Sketches on Polysigns and other Arithmetics Operators

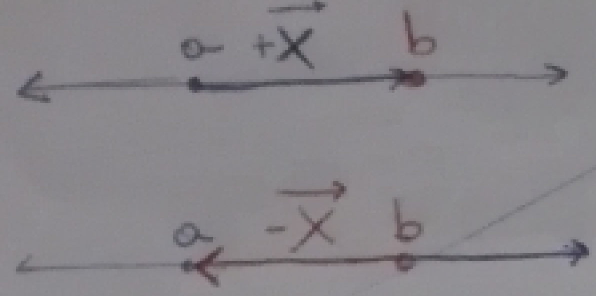
Kujonai, July 18, 2019

Abstract

A compilation of drawings sprung during a email exchanging with T. Golden, author of the Polysigns numbers. It must be mentioned that he accepts only some of the concepts presented here. Although we do share some common ground while talking about Polysigns and/or "simplexogonal" arithmetics, we do have some differences in the approach. In anycase, it is required some understanding of the Polysign Notation to fully appreciate the drawings. A few important bits arose as a direct consequence of the interaction and some are presented here, in a rather highly informal way.

Web of the Polysigns
http://www.bandtechnology.com/PolySigned/index.html

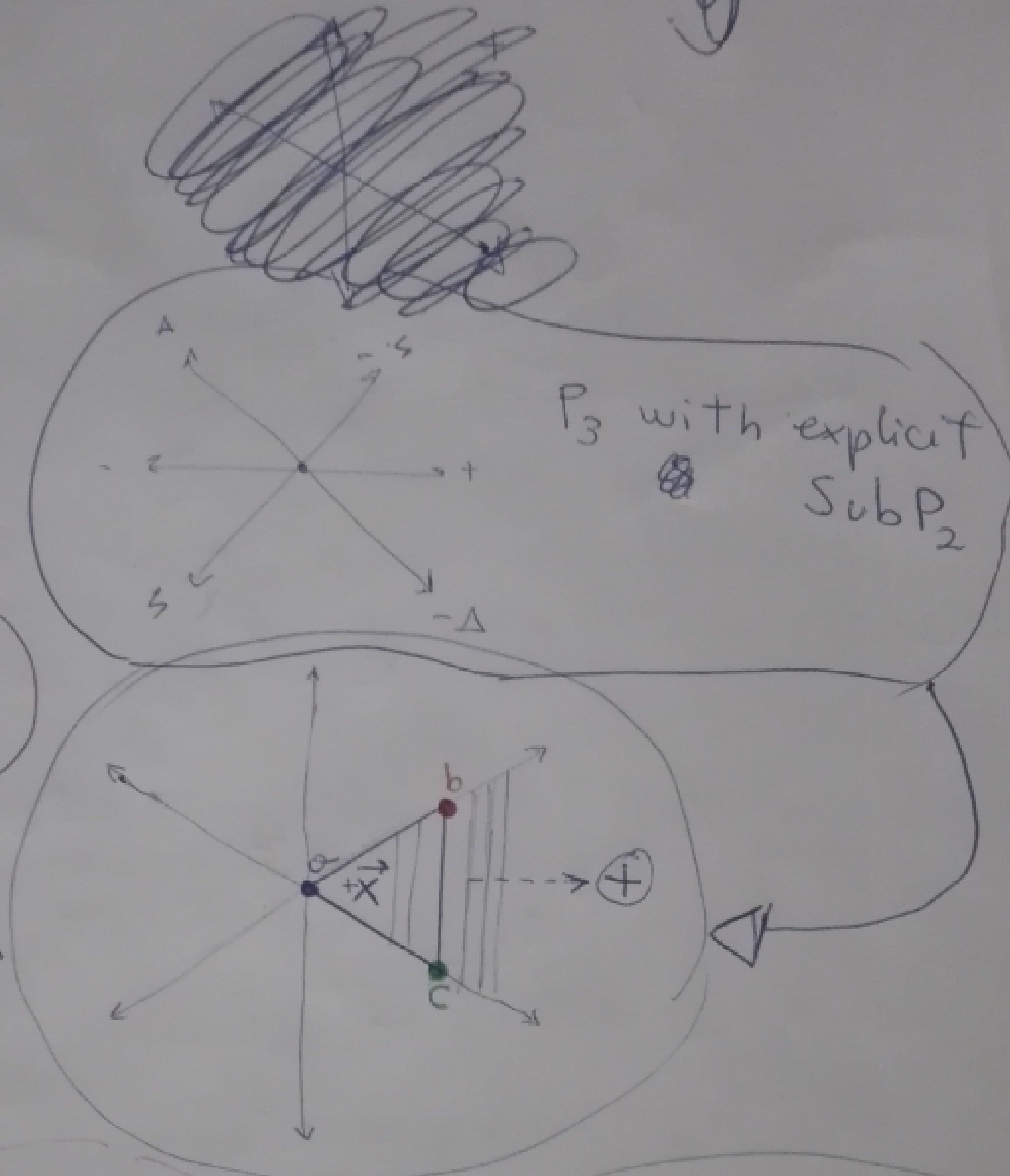
keywords : polysigns, simplex, sign, distance, equality, operator, numeral, triangle, simplexogonal, ray, opposite, inverse, binary, ternary, n-ary, cancellation, division, matrix, coordinates, successor, product, symmetry, unitary , duality, thirdness, hypergraphs, magnitude, arithmetics



