Justification for Sphere with Surface-Tension as Eddy-Model in a turbulent Fluid.

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Abstract.

Current text is to be considered as an addendum for the earlier text: "Turbulence as structured Route of Energy from Order into Chaos, by Udo E. Steinemann, vixra.com/vixra.1801.0037". The recent script introduced a sphere with surface–tension as an appropriate eddy–model in a discussion on energy–transport through a turbulent fluid–volume. Maybe this vortex–model seemed to be a bit arbitrarily chosen at the publication–time of the article mentioned above. By the current text I have tried to justify the former model–idea on account of outcomes from REYNOLDS–equations and PRANDTLs mixing–distance–theory.

1. *Introduction.*

Most Information contained in this chapter has been extracted from [1].

1.1. *Fluid properties.*

A set of properties presented in the scheme below maybe appropriate for the characterization of a turbulent fluid during subsequent discussions.

⋙density: ϱ⋘						●
⋙pressure in turbulence fluid: $a(\underline{r},t) = \hat{a}(\underline{r})+a'(\underline{r},t)$⋘					●	
⋙ speed-vector of turbulence fluid: $\underline{c}(\underline{r},t) = \underline{\hat{c}}(\underline{r})+\underline{c}'(\underline{r},t)$⋘	●	●				
∣ *composed of* ∣	↓				↓	=
⋙mean portion: $\underline{\hat{c}}(\underline{r})$⋘	●		●			
⋙mean portion: $\hat{a}(\underline{r})$⋘■⋙ϱ = const⋘					●	●
	∧				∧	
⋙stochastic portion representing fluctuation: $\underline{c}'(\underline{r},t)$⋘	●			●		
⋙stochastic portion due to fluctuation: $a'(\underline{r},t)$⋘					●	
∣ *with* ∣	↓				↓	
⋙(location-vector: $\underline{r}$) ∧ (time-variable: t)⋘	●				●	
∣ *decomposed into* ∣		↓	↓	↓		
⋙components: $c_1\wedge c_2\wedge c_3$⋘■⋙components: $\hat{c}_1\wedge\hat{c}_2\wedge\hat{c}_3$⋘■⋙components: $c'_1\wedge c'_2\wedge c'_3$⋘		●	●	●		
∣ *according to* ∣		↓	↓	↓		
⋙rectangular coordinate-system⋘		●	●	●		
∣ *with* ∣		↓	↓	↓		
⋙$(x_1\text{–axis}) \wedge (x_2\text{–axis}) \wedge (x_3\text{–axis})$⋘		●	●	●		
Properties of turbulent Fluid						

1.2. *Equations of Fluids Motion.*

As shown below there is direct way from NAVIER–STOKE equation for a non–stationary fluid to the REYNOLDS–equation, which finally will deliver fluid–tensions due stochastic fluctuations of the fluid.

