentum would oscillate periodically which violates the isotropy of physical space.

C. Physical Meanings of Potential Functions R and I

Substituting $\Psi = e^{R+iI}$ into $\frac{\partial \Psi}{\partial t} = i\Re \nabla^2 \Psi$, we equalise the real and imaginary part separately as

$$\partial_t R = -\Re(2\nabla R \cdot \nabla I + \nabla^2 I) \quad (150)$$

$$\partial_t I = \Re[(\nabla R)^2 - (\nabla I)^2 + \nabla^2 R] \quad (151)$$

Combining with previous result $\rho = |\Psi|^2 = e^{2R}$, we have

$$\partial_t \rho = 2\rho \partial_t R \quad (152)$$

$$\nabla \rho = 2\rho \nabla R \quad (153)$$

The differential equation of potential R can be turned into

$$\partial_t \rho = -2\Re \nabla \cdot (\rho \nabla I) \quad (154)$$

With $\nabla I = \frac{1}{2\Re} \vec{v}$, the differential equation of potential R is equivalent to the equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \quad (155)$$

Noticing that the classical momentum of particle is $m\vec{v} = \hbar \nabla I$, we find that the differential equation of potential I goes to

$$\partial_t(\hbar I) + \frac{(\nabla(\hbar I))^2}{2m} - \hbar \Re[(\nabla R)^2 + \nabla^2 R] = 0 \quad (156)$$

Comparing with the Hamilton-Jacobi equation from classical mechanics [26, 27] as

$$\partial_t S + \frac{(\nabla S)^2}{2m} + V(x) = 0 \quad (157)$$

which is particularly useful in identifying conserved quantities for mechanical systems. There are two crucial remarks: Firstly, potential function I is proportional to the Hamilton-Jacobi function S as $S = \hbar I$. Secondly, for a free particle, the influence of spacetime can be summed up to the spacetime potential

$$V_{ST} = -\hbar \Re[(\nabla R)^2 + \nabla^2 R] \quad (158)$$

where the spacetime potential V_{ST} will play the same role of potential V in the Hamilton-Jacobi equation. The spacetime potential V_{ST} vanishes in the classical limit $\hbar = 0$, which

is equivalent to $V = 0$ for free particles in classical mechanics. The quantum effect, which corresponding to nonzero \hbar , now is the natural result of the existence of the spacetime potential V_{ST} , induced by MIP. In principal, the moving of free particle can be described precisely by the spacetime potential V_{ST} as

$$m \frac{d^2 \vec{x}}{dt^2} = -\nabla V_{ST} = \hbar \Re \nabla [(\nabla R)^2 + \nabla^2 R] \quad (159)$$

This equation indicates that free particle moves not along straight line within interactions of SPT. It is affected by a space-time potential V_{ST} . The interactions between SPT and particle give the statistical mass to particle.

D. Space-time Random Motion of Charged Particles in Electromagnetic Field

According to the MIP, in case of low speed, electromagnetic field only serves as an external potential, which itself is not affected by random impacts of spacetime. In a electromagnetic field (\vec{E}, \vec{B}) , the charged particle will experience a Lorentz force $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$. Therefore, the average acceleration of charged particles will be

$$\vec{a} = e(\vec{E} + \vec{v} \times \vec{B})/m \quad (160)$$

where m is the inertial mass of charged particle and e is the charge. Based on the spacetime principle, we are able to derive the equation of motion of charged particle in electromagnetic field, which is finally shown to be Schrödinger equation in electromegnetic field, which is

$$i\hbar \partial_t \Psi = \frac{1}{2m} (-i\hbar \nabla - \frac{e}{c} \vec{A})^2 \Psi + e\phi \Psi \quad (161)$$

where the electromagnetic potential and the electromagnetic field are connected by

$$\vec{B} = \nabla \times \vec{A}, \vec{E} = -\partial_t \vec{A} - \nabla \phi. \quad (162)$$

We do not have average acceleration in absence of electromagnetic field. However, this is not the case when the particle have non-zero electric charge, moving in external electromagnetic field. Identifying the velocity in the Lorentz force as the classical velocity of random motion of particle in spacetime, we have

$$\partial_t \vec{v} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla) \vec{v} + (\vec{u} \cdot \nabla) \vec{u} + \Re \nabla^2 \vec{u} \quad (163)$$

In the electromagnetic field, the equation of motion of charged particle becomes coupled non-linear partial differential equations as following

$$\frac{\partial \vec{u}}{\partial t} = -\Re \nabla (\nabla \cdot \vec{v}) - \nabla (\vec{u} \cdot \vec{v}) \quad (164)$$

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} = e(\vec{E} + \vec{v} \times \vec{B})/m - (\vec{v} \cdot \nabla) \vec{v} \\ + (\vec{u} \cdot \nabla) \vec{u} + \Re \nabla^2 \vec{u} \end{aligned} \quad (165)$$

In order to solve the coupled non-linear partial differential equations, we have to linearise it firstly. Let $\Psi = e^{R+iI}$ and notice that the canonical momentum of charged particle [28] is $\vec{p} = m\vec{v} + e\vec{A}/c$, we suppose

$$\nabla R = \frac{1}{2\Re} \vec{u} \quad (166)$$

$$\nabla I = \frac{1}{2\Re} \left(\vec{v} + \frac{e\vec{A}}{mc} \right) \quad (167)$$

In order to prove Eq.(161), we expand the first term of right side of Eq.(161) as

$$\begin{aligned} \frac{1}{2m} (-i\hbar \nabla - \frac{e}{c} \vec{A})^2 \Psi = -\frac{\hbar^2 \nabla^2}{2m} \Psi + \frac{e^2 A^2}{2mc^2} \Psi \\ + \frac{i\hbar e}{2mc} (\nabla \cdot \vec{A}) \Psi + \frac{i\hbar e}{mc} \vec{A} \cdot (\nabla \Psi) \end{aligned} \quad (168)$$

Substituting $\Psi = e^{R+iI}$, it leads to

$$\begin{aligned} -\frac{\hbar^2}{2m} [\nabla^2 R + i\nabla^2 I + (\nabla R + i\nabla I)^2] \Psi + \frac{e^2 A^2}{2mc^2} \Psi \\ + \frac{i\hbar e}{2mc} (\nabla \cdot \vec{A}) \Psi + \frac{i\hbar e}{mc} (\vec{A} \cdot (\nabla R + i\nabla I)) \Psi \end{aligned} \quad (169)$$

With vector formulas

$$\begin{aligned} \nabla (\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) \\ + (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A} \end{aligned} \quad (170)$$

$$\nabla (\nabla \cdot \vec{A}) = \nabla \times (\nabla \times \vec{A}) + \nabla^2 \vec{A} \quad (171)$$

and Eq.(166), we will obtain

$$\nabla \times \vec{u} = 0 \quad (172)$$

$$\nabla \times \left(\vec{v} + \frac{e\vec{A}}{mc} \right) = 0 \quad (173)$$

Straightforwardly, we have

$$\begin{aligned}
i\hbar(\partial_t R + i\partial_t I) &= -\frac{\hbar^2}{2m}[\nabla^2 R + i\nabla^2 I \\
&\quad + (\nabla R + i\nabla I)^2] + \frac{e^2 A^2}{2mc^2} \\
&\quad + \frac{i\hbar e}{2mc}(\nabla \cdot \vec{A}) + \frac{i\hbar e}{mc}(\vec{A} \cdot (\nabla R + i\nabla I)) + e\phi
\end{aligned} \tag{174}$$

Now, let's prove that the real and imaginary parts are separately equaled as

$$\begin{aligned}
\partial_t I &= \frac{\hbar}{2m}(\nabla^2 R + (\nabla R)^2 - (\nabla I)^2) \\
&\quad - \frac{e^2 \vec{A}^2}{2mc^2} + \frac{e}{mc}(\vec{A} \cdot (\nabla I)) - \frac{e\phi}{\hbar}
\end{aligned} \tag{175}$$

$$\begin{aligned}
\partial_t R &= -\frac{\hbar}{2m}(\nabla^2 I + 2(\nabla R) \cdot (\nabla I)) \\
&\quad + \frac{e}{2mc}(\nabla \cdot \vec{A}) + \frac{e}{mc}\vec{A} \cdot (\nabla R)
\end{aligned} \tag{176}$$

Taking the gradient from both sides and the definitions $\vec{B} = \nabla \times \vec{A}$, $\vec{E} = -\partial_t \vec{A} - \nabla\phi$, we have reproduced the Eq.(164). Therefore, we have proved that both sides of Eq.(164) are at most different from a zero gradient function. It's important to notice that the choices of electromagnetic potentials are not completely determined. It allows a gauge transformation [29]

$$\vec{A}' = \vec{A} + \nabla\Lambda \tag{177}$$

$$\phi' = \phi - \partial_t\Lambda \tag{178}$$

For any function $\Lambda(\vec{x}, t)$, the electromagnetic field is invariant. Therefore, the corresponding wave function cannot change essentially, at most changing a local phase factor. Given $\psi' = \psi e^{\frac{ie\Lambda}{\hbar c}}$, Schrödinger equation of charged particle in electromagnetic field is invariant, i.e., $U(1)$ gauge symmetry. By choosing the function $\Lambda(\vec{x}, t)$ properly, we are able to eliminate the redundant zero gradient function. So we have proved Eq.(161) at the end.

E. Stationary Schrödinger Equation from MIP

Compare to the definition of classical statistical velocity as in eq.(146), it is easy to see that for the ground state, the classical statistical velocity is zero. Moreover, we can prove for all stationary states, their classical statistical velocities are zero. For a stationary state

has exact energy E , the Schrödinger equation is

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + V_c(\vec{x})\right]\Psi = E\Psi \quad (179)$$

its conjugation reads

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + V_c(\vec{x})\right]\Psi^* = E\Psi^* \quad (180)$$

here $V_c(\vec{x})$ is classical external potential. Add the above two equations, the new real wave function has to satisfy the Schrödinger equation with same eigen-energy E .

Corresponding to the classical velocity from Eq.(146), it is easy to show that the classical velocity of particles must be zero in stationary states. Within the framework of MIP, we should interpret the stationary states from quantum mechanics as a spacetime random motion with zero classical velocity. Once we have all the stationary states, we will get the general solution by linear superposition. Therefore, we are going to derive stationary Schrödinger equation from classical velocity $\vec{v} = 0$, which can provide a clear physical picture of MIP. Moreover, when $|\vec{v}|$ is large and close to velocity of light c , the generalisation of this framework is clear and will be explained in our further work.

The trajectory of random motion of particle can be understood as the superposition of classical path and fluctuated path. During time interval Δt , there are two contributions to the trajectory as

$$\delta\vec{x} = \vec{u}(\vec{x}, t)\Delta t + \Delta\vec{x} \quad (181)$$

of which distribution satisfies $\varphi(\Delta\vec{x}) = \varphi(-\Delta\vec{x})$ and

$$\int \varphi(\Delta\vec{x})d(\Delta\vec{x}) = 1$$

. The spacetime coefficient reads

$$\Re = \frac{1}{2\Delta t} \int (\Delta\vec{x})^2 \varphi(\Delta\vec{x})d(\Delta\vec{x}) \quad (182)$$

The probabilistic density $\rho(x, t)$ evolves as

$$\rho(\vec{x}, t + \Delta t) = \int \rho(x - \delta\vec{x}, t)\varphi(\Delta\vec{x})d(\Delta\vec{x}) \quad (183)$$

Expanding Taylor series of both sides, we have

$$\partial_t \rho = -\nabla \cdot (\rho\vec{u}) + \Re \nabla^2 \rho \quad (184)$$

which is consistent with Fokker-Planck equation. In any external potential $V(\vec{x})$, there are two contributions to the changing of average velocity. One is from random impacts of spacetime, another one is from acceleration provided by external potential. Therefore, the average velocity will evolve during time interval Δt as

$$\vec{u}(\vec{x}, t + \Delta t) = \frac{\int (\vec{u}(\vec{x} - \delta\vec{x}, t) - \frac{\Delta t \nabla V(\vec{x} - \delta\vec{x})}{m}) \rho(x - \delta\vec{x}, t) \varphi(\Delta\vec{x}) d(\Delta\vec{x})}{\int \rho(x - \delta\vec{x}, t) \varphi(\Delta\vec{x}) d(\Delta\vec{x})} \quad (185)$$

the denominator of eq. 185 is the normalisation factor of the probability distribution. Expanding Taylor series of both sides, we obtain

$$m \frac{d\vec{u}}{dt} = -\nabla V + \Re m \left(\frac{\nabla^2(\rho\vec{u})}{\rho} - \vec{u} \frac{\nabla^2 \rho}{\rho} \right) \quad (186)$$

From this we can see the acceleration of the quantum envelope velocity \vec{u} , whose dynamics are rooted in the joint contribution of the classical potential and the quantum potential. For the physical state with certain energy, the three-velocity decomposition $\vec{V}(\vec{x}, t) = \vec{u}(\vec{x}, t) + \vec{v}(\vec{x}, t) + \vec{v}(t)$ has clear physical meaning. The quantum envelope velocity $\vec{u}(\vec{x}, t)$ and the classical statistical velocity $\vec{v}(\vec{x}, t)$ are both velocity fields, which are functions of space-time coordinates. The classical statistical velocity field of a physical state with certain energy is zero, which can be used as a new interpretation of the steady state of quantum mechanics. The dynamic mechanism of the quantum envelope velocity field $\vec{u}(\vec{x}, t)$ has two contributions, the classical external potential field where the particle is located and the quantum potential field generated by the random collision of time-space. The diffusion velocity $\vec{v}(\vec{x}, t)$ is the background of space-time fluctuations, evenly distributed in space, and satisfies the properties of Brownian motion in time, which is the intrinsic property of space-time. The sum of these three velocities is the real velocity of the objective reality of the particles required by materialism. See appendix B where we proved these. With the condition of stationary state $\partial_t \rho = 0$, it goes to

$$\vec{u} = \Re \frac{\nabla \rho}{\rho} \quad (187)$$

$$\partial_t \vec{u} = 0 \quad (188)$$

It's important to notice that

$$\frac{d\vec{u}}{dt} = \partial_t \vec{u} + (\vec{u} \cdot \nabla) \vec{u} \quad (189)$$

The average velocity \vec{u} is not zero in the stationary state, which exactly cancel out its fluctuation velocity. Therefore, given the condition of stationary state, we are able to get

$$-2m\Re^2 \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + V(x) = \text{Const.} \quad (190)$$

We can prove this constant is exactly the average energy of particle

$$E = \int \rho \left(\frac{1}{2} m u^2 + V \right) d^3x \quad (191)$$

Now, we have derived

$$-2m\Re^2 \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} + V(x) = E \quad (192)$$

$$\psi = \sqrt{\rho} e^{-iEt/\hbar} \quad (193)$$

Let $\Re = \frac{\hbar}{2m}$ once again, we arrive at the stationary Schrödinger equation

$$-\frac{\hbar^2 \nabla^2}{2m} \psi + V\psi = E\psi \quad (194)$$

F. Ground States of Hydrogen Atoms in MIP

In the hydrogen atom system, we can take $\vec{A} = 0$ and $\phi = -\frac{e}{4\pi\epsilon_0 r}$. The stationary solution of the equation (161) satisfies

$$E\Psi = \frac{1}{2m} (-i\hbar\nabla)^2 \Psi - \frac{e^2}{4\pi\epsilon_0 r} \Psi \quad (195)$$

The lowest energy stationary state solution (ground state wave function) is $\Psi(r, \theta, \varphi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$, where $a = 5 \times 10^{-11} m$ is the Bohr radius of the hydrogen atom. Using the wave function of the ground state of a hydrogen atom, we can get its quantum envelope velocity as

$$\vec{u} = 2\Re \nabla R = -\frac{\hbar}{ma} \hat{r} = -\frac{c}{137} \hat{r} \quad (196)$$

Where c is the velocity of light in vacuum, \hat{r} is the unit vector $\hat{r} = \frac{\vec{r}}{r}$. Similarly we can get its classic average velocity

$$\vec{v} = 2\Re \nabla I = 0 \quad (197)$$

Its spacetime fluctuation rate is satisfied

$$\langle \nu_i \rangle = 0, \langle \nu_i(t) \nu_j(t') \rangle = \Re \delta_{ij} \delta(tt') \quad (198)$$

Then the electron in the ground state of the hydrogen atom has its coordinate $\vec{X}(t)$ as a random variable, and its real velocity \vec{V} satisfies the following microscopic dynamic equations.

$$\frac{d\vec{X}(t)}{dt} = \vec{V}(t) = \vec{u} + \vec{v} + \vec{v} = -\frac{c}{137}\hat{r} + \vec{v}(t) \quad (199)$$

This is the real equation of motion of the ground state electrons of a hydrogen atom in the context of MIP. The quantum envelope velocity always points to the center of hydrogen atom. The closer to the center, the greater the repulsive force generated by the spacetime potential. Because this envelope velocity is balanced out by the combination of the classical Coulomb potential and the spacetime potential, the hydrogen atom can be stabilized on the ground state.