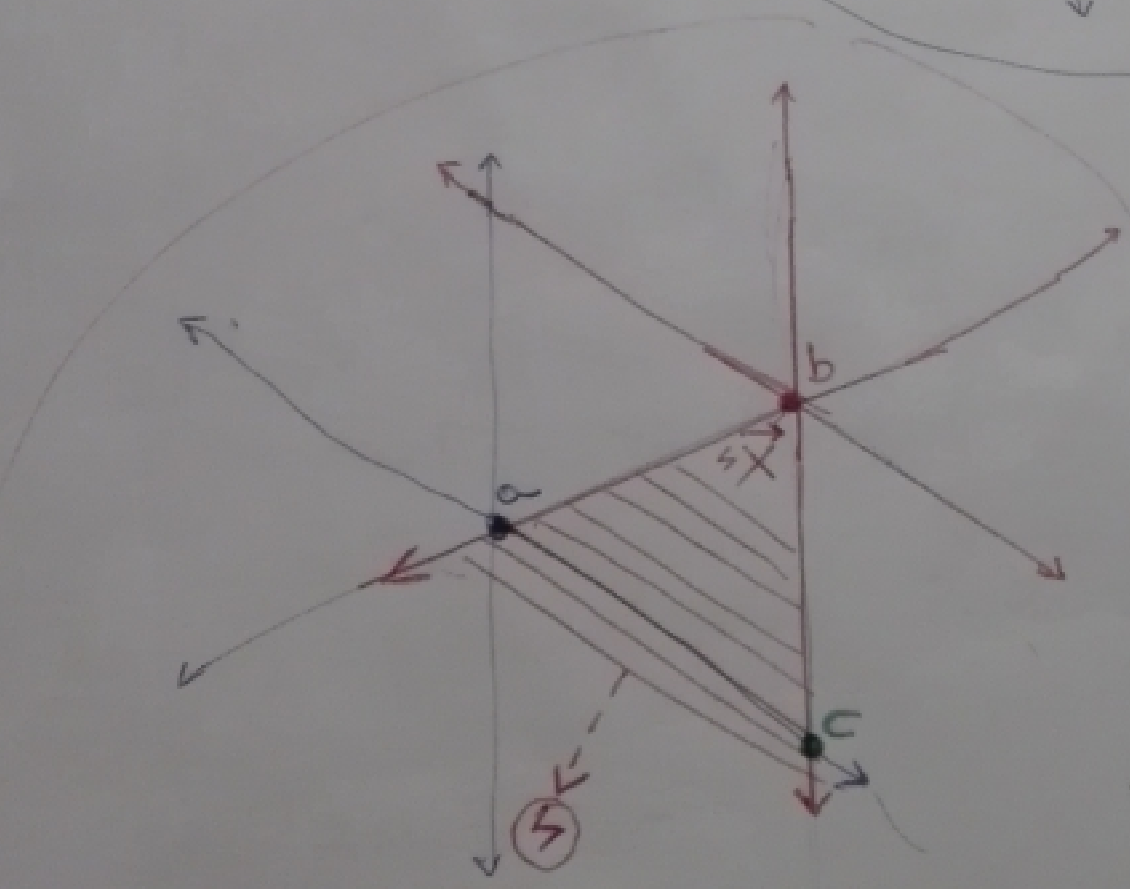
$P_{a(b)} = -P_{b(a)}$

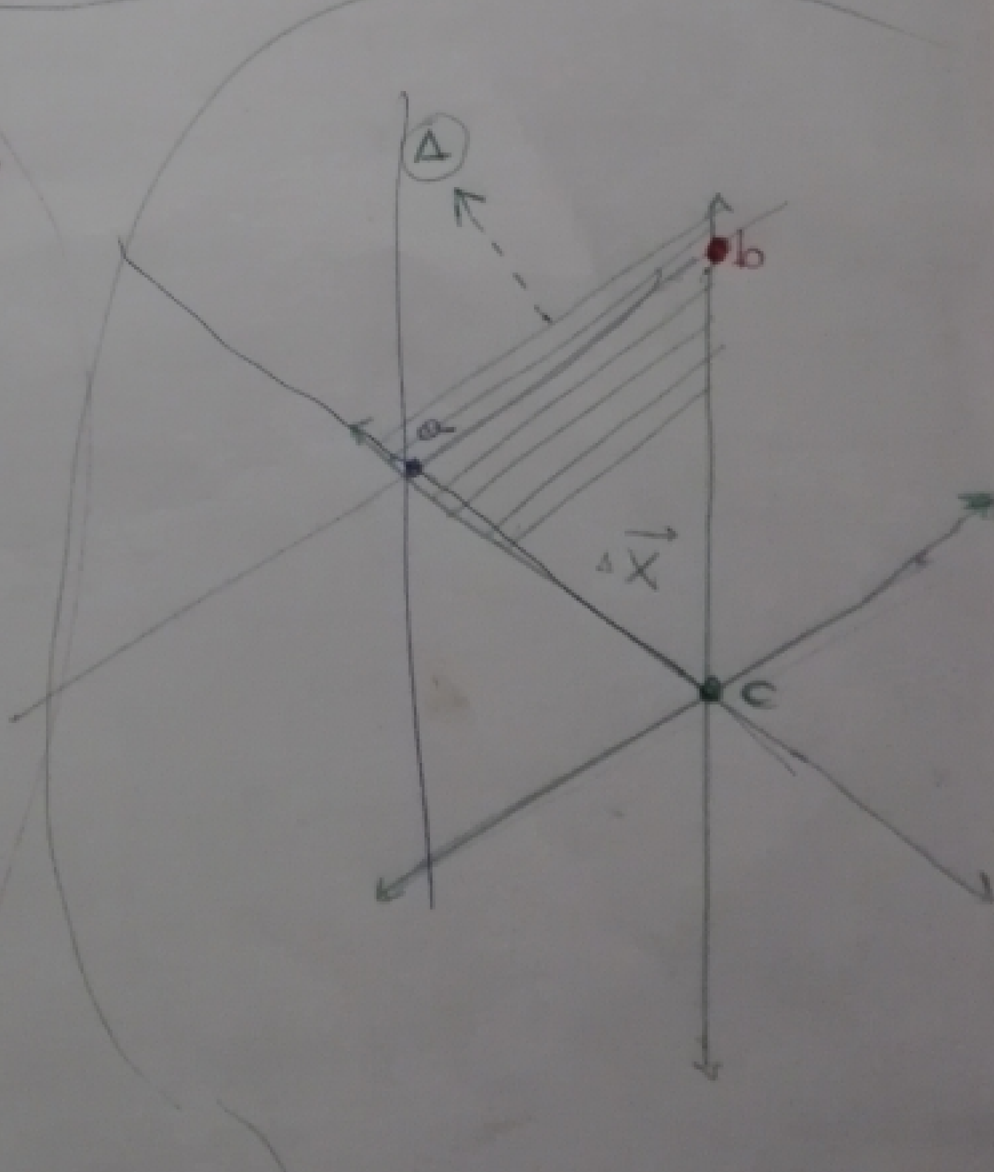
$a + x = b$

directed line → directed Area

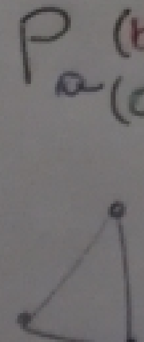


change of position and change of sign.





$P_{a(c)}^{(b)}$

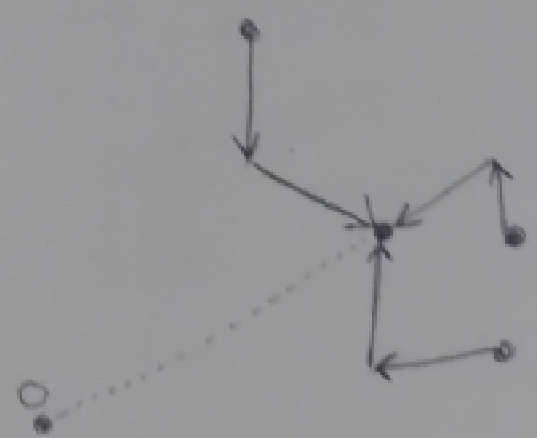
P  $P_{a\langle c\rangle}^{(b)}$

- It can be observed

$\bar{A}$: Aditive inverse of A

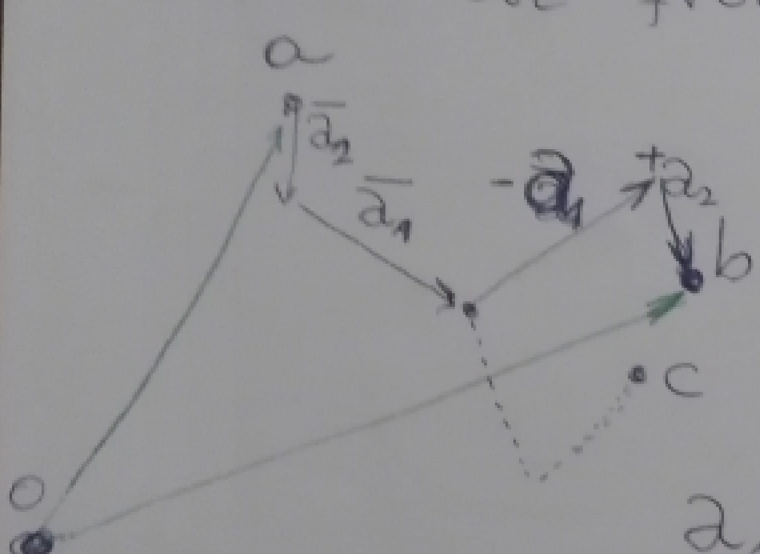
$$X_I = a_0 @ \left(\begin{smallmatrix}@\\+\\-\end{smallmatrix}\right) a_1 @ \left(\begin{smallmatrix}@\\-\\+\end{smallmatrix}\right) a_2$$

$$X_I @ \overline{\left(\begin{smallmatrix}@\\+\\-\end{smallmatrix}\right) a_1} @ \overline{\left(\begin{smallmatrix}@\\-\\+\end{smallmatrix}\right) a_2} = a_0$$



From The points, we Add The aditive inverse of the sections of The Arms, obtaining The centroid in The 3 cases, from X_0, X_1, X_2.

