Trigonometric functions – is there a problem?

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Abstract —Trigonometry is an important part of mathematics. In general, a proper understanding of trigonometric functions is a pre-requisite for understanding important topics in physics, architecture, and many branches of engineering. We commonly suppose that the mathematics of trigonometric functions is perfectly true. However, trigonometric functions have a very long and colourful history and the need to take a more precise look on the same is great. This publication provides some evidence that there are circumstances where today understanding of trigonometric functions leads to contradictions.

Keywords — *Trigonometric functions, Classical logic, Contradiction*

I. Introduction

The knowledge of the true origins of memorable discoveries [1] even if found by accident is of help to recognize the relativity even of mathematical knowledge. In this context, the early studies of triangles can be traced back to the 2nd millennium BC to Egyptian (*Rhind Mathematical Papyrus*) and Babylonian mathematics. However, a systematic study of trigonometric functions began with the Hellenistic mathematics during the second half of the 2nd century BC. The Hellenistic astronomer *Hipparchus of Nicaea* (ca. 180–ca. 125 BC) has been the first to compile a trigonometric table and is known as "*the father of trigonometry*" [2]. The Hellenistic mathematics reached India [2] where significant developments of trigonometry are ascribed especially *Aryabhata* (sixth century CE), who discovered *the sine function*. Finally, Aryabhata's table of sines reached China in 718 AD [3] during the Tang Dynasty. In the following, the studies of trigonometry continued in the Middle Ages by Islamic mathematicians [4] and led to the discovery of all six trigonometric functions. Latin translations of accumulated Arabic knowledge inspired trigonometry to be adopted in western Europe. In 1342, *Levi ben Gershon (1288-1344)* [5], known as Gersonides too, worked On Sines, Chords and Arcs [6]. Finally, the western *Age of Enlightenment* inspired and accelerated the development of modern trigonometry by Jost Bürgi (1552-1632) [7], Henry Briggs (1561-1630) [8], Isaac Newton (1643 - 1727) [9], Roger Cotes (1682–1716) [10], James Stirling (1692-1770) [11], Leonhard Euler (1707-1783) [12] and other too.

II. MATERIAL AND METHODS

A. Definitions

DEFINITION 1. (NUMBER +0)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let *i* denote an imaginary number [13]. The number +0 is defined as the expression

$$\begin{aligned} +0 &\equiv (c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \\ &\equiv +1 - 1 \\ &\equiv +i^2 - i^2 \end{aligned} \tag{1}$$

while "=" denotes the equals sign or equality sign [14, 15] used to indicate equality and "-" [14, 16, 17] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" denotes the plus [16] signs used to represent the operations of addition and the notions of positive as well.

DEFINITION 2. (NUMBER +1)

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number [13]. The number +0 is defined as the expression

$$+1 \equiv (c^2 \times \varepsilon_0 \times \mu_0) \equiv -i^2 \tag{2}$$

DEFINITION 3. (THE RIGHT-ANGLED TRIANGLE)

A right-angled triangle is a triangle in which one angle is 90-degree angle. Let ${}_RC_t$ denote the *hypotenuse*, the side opposite the right angle (side ${}_RC_t$ in the figure). The sides a_t and b_t are called legs. In a right-angled triangle ABC, the side AC, which is abbrivated as b_t, is the side which is adjacent to the angle α, while the side CB, denoted as a_t, is the side opposite to angle α. The following figure ([18], p. 117) may illustrate a right-angled triangle.

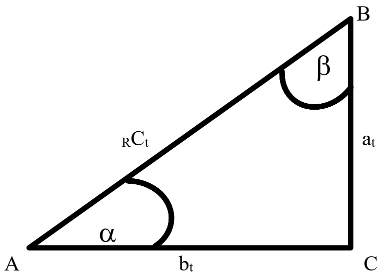

Figure 1. A right-angled triangle

DEFINITION 4. (PHYTHAGOREAN THEOREM)

The Pythagorean theorem is defined as

$$a_t^2 + b_t^2 \equiv {}_RC_t^2 \tag{3}$$

DEFINITION 5. (THE NORMALIZATION OF THE PYTHAGOREAN THEOREM)

The *normalization* [19, 20] of the Pythagorean theorem is defined as

$$\frac{a_t^2}{{}_RC_t^2} + \frac{b_t^2}{{}_RC_t^2} \equiv +1 \tag{4}$$

DEFINITION 6. (THE VARIANCE OF A RIGHT-ANGLED TRIANGLE)

The variance σ^2 of a right-angled triangle [19, 20] is defined as

$$\sigma_t^2 \equiv \frac{(a_t^2) \times (b_t^2)}{({}_RC_t^2) \times ({}_RC_t^2)} \tag{5}$$

DEFINITION 7. (SINE FUNCTION)

The sine function denoted as sin(*angle*) is a trigonometric function which relates an angle of a right-angled triangle and the ratios of two side lengths. The sine function is defined ([18], p. 117) in terms of a_t, b_t and ${}_RC_t$ as

$$\sin(\alpha) \equiv \frac{(a_t)}{({}_RC_t)} \tag{6}$$

and

$$\sin(\beta) \equiv \frac{(b_t)}{({}_RC_t)} \tag{7}$$

DEFINITION 8. (COSINE FUNCTION)