According to MIP, the real motion of electrons in the ground state of hydrogen atoms, we can calculate the average kinetic energy of electrons as

$$\langle K \rangle = \frac{m}{2} \langle \vec{V}(t)^2 \rangle = \frac{m}{2} \left(\frac{c}{137} \right)^2 + \frac{m}{2} \langle \vec{v}(t)^2 \rangle \quad (200)$$

The average of the square of the spacetime fluctuation is

$$\langle \vec{v}(t)^2 \rangle = \mathfrak{R}/T \quad (201)$$

Where T is the cumulative interaction time of the electrons. The ground state of a hydrogen atom can exist forever, that is, T tends to infinity, and thus we can obtain the average kinetic energy of the ground state electron as

$$\langle K \rangle = \frac{m}{2} \langle \vec{V}(t)^2 \rangle = \frac{m}{2} \left(\frac{c}{137} \right)^2 \quad (202)$$

We can calculate the average potential energy of the electron as

$$\langle U(r) \rangle = \left\langle -\frac{e^2}{4\pi\epsilon_0 r} \right\rangle = \left\langle -\frac{e^2}{4\pi\epsilon_0 a} \right\rangle \quad (203)$$

Where a is the Bohr radius and ϵ_0 is the vacuum permittivity. The average energy of the ground state electrons is the sum of the average kinetic energy and the average potential energy. Substituting the standard values of physical constants, we can get the numerical result of the average energy of the ground state electrons as

$$E = \langle K \rangle + \langle U \rangle = -13.6\text{ev} \quad (204)$$

We have reached the same conclusion as quantum mechanics through the microscopic equation of motion of MIP. It can be seen that quantum mechanics only reflects the statistical average nature of the real motion process and does not reflect all the physics under the framework of MIP.

1. *Deriving the amount of elementary charge from MIP*

According to MIP, the interaction between particles and STP (the basic definition of the action is the product of momentum and displacement)

$$Nh = \oint pdq \quad (205)$$

For example, the simplest uniform circular motion is

$$\oint pdq = 2\pi mvr \quad (206)$$

Consider the electrons inside the hydrogen atom. STP collisions provide random Brownian motion, and attraction from proton provides centripetal force with equilibrium conditions

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \quad (207)$$

The amount of charge can be solved as

$$e = nh\sqrt{\frac{\epsilon_0}{m\pi r}} \quad (208)$$

The exact value of the electronic charge can be accurately obtained. We know that in MIP, the exchange action is nh , where n can be any integer.

We only need to make a hypothesis that the orbit of the electron is determined by the quantum number n of STP interaction. The proof of this hypothesis is shown in the next section. That is, when $n = 1$, the electron falls on the Bohr's orbit ($r = 0.53 \times 10^{-10}m$). When $n = 2$, the electrons fall on the second orbit (by analogy). You can get important results (all values below are with international units)

$$h = 6.62 \times 10^{-34}, m = 9.11 \times 10^{-31}, \epsilon_0 = 8.85 \times 10^{-12}$$

After substituting, we obtain the amount of charge as

$$e = 1.6 \times 10^{-19}C \quad (209)$$

2. Quantum number n of STP determining the orbit of hydrogen atoms

What we want to prove is that when the electrons are in Bohr's orbit ($r = a$), the amount of exchange action of STP is just a Planck constant, ie

$$h = 2\pi mva \quad (210)$$

Using the ground state wave function of the hydrogen atom derived above

$$\psi = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \quad (211)$$

The average value of the momentum can be found as

$$Mv = p = \left| \int \psi^* (-i\hbar\nabla)\psi d\tau \right| = \frac{\hbar}{a} \quad (212)$$

The integral volume element is $d\tau = r^2 \sin\theta d\theta d\varphi dr$ and $h = 2\pi mva$.

3. Generalisation to Hydrogen-like atoms

The exchanged action between particles and STP

$$nh = \oint pdq \quad (213)$$

In uniform circular motion

$$\oint pdq = 2\pi mvr \quad (214)$$

An electron in a hydrogen-like atom with a positively charged nucleus. STP collisions provide random Brownian motion, and the attraction of the nucleus provides centripetal force with equilibrium conditions

$$\frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \quad (215)$$

The amount of charge can be solved as

$$e = nh \sqrt{\frac{\epsilon_0}{Zm\pi r}} \quad (216)$$

The Bohr-like orbital electron corresponding to $n = 1$ has a Bohr radius of $r = a/Z$, from which the elementary charge can be derived as

$$e = 1.6 \times 10^{-19} C \quad (217)$$

Starting from MIP, we have made a thorough study of free matter particles and obtained the most important conclusions of quantum mechanics. Furthermore, the most fundamental cause of atomic stability is explained by MIP, and from the first principle we calculate the basic physical quantity of electron charge unit. It can be seen that the random collision of STP does not only provide chaotic background noise, but also the stability of all matter in a seemingly chaotic background. At the most profound level, materialism interprets the physical world and the contradictions are unified.

VI. QUANTUM MEASUREMENT IN MIP

A. General Principle

There are fundamental distinctions on quantum measurement between MIP and Copenhagen interpretation. Within the framework of MIP, since matter particle is collided randomly by STP. Any measurement related to position and momentum can not be done in a time interval between two collisions, therefore any this kind of measurement cannot lead to precise result, which means we cannot make errors as small as possible in principle. Therefore, incommutable observables can not only be measured precisely at the same time, but also cannot be measured precisely separately. Theoretically, all measure values means statistical average, which include intrinsic uncertainty from spacetime besides normal measurement errors. For examples, the momentum uncertainty from MIP is due to the statistical properties of fluctuated mass. As a statistical mass, the minimum fluctuation is Δm_{st} , which roughly is one part per million of electron mass. The position intrinsic uncertainty ΔX_{st} from MIP is the mean free path between two consecutive collision by STP.

When the spacetime sensible mass is equivalent to the statistical inertial mass, the equation of motion will be determined by Schrödinger equation. In other words, moving matter particle and propagational wave are unified in spacetime. If we want to measure a matter particle, we need apparatus to interact with particle somehow. However, every such measurement has to interrupt the random motion of particle. Therefore, measurement means the end of a Markov process. When the measurement is finished, a new Markov process will begin. For the moving matter particle, the phases of wave functions before and after measurements is completely irrelevant, which cannot interfere each other. Under this framework,

it's unnecessary to introduce hypotheses of wave function collapse or multi universe.

B. EPR Paradox in MIP

In a 1935 paper[35], Einstein with Podolsky and Rosen considered an experiment in which two particles that move along the x-axis with coordinates x_1 and x_2 and momenta p_1 and p_2 were somehow produced in an eigenstate of the observables $X = x_1 - x_2$ and $P = p_1 + p_2$ (these two observables commute $[X, P] = 0$).It's easy to understand that the measurement of the position of particle 1 can interfere with its momentum, so that after the second measurement the momentum of particle 1 no longer has a definite value. However two particles are far apart, how can the second measurement interfere with the momentum of particle 2? And if it does not, then after both measurements particle 2 must have both definite position and momentum, contradicting the quantum uncertainty principle. If it does, there exist some “spooky” interaction between two far apart particles, contradicting the locality principle in the special theory of relativity. The orthodox interpretation of quantum mechanics suppose that the second measurement which gives particle 1 a definite position, prevents particle 2 from having a definite momentum, even though the two particles are far apart. The states of the two particles are so call quantum entanglement.

Let's investigate the experimental process in detailed and estimate every uncertainty relations. Suppose two particles that are originally bound in some sort of unstable molecule at rest fly apart freely in opposite directions, with equal and opposite momenta until their separation becomes macroscopically large. Their separation will evolve as

$$x_1 - x_2 = x_{10} - x_{20} + (p_1 - p_2)t/m \quad (218)$$

where x_{10}, x_{20} are initial positions of two particles. It's noticed that under MIP, every massive particle is collided randomly by STP, the initial separation of two particle cannot be measured precisely. There exists intrinsic uncertainty $\Delta X_{st} = \Delta|x_{10} - x_{20}|$ as the mean free path between two consecutive collision by STP. According to the uncertainty relation derived from MIP, the momentum difference at least has intrinsic uncertainty as $\Delta P_{st} = \Delta|p_1 - p_2| \geq \frac{\hbar}{\Delta X_{st}}$, because of the commutation $[x_1 - x_2, p_1 - p_2] = 2i\hbar$. Therefore the

uncertainty of separation will be

$$\Delta|x_1 - x_2| = \Delta X_{st} + \frac{\hbar t}{\Delta X_{st} m} \quad (219)$$

Its minimum is at $\Delta X_{st} = \sqrt{\frac{\hbar t}{m}}$, leading to

$$\Delta|x_1 - x_2| \geq 2\sqrt{\frac{\hbar t}{m}} \quad (220)$$

Similarly, the total momentum P is not strictly zero under MIP, which includes at least the intrinsic uncertainty due to

$$\Delta P = \Delta m_{st} v \quad (221)$$

where Δm_{st} is the fluctuation of statistical mass, according to MIP, roughly as one part per million of electron mass. Perform EPR experiment after the second measurement of particle 1, the uncertainty of particle 2 at least will be

$$\Delta p_2 \Delta x_2 = 2\sqrt{\frac{\hbar t}{m}} \Delta m_{st} v \quad (222)$$

More importantly, does the intrinsic uncertainty of particle 2 given by MIP contradict the uncertainty relation given by quantum mechanics? If

$$\Delta p_2 \Delta x_2 \leq \frac{\hbar}{2} \quad (223)$$

it still contradicts uncertainty relation of quantum mechanics, which means that we will observe the quantum entanglement experimentally, because we have to suppose the ‘‘spooky’’ interaction between two far apart particles to satisfy uncertainty relation. Therefore, we obtain the key criterion of quantum entanglement (momentum-position type) as

$$\frac{\Delta m_{st}^2}{m^2} \leq \frac{\lambda_d}{16\pi L} \quad (224)$$

where $\lambda_d = \frac{h}{mv}$ is de Broglie’s wavelength and L is the separation of two particles. So we can conclude that there is a characteristic separation of quantum entanglement as

$$L^* = \frac{\lambda_d}{16\pi} \left(\frac{m}{\Delta m_{st}}\right)^2 \quad (225)$$

When the separation of two particles is larger than L^* , the inequality of (8) cannot be satisfied which means we are no longer able to determine the existence of quantum entanglement from experimental results. The reason is that the intrinsic uncertainty of particle 2 given

by MIP has already satisfy uncertainty relation of quantum mechanics automatically. We cannot deduce the existence of 'spooky' interaction in this scenario. For two electrons moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 1m$. For two atoms moving at the speed of $0.01c$, the corresponding characteristic separation will be $L^* \approx 10^6m$.

VII. FROM MIP TO PATH INTEGRAL

The path integral representation of quantum mechanics is a generalization and formulation method for quantum physics, which extends from the principle of action in classical mechanics. It replaces a single path in classical mechanics with a quantum amplitude that includes the sum or functional integral of all paths between two points. The path integral expression was theoretically published by theoretical physicist Richard Feynman in 1948 [39]. Prior to this, Dirac's 1933 paper[40], had major ideas and some early results. The main advantages of the path integral expression is that it treats spacetime equally, so it is easy to generalize to the theory of relativity, which is widely used in modern quantum field theory. However, the basic assumptions of MIP tell us that the effect of each STP colliding on particles can be seen as an independent path. The weight of each independent path is related to the distribution of energy. This is essentially a process of path integration. To understand this concept more clearly, we consider a simple process as follows. Assuming that the effect of random motion of particles over time Δt is from point A to point B. According to MIP, in this process, the change of the action can only be h , $2h$, $3h$, ..., but the paths are different corresponding to each specific action change. For example, the smallest amount of action change is one h , corresponding to a linear motion from A to B, and the $2h$ change corresponds to the movement of the polyline, during which the particle is struck twice by STP, and so on. In this picture, the so-called infrared effect is naturally ruled out, that is, the process of less than one h in Δt . The effect of infinity is also ruled out because the instantaneous velocity has certain upper bound which is the speed of light. This suggests that such a path integral effect is a finite summation rather than an infinite, so there is no need to introduce a so-called renormalization procedure. We see that under the framework of MIP, the quantum properties of particles are rooted in nature as the statistical description of their random motion.

A. Path Integral of Free Particle and Spacetime Interaction Coefficient

We had argued the real velocity of free particle in space-time satisfies the decomposition as

$$\vec{V}(\vec{x}, t) = \vec{v}(\vec{x}, t) + \vec{u}(\vec{x}, t) + \vec{v}(t) \quad (226)$$

in which there are two kinetic arguments, they are classical statistical velocity \vec{v} and quantum envelope velocity \vec{u} .

There are two kinetic variables with random motion particle in spacetime, which are classical speed \vec{v} and fluctuated speed \vec{u} . The corresponding kinetic equations are

$$\frac{\partial \vec{u}}{\partial t} = -\Re \nabla (\nabla \cdot \vec{v}) - \nabla (\vec{u} \cdot \vec{v}) \quad (227)$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \cdot \nabla) \vec{v} + (\vec{u} \cdot \nabla) \vec{u} + \Re \nabla^2 \vec{u} \quad (228)$$

Setting $\Psi = e^{R+iI}$, we are able to linearise as

$$\nabla R = \frac{1}{2\Re} \vec{u} \quad (229)$$

$$\nabla I = \frac{1}{2\Re} \vec{v} \quad (230)$$

which leads to

$$\frac{\partial \Psi}{\partial t} = i\Re \nabla^2 \Psi \quad (231)$$

During an infinite small time interval ϵ , the solution can be written in terms of integrals as

$$\Psi(x, t + \epsilon) = \int G(x, y, \epsilon) \Psi(y, t) dy \quad (232)$$

which represents the superposition of all the possible paths from y to x . The critical observation of Feynman is the weight factor $G(x, y, \epsilon)$ will be proportional to $e^{iS(x, y, \epsilon)/\hbar}$, where $S(x, y, \epsilon)$ is the classical action of particle as

$$S(x, y, \epsilon) = \int L(x, y, \epsilon) dt = \int (K - U) dt = (\bar{K} - \bar{U})\epsilon \quad (233)$$

\bar{K} and \bar{U} are average kinetic energy and potential energy separately. In order to show the equivalence between path integral formulation and the spacetime interacting picture, we should derive our basic kinetic equations from the postulation of path integral $G(x, y, \epsilon) = A e^{iS(x, y, \epsilon)/\hbar}$. For a free particle in spacetime, one has $\bar{U} = 0, \bar{L} = \frac{m}{2} \left(\frac{x-y}{\epsilon}\right)^2$ and $S = \frac{m(x-y)^2}{2\epsilon}$, which leads to

$$\Psi(x, t + \epsilon) = A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon}} \Psi(y, t) dy \quad (234)$$

Setting $y - x = \xi$ and $\alpha = -\frac{im}{2\hbar\epsilon}$, it can be written in terms of

$$\begin{aligned}\Psi(x, t + \epsilon) &= A \int e^{-\alpha\xi^2} \Psi(x + \xi, t) d\xi \\ &= A \int e^{-\alpha\xi^2} \left(\Psi(x, t) + \xi \frac{\partial \Psi}{\partial x} + \frac{1}{2} \xi^2 \frac{\partial^2 \Psi}{\partial x^2} + \mathcal{O}(\xi^4) \right) d\xi\end{aligned}\quad (235)$$

With the properties of Gaussian integral

$$\int e^{-\alpha\xi^2} d\xi = \sqrt{\frac{\pi}{\alpha}} \quad (236)$$

$$\int e^{-\alpha\xi^2} \xi d\xi = 0 \quad (237)$$

$$\int e^{-\alpha\xi^2} \xi^2 d\xi = \frac{1}{2\alpha} \sqrt{\frac{\pi}{\alpha}} \quad (238)$$

we can obtain

$$\Psi(x, t + \epsilon) = A \left(\sqrt{\frac{\pi}{\alpha}} \Psi(x, t) + \frac{1}{4\alpha} \sqrt{\frac{\pi}{\alpha}} \frac{\partial^2 \Psi}{\partial x^2} + \mathcal{O}(\alpha^{-\frac{5}{2}}) \right) \quad (239)$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, we have

$$\Psi(x, t + \epsilon) - \Psi(x, t) = \epsilon \partial_t \Psi(x, t) = \frac{1}{4\alpha} \frac{\partial^2 \Psi}{\partial x^2} \quad (240)$$

From this integral, We observed that the most important contribution comes from $y - x = \xi \propto \sqrt{\epsilon}$, where the speed of particle is $\frac{y-x}{\epsilon} \propto \sqrt{\frac{\hbar}{m\epsilon}}$, we see here when $\epsilon \rightarrow 0$, the speed divergent in order $\sqrt{1/\epsilon}$. The paths involved are, therefore continuous but possess no derivative, which are of a type familiar from study of stochastic process. With the isotropy of space, we have

$$\partial_t \Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon} \nabla^2 \Psi(\vec{x}, t) \quad (241)$$

Corresponding to the Eq. (231), if one requires the equivalence between path integral formulation and MIP, there must be

$$i\mathfrak{R} = \frac{1}{4\alpha\epsilon} \quad (242)$$

$$\mathfrak{R} = \frac{1}{4i\alpha\epsilon} = \frac{1}{4i(-\frac{im}{2\hbar\epsilon})\epsilon} = \frac{\hbar}{2m} \quad (243)$$

Notice that \mathfrak{R} is only an arbitrary parameter in the Eq.(143). The consistency between path integral and MIP requires $\mathfrak{R} = \frac{\hbar}{2m}$. An arbitrary finite time interval Δt , can be cut into infinitely many pieces of infinitesimal time interval ϵ . And in each ϵ , the collisions leads to many different paths, one can pick one path and consecutively another along the time direction, this will end up a path in Δt , sum over all possible paths in Δt gives an

integration over path space, which is the celebrated historical summation or path integral. The method here can be straightforwardly generalised to the particle in the external potential as in following section.