Also can be observed. That one can move from one point, to oTher point.



$$a @ \bar{a}_2 @ \bar{a}_1 @ +a_1 @ +a_2 = b$$

From one point, we add The aditive inverses of a_1 and a_2 To reach The centroid and Then add a_1 and a_2, with the proper signs to reach point B.

As an application, it is possible To obtain. the positions of n points in function of The Arms.

$$X_I = f(x) @ \left(\begin{smallmatrix}@\\+\\-\end{smallmatrix}\right) g(x) @ \left(\begin{smallmatrix}@\\-\\+\end{smallmatrix}\right) h(x)$$

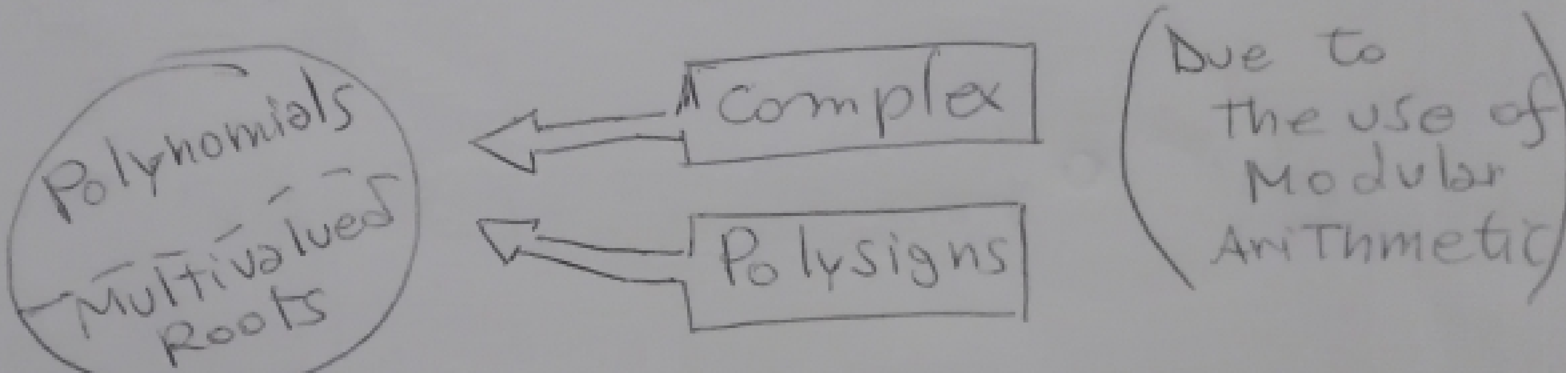
where f(x), g(x) and h(x) belong to P_3 or Positive Reals

- $\left(\frac{@}{-}\right)A$, $\left(\frac{@}{+}\right)A$ mean multivalue, does not cancelate with itself- in P_n.

- Unless one is using The "power" operator to plot curves, it is possible to "avoid" The use of Polynomials or "avoid" to codify information into Polynomial shape and use directly multivalued ariThmetic.

- While using Polynomials, THe power operator "Hide" The multivaLues but, at THe same Time affect The magnitude

$$\left(\left(\frac{@}{+}\right)A\right)^3 = A^3 = \left(\begin{smallmatrix}@\\@\\@\end{smallmatrix}\right)A^3 \text{ VS } \frac{@}{+}A$$

- 

- For The case $\omega_3^i 1$, as a "abstract root", is equal geometrically speaking in Polysigns and Complex (The only case)

- $T^{-1}\left(\frac{\Lambda}{n}T(\text{points})\right) = \text{points}$

  $\frac{\Lambda}{n}T\left(T^{-1}(\text{Arms})\right) = \text{Arms}$

- THe atraction Towards This "multi balance" shape lies in The fact That is well behaved with THe power operator, while mixing with THe "remarkables identities" acting like "remarkables identities" and ~~[illegible]~~ leveraging over magic cancelations like $(a@b)(a@-b)(a@+b) = a^3@b^3$ or similars.

- THE operator T can accept any P_n values (as long as n = prime), NOT JUST POSITIVE REALS

- especial cases:
  - colinear points.
  - All points are The same point
  - points in a "equilateral triangle" arrangement

  but still works in those cases

- Order of THe Inputs "matters"
- HELP TO BUILD DISTANCE OPERATOR FOR MULTISETS

$a_i = \frac{1}{n} T_i (x_0, x_1, \ldots, x_{n-1})$

(1) Example for P_3

$A = \{a_0, a_1, a_2\}$

(2) $\text{distance}(x_0, x_1, x_2) = 3|a_1| + 3|a_2|$ or $(3|a_1|, 3|a_2|)$

Example for P_2

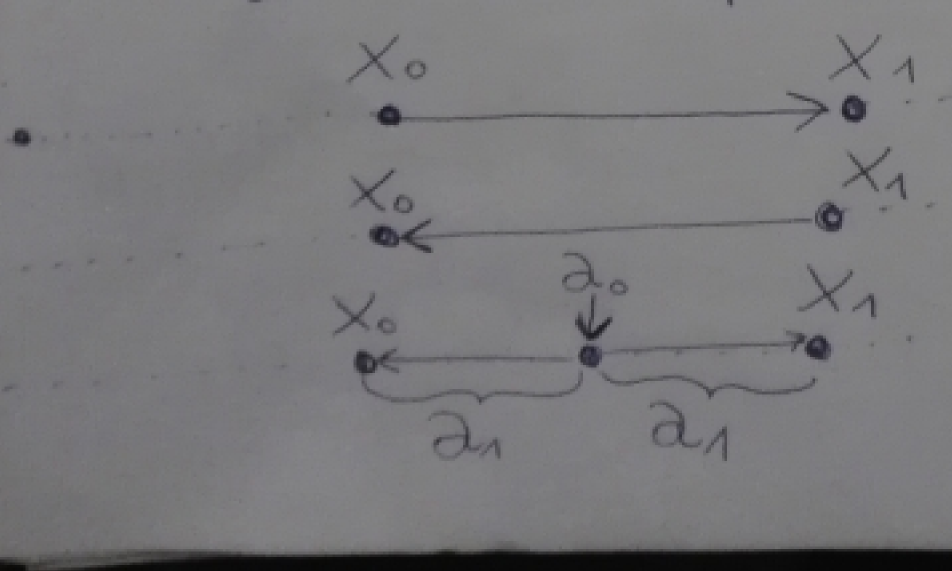
$A = \{a_0, a_1\}$

$\text{distance}(x_0, x_1) = 2|a_1|$

if you observe

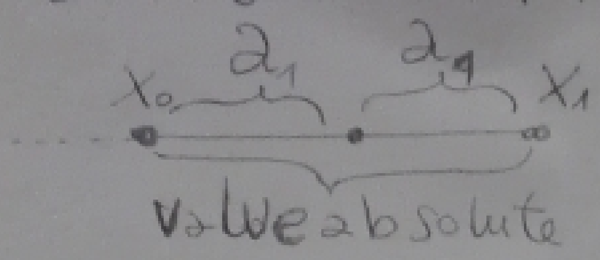
$|x_0 - x_1| = |x_1 - x_0| = 2|a_1|$

for P_2, The classical distance coincide wiTH THis operator of distance after applied THe absoLute value

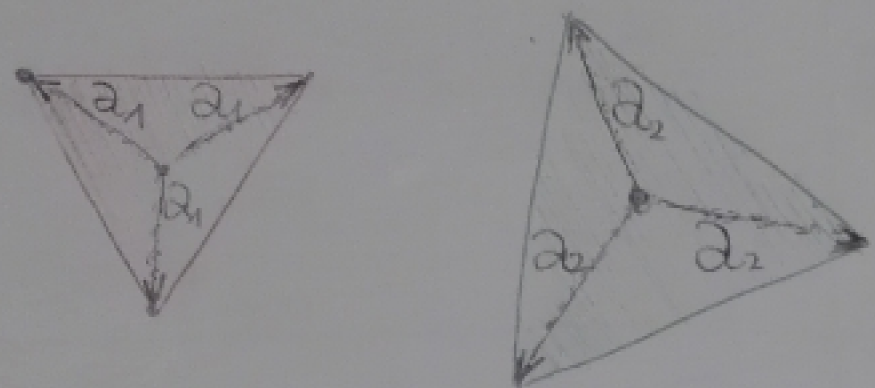


- Help To build alternative distance operator.

for P_2



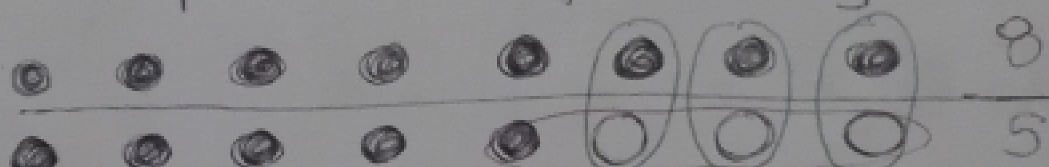
for P_3 would be:



The areas asociated with The equilateral. Triangles with circunradius $|\partial_1|$ and $|\partial_2|$ respecTively.