The sine, the cosine, and the tangent are the most familiar trigonometric functions. The cosine function is defined ([18], p. 117) in terms of a_t, b_t and ${}_RC_t$ as

$$\text{cosin}(\alpha) \equiv \frac{(b_t)}{({}_RC_t)} \tag{8}$$

and

$$\text{cosin}(\beta) \equiv \frac{(a_t)}{({}_RC_t)} \tag{9}$$

DEFINITION 9. (TANGENT FUNCTION)

The tangent function is defined ([18], p. 117) in terms of a_t and b_t as

$$tan(\alpha) \equiv \frac{(a_t)}{(b_t)} \tag{10}$$

and as

$$\text{cotan}(\alpha) \equiv \frac{(b_t)}{(a_t)} = \frac{1}{tan(\alpha)} \tag{11}$$

DEFINITION 10. (SIMPLE ALGEBRAIC VALUES)

The following table provides an overview [21] about some simplest algebraic values of trigonometric functions.

Table 1. Simple algebraic values

Degree Function:	0°	90°
sin	0	1
cosine	1	0
tan	0	∞

B. Axioms

1) Axiom I (Lex identitatis. Principium Identitatis. Identity Law)

In general, it is

$$+1 \equiv +1 \tag{12}$$

or the superposition of +0 and +1 as one of the foundations of quantum computing

$$+1 \equiv (1+0) \times (1+0) \times (1+0) \times (\ldots) \times (1+0) \tag{13}$$

2) Axiom II (Lex contradictionis. Principium contradictionis. Contradiction Law)

The (logical) contradiction is expressed mathematically as

$$+1 \equiv +0 \tag{14}$$

3) Axiom III (Principium negationis)

In general, it is

$$\frac{+1}{+0} \approx +\infty \tag{15}$$

III. RESULTS

THEOREM 3.1 (THE CONSEQUENCES OF COS(α=0) = 1)

CLAIM.

Under conditions where cos (α = 0) = 1 it is

$$a_t = 0 \tag{16}$$

PROOF.

In general, according to the rules of trigonometry, the cosine function is defined as

$$\text{cosine}(\alpha) \equiv \frac{(b_t)}{({}_RC_t)} \tag{17}$$

However, it is accepted as correct that **cos (α = 0) = 1** [21]. In this case it is

$$\text{cosine}(\alpha = 0) \equiv \frac{(b_t)}{({}_RC_t)} = +1 \tag{18}$$

In other words, it is

$$\frac{(b_t)}{({}_RC_t)} = +1 \tag{19}$$

or

$$(b_t) = ({}_RC_t) \tag{20}$$

or

$$(b_t^2) = ({}_RC_t^2) \tag{21}$$

Even under these circumstances, Pythagorean theorem as

$$a_t^2 + b_t^2 \equiv {}_RC_t^2 \tag{22}$$

is valid. Rearranging, we obtain

$$a_t^2 + {}_RC_t^2 \equiv {}_RC_t^2 \tag{23}$$

or

$$a_t^2 \equiv 0 \tag{24}$$

or

$$a_t \equiv 0 \tag{25}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3.2 (THE DEFINITION COS(α=0) = 1 LEADS TO CONTRADICTIONS)

CLAIM.

The definition **cos (α = 0) = 1** is logically inconsistent because the same reduces the value of b_t only to

$$b_t = +1 \tag{26}$$

PROOF.

In general, according to the rules of trigonometry it is

$$\frac{cos(\alpha)}{sin(\alpha)} \equiv \frac{\frac{b_t}{{}_RC_t}}{\frac{a_t}{{}_RC_t}} = \frac{b_t}{a_t} \tag{27}$$

This relationship is claimed to be valid even if α = 0. In this case, it is

$$\frac{cos(0)}{sin(0)} = \frac{+1}{+0} = \frac{\frac{b_t}{{}_RC_t}}{\frac{a_t}{{}_RC_t}} = \frac{b_t}{a_t} \tag{28}$$

In other words, it is

$$\frac{+1}{+0} = \frac{b_t}{a_t} \tag{29}$$

However, if **cos (α = 0) = 1** then $a_t = 0$ as proofed by the theorem before and we obtain

$$\frac{+1}{+0} = \frac{b_t}{+0} \tag{30}$$

Under these circumstances, the division by zero doesn't matter at all. In particular, whatever the division by 0 may be, cos (α = 0) = 1 demands that

$$b_t = +1 \tag{31}$$

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IV. DISCUSSION

Trigonometric functions [22] are widely used in science and our trust into the same appears to be limitless. In one way or another trust is important in science but can be dangerous too. What we risk while trusting without a clear proof of the correctness of something is among other that contradictions may take root in science to such an extent that one definition after the other is necessary to rescue what can and must be rescued.

In this context, it is important to note, that we cannot rely on trigonometric functions any longer to the extent which is necessary. Especially under conditions, where **cos(α = 0) = 1,** there is a contradiction. The side b_t of a right-angled triangle can take values different from 1, especially if **α = 0**. However, *cos(α = 0) = 1* demands under these conditions that b_t must be equal to +1, which is a non-acceptable contradiction.

V. CONCLUSION

It is necessary to review the general validity of the trigonometric functions in detail.

Financial support and sponsorship: Nil.

Conflict of interest statement:

The author declares that no conflict of interest exists according to the guidelines of the International Committee of Medical Journal Editors.

Acknowledgements

The open source, independent and non-profit Zotero Citation Manager has been used to create and manage references and bibliographies.

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