B. Path Integral of Particle in an External Potential and Spacetime Interaction Coefficient

In an external potential U , one has $\bar{U} = U(\frac{x+y}{2})$ and $\bar{L} = \frac{m}{2}(\frac{x-y}{\epsilon})^2$, which leads to the action

$$S = \frac{m(x-y)^2}{2\epsilon} - U(\frac{x+y}{2})\epsilon \quad (244)$$

According to the path integral formulation, it must satisfy

$$\begin{aligned} \Psi(x, t + \epsilon) &= A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon} - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}} \Psi(y, t) dy \\ &= A \int e^{\frac{im(x-y)^2}{2\hbar\epsilon}} \left(1 - \frac{iU(\frac{x+y}{2})\epsilon}{\hbar}\right) \Psi(y, t) dy \end{aligned} \quad (245)$$

To the lowest order of ϵ , it shows

$$U(\frac{x+y}{2})\epsilon = U(x + \frac{\xi}{2})\epsilon = U(x)\epsilon \quad (246)$$

$$\Psi(x, t + \epsilon) = A \int e^{-\alpha\xi^2} \left(1 - \frac{iU(x)\epsilon}{\hbar}\right) \Psi(x + \xi, t) d\xi \quad (247)$$

From the properties of Gaussian integral in the previous section, we obtain

$$\Psi(x, t + \epsilon) = A \left(1 - \frac{iU(x)\epsilon}{\hbar}\right) \sqrt{\frac{\pi}{\alpha}} \Psi(x, t) + A \frac{1}{4\alpha} \sqrt{\frac{\pi}{\alpha}} \frac{\partial^2 \Psi}{\partial x^2} \quad (248)$$

Setting $A = \sqrt{\frac{\alpha}{\pi}}$, $\epsilon \rightarrow 0$, we have

$$\partial_t \Psi(\vec{x}, t) = \frac{1}{4\alpha\epsilon} \nabla^2 \Psi(\vec{x}, t) + \frac{1}{i\hbar} U \Psi(\vec{x}, t) \quad (249)$$

To be consistent with the case of free particle, let's take $\mathfrak{R} = \frac{\hbar}{2m}$ which leads to

$$\partial_t \Psi(\vec{x}, t) = i\mathfrak{R} \nabla^2 \Psi(\vec{x}, t) + \frac{1}{i\hbar} U \Psi(\vec{x}, t) \quad (250)$$

Therefore we have derived the equation of motion from MIP.

VIII. SPIN WITHIN THE FRAMEWORK OF MIP

A. MIP and Spin 1/2

According to the MIP, let us consider the movement of a free particle in a closed path. The total amount of STP collision exchanged action in this process is

$$\oint \vec{p} \cdot d\vec{l} = \sum nh = \zeta(0)h = -h/2 \quad (251)$$

The particle's momentum \vec{p} consists of classical momentum and fluctuation momentum. The average contribution of the Brownian movement is zero, that is

$$\oint \vec{v} \cdot d\vec{l} = -2\pi\Re \quad (252)$$

Where \vec{v} is the classic statistical velocity of the particle. The wave function emerges from the particle collision of STP is $\Psi = e^{iI}e^R$, where I is the phase of the particle wave function and the classical statistical velocity of the particle is $\nabla I = \frac{\vec{v}}{2\Re}$. Substituting the equation into (252), we get

$$\oint \nabla I \cdot d\vec{l} = I_i - I_f = -\pi \quad (253)$$

Where I_i is the phase of the particle's starting point and I_f is the phase back to the starting point. When the movement of a closed path is completed, the wave function of the particle is changed from Ψ_i to $\Psi_f = -\Psi_i$, therefor the spin of massive particles are constrained by the MIP. Its spin can only be 1/2, belonging to the fermion. Physically, the potential function R characterizes the fluctuation motion of particles caused by STP collisions, corresponding to statistical mass. The potential function I characterizes the nature of the particle's rotation in spacetime, corresponding to the spin of the particle. The mass and spin are combined by the potential functions R and I to fully describe the motion properties of the particles in spacetime.

According to quantum mechanics, when a particle of spin 1/2 rotates the θ angle around the z-axis, its wave function becomes

$$e^{iS_z\theta/\hbar}\Psi = e^{iS_z\theta/\hbar}(\alpha\Psi_+ + \beta\Psi_-) = e^{i\theta/2}\alpha\Psi_+ + e^{-i\theta/2}\beta\Psi_- \quad (254)$$

The particle rotation $\theta = 2\pi$ is taken into the above formula as

$$e^{iS_z2\pi/\hbar}\Psi = -(\alpha\Psi_+ + \beta\Psi_-) = -\Psi \quad (255)$$

That is, the wave function of the particle changes from Ψ_i to $\Psi_f = -\Psi_i$. It can be seen that under the MIP framework, we obtain the same statistical properties as the 1/2 spin wave function of quantum mechanics. In the framework of MIP, mass is no longer a classic concept. At the same time, the linear velocity of the particle surface is no longer a classic concept. Therefore, Pauli's criticism based on the classical mechanical angular momentum formula is irrelevant in the MIP framework. Particles do not have mass properties when they are not impacted by STP. Subjected to STP impact, its mass is not evenly distributed, but a statistical attribute.

According to the MIP, each random collision of STP will give the electrons Planck constant multiples action. The statistical effect of multiple impacts on the one hand produces the quantum wave function of the electron translation, and on the other hand, the quantum spin property of the electron rotation.

In the picture of MIP, let us consider two identical free particles. Without loss of generality, let each particle have only two possible states as $|k_1 \rangle$ and $|k_2 \rangle$. Regardless of the random collision of STP, the total possible state of the two-particle system. There are four types: $|k_1 \rangle|k_2 \rangle, |k_2 \rangle|k_1 \rangle, |k_1 \rangle|k_1 \rangle, |k_2 \rangle|k_2 \rangle$. Two identical free particles have the same mass, considering STP random collision, $|k_1 \rangle|k_2 \rangle$ and $|k_2 \rangle|k_1 \rangle$ states will be absolutely indistinguishable, describing the same physical state. Considering the following process, STP collides particle 1 to particle 2 and particle 2 to particle 1. Taking the second particle as the frame of reference, this process is equivalent to particle 1 rotating around particle 2 by 2π . According to the equation (255), The wave function of this two-particle system will obtain a negative sign. So for any state x and y we have

$$|x \rangle |y \rangle = -|y \rangle |x \rangle \quad (256)$$

Thus, $|x \rangle |x \rangle = 0$ is obtained, indicating that two identical fermions cannot be in the same quantum state, which is the Pauli exclusion principle. The wave function of the identical particle system with spin 1/2 must satisfy antisymmetry, which is an extremely important axiom of quantum mechanics. If the sign of the wave function of any two particles is changed, then the wave function is completely antisymmetric. This means that the two fermions can never occupy the same state in the same system. Since the identical particles are randomly impacted by STP, they cannot be distinguished as the classic particles which can be tracked by the mark, therefore they are absolutely indistinguishable. If the two fermions are in

exactly the same state, then the value of the wave function should not change after exchange. The only way out is that the value of the wave function is zero, that is, the probability is zero. It must be emphasized that this important logic of quantum mechanics is a natural inference of MIP.

For two identical bosons, without random collisions of STP, there are three possible states: $|k_1 \rangle |k_1 \rangle, |k_2 \rangle |k_2 \rangle, |k_1 \rangle |k_2 \rangle + |k_2 \rangle |k_1 \rangle$. Furthermore, from the extension of two particles to the multi-particle system, it can be seen that under the framework of MIP, the essential difference between the Bose-Einstein distribution and the Fermi-Dirac distribution is naturally interpreted.

The main focus of this paper is to clarify the physical nature of single particles subjected to random collisions with STP. However from the above discussion, the theoretical framework of this paper can be extended to multi-particle systems to describe the quantum statistical behavior of multi-particle systems.

In modern quantum physics, spin is interpreted as the intrinsic property of particles. On the other hand, from the point of view of group theory, it is also seen as a representation of the Lorentz group. For example, a particle with spin $1/2$ corresponds to the minimal representation of the Lorentz group, and a particle with a spin of 1 corresponds to the vector representation of the Lorentz group. These two expressions are actually incompatible. Because it is the representation of the Lorentz group, which means that the particles present in the spacetime, and can not be said to be intrinsic. On the other hand, if the spin of a particle is not intrinsic, then under the framework of the perturbation theory of modern quantum physics, the state of motion can always change due to perturbations. Since the spin does not change with the state of motion, it cannot be a motional property under the framework of perturbation theories.

B. **Statistical Properties of Spin**

In the framework of the MIP, the spin of the particles needs to be reconsidered. We believe that the motion of particles in spacetime will be impacted by STP anytime and anywhere, and its motion can be described by the statistical wave function introduced in this paper. It should be noted that our statistical wave function does not contain a relativistic effect. The spin as a representation of the Lorentz group is obviously a relativistic property. In

the framework of modern quantum physics, the behavior of relativistic particles and non-relativistic particles is quite different. However, under the framework of MIP, the statistical movement velocity of particles and the fluctuation velocity of particles are mathematically independent, and in fact are physically inseparable. The latter is accompanied by the former. So it is impossible to split the two movements. Therefore, under the framework of MIP, non-relativistic statistical movements are naturally accompanied by relativistic behavior. This explains precisely that the particle has a "inner spin", and this "inner spin" is the property of the relativistic Lorentz transformation. Therefore, under the framework of MIP, the classical, quantum and relativistic behavior of particles is always perfectly unified.

C. Spin Dynamics in MIP

We now consider the dynamics of spin under the MIP framework. Considering the statistical behavior of particles, their fluctuation motion is generally considered to have no statistical average effect. The general behavior here does not include non-perturbative fluctuations, however non-perturbative fluctuations bring new effects. We can classify the fluctuations of particles as the perturbation fluctuation, which corresponds to the tiny Lorentz transformation of the particle. The other type is non-perturbative fluctuations, which reflect the topological properties of the Lorentz group. It requires a topological subgroup of the Lorentz group for further classification.

1. Topological Subgroup of Lorentz Group

Lorentz group $SO(3, 1)$'s complex algebra $so(3, 1)_{\mathbb{C}}$ can be equivalent to the direct product of two $su(2)$, ie

$$so(3, 1)_{\mathbb{C}} \simeq su(2)_L \otimes su(2)_R \quad (257)$$

But each $su(2)$ Lie algebra has a sub Lie algebra of $u(1)$, so the maximal ring of the Lie algebra of the Lorentz group is $u(1)_L \otimes u(1)_R$ which is topologically equivalent to a torus. It can be seen that the topology subgroup of the Lorentz group is $U(1)_L \otimes U(1)_R$ which can be used to classify the non-perturbative fluctuations described above.

2. *Topological Classifications of Non-Perturbative Fluctuations*

With the first order homotopy group

$$\pi_1(U(1)_L \otimes U(1)_R) = \mathbb{Z} \oplus \mathbb{Z} \quad (258)$$

two integers can be used to classify all the closed paths of the Lorentz group that are not topologically equivalent. For example, (1,0) means a path around $U(1)_L$ without a path around $U(1)_R$. And (1,1) It means a path around each of $U(1)_L$ and $U(1)_R$.

3. *Non-Perturbative Path and Statistical Origins of Spin*

Non-perturbative paths are a very special class of extreme relativistic paths that correspond to the topology of spacetime. Considering that if the relativistic non-perturbative path of a particle is a path around the left-hand topological subgroup $U(1)_L$, then forming this path requires the particle to undergo a statistically infinite number of STP collision. Each collision will increase or decrease the particle's action by a Planck constant of h . Then the total amount of exchanged action between the particle and the STP will be

$$\varphi = \oint p \cdot dl = \sum_{n=1}^{\infty} 1 \times h = -\frac{1}{2}h \quad (259)$$

This contribution to the statistical wave function of the particle is significant because the wave function of the particle is in fact a partition function of the particle state. So this path-dependent statistical phase will transform the wave function Ψ into

$$\Psi \times e^{i\varphi/h} = \Psi \times e^{-i\pi} = -\Psi \quad (260)$$

which shows that the particles of the (1,0) path classification will statistically equivalent to a spin 1/2 particle.

Similarly, it can be proved that the particles of the (0,1)-path also statistically equivalent to a spin 1/2 particle. In order to distinguish between these two topological properties, one is the left-handed spin 1/2 fermion, and the other is the right-handed spin 1/2 fermion.

For the particle represented by the path (1,1),

$$\varphi = \oint_{L+R} p \cdot dl = \sum_{n=1}^{\infty} 1 \times h + \sum_{n=1}^{\infty} 1 \times h = -h \quad (261)$$

Its wave function remains unchanged under the phase transformation, which actually depicts a particle with a spin of 1.

From this classification, the non-perturbative path corresponding to the particles of any spin can be derived. Since it is a topological property, it will not change under the local Lorentz transformation, which coincides with the nature of the spin exactly.

IX. ELECTROMAGNETISM: AN MIP APPROACH

A. Essential Properties of Electronic Charge In Modern Physics

In framework of modern physic, fundamental matter particles are all electric charged. The fundamental electric charge is defined as the amount of charge of an electron or a positron.

For electric charge [50], there are five fundamental properties. Firstly, there are only two kinds of charges, as known as the positive and negative charges. The characteristic quantum numbers of positron and electron are 1 and -1. Secondly, same charges repel each other, different charges attract each other. Thirdly, the interaction between charges is known as the Coulomb force, obeys the inverse square law. Electron and positron can annihilation each other, emit photons. Forthly, in an isolated system, the algebraic amount of charges are conserved. Finally, the amount of fundamental charge is $1.6 \times 10^{-19}C$.

Dynamics of STP revisited

Since there are no interactions between STP, the differential dynamics of STP is discribed by a massless free scale field theory, its Lagrangian is:

$$\mathcal{L}_{ST} = \partial_\mu \phi \partial^\mu \phi. \quad (262)$$

The dynamic equation is the 3+1 dimentional Klein-Gordon equation,

$$\partial_\mu \partial^\mu \phi = 0, \quad (263)$$

the solution of above equation is a wave solution, it can be written as follow

$$\phi(\vec{x}, t) = \sum_{E^2 = \sum_{i=1}^3 p_i^2} f(E, \vec{p}) \exp(iEt - i\vec{p} \cdot \vec{x}), \quad (264)$$

in which $f(E, \vec{p})$ is an analytic function in momentum space.

Now let us consider putting a particle into space-time. The impact of introducing the matter particle into space-time scalar field, is somehow like dropping a cobble into the water surface of a peaceful lake, leads to the ripple effect. Compare to the fluctuation of space-time, the matter particle introduces a non-perturbative effect, which will bring into the space-time a strong potential. The reason that the matter particle results a strong potential is as follows Any perturbative disturbance will be get drowned out by the fluctuation of microscopic space-time energy fluctuation. In general, strong perturbation will lead to nonlinear effects, especially non-perturbative soliton effect. The soliton effect is steady and relatively large than STP. We know STP are local excitation of space-time energy, obviously, a cluster of STP describes a “huge” excitation of space-time energy. So it is nature to introduce solitons into space-time field since a local non-perturbative energy disturbance leads a local space-time soliton, discribing a cluster effect of STP.

B. 2+1-dim Complex Scalar Space-time field

In modern quantum field theory, the microscopic energy can be non-conserved locally, which is saying the vacuum can excit any pair of virtual conjugated particles. In framework of MIP, the fluctuations of space-time energy are STP. The non-conservation nature of local space-time energy is saying the number of STP are locally non-conserved. However, in a global viewpoint, the energy of STP are conserved.

In framework of MIP, we introduce a local companion for STP field, which is a local field that can interact with STP. However, in global, the companion field will decouple with the STP field. The existence of the local companion field also implies in local there is a kind of local symmetry, which is broken in global. In fact, when the local symmetry is $U(1)$, STP are excitations of a complex scalar field.

In framework of MIP, matter particle experiences quantum Brownian motion, which essentially is a Markov progress. This implies the past and future of the matter particle are causal irrelated. So at an arbitrary point of time, one can cut the slice vertical to the direction of the velocity of the matter particle, as known as the normal slice. The dynamics of matter particle on normal slice is a 2+1-dim dynamics. The whole 3+1-dim dynamics could be extented from the dynamics on slices. Notice there are two kinds of dynamics on

the 2+1-dim normal slice, one for matter particle, the other for STP, respectively.