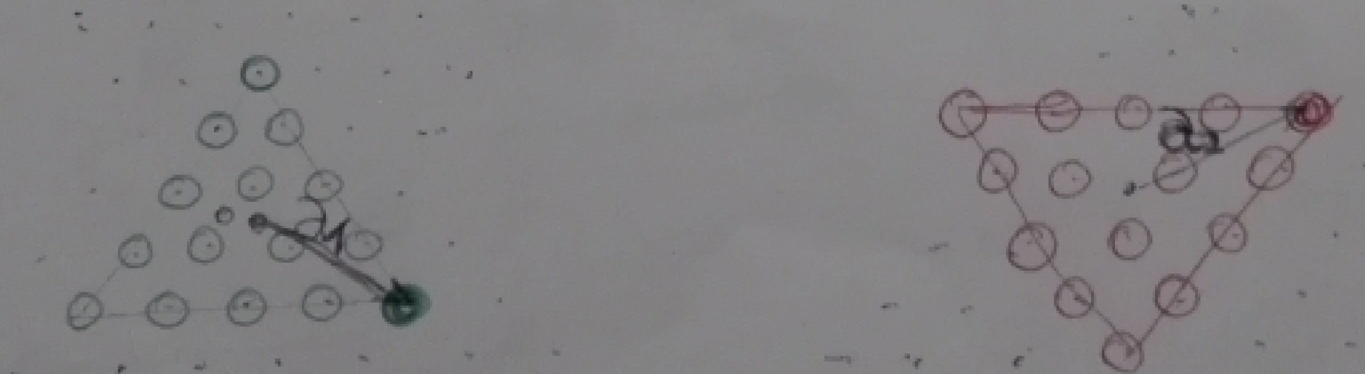
- AnoTher distance operator for P_3 Integers

for P_2 Integers



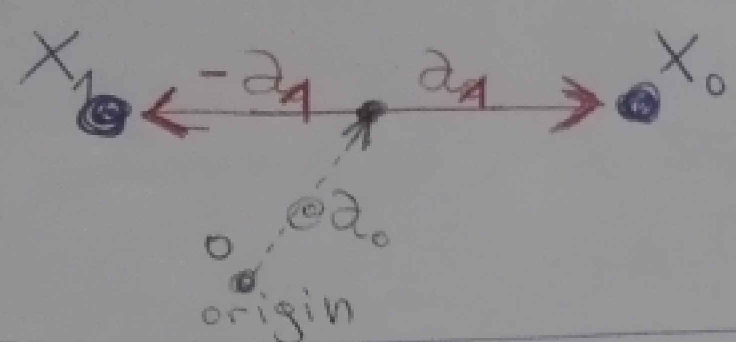
distance(8,5) = |(8)@-(5)| = | -(5)@(8)| = 3

for P_3 Integers.

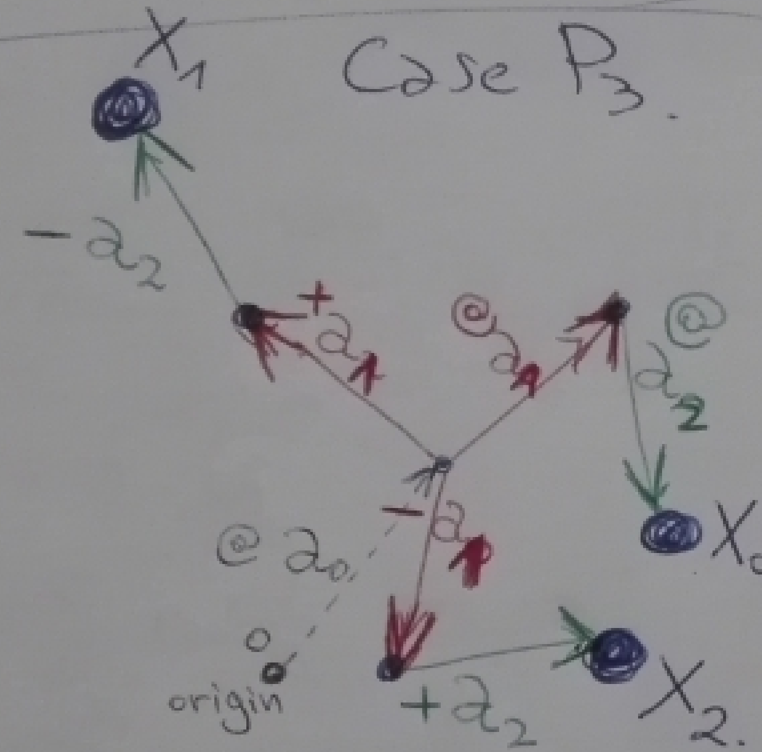


One count The numbers of integer inside The area generated for The vertices @∂_1, $-\partial_1$, $+\partial_1$ in The case of the firsT Δ and @∂_2, $-\partial_2$, $+\partial_2$ in The case of The second Δ