⋙NAVIER-STOKE-equation for non-stationary fluids⋘	●							
∣ *represented by* ∣	↓							
⋙$d\underline{c}/dt = (\partial\underline{c}/\partial t)+\underline{c}(\nabla\cdot\underline{c}) = \underline{f}-\varrho^{-1}(\nabla a)+\nu(\Delta\underline{c})$⋘	●							
	∨							
⋙$dc_j/dt = (\partial c_j/\partial t)+c_k(\partial c_j/\partial x_k) = f_j-\varrho^{-1}(\partial a/\partial x_j)+\nu(\partial^2 c_j/\partial x_k^2)$⋘	●	●	●					
∣ *where* ∣	↓							
⋙$[j,k = (1,2,3)] \wedge [f_j = \text{external forces}] \wedge [\nu = \text{viscosity}]$⋘	●							
∣ *takes into consideration* ∣ ♦ ∣ *leads to* ∣		↓	↓					
⋙fluctuation-property: $\underline{c} = \underline{\hat{c}}+\underline{c}'$⋘		●						
		∧						
⋙time-everage of a property: ⟪…⟫⋘		●						
⋙$(\partial\hat{c}_j/\partial t)+\hat{c}_k(\partial\hat{c}_j/\partial x_k) = f_j-\varrho^{-1}(\partial\hat{a}/\partial x_j)+\nu(\partial^2\hat{a}_j/\partial x_k^2)-\langle\!\langle c_k'(\partial c_j'/\partial x_k)\rangle\!\rangle$⋘			●	●				
∣ *with* ∣				↓				
⋙$\langle\!\langle c_k'(dc_j'/dx_k)\rangle\!\rangle = \langle\!\langle d(c_k'c_j')/dx_k\rangle\!\rangle-\langle\!\langle c_j'(dc_k'/dx_k)\rangle\!\rangle$⋘				●				
				∧				
⋙continuity-equation: $\partial c_j'/\partial x_j = 0$⋘				●				
∣ *leads to* ∣				↓				
⋙REYNOLDS-equation⋘				●				
∣ *represented by* ∣				↓				
⋙$(\partial\hat{c}_j/\partial t)+\hat{c}_k(\partial\hat{c}_j/\partial x_k) = f_j-\varrho^{-1}(\partial\hat{a}/\partial x_j)+\nu(\partial^2\hat{a}_j/\partial x_k^2)-\langle\!\langle d(c_k'c_j')/dx_k\rangle\!\rangle$⋘				●	●			
∣ *with* ∣					↓			
⋙$[\nu(\partial^2\hat{a}_j/\partial x_k^2) = \varrho^{-1}(\partial\tau_{jk}/\partial x_k)] \wedge [\langle\!\langle d(c_k'c_j')/dx_k\rangle\!\rangle = (d\langle\!\langle c_k'c_j'\rangle\!\rangle/dx_k)]$⋘					●			

/ *leads to* /					↓			
≫$(\partial\hat{c}_j/\partial t)+\hat{c}_k(\partial\hat{c}_j/\partial x_k) = f_j-\varrho^{-1}(\partial\hat{a}/\partial x_j)+\varrho^{-1}(\partial/\partial x_k[\tau_{jk}-\varrho\langle\langle c_j'c_k'\rangle\rangle])$					●	●		
/ *results in* /						↓		
≫stress-tensor: $-\varrho\langle\langle c_j'c_k'\rangle\rangle = (-\varrho)\cdot\begin{matrix}\langle\langle(c_1')^2\rangle\rangle\langle\langle c_1'c_2'\rangle\rangle\langle\langle c_1'c_3'\rangle\rangle\\ \langle\langle c_2'c_1'\rangle\rangle\langle\langle(c_2')^2\rangle\rangle\langle\langle c_2'c_3'\rangle\rangle\\ \langle\langle c_3'c_1'\rangle\rangle\langle\langle c_3'c_2'\rangle\rangle\langle\langle(c_3')^2\rangle\rangle\end{matrix}$≪						●	●	●
/ *gives* /							↓	↓
≫normal tensions: $\{(-\varrho\cdot\langle\langle(c_j')^2\rangle\rangle) \rightarrow (j = 1,2,3)\}$≪							●	
≫shear-tensions: $\{(-\varrho\cdot\langle\langle c_p c_q'\rangle\rangle) \rightarrow (p,q = 1,2,3)\}$≪								●
REYNOLDS-Tensions								

1.3. *Physical Interpretation of the REYNOLDS–Tensions.*

Obviously exist an analogy – as demonstrated by scheme below – between tensions as they exist e.g. in mechanics and those entities introduced by O. REYNOLDS, which can rightly be called tensions.