We now consider the 2+1 dimensional dynamics of STP. As is stated above, the matter particle drops a cobble into the STP lake and results a period potential. We denote the potential as $V(\phi, \phi^*)$, thus the Lagrangian of complex STP field now becomes

$$\mathcal{L}_{ST} = -\frac{1}{2}\partial_j\phi\partial^j\phi^* + V(\phi, \phi^*), \quad j = 0, 1, 2. \quad (265)$$

C. Abrikosov-Nielsen-Olesen-Zumino Vortex

In 2+1 dimension, the famous non-pertubative solution for a complex scalar field is the Abrikosov-Nielsen-Olesen-Zumino(ANOZ) vortex solution [44][45]. The Lagrangian supports the ANOZ vortex is

$$\mathcal{L} = -\partial_j\phi^*\partial^j\phi - \frac{\lambda}{2}(\phi^*\phi - F^2)^2 \quad (266)$$

The minimum of the potential is obvious, it is

$$\phi = F \cdot e^{i\varphi}$$

which is a cycle with radius F . Notice this configuration is compatible with the ‘‘ripple’’ effect of matter particle acting on STP field. It also introduces a symmetry $U(1)$. Since this $U(1)$ now is a local symmetry, it implies there should be a gauge field companion with the STP field. The soliton solution is obtained when introducing the boundary condition at infinity, that is

$$|x| \rightarrow \infty : \quad \vec{\phi} \rightarrow F \frac{\vec{x}}{|x|}, \quad \phi \rightarrow F e^{i\varphi}. \quad (267)$$

However, the soliton solution suffers an energy divergence because

$$E = \int d^2x \left(\vec{\partial}\phi^*\vec{\partial}\phi + V(\phi, \phi^*) \right) \quad (268)$$

goes to infinity. One can check this as follows

$$\begin{aligned} |x| \rightarrow \infty : \quad \partial_i\phi_j &\rightarrow \frac{F}{|x|} \left(\delta_{ij} - \frac{x_i x_j}{|x|^2} \right) \\ \sum_{i,j=1}^2 (\partial_i\phi_j)^2 &\rightarrow \frac{F^2}{|x|^2} \\ \int d^2x \vec{\partial}\phi^*\vec{\partial}\phi &\rightarrow 2\pi \int_0^\infty d|x| \frac{F^2}{|x|} : \quad \text{Log divergent.} \end{aligned} \quad (269)$$

We saw the energy of the vortex is divergent at spatial infinity, this is unphysical since it implies there is an infinity energy source at spatial infinity. To avoid this divergence, the way out is to introduce a gauge vector field to smear the infinity energy on whole 2+1-dim normal slice. In fact, the local non-conservation of space-time energy implies we need a companion field for STP field in the first place. Here it is clear that the field is a gauge field. To do so, we need introduce the covariant derivative for STP field, instead of original derivative, as well as a kinetic term for the gauge field. Now the Lagrangian is

$$\mathcal{L} = -\frac{1}{2}D_\mu\phi^*D^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi, \phi^*) \quad (270)$$

$$D_\mu\phi = \partial_\mu - igA_\mu \quad (271)$$

The complex STP field degenerates into a real scalar field. This is because the energy non-conservation is recovered in global. The complexity of the STP field reflects the local property of STP. At spatial infinity,

$$\phi \rightarrow Fe^{i\varphi}|_{\varphi=0} = F \quad (272)$$

the gradient of STP field is

$$\vec{\partial}\phi = (\partial_r\phi\vec{e}_r + \partial_\varphi\phi\vec{e}_\varphi)|_{\varphi=0} = iF/r \quad (273)$$

and the gauge field becomes pure gauge field (with vanishing field strength), that is

$$\vec{A} \rightarrow \frac{1}{ig}\phi^{-1}\vec{\partial}\phi \quad (274)$$

In form of polar coordinates,

$$A_r = 0, \quad A_\varphi = \frac{1}{gr} \quad (275)$$

In general, we can not let a complex scalar field directly equals to a real scalar field at an arbitrary spatial point. However, we can let them equals to each other up to a gauge transformation, say

$$\phi \rightarrow \Omega F, \quad \Omega(\vec{x}) = e^{i\varphi(\vec{x})} \quad (276)$$

and then we have

$$\vec{A} \rightarrow -\frac{1}{ig}\Omega\vec{\partial}\Omega^{-1} \quad (277)$$

Actually, under this general configuration, the divergence of energy will be strictly vanished, as

$$\vec{D}\phi \rightarrow \left(\vec{\partial}\Omega + \Omega(\vec{\partial}\Omega^{-1})\Omega\right) F = \Omega\vec{\partial}(\Omega^{-1}\Omega)F = 0 \quad (278)$$

In terms of component, the gauge field reads

$$A_i = -\frac{1}{g}\epsilon_{ij}\frac{x_j}{r^2} \quad (279)$$

From the Stokes theorem, we have

$$\Phi \equiv \oint_{C=n\cdot\partial\Sigma} \vec{A}d\vec{x} = \int_{\Sigma} \vec{B}d\vec{\sigma} = \frac{2\pi n}{g} \equiv g_m \quad (280)$$

here we recognize the famous Dirac quantization condition [43] for electronic charges, say

$$g \cdot g_m = 2\pi n, \quad n \in \mathbb{Z} \quad (281)$$

This implies if there was an ANOZ vortex solution, the electronic charge is quantized. When n is a negative integer, it describes an opposite spinning vortex solution and also describes a negative charge. In modern physics, there should be a Dirac monopole to support the Dirac quantization condition of charges. In framework of MIP, the only origin of quantized charge is the STP field.

D. From 2+1-d to 3+1-d

In 3+1 Minkowski space-time, the local space-time symmetry is Lorentz symmetry, denoted by $SO(3, 1)$. In Lie group theory, $SO(3, 1)$ is algebraic isomorphism to $SU(2) \times SU(2)$, that is

$$so(3, 1) \cong su(2) \times su(2) \cong so(3) \times so(3). \quad (282)$$

In fact, this isomorphism reveals locally, the 3+1-dim space-time equals to cross extension of two 2+1-dim space-time.

Now we consider how this local extension of dimension can be done from Lie algebra. Notice the six generator of Lorentz group can be written explicitly as

$$K_i \equiv L_{0i} = t\partial_i - x_i\partial_t \quad i, j, k \in [1, 2, 3] \quad (283)$$

$$R_k = \epsilon^{ij}_k L_{ij} = \epsilon^{ij}_k x_i\partial_j \quad (284)$$

The two algebra $su(2)$ are isomorphic to $so(3)$, in terms of derivative, they are

$$S_a = \epsilon^{abc} r_a \partial_{r_b} \quad (285)$$

$$\tilde{S}_a = \epsilon^{abc} l_a \partial_{l_b} \quad (286)$$

in which there are six degrees of freedom, in the meaning of linear space, they are

$$r_1, r_2, r_3, \quad l_1, l_2, l_3$$

Though the Lie algebras of $SO(3, 1)$ and $SU(2) \times SU(2)$ is isomorphic to each other, from the viewpoint of degree of freedom, they are not the same. Notice there is a hidden duality, which maps 2-dim surface to 1+1-dim surface and vice versa, as follows

$$\begin{aligned} \star : e_0 \otimes e_i &\rightarrow \epsilon_{0i}^{jk} e_j \otimes e_k \\ \star : e_j \otimes e_k &\rightarrow \epsilon_{jk}^{0i} e_0 \otimes e_i \end{aligned} \quad (287)$$

This duality is actually the Hodge duality in differential geometry. It implies extension rules should be followed when extending a theory from 2+1-dim to 3+1-dim.

In conclude, we know the rule guiding the extension from 2+1-dim to 3+1-dim is Hodge duality. In the vortex situation considered at hand, the Hodge duality actually corresponds to a resolving of singularity. The vortex has a singular tube which shrinks to a point when goes to its center. If one wants to resolving the singularity, the general way in differential topology is to introduce a finite size sphere instead of the singularity. The resolving operation can be done by two steps: cut the vortex tube at a finite size, which will be a circle, then rotate the circle into a sphere. This rotation was been done in 3+1-dim and is the physical saying of the Hodge duality.

E. The Origin of Photon from ANOZ Vortex

In discussion of ANOZ Vortex, we obtained the gauge constraint and the quantization condition of electric charge, however, we didn't obtain the dynamics of the vortex. Because vortex is not a fundamental excitation, its dynamics can not be analytically achieved from fundamental SPTs. So in order to obtain the vortex dynamics. We need to introduce the Lagrangian for vortices.

1. Dynamics on normal slice

For the kinetic part of SPTs field, say,

$$\mathcal{L}_\phi = \frac{1}{2} \vec{D}_i \phi^* \vec{D}^i \phi = \frac{1}{2} |(\partial_i - igA_i)\phi|^2 \quad (288)$$

in this subsection, $i, j, k, l, m, n = 0, 1, 2$ label indices on the 2+1-dim normal slice. We only consider the excitations nearby the vortex potential, which is $\phi = Fe^{i\varphi}$. The above STP field kinetic Lagrangian can be written as

$$\mathcal{L}_\phi = \frac{1}{2}F^2 (\partial_i\varphi - gA_i)^2 \quad (289)$$

After a simple square matching operation, we arrive a linear form

$$\mathcal{L}_\phi = -\frac{1}{2F^2}\xi^i\xi_i + \xi_i(\partial^i\varphi - gA^i) \quad (290)$$

here ξ^i is a static auxillary field. Notice that for vortex solution, the phase angle field φ is singular at the vortex center, we now separate the phase angle into two parts, one is smooth and the other is for vortex, say,

$$\varphi = \varphi_0 + \varphi_{vortex} \quad (291)$$

The smooth part does not have a significant effect on what we concerned, we integral it out and it results a constraint equation for the auxillary field,

$$\partial_i\xi^i = 0 \quad (292)$$

This reveals the auxillary field is a 2+1-dim sourceless field, and it can be rewritten as a pure curl as

$$\xi^i = \epsilon^{ijk}\partial_j a_k \quad (293)$$

On the other hand, the equation of motion of auxillary field ξ can also be obtained from Euler-Lagrange equation, it reads

$$\xi^i = F^2(\partial^i\varphi - gA^i) \quad (294)$$

The above two equations define a hidden duality as follow

$$F^2(\partial^i\varphi - gA^i) = \epsilon^{ijk}\partial_j a_k \quad (295)$$

Substitute it into equation (290), we obtain

$$\begin{aligned} \mathcal{L}_\phi &= \frac{1}{2F^2}\xi^i\xi_i = \frac{1}{2F^2}\epsilon^{ijk}\partial_j a_k \epsilon_{imn}\partial^m a^n \\ &= \frac{1}{2F^2}f^{jk}f_{jk} \end{aligned} \quad (296)$$

here

$$f_{jk} = \partial_j a_k - \partial_k a_j \quad (297)$$

is the field strength of a field. Here we saw the dynamics of the STP field on normal slice is fully equivalent to a vector field a . Recall the kinetic term of gauge field A , we obtain a effective Lagrangian on normal slice

$$\mathcal{L}_{total} = \mathcal{L}_A + \mathcal{L}_\phi = -\frac{1}{4g^2} F^{jk} F_{jk} + \frac{1}{2F^2} f^{jk} f_{jk} \quad (298)$$

2. The Hodge duality

Notice in the dynamics of 2+1-dim vortex, the singularity of the phase angle is essential, which results that the corresponding gauge field A is also singular at the center of the vortex. This singularity could be resolved in higher dimension, for example, in 3+1-dim space-time, we can extend the 2+1-dim Hodge duality (295) to 3+1-dim. This 3+1-dim Hodge duality reflects the local duality of 3+1-dim Lorentz group, as revealed in last subsection. In 3+1-dim, the complex SPTs field becomes real because the phase angle is fixed to zero and has no dynamics at all, leads a free STP scalar field in 3+1-dim. Actually, in 3+1-dim, we can define the Hodge duality of a field as:

$$F'^{\alpha\beta} = \sqrt{2}g F i \epsilon^{\alpha\beta}_{ij} f^{ij} \quad (299)$$

from which we has defined a gauge field A' , its field strength is

$$F'^{\alpha\beta} = \partial^\alpha A'^\beta - \partial^\beta A'^\alpha \quad (300)$$

It is an extension of a field in 3+1-dim and on any 2+1-dim sub-manifold of the 3+1-dim space-time, its dynamics is equivalent to field a . In total, we know

$$\mathcal{L}_{total} = -\frac{1}{4g^2} F^{jk} F_{jk} - \frac{1}{4g^2} F'^{\alpha\beta} F'_{\alpha\beta} \quad (301)$$

Actually, in 3+1-dim, the two parts of above Lagrangian can be written as a single term when defined a new field \tilde{A} satisfying

$$\frac{1}{g} \tilde{F}_{ij} = F_{ij}, \quad \frac{1}{g} \tilde{F}_{\alpha\beta} = F'_{\alpha\beta} \quad (302)$$

Notice the above equations are six equations, which are

$$\partial_0 \tilde{A}_1 - \partial_1 \tilde{A}_0 = g(\partial_0 A_1 - \partial_1 A_0) \quad (303)$$

$$\partial_0 \tilde{A}_2 - \partial_2 \tilde{A}_0 = g(\partial_0 A_2 - \partial_2 A_0) \quad (304)$$

$$\partial_1 \tilde{A}_2 - \partial_2 \tilde{A}_1 = g(\partial_1 A_2 - \partial_2 A_1) \quad (305)$$

$$\partial_0 \tilde{A}_3 - \partial_3 \tilde{A}_0 = g(\partial_0 A'_3 - \partial_3 A'_0) \quad (306)$$

$$\partial_1 \tilde{A}_3 - \partial_3 \tilde{A}_1 = g(\partial_1 A'_3 - \partial_3 A'_1) \quad (307)$$

$$\partial_2 \tilde{A}_3 - \partial_3 \tilde{A}_2 = g(\partial_2 A'_3 - \partial_3 A'_2) \quad (308)$$

On 0-1-2 normal slice, we can assume

$$\tilde{A}_0|_{\Sigma=(t,x_1,x_2)} = gA_0, \quad \tilde{A}_1|_{\Sigma=(t,x_1,x_2)} = gA_1, \quad \tilde{A}_2|_{\Sigma=(t,x_1,x_2)} = gA_2 \quad (309)$$

here $\tilde{A}_i|_{\Sigma=(t,x_1,x_2)}$ denotes the reduced field of the four dimensional gauge field \tilde{A} onto normal slice $\Sigma = (t, x_1, x_2, 0)$. Hence from eq.(306-308) we see, the constraint equations require that on x_3 direction, $\tilde{A}_0, \tilde{A}_1, \tilde{A}_2$ should coincide with A'_0, A'_1, A'_2 ,

$$A_i(0, 0, 0, x_3) = A'_i(0, 0, 0, x_3), \quad i = 0, 1, 2 \quad (310)$$

then we obtain

$$\tilde{A}_3(t, x_1, x_2, x_3) = gA'_3(t, x_1, x_2, x_3) \quad (311)$$

Actually, the A'_3 is a new component of the gauge field results from the Hodge duality, it is unique up to a pure gauge with vanishing field strength. Now we see how to extend the gauge field on 2+1-dim to 3+1-dim guiding by the Hodge duality. A simple extension is

$$\tilde{A}_i(t, x_1, x_2, x_3) = g(A_i(t, x_1, x_2, 0) + A'_i(0, 0, 0, x_3)), \quad i = 0, 1, 2 \quad (312)$$

$$\tilde{A}_3(t, x_1, x_2, x_3) = gA'_3(t, x_1, x_2, x_3) \quad (313)$$

Under this extension, we arrive a simple Lagrangian

$$\mathcal{L}_{3+1d}^{eff} = -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \mu, \nu = 0, 1, 2, 3 \quad (314)$$

it is the famous Lagrangian for 3+1-dim gauge field, the field strength is the same as Maxwell field strength. In three dimensional form, the field strength can be written as electric and magnetic field strengths as

$$E_i = \tilde{F}_{0i}, \quad B_i = \epsilon_{ijk} \tilde{F}^{jk}, \quad i, j, k = 1, 2, 3 \quad (315)$$

In above derivation, we saw that the dynamic effects of STP ANOZ vortex and 3+1-dim electromagnetic field are completely equivalent. This reveals an important assertion: photons are companion particles of STP vortices. In 3+1-dim space-time, Maxwell field strength is a derived result because of vanishing of the ANOZ vortex singularity.

In conclusion, when introducing the third spatial dimension, the singularity of ANOZ vortex is vanished. Meanwhile the equation of motion for ANOZ vortices is equivalent to 3+1-dim Maxwell equations, they are

$$\vec{\nabla} \cdot \vec{E} = 0 \quad (316)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (317)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (318)$$

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \quad (319)$$

Here what we obtained is the source-free Maxwell equations because we didn't consider the effect of matter particles, which will couple to gauge field as will considered in next subsection.

F. The Coulomb Force

We now consider the force between two matter particles. In hydrodynamics, two vortices will repel each other if their handing of spins are the same, and will attract each other if their handing of spins are different. This is a nature derivation from Bernoulli principle. There are only two kinds of charity for 2+1-dim vortices, left and right, respectively.

More than two decades ago, people had already found the correspondence between equations of motions of hydrodynamics and Maxwell eletromagenetism [46]. This correspondence was supported by [47] with a detailed derivation. The correspondence between hydrodynamics and eletromagnetism is much more like a coincidence in previous researches. However, in framework of the STP vortex, the fluid-eletromagnetism correspondence now has a concrete theoretic origin.