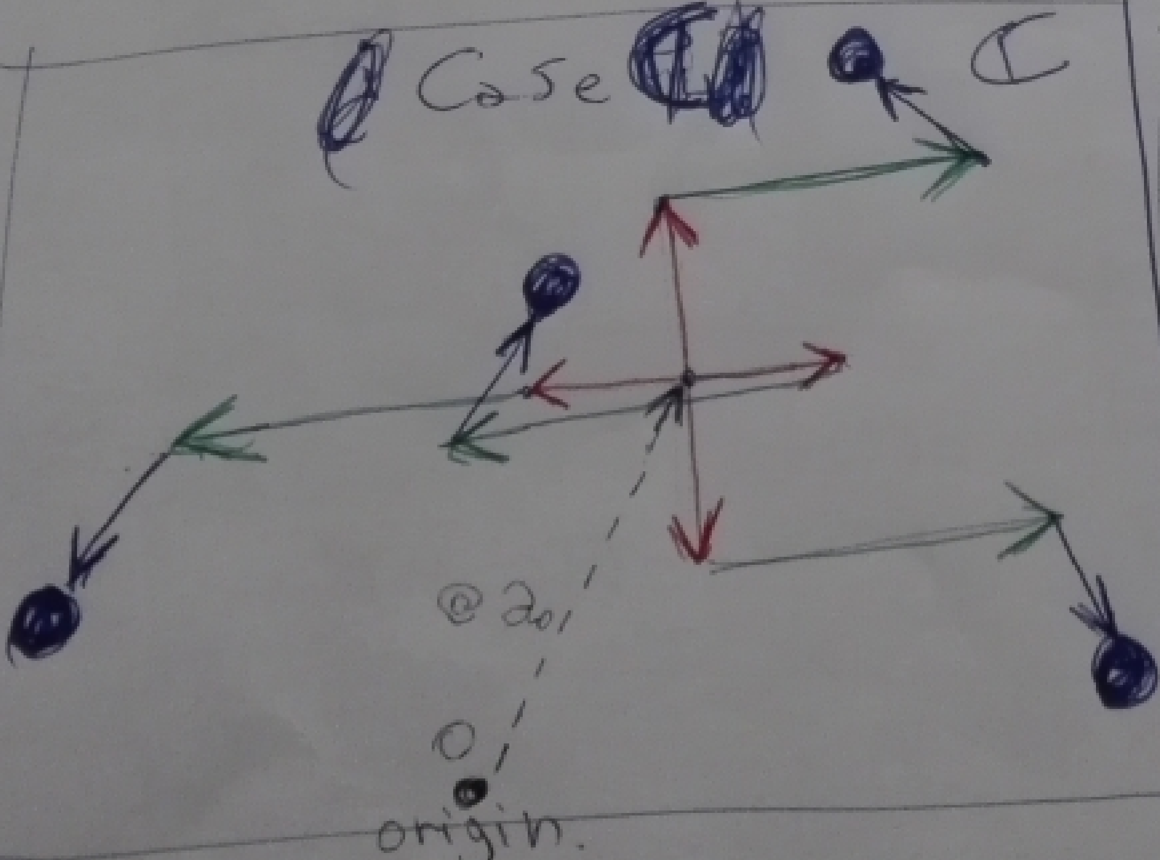
Case P_2



Case P_3.



Case $\mathbb{C}$



CASE P_3

ARMS

$$a_0 = \frac{1}{3}(X_0 @ X_1 @ X_2)$$

$$a_1 = \frac{1}{3}(X_0 @ -X_1 @ +X_2)$$

$$a_2 = \frac{1}{3}(X_0 @ +X_1 @ -X_2)$$

Points.

$$X_0 = a_0 @ a_1 @ a_2$$

$$X_1 = a_0 @ +a_1 @ -a_2$$

$$X_2 = a_0 @ -a_1 @ +a_2$$

WITH

$X_0, X_1, X_2, a_0, a_1, a_2 \in P_3$

$\frac{1}{3}(A+B+C) \equiv$ centroide.

case P_n.

$x_n^i \longleftrightarrow x_i$: i-TH point of THE SET.
$i \in \{0, 1, \cdots, n-1\}$.

$a_n^i \longleftrightarrow a_i$: i-TH ARM of THE MULTIBALANCE.
$i \in \{0, 1, \cdots, n-1\}$

$\omega_n^i \longleftrightarrow \omega_i$: i-TH sign of Polisigns case n.
$i \in \{0, 1, \cdots, n-1\}$

$a_i = f(x_i)$ Arms = f(points)

$$a_i = \frac{1}{n} \sum \omega_{j \cdot i} \cdot x_j = \frac{1}{n} T_i(x_j)_j = \frac{1}{n} T_i(x_0, x_1, \cdots, x_{n-1})$$

$x_i = f^{-1}(a_i)$ Points = f^{-1}(Arms)

$$x_i = \sum \omega_{j \cdot (n-i)} \cdot a_j = T_i^{-1}(a_j)_j = T_i^{-1}(a_0, a_1, \cdots, a_{n-1})$$

a_0 = "POSITION OF THE MULTIBALANCE"

$$\omega_n^i \equiv \omega_n^{kn+i} \longleftrightarrow i \equiv kn+i \pmod{n}$$

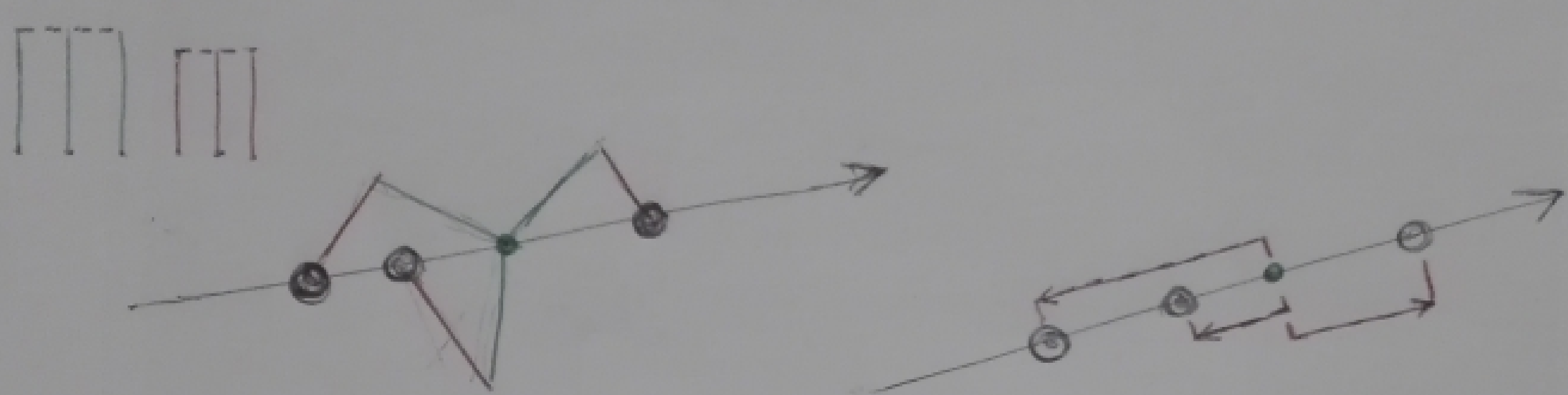
$k \in$ positive integer.

It works fine for P_n with n being a prime number.

- In statistic, It's never used:

$$\sum (x_i - \text{Average}) = 0$$

because outputs zero, The geometrical meaning behind that (case p=3)



- ¿ Does it matter where polinomials are being used? Yes & No

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- What is The geometrical meaning of Vieta formulas in Polysigns?

Arms = f(Points) ⟷ Points = f^{-1}(Arms)

Case P_3

$a_0 = \frac{1}{3}(x_0 @ x_1 @ x_2)$ | $a_1 = \frac{1}{3}(x_0 @ -x_1 @ +x_2)$ | $a_2 = \frac{1}{3}(x_0 @ +x_1 @ -x_2)$

$$a_0 @ a_1 @ a_2 = \frac{1}{3}(x_0 @ x_1 @ x_2 @ x_0 @ -x_1 @ +x_2 @ x_0 @ +x_1 @ -x_2)$$

$$= \frac{1}{3}(3x_0) = x_0$$

$$a_0 @ +a_1 @ -a_2 = \frac{1}{3}(x_0 @ x_1 @ x_2 @ +x_0 @ x_1 @ -x_2 @ -x_0 @ x_1 @ +x_2)$$

$$= \frac{1}{3}(3x_1) = x_1$$

$$a_0 @ -a_1 @ +a_2 = \frac{1}{3}(x_0 @ x_1 @ x_2 @ -x_0 @ +x_1 @ x_2 @ +x_0 @ -x_1 @ x_2)$$

$$= \frac{1}{3}(3x_2) = x_2$$

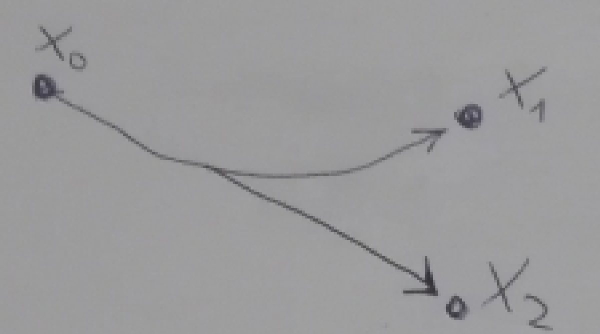
- Help to build Diference operator for multisets

$$X_I = T_I^{-1}(a_0, a_1, a_2) = W_3^{-0\cdot I} a_0 + W_3^{-1\cdot I} a_1 + W_3^{-2\cdot I} a_2$$

