# Bernoulli Numbers and Pi

Edgar Valdebenito

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ABSTRACT. We give a formula for Pi.

keywords: bernoulli numbers , number Pi

## I. Bernoulli Numbers .

The Bernoulli numbers are defined by

(1) $n \geq 1\,,\ B_n = \left\{\frac{1}{6},\ \frac{1}{30},\ \frac{1}{42},\ \frac{1}{30},\ \frac{5}{66},\ \frac{691}{2730},\ \frac{7}{6},\ \frac{3617}{510},\ \frac{43867}{798},\ \ldots\right\}$

## II. Pi Constant .

Pi constant is defined by

(2) $\pi = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\right) = 3.141592\ldots$

## III. Pi Formula .

(3) $\pi = 3+2\left(\tan^{-1}\left(\frac{1}{6}\right)-\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2\times 3^{2n+1}B_{n+1}}{(2n+2)!\,(1+u_n\,u_{n+1})}\right)\right)$

where $u_{n+1} = u_n - \frac{2\times 3^{2n+1}B_{n+1}}{(2n+2)!}\ ,\ u_1 = \frac{1}{6}\ ,\ n \geq 1$

(4) $\pi = 3+2\left(\tan^{-1}\left(\frac{1}{6}\right)-\tan^{-1}\left(\frac{54}{731}\right)-\tan^{-1}\left(\frac{3240}{202\,997}\right)-\tan^{-1}\left(\frac{58\,320}{16\,215\,757}\right)-\ldots\right)$

## IV. References .

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