In previous subsections, we only considered dynamics of STP and gauge fields, leaving the matter particle as a source of potential. It is nature to consider the interaction between matter field and gauge field as well. To do so, we introduce the matter field in Lagrangian

as follow

$$\mathcal{L}_{total} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - i\bar{\psi}\gamma^\mu\tilde{D}_\mu\psi + m\bar{\psi}\psi \quad (320)$$

$$\tilde{D}_\mu \equiv \partial_\mu + ie\tilde{A}_\mu \quad (321)$$

This interaction can be understood as an effective representation of the collision between matter particle and STP vortices, though there are no terms representing vortices in the Lagrangian. This is because the dynamics of vortices now is equivalent to gauge field in 3+1-dim. Other collisions between matter particle and STP are not considered in this section, as we will see, they also play important roles in deriving gravity between matter particles.

In global, the STP and gauge field are decoupled, hence all local dynamics have been reduced to gauge field dynamics in 3+1-dim space-time. Notice the Lagrangian we obtained above is the same as that in famous QED [15]. Under standard calculation, the interaction between matter particles will be the Coulomb interaction. However, in framework of MIP, the gauge field is not originated from matter field, but from STP vortices. This is an essential difference between modern quantum field theory and the MIP proposed in this article.

Define the four dimensional current as

$$j^\mu \equiv i\bar{\psi}\gamma^\mu\psi \quad (322)$$

we can explicitly see the minimal couple between gauge field \tilde{A} and the electronic current j . The equation of motions now becomes the famous sourced Maxwell equation, as known as

$$\vec{\nabla} \cdot \vec{E} = j_0 \quad (323)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad (324)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (325)$$

$$\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} \quad (326)$$

G. Another Derivation of EoM of Photons

In framework of MIP, we had obtained four properties of charges, they are: 1. There are only two kinds of charges corresponding to left and right chiralities of STP vortices.

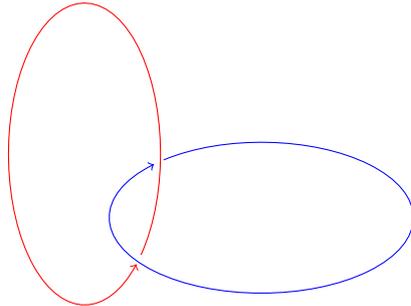


Figure 5: Photon as a topological excitation: a Hopf link

Same charges repel each other while different ones attract each other. 2. The charges are quantized guiding by the Dirac quantization condition derived from STP vortex. 3. Force between charges are mediated by photons. 4. The force between charges is the Coulomb force.

Based on calculations in previous subsections and discussion of the Hodge duality, we know some properties of photons in framework of MIP. At first, it companies with the non-perturbative soliton solution, as known as the vortex solution. Secondly, it is a gauge field in 2+1-dim normal slice on which another effective auxiliary gauge field lives as well. Thirdly, the 3+1-dim Hodge duality acts on the effective auxiliary gauge field does not only resolve the phase singularity of the STP vortex, it introduces the dual part of 2+1-dim gauge field. So the 2+1-dim gauge field and its Hodge dual merged into a 3+1-dim gauge field, which is the photon field, which means on 3+1-dim space-time, the photon field can be understood as topological excitations of 2+1-dim gauge field, the topological configuration is known as the Hopf link excitation. We now clarify the conclusion in detail since it is very important to understand the spin of photon, which has a zero mass.

In framework of STP vortex, the vortex tube is made of two fields, one is the STP field ϕ , whose gradient defines the flow direction of the vortex, the other is 2+1-dim gauge field A whose field strength characterizes the spinning direction of the vortex. So in this picture, A describes the rotation and ϕ the flowing. Under the Hodge duality, the dynamics of the soliton part of STP field is equivalent to another gauge field A' , which is Hodge dual to A . Topologically, the vortex tube represents a Wilson loop, its Hodge duality is t'Hooft loop. Put them together forms a famous topological object, the Hopf link, as shown in fig. IX G. The Hopf link is obvious a non-local object. The topological stability

of the Hopf link protects it from perturbative destruction, so it can propagate in space-time without dissipation unless it meets another vortex. This is very similar to what happens in electromagnetic interaction, where photons propagate the interaction between charges. We had seen the equation of motion of the \tilde{A} , aka the joint representation of A and A' , is nothing but the Maxwell equations. The \tilde{A} field is an effective representation of the Hopf link.

There are two circles in a Hopf link, they wind the topological subgroup (mathematically, the minimal torus) of Lorentz group separately. As we knew in previous section, they are left and right hand topological circles, each corresponds to a spinor fiber. However, in physics, there are no purely topological objects. So we need to consider the dynamics of the Hopf link, say, the effect resulted from deformation of either circle.

Consider an arbitrary deformation on one of the two circles, it will affect the whole Hopf link and defines a self isomorphism as follow

$$A : \Lambda_L \otimes \Lambda_R \rightarrow \Lambda_L \otimes \Lambda_R \quad (327)$$

here A denotes the self isomorphism on $\Lambda_L \otimes \Lambda_R$, Λ_L and Λ_R are left and right spinor fibers respectively. In appendix E, we proved that such a self isomorphism should be a vector map. Relatively, all derivatives should be changed into covariant derivatives, as

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + igA_\mu \quad (328)$$

This leads to non-trivial local transmission that

$$[D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu = ig(\partial_\mu A_\nu - \partial_\nu A_\mu). \quad (329)$$

This reflects the local homomorphism deformation. The strength of the deformation is described by the coefficient g , which relates to charge of matter particle. So we could propose an assertion: the amount of electric charge reflects the strength of local deformation of local space-time. The RHS of above equation is nothing but a field strength of four dimensional gauge field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (330)$$

Since

$$\begin{aligned}
D^\mu[D_\mu, D_\nu] &= ig\partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2 A^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) \\
&= -ig\Box A_\nu + ig\partial_\nu(\partial^\mu A_\mu) - g^2\partial_\mu(A^\mu A_\nu) + g^2(\partial_\mu A^\mu)A_\nu \\
&= \frac{1}{2}g^2\partial_\nu(A_\mu A^\mu)
\end{aligned} \tag{331}$$

under Lorentz gauge $\partial_\mu A^\mu = 0$, the above equation only have pure derivation contributions, with vanishing contributions for no-boundary free field. So this equation can be simply written as

$$D^\mu F_{\mu\nu} = 0 \tag{332}$$

In three dimensional form, it can be written as

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{333}$$

$$\partial_t E - \vec{\nabla} \times \vec{B} = 0 \tag{334}$$

In another way, because the Hopf link configuration is unchanged under left-right flop symmetry, this leads to a electromagnetic duality for field strength $F_{\mu\nu}$. The left-right flop symmetry actually means a flop between pair of indices $(0, i) \leftrightarrow (j, k)$, this can be achieved by introducing the Levi-Cevita connection

$$\epsilon^{0ijk} : (0, i) \rightarrow (j, k) \tag{335}$$

thus for the field strength $F_{\mu\nu}$, we have the following dual relation

$$\tilde{F}_{\alpha\beta} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu} \tag{336}$$

The Levi-Cevita connection flip electric and magnetic fields in three dimensions, and the above dual relation reads

$$\vec{E} \rightarrow \vec{B}, \quad \vec{B} \rightarrow -\vec{E} \tag{337}$$

The dual equation in four dimensional is written as

$$D_\mu \tilde{F}^{\mu\nu} = 0 \tag{338}$$

In three dimension, it becomes two equations

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{339}$$

$$\partial_t \vec{B} - \vec{\nabla} \times \vec{E} = 0 \tag{340}$$

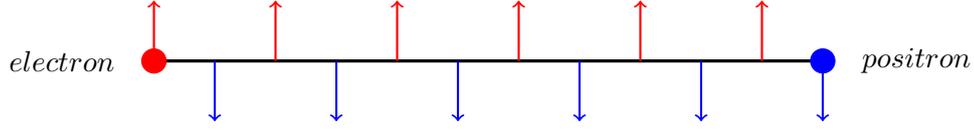


Figure 6: Photons deliver the interaction between electron and positron

Equations (333,334,339,340) are Maxwell equations for source-free electromagnetic fields, which proves in 3+1-dim, the Hopf link transforms the local deformation just the same as photon propagates in space-time.

The figure fig.IX G shows how a deformation propagates from an electron to a positron, where red upper arrows denote left topological circles and blue downer arrows denote right topological circles.

H. Photon and vortex tube

We had already known that in framework of MIP, the spins of matter particles are originated from collisions between them and STP along topological circles in local space-time. Now we knew the photon could be represented as a Hopf link, which also is winding topological circles in 3+1-dim local space-time. So it is possible the spin of photons are also originated from STP.

In case of matter particles, for examples, electron and positron, their spins are sourced from local winding along left and right topological circles $U(1)_L$ and $U(1)_R$ in local space-time, respectively. At arbitrary moment, electron or positron has a phase angle φ_L or φ_R . These two phase angles are undetermined. It means electron or positron has a local phase angle symmetry, which is $U(1)$ symmetry. Because it is deduced from local space-time symmetry, it is a gauge symmetry.

Let us choose the phase angle be θ . The identical principle for fundamental particles requests the following equations

$$\psi \rightarrow \psi e^{-i\varphi_L} \equiv \psi e^{-i\phi}, \quad \bar{\psi} \rightarrow \bar{\psi} e^{-i\varphi_R} \equiv \psi e^{i\phi} \quad (341)$$

from which we know

$$\varphi_L = -\varphi_R = \phi \quad (342)$$

It means the gauge group $U(1)$ is the diagonal subgroup of $U(1)_L \times U(1)_R$, with transition

matrix be -1. This perspective could be extend to higher dimensional transition matrices, which will leads to non-Abelian gauge groups, for example, $SU(2)$ or $SU(3)$.

In this picture, photon is represented as a Hopf link of 2+1-dim gauge fields, it is massless. However, it carries the information of collisions between matter particle and STP vortices. So it will also record the motion of the matter particle, as well as its spin. Since it is a $(1, 1)$ representation of the topological subgroup of Lorentz group. Therefore, from the Hopf link proposition, we obtained photon has spin 1, and massless, and satisfies Maxwell equations. It actually explains how a massless photon has non-zero spin.

I. Conclusion of the section

In this section, from the MIP picture, we explained the origin of electromagnetic interaction in detail. In framework of MIP, the 3+1-dim electromagnetic field represents itself as a Hopf link excitation made of 2+1-dim gauge field and its Hodge dual partner. It is a topological state. From this topological configuration, we obtained the Maxwell equations in two different ways, also from which, we explained why massless photons have spin 1. In this section, we studied four properties of electric charges, say, positive and negative, quantization, repelling and attracting , Coulomb inverse square law ,and equations of motions of photons, which propagates the Coulomb interaction between charged particles. In addition, together with the charge amount calculated in section 5, we obtained all five properties of the electric charge.

There is one additional expression for the STP vortex configuration. In this section, we only considered the non-pertubative potential came from matter particle. However, a non-pertubative disturbance of space-time energy does not only have such a single origin in our universe. In early universe, the disturbance is very large and STP vortices could also be generated as well as its partner field, the photon field. It implies in early time, the universe was dominated by radiation, which is coincide with observations in cosmology. Another example for non-pertubative potential is black holes, near the horizon of a black hole, the space-time energy disturbance is quite large, and it will also generate electromagnetic radiation. This kind of radiation has a completely different origin comparing with Hawking radiation. This may offers quite a lot of new perspectives on black hole and cosmology researches.

X. MIP AND SPECIAL RELATIVITY

Under the framework of MIP, STP itself has no self-interaction. The speed of STP is constant at v_{st} . The effect of STP on particles is a stochastic dynamics problem, which makes the particle's time derivative $d\vec{x}/dt$ not well defined. Under this framework, the classical speed of particles only has clear meaning under statistical average. Because any experimental results can't isolate the effect of STP which are generated at the time that cannot be accurately known, it can't be determined from the beginning that the initial velocity of the particles is the classic speed in textbooks. This is actually just an ideal concept, and there is actually no such so-called classic speed. In the framework of MIP, the so-called "classic speed" only has a statistical meaning, which actually represents the statistical average speed of particles in spacetime. In the following, when we say classical speed, the actual meaning refers to the statistical average speed of the particles.

A. Equivalence between Inertial Reference Systems

Under the MIP framework, the real moving speed of a material particle is

$$\vec{V} = \vec{v} + \vec{u} + \vec{v} \quad (343)$$

The quantum envelope velocity of free material particles can be obtained by combining the wave function corresponding to the particle of matter, $\psi = e^{i(\vec{p}\cdot\vec{x}-Et)/\hbar}$

$$\vec{u} = 2\Re\nabla R = 0 \quad (344)$$

Classic statistical speed of matter particles

$$\vec{v} = 2\Re\nabla I = \frac{\vec{p}}{m} \quad (345)$$

From the three-speed decomposition process, we know that the classic statistical speed \vec{v} and the fluctuation speed \vec{v} are independent, so the fluctuation speed of \vec{v} is completely caused by STP collision as

$$\vec{v} = f(\{\vec{v}_{st}\}) \quad (346)$$

Where $\{\vec{v}_{st}\}$ is the speed of all STP and satisfies $f(0) = 0$, from which we infer the STP speed \vec{v}_{st} . The classic statistical speed of material particles, \vec{v} , is also independent.

Regardless of the classical velocity at which the free matter particles move, the STP velocity \vec{v}_{st} is a constant, so equivalence between the inertial reference systems is no longer a basic assumption, but a natural results of our theory.

The interaction of STP on particles causes the particles to perform random fluctuations. The speed of fluctuational movement is very different from that of the classic speed. It is essentially a relative speed that is constant under time reversal. In the Appendix B, we prove that this random Markov fluctuation is not related to the classical motion, so it is also invariant under the transformation of the inertial reference system. From this perspective, the equivalence between inertial reference systems is a natural inference under the MIP framework.

B. STP Collision and Particle Mass

In a random process in which particles are collided by STP, the instantaneous velocity of the particle is equal to its classical statistical speed \vec{v} , the fluctuation velocity is \vec{v}' , and the quantum envelope velocity is \vec{u} overlay, ie the eq.(343) . We know that under time reversal T ($T : (t, \vec{x}) \rightarrow (-t, \vec{x})$), there is

$$T : \vec{v} \Rightarrow -\vec{v}, \quad T : \vec{u} \Rightarrow \vec{u}, \quad T : \vec{v}' \Rightarrow \vec{v}' \quad (347)$$

This is because for a continuous time Markov process under time reversal, it is still a Markov process. The quantum envelope velocity \vec{u} and the fluctuation velocity caused by the STP collision \vec{v}' are actually the consequences of the STP collision on the particles. Both the quantum envelope velocity and the fluctuation velocity are independent of the classical statistical velocity under the time inversion transform. Therefore, we can always consider the scenario where the particles with zero classic velocity are collided by STP. At this point, the relative speed of STP to particles is v_{st} and will always be v_{st} .

Because of STP, absolutely free particles do not exist. We refer to particles whose classical statistical speed is constant as "free particles." For free particles, the quantum envelope velocity is $\vec{u} = 0$, and \vec{v}' is a random variable. As can be seen from the probability distribution of \vec{v}' , \vec{v}' has nothing to do with the classic statistical speed \vec{v} . The true speed of such free

particles is \vec{V} which consists of two parts that are independent of each other, namely

$$\vec{V} = \vec{v} + \vec{v} \quad (348)$$

When the particle's classic speed \vec{v} changes, \vec{v} does not change. It shows that the STP background does not change with the change of the classical speed of the particles themselves. This is actually the equivalence of the inertial reference system.

On the other hand, we already know that the statistical mass of a particle actually induced by impact of STP. The more particles are hit by STP per unit time, the greater the statistical mass of the particles.

First let's consider the static mass of the particles. Particles in spacetime are always subject to random collisions of STP. We know that the mass of a particle reflects the statistical properties of the motion which manifests in STP impacts. This does not mean that there is no statistical mass in the reference frame where the particle's classical velocity is zero. Because in the stationary reference frame, the motion of the particle is still a random quantum Brownian motion, except that its statistical position is at the origin. Therefore, when considering the motion of particles, we should first separate the mass in the stationary reference frame and then consider the change in the number of relative collisions due to motion. If the stationary particles are subjected to N_0 in the z direction in the z direction unit time, and the moving particles are subjected to N collisions in z direction per unit time. . At this point, the number of impacts on the particle in the same time period on the $x - y$ plane still appears to be N_0 , so the mass is not a scalar property. But in fact, if we want to guarantee the principle of relativity is right, then the number of collisions in any direction will increase relatively.

$$m = m_0 N / N_0 \quad (349)$$

Where N is the number of times which the particle is hit by STP per unit time observed by the laboratory observer, and m_0 is the static mass of the particle.

C. Time Dilation Effect

From the relative speed constant assumption, the distribution of STP under the reference system transformation will not change. If the distribution of the STP is uniform and isotropic in the rest frame , then because the speed of STP relative to the particle does not change,

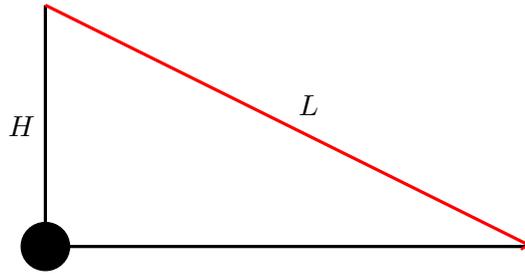


Figure 7: Collision between STP with matter particle.

in the frame of relative velocity \vec{v} . The distribution of STP is still uniform and isotropic. It should be noted, however, that the time costs for the same collision process in different reference frames are different. This can be explained by the following explanation.