≫τ_T: shear-tensions: $\{(-\varrho\cdot\langle\langle[c_p'c_q'\rangle\rangle) \wedge (p,q = 1,2,3)\}$≪			●		
≫normal tensions: $\{(-\varrho\cdot\langle\langle[c_j']^2\rangle\rangle) \wedge (j = 1,2,3)\}$≪	●	●			
/ *represent* / ♦ / *considered as* / ♦ / *acting as* /	↓	↓	↓		
≫macroscopic anisotropic analogy≪■≫pendant≪	●	●			
≫parallel-motion in (x_1,x_2)-plane≪			●		●
/ *intersecting* /			↓		
≫A-plane within (x_2,x_3)-plane≪			●	●	
/ *attacked by* / ♦ / *causes* /				↓	↓
≫shear-tensions: τ_T≪■≫shear-force: $F = A\cdot\tau_T = \varrho A\langle\langle c_2'c_1'\rangle\rangle$≪				●	●
/ *of* /	↓	↓			
≫molecular-motions in kinematic gas-theory≪	●	●			
/ *where is specified* /	↓				
/ *leads to* /		↓			↓
≫statistical pressure: $p = 3^{-1}m\cdot n\langle\langle s^2\rangle\rangle$≪■≫$\tau_T = \varrho(A/F)\langle\langle c_2'c_1'\rangle\rangle$≪	●				●
≫in x_j-direction: $\{(p_j = \varrho\cdot\langle\langle(c_j')^2\rangle\rangle) \wedge (j = 1,2,3)\}$≪		●			
/ *with* /	↓				
≫number of molecules per unit-volume: n≪	●				
	∧				
≫molecule-mass: $m \rightarrow (m\cdot n = \varrho)$≪	●				
	∧				
≫mean kinetic-energy per molecule-mass: $\langle\langle s^2\rangle\rangle$≪	●				
Physical Interpretation of the REYNOLDS-Tensions					

1.4. *Energy–Balance of turbulent Fluid–Motion.*

Local non–stationary time–modifications of energy in a turbulent fluid–volume are due to interactions of four different time–dependent effects: production, dissipation, convection and diffusion. Two of them – production and dissipation – have to be considered as source and sink of turbulent energy, the other two effects – convection and diffusion – are responsible for transportation of the energy through the turbulent fluid–volume. While production is strongly related with REYNOLDS–tensions and creates order in fluid–volume on this base, dissipation on the other hand transforms turbulent energy by fiction into heat and creates chaos thereby. Production and dissipation – equally sized – turn out to be counterparts in creation and destruction of order.

≫in complete flow-area of the fluid≪										●
≫local non-stationary time-modification of turbulent energy≪	●									
/ *contains* / ♦ / *is constantly fulfilled* /	↓									↓
≫terms≪	●									
/ *for* /	↓									
≫production: acceptance of turbulent-energy from tensions≪	●	●	●					●	●	●
/ *due to* /	∧	↓	↓							

⋙normal-tensions: $-{}_{j=1}\Sigma^3[\langle\langle(c'_j)^2\rangle\rangle](\partial\hat{c}_j/\partial x_j)]$⋘		●		●						
| as | ♦ | can be compared with |				∧				↓	↓	=
⋙shear-tensions: $-\langle\langle c'_1c'_2\rangle\rangle[(\partial\hat{c}_1/\partial x_2)+(\partial\hat{c}_2/\partial x_1)]-$ $\langle\langle c'_1c'_3\rangle\rangle[(\partial\hat{c}_1/\partial x_3)+(\partial\hat{c}_3/\partial x_1)]-$ $\langle\langle c'_2c'_3\rangle\rangle[(\partial\hat{c}_2/\partial x_3)+(\partial\hat{c}_3/\partial x_2)]$⋘			●	●						
⋙source⋘								●		
| can be combined to |				↓				∧		
⋙$_{j=1}\Sigma^3\langle{}_{k=1}\Sigma^3[\tau_{jk}(\partial\hat{c}_j/\partial x_k)]\rangle$⋘										
⋙dissipation: waste of turbulent-energy by transition into heat⋘	●			●	●			●	●	●
| specified by | ♦ | as | ♦ | in specific sense of |	∧				↓			↓	↓	
⋙$\nu\{2[{}_{j=1}\Sigma^3\langle\langle(\partial c'_j\partial x_j)^2\rangle\rangle+$ $\langle\langle[(\partial c'_1/\partial x_2)+(\partial c'_2/\partial x_1)]^2\rangle\rangle+\langle\langle[(\partial c'_1/\partial x_3)+(\partial c'_3/\partial x_1)]^2\rangle\rangle+$ $\langle\langle[(\partial c'_2/\partial x_3)+(\partial c'_3/\partial x_2)]^2\rangle\rangle$⋘					●					
⋙sink⋘■⋙energy-transition from order into chaos⋘								●	●	
⋙convection: transportation of turbulent-energy due to mean-motion⋘	●					●				
| specified by | ♦ | represent |	∧					↓		↓		
⋙$-½\{{}_{j=1}\Sigma^3(\partial\hat{c}_j\langle\langle{}_{k=1}\Sigma^3(c'_k)^2\rangle\rangle/\partial x_j)\}$⋘						●				
⋙diffusion: transportation of turbulen]t-energy due to fluctuations⋘	●						●			
| specified by |							↓			
⋙$-{}_{j=1}\Sigma^3(\partial\langle\langle c'_j\{p'/\varrho+½[{}_{k=1}\Sigma^3(c'_k)^2]\}\rangle\rangle/\partial x_j)]$⋘							●			
⋙energy-changes in the considered fluid-volume⋘								●		
Production for Creation of Order and Dissipation for Destruction into Chaos playing the roles of Counterparts in turbulent Fluid-Volume										