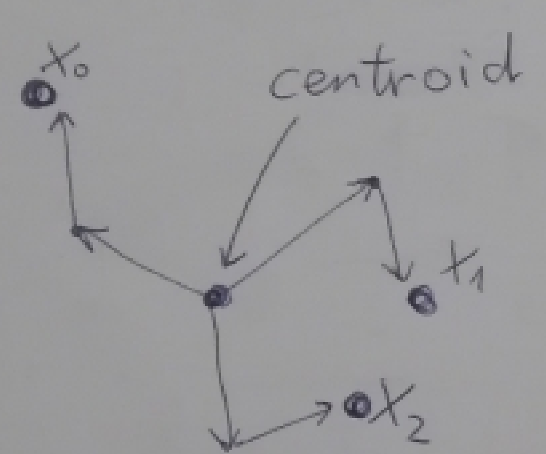
$$X_I = \begin{pmatrix} @ \\ @ \\ @ \end{pmatrix} a_0 @ \begin{pmatrix} @ \\ + \\ - \end{pmatrix} a_1 @ \begin{pmatrix} @ \\ - \\ + \end{pmatrix} a_2 = a_0 @ \overset{@}{-}{+} a_1 @ \overset{@}{+}{-} a_2$$

$$\boxed{\omega_n^{-i} = \omega_n^{n-i}}$$

where instead of this in P_3



one define the diference "from" the centroid



The descomposition into 2 variables means that a ~~"[illegible]"~~ "Equilateral triangular arrangement" can not contain all the points in the plane, and instead, one has to de-compound a irregular 3-point arrangement into regular 3-point arrangements

For P_3, the diference:

(1) $\text{diference}(X_0, X_1, X_2) = \begin{pmatrix} @ \\ + \\ - \end{pmatrix} a_1 @ \begin{pmatrix} @ \\ - \\ + \end{pmatrix} a_2$.

~~[illegible]~~

$$a_i = \frac{1}{n} \Pi_i^j (x_j)_j = \frac{1}{n} \sum_{i=0}^{n-1} (w_i)^j x_j = \frac{1}{n} \sum_{i=0}^{n-1} w_{ij} x_j$$

"Reflected or Twisted sums

"Centroid

Abstract Roots / Abstract Polinomials.

$w_n^{I} 1$ $\qquad$ $x^n + \bar{1} = 0$

$w_n^1 1$ → (-1) Polysign.
→ $e^{2i\pi/n}$ complex

$w_n^0 1$ → $@1$ Polysign.
→ $+1$ complex

MultivaLued Arithmetic

$$a_0 = \frac{1}{3}(w_3^0 x_0 + w_3^0 x_1 + w_3^0 x_2) = \frac{1}{3}(x_0 @ x_1 + @ x_2)$$

a_0 : Section zero

$$a_1 = \frac{1}{3}(w_3^0 x_0 + w_3^1 x_1 + w_3^2 x_2) = \frac{1}{3}(x_0 @ - x_1 @ + x_2)$$

a_1 : Section one.

$$a_2 = \frac{1}{3}(w_3^0 x_0 + w_3^2 x_1 + w_3^1 x_2) = \frac{1}{3}(x_0 @ + x_1 @ - x_2)$$

a_2 : Section two

$$a_i = \frac{1}{3}(w_3^{0 \cdot j} x_0 + w_3^{1 \cdot j} x_2 + w_3^{2 \cdot j} x_2)$$

a_i : Section "ANY"

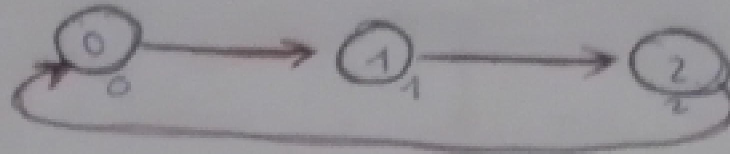
$$A_I = \frac{1}{3}(w_3^{0 I} x_0 + w_3^{1 I} x_1 + w_3^{2 I} x_2) = \frac{1}{3}(^{@}_{@ @}1 \; @ ^{@}_{- +}1 \; @ ^{@}_{+ -}1)$$

A_i : Section "ALL" (multivalued)

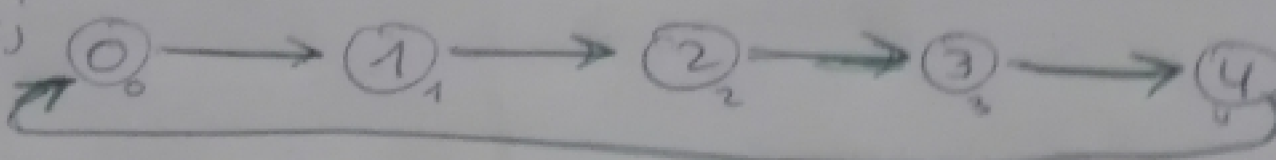
similar to say in P_2 $\pm A$

$(w_2^{I} A)^2 = A^2$	$(\pm A)^2 = A^2$	$(^{@}_{+}- A)^3 = A^3$,	$(w_3^{I} A)^3 = A^3$	$(^{@}_{+}- A)^2 = ^{@}_{-+} A^2$