In the rest frame, the STP is at a constant speed v_{st} , moving toward the particle from the distance H . After time t , it will collide with the particle, so the time

$$t = H/v_{st}. \quad (350)$$

However, in the moving frame with constant velocity v , after time t' . The distance between the time and space will be L . And the distance from the particle is $\sqrt{L^2 - H^2}$, Then $t' = L/v_{st}$, the following formula pops

$$\frac{L}{v_{st}} = \frac{\sqrt{L^2 - H^2}}{v} \quad (351)$$

hence

$$t' = \sqrt{t'^2 - t^2} \frac{v_{st}}{v} \Rightarrow T = t' \sqrt{1 - \frac{v^2}{v_{st}^2}} \quad (352)$$

As long as the speed of STP v_{st} equal to the speed of light c , the above equation returns to the relativity of simultaneity in special relativity. Because of zero static mass of photon, the STP and photons have no interaction. On the other hand, due to the isotropy of STP, there can always exist STP moving parallel to the photon, so there are no relative movement between such STP and the photon. Hence the speed of light should be essentially equal to the speed of STP, that is $c = v_{st}$, which is a rigorous conclusion. and is the physical origin of the axiom of invariance of speed of light in special relativity.

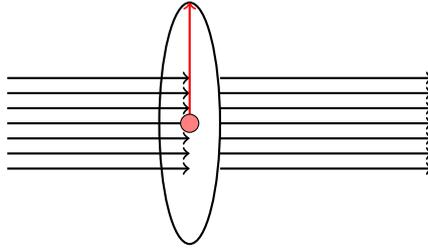


Figure 8: Flux of STP cross a disk.

D. Relativistic Mass Effect

Now we consider the expression of the particle mass under the frame transformation. Due to the homogeneity and isotropy of STP distribution, we can assume the density of the STP is ρ_0 , and in moving frame with constant velocity v , the particle moves along the horizontal direction .

Then in the rest frame, the number of STP passing through the disc on the vertical direction is

$$N_0 = \rho_0 \pi r^2 v_{st} \Delta t \quad (353)$$

The mass of the particle is

$$m_{st} = k_{st} \rho_0 \pi r^2 v_{st} \Delta t \quad (354)$$

where k_{st} is the propotional coefficient. In the moving frame with constant velocity v , the number of STP passing through the same disc is

$$N = \rho_0 \pi r^2 v_{st} \Delta t' \quad (355)$$

Notice that in moving frame, the time interval $\Delta t'$ is coincident with that in rest frame. The time dilation is universal along all directions, not only along the moving direction. Therefore, numbers of collisions on three spatial directions are uniformal N .

Since

$$\Delta t' = \Delta t / \sqrt{1 - v^2/v_{st}^2} \quad (356)$$

we have

$$m'_{st} = k_{st} N = k_{st} \rho_0 \pi r^2 v_{st} \Delta t' = m_{st} / \sqrt{1 - v^2/v_{st}^2} \quad (357)$$

When $v_{st} = c$, it is the same expression of relativistic mass as in special relativity.

E. Length Contraction Effect

Next, we consider the relativity of spatial distance, which is the length contraction effect in the special theory of relativity. In the framework MIP, we have two independent methods to derive this effect.

The first derivation is a natural inference of time dilation and mass enhancement effects, which we have derived in the above two sections. In each direction, STP and the particle's unit cross-section have the same collision number N . The time dilation is independent of the spatial direction (that is, the orthogonality is guaranteed). Based on these two points, in the parallel direction and the vertical direction of the particle motion, each of the collisions of a rectangular cross section is considered. In the case where the particles are stationary, two rectangular sections are both a long, the width is also b .

For a rectangular section in the parallel direction of particle motion (which is perpendicular to the direction of particle motion), the increase in the number of collisions N is due to the dilation of time, see the equation (357). In the view of a stationary observer, if the area of the rectangle is constant, then the rectangular section in the vertical direction of the particle motion (which is parallel to the direction of particle motion) will have more STP passes. Similar to calculation of time dilation, we can get it at $\Delta t'$. The number of STP passing the same area are:

$$\tilde{N} = \rho_0 ab v_{st} \Delta t' / \sqrt{1 - v^2/v_{st}^2} = N / \sqrt{1 - v^2/v_{st}^2}. \quad (358)$$

Then this result will show that the mass is not isotropic, which clearly violates the definition of statistical mass. The only way to resolve this contradiction is to make the length in the particle's direction of motion contract, and its contraction ratio is exactly equal to $\sqrt{1 - v^2/v_{st}^2}$. This makes $a \rightarrow a' = a\sqrt{1 - v^2/v_{st}^2}$, $\tilde{N} = N$. Therefore, from the inherent self-consistency of the theory, the moving ruler under the framework of MIP must be contracted.

The second derivation is discussed below.

We consider the relativity of the spatial distance, that is, the measure effect in special relativity. We first consider the rest reference system, the length of the ruler is

$$l_0 = x_B - x_A \quad (359)$$

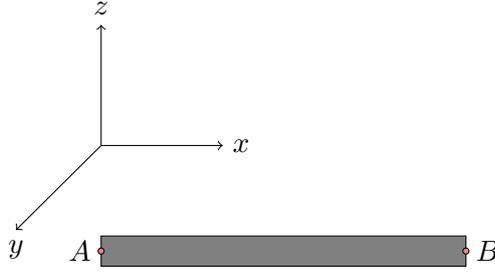


Figure 9: Ruler in rest reference frame.

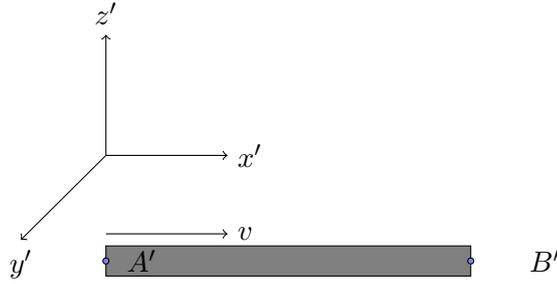


Figure 10: Ruler in moving reference frame.

When the ruler is moving along x - direction in speed of v , as shown in the following figure.

The spacetime coordinates at both ends of the ruler are

$$(x'_A, t'_A), \quad (x'_B, t'_B),$$

as a rigid body, it requires $t'_A = t'_B$. In this coordinate system, the special relativistic transformation is

$$X_A = \frac{x'_A + vt'_A}{\sqrt{1 - \frac{v^2}{v_{st}^2}}}, \quad x_B = \frac{x'_B + vt'_B}{\sqrt{1 - \frac{v^2}{v_{st}^2}}} \quad (360)$$

Hence

$$x_B - x_A = \frac{x'_B - x'_A}{\sqrt{1 - \frac{v^2}{v_{st}^2}}} \quad (361)$$

and

$$l = x'_B - x'_A = l_0 \sqrt{1 - \frac{v^2}{v_{st}^2}} \quad (362)$$

In the framework of MIP, we consider the differential distance dx' The MIP requires $\delta(px) = nh = \delta(p'x')$, $n \in \mathbb{Z}$. In all inertial frames, each time the STP acting on matter particle ,

the changing of action is nh . The basic principle will remain the same regardless of inertial reference frames.

In the motion reference frame, we know that the mass $m' = m_0/\sqrt{1 - v^2/v_{st}^2}$, thus inducing $\delta p' = m'\delta v$. In the rest reference frame $\delta p = m\delta v_0$, we can easily see that to ensure the MIP is independent of reference frame transformation, there must exist the relation

$$dx' = dx\sqrt{1 - v^2/v_{st}^2} \quad (363)$$

The length now is the integral of the above formula, and we have

$$l = \int_A^B dx' = l_0\sqrt{1 - v^2/v_{st}^2} \quad (364)$$

Thus we have derived the same result as in special relativity. However, its intrinsic meaning is not the same as in special relativity. Since we study within the framework of MIP, in which the STP's relative movement to the particle does not change under reference frame transformation. Distinct from the macro length contraction effect of special relativity, the differential distance is also contracted under MIP, which precisely reflects the universal applicability of the MIP.

F. An Alternative Method of Deriving Special Relativity

From the nature of time and space to study the transformation between inertial systems, we don't have to discuss the nature of light from beginning to end. We only need: the speed of STP is the maximum possible speed, and we can obtain special relativity. In Model 1 of Chapter 3 of this paper, we discuss examples of discrete space-time, where the maximum STP speed is not an assumption.

Consider two inertial systems K, K' . When $t = 0$ is set, the two origins of inertial systems coincide. K' relative K with speed ν . Simplifying the derivation of symbols, using 1 + 1 dimension space-time. Most commonly, the time and space of K and K' are transformed as follows:

$$x' = X(x, t, \nu), \quad t' = T(x, t, \nu), \quad (365)$$

where X and T are two universal functions. We will determine the form of X, T by the nature of space and time in the following.

Consider that the space is uniform. That is, a ruler is in K and the endpoints are in x_1 and x_2 . It must be the same length as the endpoints in $x_1 + \Delta x$ and $x_2 + \Delta x$. With spatial invariance, also in the K' system:

$$X(x_2 + \Delta x) - X(x_2) = X(x_1 + \Delta x) - X(x_1). \quad (366)$$

Get $\Delta x \rightarrow 0$, you can get:

$$\frac{\partial X(x, t, v)}{\partial x} \Big|_{x_1} = \frac{\partial X(x, t, v)}{\partial x} \Big|_{x_2}. \quad (367)$$

With the arbitrariness of x_1 and x_2 , we know that the equation must be constant at both ends, i.e. X is a linear function of x . By the same token, a linear function of T is t can be obtained from the homogenous of time.

With $t = 0 = t'$, the origin coincides with $x = 0 = x'$, we can get

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} a_v & b_v \\ C_v & d_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (368)$$

where a_v, b_v, c_v, d_v are functions of ν and the diagonal elements are dimensionless. When $x = vt$ $x' = 0$ gets $b_v = -va_v$.

Let's consider the spatial homogenous. In the 1+1 dimension spacetime, it can be understood as the inversion of the x axis. $x \rightarrow -x, v \rightarrow -v, x' \rightarrow -x'$, the spatio-temporal transformation is unchanged, that is, union:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} a_v & b_v \\ C_v & d_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (369)$$

as well as

$$\begin{pmatrix} -x' \\ t' \end{pmatrix} = \begin{pmatrix} a_{-v} & b_{-v} \\ C_{-v} & d_{-v} \end{pmatrix} \begin{pmatrix} -x \\ t \end{pmatrix} \quad (370)$$

Can get:

$$a_{-v} = a_v, b_{-v} = -b_v, c_{-v} = -c_v, d_{-v} = d_v. \quad (371)$$

By definition, the transformation of K to K' must be equivalent to the transformation of

K' to K with a relative speed of $-\nu$, ie:

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} a_{-\nu} & b_{-\nu} \\ C_{-\nu} & d_{-\nu} \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \quad (372)$$

Lianli available:

$$d_{\nu} = a_{\nu}, \quad c_{\nu} = \frac{a_{\nu}^2 - 1}{b_{\nu}}. \quad (373)$$

which is,

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} a_{\nu} & -\nu a_{-\nu} \\ -\frac{a_{\nu}^2 - 1}{\nu a_{\nu}} & a_{\nu} \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (374)$$

Very importantly, we generally determined that the diagonal elements of the spacetime transformation must be equal.

With the definition of the inertial system, there is a K'' inertial system relative to K' to w movement, then K'' relative K Must be an inertia transformation. This can be formulated as:

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = a_w a_{\nu} \begin{pmatrix} 1 + w \frac{a_{\nu}^2 - 1}{\nu a_{\nu}^2} & -(w + \nu) \\ -\frac{a_w^2 - 1}{w a_w^2} - \frac{a_{\nu}^2 - 1}{\nu a_{\nu}^2} & 1 + \nu \frac{a_w^2 - 1}{w a_w^2} \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (375)$$

Combining the above two equations, we get an important relationship:

$$1 + w \frac{a_{\nu}^2 - 1}{\nu a_{\nu}^2} = 1 + \nu \frac{a_w^2 - 1}{w a_w^2} \quad (376)$$

$$\frac{v^2 a_{\nu}^2}{a_{\nu}^2 - 1} = \frac{w^2 a_w^2}{a_w^2 - 1} = G. \quad (377)$$

Because w, ν is any speed, this formula must be equal to both constants and set to G .

The following proves that this formula must be no less than zero:

Available

$$a_{\nu} = \frac{1}{\sqrt{1 - \frac{v^2}{G}}}, \quad (378)$$

Since $v = 0$, $a_{\nu} = 1$ is equivalent to no transformation, so we can remove the negative root.

It can be seen that for any speed v , there must be $a_{\nu} > 0$. Using (375), after two inertial transformations, it is still an inertial system transformation. So have

$$1 + w \frac{a_{\nu}^2 - 1}{\nu a_{\nu}^2} > 0 \quad (379)$$

If $a_v < 1$, when $w \gg v$, the above formula cannot be maintained. So we prove that $a_v \geq 1$ is equivalent to $G \geq 0$. From this we can set

$$\frac{v^2 a_v^2}{a_v^2 - 1} = \frac{w^2 a_w^2}{a_w^2 - 1} = \theta^2. \quad (380)$$

In summary, the final transformation can be obtained:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{\theta^2}}} \begin{pmatrix} 1 & -v \\ \frac{-v}{\theta^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \quad (381)$$

From this formula, you can see that $v < \theta$. (space-time coordinates cannot be imaginary).

The joint transformation satisfies:

$$\omega = \frac{w + v}{1 + \frac{wv}{\theta^2}}. \quad (382)$$

Because $w < \theta$, $v < \theta$, no matter how we combine, we must have $\omega < \theta$. In summary, θ must be the maximum possible speed allowed for time and space. Combined with our discrete space-time model, there is a minimum length in time and space l and a minimum time τ , and STP speed is defined as the ratio of the minimum length and time.

Let's prove $\theta = \frac{l}{\tau} \equiv v_{st}$. as follows: If there was a speed $V > \theta$, it would move τ time interval, The distance must be greater than l , which is no problem and will not cause any contradiction. But it travels through the shortest distance l . It only takes $\frac{l}{V} < \tau$ time, which contradicts the definition of the shortest time.

Therefore we prove that the STP speed $\theta = v_{st}$ is the maximum possible speed in space and time. Time and space of this nature must have the properties derived above. When the STP speed is exactly equal to the propagation speed of light in a vacuum, our results are equivalent to the special relativity.

XI. MIP AND GENERAL RELATIVITY

Gravitation refers to the attraction of objects with mass, and together with electromagnetic force, weak interaction force and strong interaction force constitute the four fundamental interactions of nature. Among these four fundamental interactions, gravity is the weakest, but it is also a universal long-range attraction. Whenever is an attraction between any two particles, the magnitude of the force is inversely proportional to the square of the

distance, and proportional to the product of mass, we can conclude that this force must be gravitational. In the framework of the mass principle, we naturally derive the gravitation from the interaction between the massive fermion and STP.

A. Electron and STP

The electrons are described by the spinor field ψ , and the dynamics of free electrons are determined by the amount of Dirac Lagrangian:

$$\mathcal{L}_D(\psi) = \bar{\psi}(i\partial - m)\psi \quad (383)$$

Where m is the electronic mass.