1.5. *Measure for Sizes of energetic Vortices and dissipating Vortices in a Dissipation–State independent of REYNOLDS–Numbers.*

Dissipation in turbulent fluid for large REYNOLDS—numbers enables estimates about measures of average—sizes (L) for energetic vortices and (λ) for dissipating vortices as well. This is made obvious in the following scheme:

⋙dissipation: waste of turbulent-energy by transition into heat⋘	●				
| as specified by |	↓				
⋙$\nu\{2[{}_{j=1}\Sigma^3\langle\langle(\partial c'_j\partial x_j)^2\rangle\rangle+$ $\langle\langle[(\partial c'_1/\partial x_2)+(\partial c'_2/\partial x_1)]^2\rangle\rangle+\langle\langle[(\partial c'_1/\partial x_3)+(\partial c'_3/\partial x_1)]^2\rangle\rangle+$ $\langle\langle[(\partial c'_2/\partial x_3)+(\partial c'_3/\partial x_2)]^2\rangle\rangle$⋘	●				
| written more densely |	↓				
⋙$\nu[{}_{j,k=1}\Sigma^3\langle\langle[(\partial c'_j\partial x_k)+(\partial c'_k\partial x_j)](\partial c'_k\partial x_j)\rangle\rangle$⋘	●	●			
| leads to |		↓			
⋙$\sim\nu\langle\langle(\partial \underline{c}')^2\rangle\rangle/\lambda^2$⋘		●	●		
| if | ♦ | where |		↓	↓		
⋙turbulent state independent of REYNOLDS-numbers⋘		●			●
⋙$\lambda = {}_{j=1}\Sigma^3\langle\langle(c'_1)^2/(\partial c_1/\partial x_j)^2\rangle\rangle^{1/2}$⋘			●	●	
| except for | ♦ | to be considered as | ♦ | becomes independent for |		↓		↓	↓
⋙small structures strongly influenced by: ν ⋘■⋙typical size (micro-scale) of dissipating vortices⋘		●		●	
⋙REYNOLDS-number: $\mathrm{Re}_L \approx (L/\lambda)^2$⋘					●
| where |					↓
⋙L: "integral correlation-lengths" or typical size of energetic vortices⋘					●
Measures for Mean-Sizes of Vortices in a Dissipation-State independent from REYNOLDS-Numbers					

Further measures were added by PRANDTL on base of his ″mixing distance hypothesis″. Under assumptions:

- $\hat{c}_1 = \hat{c}_1(x_2) \wedge \hat{c}_2 = \hat{c}_3 = 0 \wedge c'_{(j=1\rightarrow 3)} \neq 0$

he developed an impulse—exchange—model for turbulent shear—tensions. Starting from kinetic gas—theory he specified a molecular viscosity as product of molecular speed and average—free—distance of the molecules and proposed for the pendant – the turbulent motion – a similar connection will have to exist. This means, he proposed for vortices a viscosity as product of a characteristic velocity of the turbulent flow and a length (the so–called mixing–distance length). Details of PRANDTLs theory are sketched shortly by the scheme below:

≫transported quality≪					●			
≫turbulent motion of a fluid≪		●		●				
/assumed to be/ ♦ /transports/ ♦ /becomes/		↓		↓	↓			
≫macroscopic pendant≪■≫quality: $q(x_2)$≪		●		●				
≫$Q = \langle\langle c'_2[\langle\langle q(x_2)_2\rangle\rangle - q(x_2)_1]\rangle\rangle$≪					●	●		
/of/		↓		↓				
/with/						↓		
≫kinetic gas-theory≪	●	●						
≫a turbulence-ball≪■≫$q(x_2)_1-\langle\langle q(x_2)_2\rangle\rangle = q(x_2+\Delta x_2)_1-\langle\langle q(x_2)_2\rangle\rangle$≪				●		●		
/expanded into/						↓		
≫TAYLOR-series≪						●		
/leads to/	↓					↓		
/with/		↓		↓				
≫molecular viscosity: $\nu = \lambda_m\langle\langle s^2\rangle\rangle^{1/2}$≪■≫velocity: c'_2≪	●			●				
≫$Q = \langle\langle c'_2\Delta x_2\rangle\rangle\langle\langle dq/dx_2\rangle\rangle\vert_2+½\langle\langle c'_2(\Delta x_2)^2\rangle\rangle\langle\langle d^2q/d(x_2)^2\rangle\rangle\vert_2+\ldots$≪						●		
≫a similar correlation≪		●	●					
/where/ ♦ /means/ ♦ /across/ ♦ /leads to/	↓		↓	↓		↓		
≫mean distance between molecules: λ_m≪■≫vortex-viscosity≪	●		●					
≫$Q = \langle\langle c'_2\Delta x_2\rangle\rangle\langle\langle dq/dx_2\rangle\rangle\vert_2$≪						●	●	
/becomes/ ♦ /for/ /leads to/	∧		↓			↓	↓	
≫speed of a molecule: $\underline{s}$≪■≫product≪■≫very small: Δx_2≪	●		●			●		
/of/			↓					
≫characteristic speed≪■≫$Q = -l^*\langle\langle(c'_2)^2\rangle\rangle^{1/2}$≪			●				●	
/where/			∧				↓	
≫characteristic length≪■≫$\Delta x_2 = (x_2)_2-(x_2)_1$≪			●	●				
≫$-\langle\langle c'_2\Delta x_2\rangle\rangle = l^*\langle\langle(c'_2)^2\rangle\rangle^{1/2}$≪							●	●
/for/ ♦ /where/							↓	↓
≫$c'_2\Delta x_2 < 0$≪■≫exchange-length: l^*≪							●	●

Overview of PRANDTL's Mixing-Distance-Hypothesis

As outcome – in connection with the above considerations – a length (l_m = mixing–distance–length) can be estimated, which informs about the average–distance a turbulent–ball (vortex) must travel until it loses its individuality – being transformed into another vortex or due to viscosity into heat. This is further demonstrated in the following scheme:

≫$[c'_1 = \Delta x_2(dc_1/dx_2)] \wedge [\langle\langle(c'_1)^2\rangle\rangle = \langle\langle\Delta x_2^2(dc_1/dx_2)^2\rangle\rangle] \wedge [\langle\langle c'_1\rangle\rangle \sim \langle\langle c'_2\rangle\rangle]$≪			●	●		
≫quality: $q(x_2)$≪	●					
/identified by/ ♦ /leads to/	↓		↓	∧		
≫impulse: $\langle\langle p\rangle\rangle = \varrho\langle\langle c_1\rangle\rangle$≪	●	●				
≫$\langle\langle(c'_2)^2\rangle\rangle^{1/2} \sim \langle\langle\Delta x_2^2(dc_1/dx_2)^2\rangle\rangle^{1/2} = \langle\langle\Delta x_2\rangle\rangle^{1/2}\vert\langle\langle dc_1/dx_2\rangle\rangle\vert$≪			●	●		
/leads to/	↓			↓		
/where/		↓				
≫$c'_1 = \langle\langle c_1(x_1)\rangle\rangle - \langle\langle c_1(x_2)\rangle\rangle$≪		●				
shear-tension: $\tau_T = -\varrho\langle\langle c'_1c'_2\rangle\rangle = -\varrho l^*\langle\langle\Delta x_2^2\rangle\rangle^{1/2}\vert\langle\langle dc_1/dy\rangle\rangle\vert\langle\langle dc_1/dy\rangle\rangle$≪	●			●		
				∨		
≫$\tau_T = -\varrho l_m^2\vert\langle\langle dc_1/dy\rangle\rangle\vert\langle\langle dc_1/dy\rangle\rangle$≪				●	●	
/where/					↓	
≫$l_m^2 = l^*\langle\langle\Delta x_2^2\rangle\rangle^{1/2}$≪					●	●
/specifies/						↓
≫measure for distance where in transported entity loses its individuality≪						●

Consequences from Mixing-Distance-Hypothesis

2. *Effects on Vortex–Model in Discussions* [2].

2.1. *About Eddies shaped as spherical Fluid–Elements.*

The existence of REYNOLDS–tensions within a turbulent fluid–volume give rise to a picture of sub–structures within the fluid–volume (e.g. shaped as spheres or balls as proposed by PRANDTL in the development of his mixing–distance–theory). The sub–structures are separated from each other by complicated surfaces with individual surface–tensions, directly or indirectly related to the REYNOLDS–tensions. The spheres are filled with certain amounts of turbulent translation– and rotation–energy and due to the dynamic of the turbulence permanent forces will act on their surfaces, which finally cause a cascade of splitting–steps.

2.2. *Measures relevant for Sizes of the Splitting–Cascade.*

Discussions [2] are relevant in a turbulence–range with dissipation independent from REYNOLDS–numbers; the REYNOLDS–equations enable these numbers to be estimate (as shown in chapter 1). Additionally typical size–measures:

- L: for energetic vortices and
- λ: for dissipating vortices

could be obtained from REYNOLDS–equations as well; these estimates are of relevance in discussions [2] because:

- The splitting–cascade starts with a vortex of size (L) and
- Difference between (L) and (λ) is decisive for the step–number of the splitting–cascade.

A final parameter (l_m) of turbulence could be estimated from PRANDTLs "mixing–distance–theory" and is decisive for a measure where a vortex loses its individuality under the actual turbulence–conditions:

- Measure for the distance where energetic vortices will split into follower–vortices and
- Measure where dissipating vortices are transformed into heat on account of the fluids viscosity (ν).

2.3. *Concluding Remarks with regards to Discussion* [2].

From the proceeding explanations in connection with the statements of chapter 1, it becomes obvious that the assumption of discussion [2] seems to be appropriate, to consider eddies in turbulent flow as spheres. The assumption seems appropriate because it harmonizes with turbulent–tensions and measures as outcomes from REYNOLDS–equations and PRANDTLs "mixing–distance–theory". Moreover is an existence of a splitting–cascade – from energetic to dissipating vortices with the final dissolution of the latter ones into heat – supported by PRANDTLs "mixing–distance–theory".

3. *References.*

[1] Fiedler, H.E. Turbulente Strömungen, Vorlesungsskript, TU Berlin (Hermann–Föttinger–Institut) & TU Braunschweig (Institut für Strömungsmechanik), 2003

[2] Steinemann, U. E. Turbulence as structured Route of Energy from Order into Chaos, 2018, http://vixra.org/vixra.1801.0037