$A = [a_i]_3 \quad a_i := i$

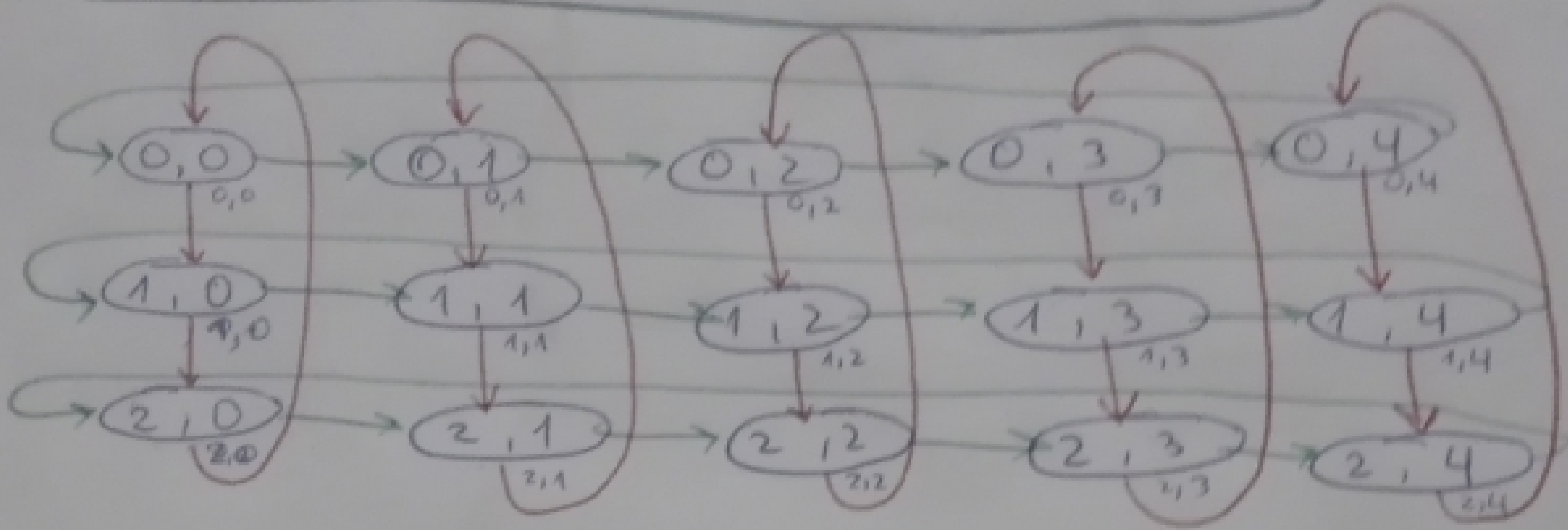


$B = [B_j]_5 \quad b_j := j$



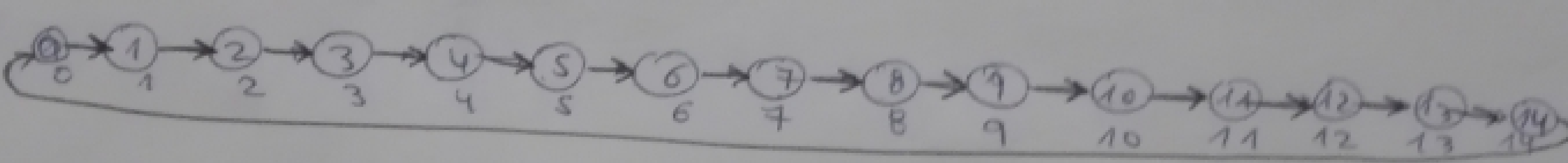
$\Psi = [\psi_{i,j}]_{3,5}$

$\psi_{i,j} = (i, j)$



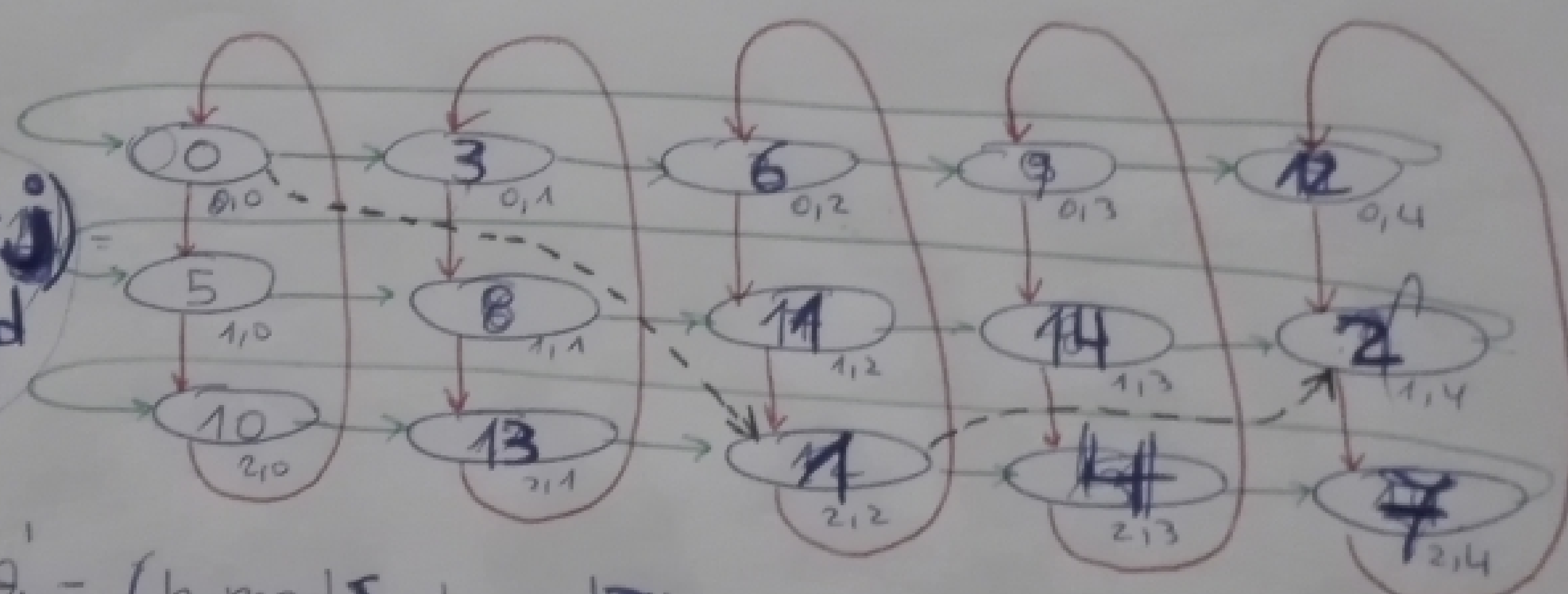
$\Theta = [\theta_h]_{15}$

$\theta_h = h$

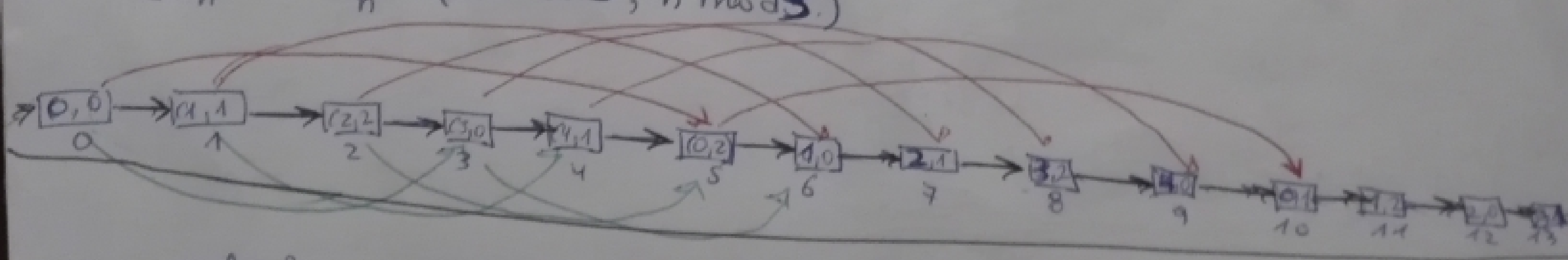


$\Psi' = [\psi'_{i,j}]_{3,5}$

$\psi'_{i,j} = (5i + 3j) \bmod 15$

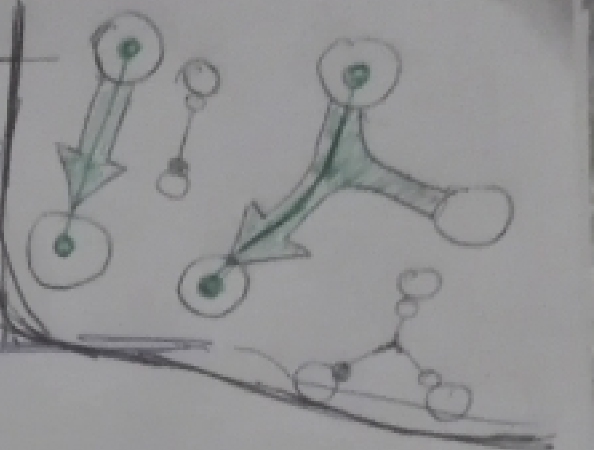


$\Theta' = [\theta'_h] \quad \theta'_h = (h \bmod 5, h \bmod 3)$



- usar el formato de uno con el contenido de otro.
- mapeo de uno hacia el otro.
- otorgarle ~~el salto de~~ la propiedad de salto uno a otro.
- codificar la información de 2 isoanillos primos en 1 isoanillos compuesto
- codificar la información de 1 isoanillo compuesto, en 2 isoanillos primos
- propiedad de Mutualidad o anillos isomutuales.
- relación entre $\theta'_h = (h \bmod 5, h \bmod 3)$ y $\theta''_h = (h \bmod 3, h \bmod 5)$

$S_\varnothing$ = Neutral sign with loopy path

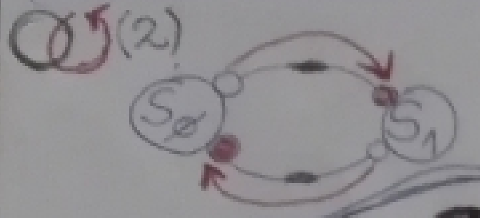


FAMILY I

N-CYCLE WITH SIZE: N (N-DIRECTED)