STP is described by the massless scalar field ϕ , and the dynamics of free STP is determined by Klein Gordon Lagrangian:

$$\mathcal{L}_{sp}(\phi) = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi \quad (384)$$

The interaction between STP and electrons is determined by the Lagrangian:

$$\mathcal{L}_{int}(\phi, \psi) = -\lambda\bar{\psi}\phi\psi \quad (385)$$

According to MIP, the interaction strength of STP and electrons λ must be positively correlated with the mass m of the electron and

$$\lim_{m \rightarrow 0} \lambda(m) = 0 \quad (386)$$

What needs to be studied here is the lowest order behavior of the interaction strength λ when the electron mass tends to zero. It is very subtle that this behavior cannot be (A is a constant coefficient):

$$\lambda(m) = Am \quad (387)$$

When we replace all the electronic masses in the system with $-m$, the interaction strength λ , all physical laws must remain the same. Therefore, the lowest order behavior is

$$\lambda(m) = A\sqrt{m^2} \quad (388)$$

At $m = 0$ is the branch point of the function, which corresponds to the singularity implied by MIP, that is the disappearance of interaction with STP. Comparing the description of

photon frequencies and wavevectors by relativistic quantum mechanics $\omega = ck$, its correct interpretation should be:

$$\omega = c\sqrt{k^2} \quad (389)$$

This leads to the important conclusion that λ^2 (instead of λ) is an analytic function for m^2 . The correct expansion must be of the form:

$$\lambda^2 = C_0 + C_2m^2 + C_4m^4 + \dots \quad (390)$$

According to the MIP, there must be $C_0 = 0$. The electron mass is very small compared to the natural Planck mass. we can almost ignore the contribution of all high-order terms, leaving only the lowest-order behavior:

$$\lambda^2 = C_2m^2 \quad (391)$$

In summary, the dynamics of the entire system are composed of STP and electrons, which are determined by the following action:

$$S[\phi, \psi] = \int d^4x [\mathcal{L}_{\text{sp}}(\phi) + \mathcal{L}_{\text{D}}(\psi) + \mathcal{L}_{\text{int}}(\phi, \psi)] \quad (392)$$

Substituting a concrete expression, the total amount of action of the system is:

$$S[\phi, \psi] = \int d^4x \left[\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \bar{\psi} (i\cancel{\partial} - m) \psi - \lambda \bar{\psi} \phi \psi \right] \quad (393)$$

According to path integration, the total partition function of this system is:

$$Z = \int D\phi D\psi e^{iS[\phi, \psi]} = \int D\psi e^{iS_{\text{eff}}[\psi]} \quad (394)$$

Let $J = -\lambda \bar{\psi} \psi$, the result for the STP field ϕ is

$$Z = \int D\phi \exp\left(\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + J\phi\right) = e^{iW(J)} \quad (395)$$

When the fermion is at static limit, $J(\vec{x}_i) = \lambda \delta(\vec{x} - \vec{x}_i)$, we have

$$W(J) = -\frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \frac{J^*(k)J(k)}{k^2} = \int dx^0 \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\lambda^2 e^{i\vec{k}\cdot\vec{r}}}{\vec{k}^2} \quad (396)$$

Using $Z = e^{iW(J)} = e^{-iV_{\text{eff}}T}$ and $T = \int dx^0$, we can get the effective interaction between electrons as:

$$V_{\text{eff}}(r) = - \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\lambda^2 e^{i\text{Vec}\vec{k}\cdot\vec{r}}}{\vec{k}^2} = -\frac{\lambda^2}{4\pi r} = -\frac{C_2m^2}{4\pi r} \quad (397)$$

The effective interaction forces between the corresponding electrons are:

$$F_{eff}(r) = -\frac{\lambda^2}{4\pi r^2} = -\frac{C_2 m^2}{4\pi r^2} \quad (398)$$

Where $r = |\vec{r}|$ is the distance between the electrons, and the preceding negative sign indicates this is an attractive interaction. This universal attraction force is inversely proportional to the square of the distance and proportional to the product of mass. We can conclude that it can only be gravitational. So we can compare the gravitational formula and determine the scale factor $C_2 = 4\pi G$, that is:

$$F_{eff}(r) = -\frac{Gm^2}{r^2} \quad (399)$$

Where G is the gravitational constant, and the interaction force between the electrons induced by STP is universal gravitation.

B. Universal Gravity among Macroscopic Bodies

Let us consider a system with three components: free protons, free neutrons, STP. The nucleon is described by the spinor field ψ_i (subscript $i=1$ is denoted as proton, $i=2$ is denoted as neutron, repeating indicator is summed), and the dynamics of free nucleus is

$$\mathcal{L}_N(\psi_i) = \bar{\psi}_i(i\hat{\phi} - m_i)\psi_i \quad (400)$$

Where m_1 is the proton mass and m_2 is the neutron mass.

The description of the free STP section is as described in the previous section. The interaction between STP and nucleon is determined by the Lagrangian quantity \mathcal{L}_{int} :

$$\mathcal{L}_{int}(\phi, \psi) = -\lambda_{ij}\bar{\psi}_i\phi\psi_j \quad (401)$$

The repeated index are automatically summed, and there are four terms in this formula. According to MIP, the interaction strength of STP and electrons λ_{ij} must be positively related to the mass of the nucleus $m_i m_j$, and must have:

$$\lim_{m_1 \rightarrow 0} \lambda_{11} = \lambda_{12} = \lambda_{21} = 0 \quad (402)$$

$$\lim_{m_2 \rightarrow 0} \lambda_{22} = \lambda_{12} = \lambda_{21} = 0 \quad (403)$$

With the same reasoning of the previous section, according to the analytical nature of the function and the limitations of the MIP, the lowest order behavior must be:

$$\lambda_{ij}^2 = C_2 m_i m_j \quad (404)$$

In summary, the dynamics of the entire system are composed of STP and nucleon, which are determined by the following action:

$$S[\phi, \psi] = \int d^4x [\mathcal{L}_{\text{sp}}(\phi) + \mathcal{L}_{\text{N}}(\psi) + \mathcal{L}_{\text{int}}(\phi, \psi)] \quad (405)$$

Substituting a concrete expression, the total amount of action of the system is:

$$S[\phi, \psi] = \int d^4x \left[\frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \bar{\psi}_i (i \not{\partial} - m_i) \psi_i - \lambda_{ij} \bar{\psi}_i \phi \psi_j \right] \quad (406)$$

According to path integration, the total partition function of this system is:

$$Z = \int D\phi D\psi_1 D\psi_2 e^{iS[\phi, \psi_1, \psi_2]} = \int D\psi_1 D\psi_2 e^{iS_{\text{eff}}[\psi_1, \psi_2]} \quad (407)$$

The result of accumulating the STP field ϕ can further obtain the effective interaction potential between the nucleons:

$$V_{ij}(r) = -\frac{\lambda_{ij}^2}{4\pi r} = -\frac{C_2 m_i m_j}{4\pi r} \quad (408)$$

Same as the previous section, let the lowest order proportional coefficient $C_2 = 4\pi G$, where G is the gravitational constant. This leads to the important conclusion that the interaction between the nucleus induced by STP is universal gravitation:

$$F_{ij}(r) = -\frac{G m_i m_j}{r^2} \quad (409)$$

We obtained the correct expression of the gravitational potential between protons and protons, protons and neutrons, neutrons and neutrons. Considering that the gravitational potential is a scalar potential, a direct summation superposition can be performed, thereby obtaining the gravitational force between the macroscopic objects, and mathematically proves that the sum of the gravitational forces between the constituent elements of the two macroscopic objects is equivalent to mass center. This is the classic proof of Newton and will not be repeated here.

C. MIP and Equivalence Principle

The principle of equivalence plays a very important role in the general relativity. The principle of equivalence means that the observer cannot distinguish the inertial force generated by the acceleration and the gravitational force generated by the mass, which is derived from the fact that the gravitational mass and the inertial mass are strictly equal. In the theoretical framework of general relativity, the gravitational mass and the inertial mass are strictly equal, which is a postulation of the theory. Aparting from the support of empirical facts, there is no theoretical further explanation.

Within the framework of MIP, we have the microscopic origins of gravitation in the above two sections, which implies important physical results: in the universal gravitational formula induced by STP and fermion interaction, the mass of fermion from Dirac equation is inertial mass. From the overall perspective of modern physics, we can determine that this force must be gravitational, and it can be inferred that the inertial mass of the fermion must be equal to its gravitational mass. Of course, this conclusion cannot completely replace the principle of equivalence, and does not solve the problem of whether the mass of the boson in the natural world is equal to the mass of gravity. However, it is still an extremely important conclusion, because the principle of equivalence and general relativity are mainly applied to macroscopic objects in nature (such as various celestial bodies), and their mass components are completely derived from fermions. It can be said that when general relativity is applied in macroscopic fields such as cosmology and astrophysics, the equivalence principle is no longer a hypothesis, but a property that can be derived from microscopic STP dynamics.

Starting from the equivalent principle we derived, combined with Einstein's elevator thought experiment, we can demonstrate the inevitable bending of time and space with gravitational source. Describing the curve time and space, mathematically we have to use the metric field of Riemannian geometry. The core of general relativity is the physical equation that is satisfied in this Riemannian background. To get the Einstein field equation furthermore, we must add some constraints. The important ones are as follows:

1. The Newton gravitational potential is the classical limit. The static gravitational field

is determined by the mass density distribution of matter, which is a component of the momentum energy tensor. Extending to the general gravitational field and determining the momentum energy tensor of the gravitational field must correspond to this limit.

2. Referring to the Newtonian gravitational potential equation, we also require that the differential equations satisfied cannot have more than two orders of derivatives. This is combined with a theorem of Riemannian geometry [51] and the classical limit of Newton's gravitation, the form of the gravitational field equation can be determined. The coefficient of the cosmological constant term has not been completely determined. This item has no effects on this paper and will not be discussed here.

Furthermore, according to MIP, inertial mass and gravitational mass are no longer fundamental physical quantities. The two remain equal because both come from the statistical mass of STP collisions.

XII. SUMMARY

Starting from the fundamental concept innovation of statistical mass, this paper proposes MIP: material particles will be subjected to random collision of STP which is ubiquitous in space and time to make frictionless quantum Brownian motion. The change of the action of material particles in each collision is integer multiple of Planck constant h . From MIP, we can prove all the important results of the special theory of relativity. The speed of light in a vacuum no longer has a special physical meaning, but instead the speed of STP represents the upper limit of the speed of physical information propagation in spacetime. The constant speed of light is a natural consequence of MIP. The relative invariance of the speed of light actually reflects that the speed of STP relative to the particle of matter is always relatively constant. The quantum theory obtained under the framework of MIP is fully compatible with the existing quantum theory. The advantage of this new framework is that it does not require the introduction of additional wave function assumptions, which can directly derive the Schrödinger equation. The interpretation of Bonn is also a natural consequence in our theory. In particular, the concept of wave pack collapse is not required to be introduced under our MIP framework. The Heisenberg uncertainty principle no longer has a fundamental position but a natural inference under the MIP framework. From the statistical uncertainty between inertial mass and space-time diffusion coefficient, the most basic coordinate momentum

uncertainty relationship of quantum mechanics can be derived. Therefore, it is proved that the wave-particle duality is a property exhibited by the STP colliding particles under the MIP framework. Furthermore, we apply MIP to quantum measurement problems, and have a new breakthrough interpretation of the EPR paradox problem that has confused physics for nearly a century. The STP colliding matter particles is a zero-spin scalar particle without mass. According to MIP, the topological properties and dynamic properties of STP can explain the nature of photons, and thus naturally obtain the complete electromagnetic theory and all important properties of charge. Furthermore, from the topology of space time, we obtain the origin of the spin and the relationship between spin and mass. From the microscopic behavior of a large number of STP, the macroscopic gravitational effect can be derived, and the Newton's universal gravitation formula are obtained. Inertia mass and gravitation mass are no longer basic physical quantities. The real root of the equivalence principle is that both come from the statistical mass of STP collisions. In summary, MIP systematically solves all the basic problems of modern physics, which is the common origin of special relativity, general relativity, electromagnetic theory and quantum nature. Starting from the only one principle hypothesis, we can reconstruct the foundation of modern modern physics and unify all important areas of modern physics.

Appendix A: Brown Motion and Markov Process

When the displacement of the material particle $X(t)$ satisfies the following conditions, we call the material particle doing Brownian motion: 1. $X(0) = 0$. 2. On any finite disjoint interval set $(s_i, s_i + t_i)$, the displacement of the particle is $X(s_i + t_i) - X(s_i)$, which are random variables that are independent of each other. 3. For each $s \geq 0, t \geq 0$, $X(s+t) - X(s)$ obeys the normal distribution $N(0, t)$.

For each constant a , the process $X(t) + a$ is called the Brownian motion starting from a . For the Brownian motion that is physically free of friction, we call it the quantum Brownian motion in this paper.

Consider any past set of times (\dots, p_2, p_1) , any "current time" s , and any "future time" t , all of which are within the range of X , if any

$$\dots < p_2 < p_1 < s \tag{410}$$

Then the Markov property is established, and the process is a Markov process, but only if:

$$\begin{aligned} \Pr [X(t) = x(t) | X(s) = x(s), X(p_1) = x(p_1), X(p_2) = x(p_2), \dots] \\ = \Pr [X(t) = x(t) | X(s) = x(s)] \end{aligned} \quad (411)$$

Set up for all time sets. Then calculate the conditional probability

$$\Pr [X(t) = x(t) | X(s) = x(s), X(p_1) = x(p_1), X(p_2) = x(p_2), \dots] \quad (412)$$

Future state is independent of any historical state and is only relevant to the current state.

In summary, the quantum Brownian motion studied in this paper is a Markov process.

Appendix B: Decomposition of Random Variables

In the Langevin equation, the true velocity of particle motion \vec{V} contains three parts: the classic statistical velocity \vec{v} , quantum envelope velocity \vec{u} and Gaussian noise $\vec{\nu}$

We do not consider the impact of classic statistical velocity. Then the random motion of the particles will be determined by the quantum envelope motion and Gaussian noise. The fact that we need to prove is that we can distinguish the quantum envelope motion \vec{u} in the strict mathematical differential sense. The quantum envelope motion corresponds to the smooth continuous part of the random motion, and the Gaussian noise corresponds to the continuous non-differentiable part of the random motion.

First, for any random variable $r(x, t)$, if a smooth function $f(x, t)$ is superimposed, the result is still a random variable. which is a random variable, as

$$w(x, t) = r(x, t) + f(x, t) \quad (413)$$

But if $r(x, t)$ or $w(x, t)$ has a finite order autocorrelation association, then theoretically we can strictly distinguish $w(x, t)$ and other two random variables of $r(x, t)$, which is:

$$\langle r(x_1, t_1)r(x_2, t_2) \cdots r(x_n, t_n) \rangle_r = \mathcal{F}_n(\vec{x}, \vec{t}), \quad \text{mod}(n, N) \equiv 0 \quad (414)$$

$$\langle r(x_1, t_1)r(x_2, t_2) \cdots r(x_n, t_n) \rangle_r = 0, \quad \text{mod}(n, N) \neq 0 \quad (415)$$

Then there is

$$\langle w(x_1, t_1)w(x_2, t_2) \cdots w(x_N, t_N) \cdots w(x_n, t_n) \rangle_r \neq 0, \quad n > N \quad (416)$$

Therefore, it can be strictly distinguished mathematically. In the case we considered, Gaussian noise $\vec{\nu}$ has a second-order correlation

$$\langle \nu_i(t) \nu_j(t') \rangle = \Omega \delta_{i,j} \delta(t - t') \quad (417)$$

And all odd-order associations are zero

$$\langle \nu(t) \rangle_\nu = 0$$

So obviously

$$\vec{w}(t) = \vec{u}(t) + \vec{\nu}(t)$$

The odd-order correlation is not zero. So you can strictly distinguish between $\vec{w}(t)$ and $\vec{\nu}(t)$. Due to the MIP, there is only one kind of Gaussian noise, and there is no other noise source. So continuous functions other than noise are smooth and differentiable functions. So \vec{u} is a smooth function.

Appendix C: From MIP to the Uncertainty Principle

We believe that the uncertainty principle comes from the kinematic equation of stochastic spacetime motion, which is rooted in the non-differentiable motion path, i.e. the particle coordinate $\vec{x}(t)$ derivative of time $d\vec{x}/dt$ does not exist. Therefore, it must be noted that the particle's momentum $\vec{p} = m d\vec{x}/dt$ cannot be well defined. The momentum is defined as follows

$$\vec{p} = m D\vec{x} = m\vec{v} + m\vec{u} \quad (418)$$

Kinematic equation

$$\vec{u} = \Re \frac{\nabla \rho}{\rho} \quad (419)$$

For the sake of simplicity, the following discussion uses only one component in the x direction, and all vector equations become equations of one component. For any random variable O , the statistical average is $\langle O \rangle = \int O \rho(x) dx$. Multiplying both sides of the equation by ρ and integrate x, we can get the x and u_x covariance

$$\sigma(x, u_x) = \langle (x - \langle x \rangle)(u_x - \langle u_x \rangle) \rangle = -\Re \quad (420)$$

The covariance represents the total error of two variables, which is different from the variance that only represents the error of one variable. If two variables change in the same directions, then the covariance between two variables is positive. If two variables change in opposite directions, the covariance between two variables is negative. For any two real random variables A and B, there is the Schwarz inequality $|\sigma(A, B)| \leq \sigma(A)\sigma(B)$, which leads to

$$\sigma(x)\sigma(u_x) \geq \Re = \hbar/2m \quad (421)$$

The statistical definition of uncertainty is

$$\sigma(x) = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \quad (422)$$

$$\sigma(u_x) = \sqrt{\langle u_x^2 \rangle - \langle u_x \rangle^2} \quad (423)$$

So far we have proved the uncertainty relationship between the position of random spacetime moving particles and the fluctuation speed. Further, if the uncertainty of momentum has two parts of contributions

$$\sigma^2(p) = m^2(\sigma^2(v) + \sigma^2(u)) \quad (424)$$

That is, $\sigma(p) \geq m\sigma(u)$, the uncertainty of the position and the fluctuation speed can be obtained.

$$\sigma(x)\sigma(p_x) \geq \hbar/2 \quad (425)$$

The proof of our paper interprets Heisenberg's uncertainty principle as the uncertainty relationship between random spacetime moving particle position and fluctuation speed. The random spacetime motion has no friction and no irreversible dissipation.

The uncertainty of the fluctuation speed is entirely from spacetime fluctuations. According to Heisenberg's original statement, the measured action inevitably interferes with the state of the particles being measured, thus creating uncertainty. Later that year, Kennard gave another statement. The following year, Herman also obtained this result independently. According to Kennard's statement, the uncertainty of position and the uncertainty of momentum are the nature of the particle, and cannot be suppressed below a certain limit, regardless of the measured action. Thus, for the principle of uncertainty, there are two completely different interpretations. Landau believes that the two interpretations are equivalent, so one expression can be derived from another expressions (Ref. quantum mechanics of Landau). However, in the latest experimental progress, Japanese scholars published on

January 15, 2012, the empirical results of the Heisenberg uncertainty principle. They used two instruments to measure the spin angle of the neutron and obtained a smaller measurement than the Heisenberg limit, which proves the measurement interpretation by Heisenberg is wrong. However, the principle of uncertainty is still correct, because this is the quantum nature of the particle.

The derivation process of this paper has nothing to do with the measurement theory, and it has nothing to do with the internal properties of the particles. It is believed that the uncertainty principle is rooted in the fluctuation of spacetime. Under the non-relativistic framework, spacetime fluctuations are only related to the mass of the particles. The mass of a particle is the only perceptible property of the particle in spacetime.

Appendix D: Additional Physics Example with Three-speed Decomposition

The superposition of orbitals and the formation of chemical bonds, which are common in chemistry, involves quantum superposition states. In the simplest case, the ground state of the hydrogen atom and the first excited state are superimposed with equal probability as

$$\psi(r, t) = \psi_{100}e^{-iE_1t} + \psi_{200}e^{-iE_2t} \quad (426)$$

Where $E_1 = -13.6ev$, $E_2 = -13.6/4ev = -3.4ev$, the wave function of the ground state of the hydrogen atom and the first excited state are

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a} \quad (427)$$

$$\psi_{200} = \frac{1}{\sqrt{2a^3}}e^{-r/2a}\left(1 - \frac{r}{2a}\right) \quad (428)$$

Where a is the Bohr's radius $a = 0.529 \times 10^{-10}m$.

With the Euler formula, we can write the superimposed wave function as

$$\psi = [\psi_{100}\cos(E_1t) + \psi_{200}\cos(E_2t)] - i[[\psi_{100}\sin(E_1t) + \psi_{200}\sin(E_2t)]] \quad (429)$$

From the real and imaginary part, the two potential functions R and I of the superposition wave function can be further determined. It is found by equation (145) and (146) that the electrons u and v are not zero in this state.

This physics example is not a special case, and has general physical meaning. When the quantum state has definite energy, its classical statistical velocity v must be zero. Generally

speaking, the particle is in the superposition state of the energy eigenstate, and its three speeds are not zero which has clear physical meaning.

Appendix E: Self Isomorphism on Direct Product Spin Clusters

We hope to prove the following conclusions in this appendix:

Theorem 1. *Given any topological excited state deformation: $A : \Lambda_L \otimes \Lambda_R \mapsto \Lambda_L \otimes \Lambda_R$, where A For automorphism mapping, Λ_L, Λ_R represent the left-hand spin cluster and the right-hand spin cluster, respectively, and A is the vector map.*

Proof: First of all, from the symmetry of the spin structure, it is not difficult to know that we only need to prove arbitrary automorphism: $A : \Lambda_L \mapsto \Lambda_L$ Both are vector maps. This is because if we can determine that A is a vector map, we can get it through conjugate expansion: $\tilde{A} : \Lambda_L \otimes \Lambda_R \mapsto \Lambda_L \otimes \Lambda_R$ for vector mapping.

To prove that any automorphism: $A : \Lambda_L \mapsto \Lambda_L$ is a vector map, we need to consider the model on the left-handed spin sector, which is corresponding to the *Clifford* algebra. Proposition 1.3.2 by [48] It can be seen that for the finite form *Clifford* algebra, the following forms are isomorphic:

$$Cl_{r,s} \cong Cl_1 \hat{\otimes} \dots \hat{\otimes} Cl_1 \hat{\otimes} Cl_1^* \dots \hat{\otimes} Cl_1^*.$$

Among them, the number of Cl_1 corresponds to r , and the number of Cl_1^* corresponds to s .

From the theorem 1.5.4 of [48], all *Clifford* K - means that ρ can be decomposed into straight sums of irreducible algebra representations of the following form:

$$\rho = \rho_1 \oplus \dots \oplus \rho_m.$$

The feature subspace W_i corresponding to ρ_i is the smallest subspace.

In additions, by the *Bott* cycle law theorem [48], we can get the algebraic representation of all Cl_m , ($m = 1, \dots, 8$), and the representation follows the indicator m Repeated with a period of 8. That is: we can get the algebra of any Cl_m as follows:

$$\begin{aligned} Cl_1 &= \mathbb{C}, \quad Cl_2 = \mathbb{H}, \quad Cl_3 = \mathbb{H} \oplus \mathbb{H}, \quad Cl_4 = \mathbb{H}(2), \\ Cl_5 &= \mathbb{C}(4), \quad Cl_6 = \mathbb{R}(8), \quad Cl_7 = \mathbb{R}(8) \oplus \mathbb{R}(8), \quad Cl_8 = \mathbb{R}(16). \end{aligned} \quad (430)$$

For any combination of the above forms, the straight and broken parts ρ_i Can be split into direct product form:

$$Cl_{r,s} \cong Cl_1 \widehat{\otimes} \dots \widehat{\otimes} Cl_1 \widehat{\otimes} Cl_1^* \dots \widehat{\otimes} Cl_1^*.$$

The automorphism mapping between any part of the above direct product form can be made by $Cl_1 = \mathbb{C}, \dots, Cl_8 = \mathbb{R}$ (16) Algebraic combination between parts. Since the above parts are all vector spaces, the automorphism must be a vector mapping, that is, the automorphism of ρ_i must correspond to the matrix form.

In addition, from the algebraic decomposition process described above, it is not difficult to know that the homomorphic mapping between all corresponding different sub-blocks is also a vector mapping. Finally, we will be $\rho_i, i = 1, \dots, 8$ All of them are combined together in a straight form, and we can get the automorphism $A : \Lambda_L \mapsto \Lambda_L$ when $i = 1, \dots, 8$ for vector mapping. When the indicator i is greater than 8, by the *Bott* cycle law, we can still get the automorphism mapping by the above process. A is the vector map. The conclusion is proved.

Appendix F: Field Theory Calculations for Fermionic Loop Integral

We consider the following Fermion loop momentum integrals

$$\int \frac{d^d k}{D_1^{n_1} D_2^{n_2}} = i\pi^{d/2} (-p^2)^{d/2 - n_1 - n_2} G(n_1, n_2), \quad D_1 = -(k+p)^2, \quad D_2 = -k^2 \quad (431)$$

Noting that in the denominator, D_1, D_2 should actually have an infinitesimal analytic continuation ($-i0^+$). But for the sake of simplicity, we don't explicitly write it out. After analysing the continuation, we need to consider the contribution of $p^2 < 0$, and the power contribution of $-p^2$ can be easily obtained from dimensional analysis. In fact, what needs to be calculated now is the dimensionless function $G(n_1, n_2)$; to simplify the calculation, we can let $-p^2 = 1$. When $n_1 \leq 0$ or $n_2 \leq 0$, the score can be strictly calculated and $G(n_1, n_2) = 0$ can be obtained.

Using Wick rotation and α parameterization, we can rewrite $G(n_1, n_2)$ as:

$$G(n_1, n_2) = \frac{\pi^{-d/2}}{\Gamma(n_1)\Gamma(n_2)} \int e^{-\alpha_1(\mathbf{k}+\mathbf{p})^2 - \alpha_2 \mathbf{k}^2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1 d\alpha_2 d^d \mathbf{k}. \quad (432)$$

Let

$$\mathbf{k}' = \mathbf{k} + \frac{\alpha_1}{\alpha_1 + \alpha_2} \mathbf{p},$$

We can get

$$\begin{aligned} G(n_1, n_2) &= \frac{\pi^{-d/2}}{\Gamma(n_1)\Gamma(n_2)} \int \exp\left[-\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\right] \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1 d\alpha_2 \int e^{-(\alpha_1+\alpha_2) Bmk^2} d^d\mathbf{k} \\ &= \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int \exp\left[-\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\right] (\alpha_1 + \alpha_2)^{-d/2} \alpha_1^{n_1-1} \alpha_2^{n_2-1} d\alpha_1 d\alpha_2. \end{aligned} \quad (433)$$

Using the substitution $\alpha_1 = \eta x$, $\alpha_2 = \eta(1 - x)$, the above formula can be rewritten as

$$\begin{aligned} G(n_1, n_2) &= \frac{1}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 x^{n_1-1} (1-x)^{n_2-1} dx \int_0^\infty e^{-\eta x(1-x)} \eta^{-d/2+n_1+n_2-1} d\eta \\ &= \frac{\Gamma(-d/2 + n_1 + n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 x^{d/2-n_2-1} (1-x)^{d/2-n_1-1} dx. \end{aligned} \quad (434)$$

The integrand is an Euler B function, so we can get the final result

$$G(n_1, n_2) = \frac{\Gamma(-d/2 + n_1 + n_2)\Gamma(d/2 - n_1)\Gamma(d/2 - n_2)}{\Gamma(n_1)\Gamma(n_2)\Gamma(d - n_1 - n_2)}. \quad (435)$$

For all positive integers $n_{1,2}$ they are proportional to

$$G_1 = G(1, 1) = -\frac{2g_1}{(d-3)(d-4)}, \quad g_1 = \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}, \quad (436)$$

The scale factor is a rational function of d .

Noting that at $k \rightarrow \infty$, the denominator part of (431) behaves as $(k^2)^{n_1+n_2}$. Therefore, this integral is divergent when $d \geq 2(n_1 + n_2)$.

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[1] L. Erdos, Lecture Notes on Quantum Brownian Motion, arXiv: 1009.0843, (2010)

[2] N. Wiener, Differential space. J. Math and Phys. 58, 131-174 (1923)

- [3] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (4-th edition), Oxford University, (2002)
- [4] S. Ross, *A First Course in Probability* (8th Edition), Pearson Prentice Hall, (2009)
- [5] F. Reif, *Fundamentals of Statistical and Thermal Physics*, Waveland, (2009)
- [6] A. Einstein, *Investigations on the theory of the brownian movement*. Dover Edition(1956).
- [7] E. Nelson, *Dynamical theories of Brownian motion*, (Princeton University Press, Princeton, 1967).
- [8] E. Nelson, *Quantum Fluctuations*, (Princeton University Press, Princeton, 1985).
- [9] S. Weinberg, *Cosmology*, Oxford University, (2008)
- [10] F. W. Stecker “Constraining Superluminal Electron and Neutrino Velocities using the 2010 Crab Nebula Flare and the IceCube Pev Neutrino Events”. *Astroparticle Physics* 56: 16?C18. arXiv:1306.6095
- [11] K. Kodama et al. (DONUT Collaboration; Kodama; Ushida; Andreopoulos; Saoulidou; Tzanakos; Yager; Baller; Boehnlein; Freeman; Lundberg; Morfin; Rameika; Yun; Song; Yoon; Chung; Berghaus; Kubantsev; Reay; Sidwell; Stanton; Yoshida; Aoki; Hara; Rhee; Ciampa; Erickson; Graham; et al. (2001). “Observation of tau neutrino interactions”. *Physics Letters B* 504 (3): 218. arXiv:hep-ex/0012035.
- [12] R. A. Battye and A. Moss, “Evidence for Massive Neutrinos from Cosmic Microwave Background and Lensing Observations”. *Physical Review Letters* 112 (5): 051303 (2014). arXiv:1308.5870v2.
- [13] Planck Collaboration, P. A. R.; Ade, P. A. R.; Aghanim, N.; Armitage-Caplan, C.; Arnaud, M.; Ashdown, M.; Atrio-Barandela, F.; Aumont, J.; Baccigalupi, C.; Banday, A. J.; Barreiro, R. B.; Bartlett, J. G.; Battaner, E.; Benabed, K.; Benot, A.; Benoit-L’vy, A.; Bernard, J.-P.; Bersanelli, M.; Bielewicz, P.; Bobin, J.; Bock, J. J.; Bonaldi, A.; Bond, J. R.; Borrill, J.; Bouchet, F. R.; Bridges, M.; Bucher, M.; Burigana, C.; Butler, R. C.; et al. (2013). “Planck 2013 results. XVI. Cosmological parameters”. *Astronomy & Astrophysics* 1303: 5076. arXiv:1303.5076. Bibcode:2013arXiv1303.5076P. doi:10.1051/0004-6361/201321591.
- [14] S. Weinberg, *Lectures on Quantum Mechanics*, Cambridge University, (2012)
- [15] S. Weinberg, *The Quantum Theory of Fields*, vol 2, Cambridge University, (1995)
- [16] S. Chandrashekhar, *Rev. Mod. Phys.*, 15,1(1943).
- [17] J. Doob, *Stochastic processes*. Wiley: New York, (1953).

- [18] Wold, H. (1954) *A Study in the Analysis of Stationary Time Series*, Second revised edition, with an Appendix on "Recent Developments in Time Series Analysis" by Peter Whittle. Almqvist and Wiksell Book Co., Uppsala.
- [19] G. E. Uhlenbeck and L. S. Ornstein, *Rev. Mod. Phys.*, 36,823(1930)
- [20] M. C. Wang and G. E Uhlenbeck, *Rev. Mod. Phys.*, 17,323(1945).
- [21] L. de Broglie, *C.R. Acad. Sci.* 264B, 1041 (1967).
- [22] M. Davidson, *Lett. Math. Phys.* 3, 271 (1979).
- [23] M. Davidson, *Lett. Math. Phys.* 3, 367 (1979)
- [24] Max Born, *Zur Quantenmechanik der Sto vorg nge*, *Zeitschrift f'r Physik*, 37, #12 (Dec. 1926), pp. 863-C867 (German); English translation, *On the quantum mechanics of collisions*, in *Quantum theory and measurement*, section I.2, J. A. Wheeler and W. H. Zurek, eds., Princeton, New Jersey: Princeton University Press, 1983, ISBN 0-691-08316-9.
- [25] L. Kadanoff , *Statistical Physics: statics, dynamics and renormalization*. World Scientific Press, (2000).
- [26] L. Landau and E. Lifshitz , *Courses in theoretical physics*, vol 1, *Mechanics*. Butterworth-Heinemann. (1976)
- [27] H. Goldstein, *Classical Mechanics (3rd Edition)*, Addison-Wesley, (2001)
- [28] J. D. Jackson, *Classical Electrodynamics (3-rd Edition)*, Wiley, (1998)
- [29] L. Landau and E. Lifshitz , *Courses in theoretical physics*, vol 3, *Quantum Mechanics*. Pergamon Press, (1977)
- [30] R.J. Glauber, *Quantum optics and electronics*, edited by C. De Witt (Gordon-Breach New York, 1965); H. Haken, in *Encyclopedia of Physics* (Springer, New York, 1976).
- [31] G. Birkhoff and J. von Neumann, *Ann. Math.* 37, 823 (1932).
- [32] H. Haken and W. Weidlich, *Z. Phys.* 205, 96 (1967).
- [33] E. H. Kennard, "Zur Quantenmechanik einfacher Bewegungstypen". *Zeitschrift f'r Physik* 44 (4-C5): 326 (1927)
- [34] T.D.Lee, *Particle physics and introduction to field theory*, Harwood Academic Publishers , 1981.
- [35] Einstein, A; B Podolsky; N Rosen. *Physical Review.* 47 (10): 777-780.
- [36] H. Weyl, *The Theory of Groups and Quantum Mechanics*, Dover Edition, (1950)
- [37] L. A. Rozema, A. Darabi, D. H. Mahler, A. Hayat, Y. Soudagar, and A. M. Steinberg, *Phys.*

- Rev. Lett. 109, 100404 (2012)
- [38] L. A. Rozema, A. Darabi, D. H. Mahler, A. Hayat, Y. Soudagar, and A. M. Steinberg, Phys. Rev. Lett. 109, 189902 (2012)
- [39] R. P. Feynman, Ph.D thesis. Princeton Press (1942).
- [40] P.A.M. Dirac, *Physikalische Zeitschrift der Sowjetunion*, Band 3, Heft 1 , pp. 64-72 (1933)
- [41] P. Blanchard, S. Golin and M. Serva, Phys. Rev. D34, 3732 (1986).
- [42] P. Blanchard and M. Serva, Phys. Rev. D51, 3132 (1995).
- [43] P. A. M. Dirac, Quantised Singularities in the Electromagnetic Field, *Proc. Roy. Soc. A* **133**, 60 (1931)
- [44] G. 't Hooft and F. Bruckmann, Monopoles, Instantons and Confinement. arXiv:hep-th/0010225
- [45] N. K. Nielsen and P. Olesen, Vortex-Line Models for Dual Strings, *Nucl. Phys. B* **61**, 45 (1973)
- [46] L. Mario. "Hydrodynamic theory of electromagnetic fields in continuous media." *Physical Review Letters* 70.23(1993):3580-3583.
- [47] Martins, Alexandre A , Pinheiro, et al. Fluidic electrodynamics: Approach to electromagnetic propulsion[C]// Aip Conf Proc. American Institute of Physics, 2009.
- [48] H. B. Lawson. *Spin Geometry* [M]. Princeton University Press, New Jersey, 1994.
- [49] After we finished our manuscript, we found that this three-velocity decomposition is in fact consistent with Wold's decomposition theorem of the stochastic process in [18].
- [50] We will use charge instead of electric charge in this section, for simplicity.
- [51] Wenberg, "Gravity Theory and Cosmology", page 150, the uniqueness of curvature tensor is a very important mathematical theorem in Riemannian geometry.