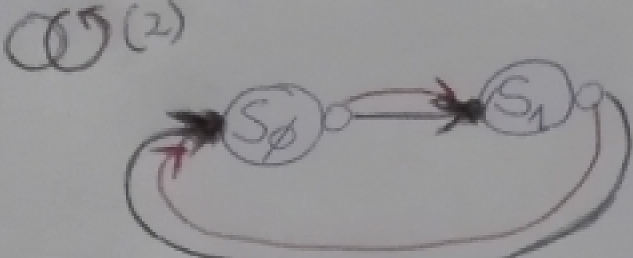
FAMILY II

2-CYCLE SIZE: N

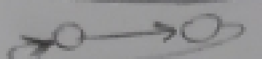


HAMILTONIAN AND CONMUTATIVE

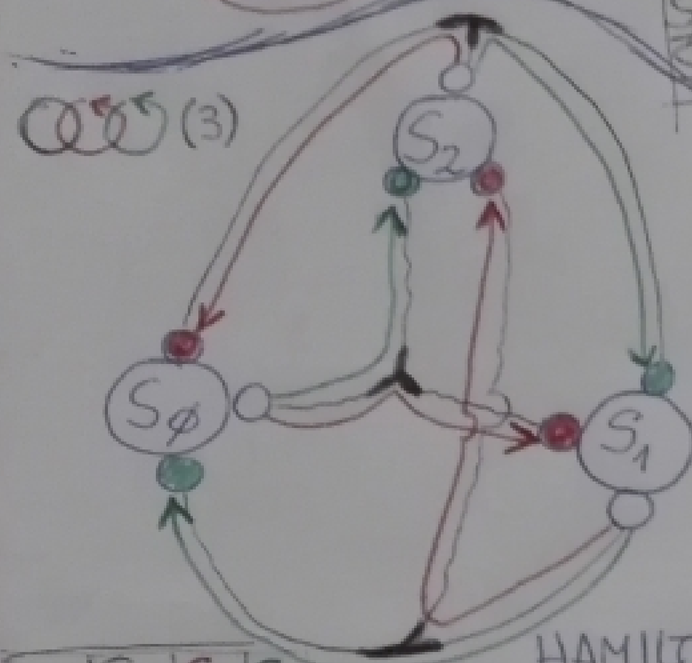
	$S_\varnothing$	S_1
$S_\varnothing$	$S_\varnothing$	S_1
S_1	S_1	$S_\varnothing$



HAMILTONIAN AND CONMUTATIVE

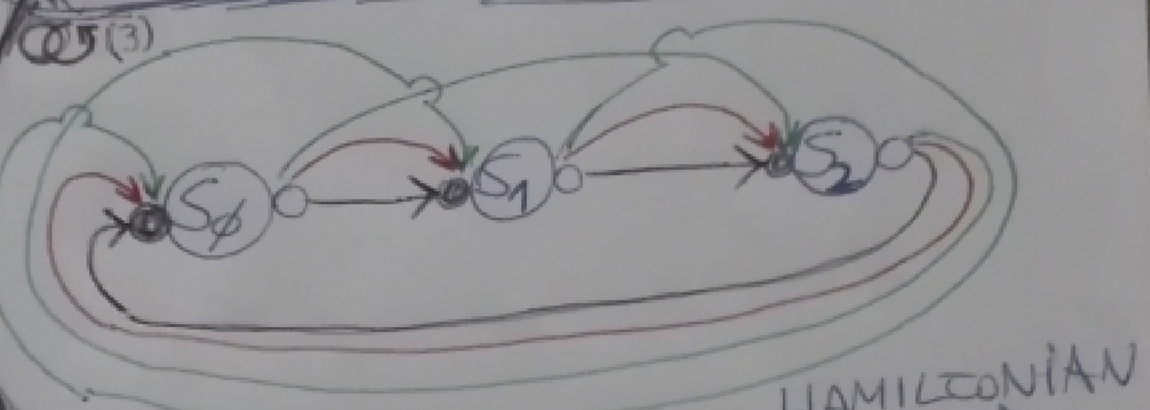


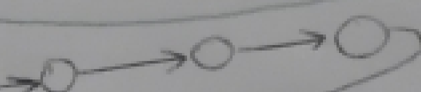
MOD2



HAMILTONIAN AND CONMUTATIVE

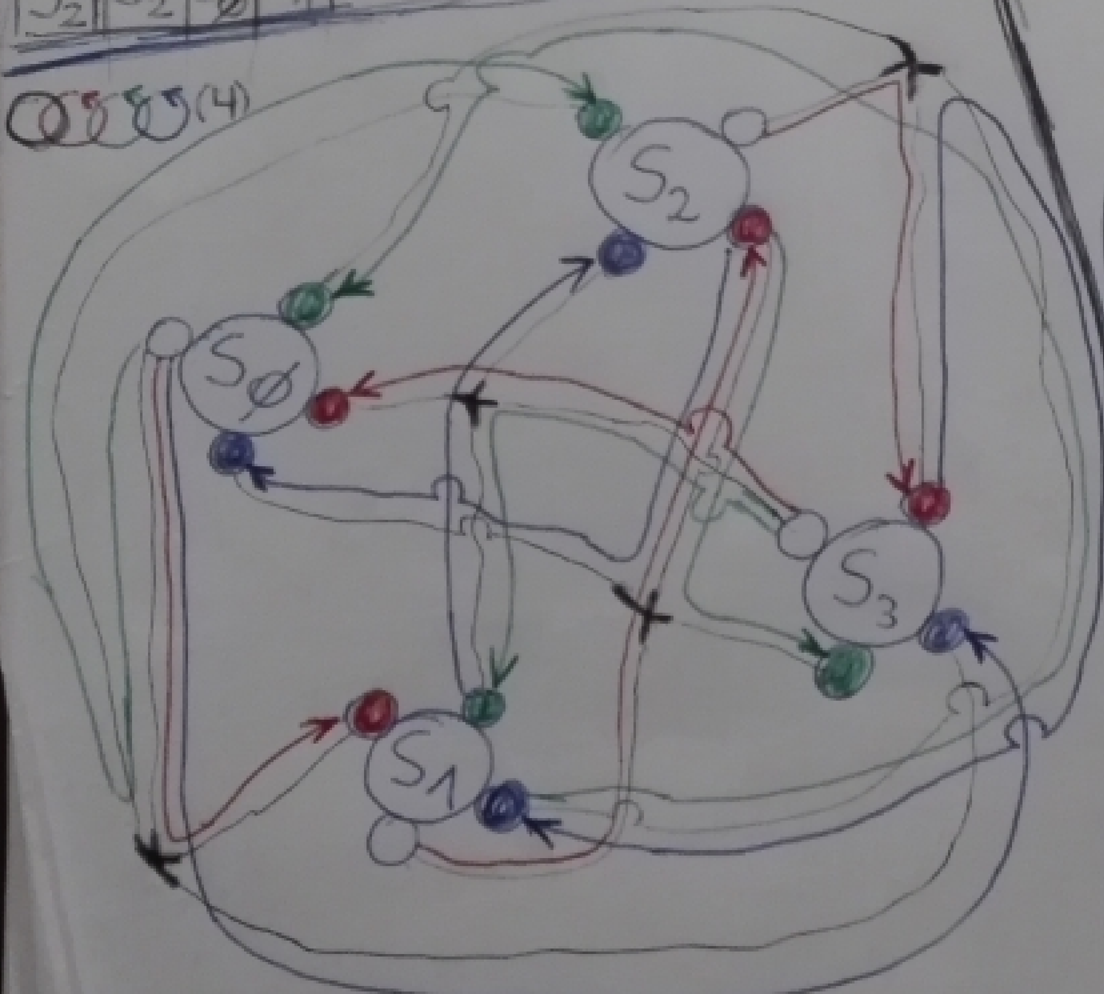
	$S_\varnothing$	S_1	S_2
$S_\varnothing$	$S_\varnothing$	S_1	S_2
S_1	S_1	S_2	$S_\varnothing$
S_2	S_2	$S_\varnothing$	S_1





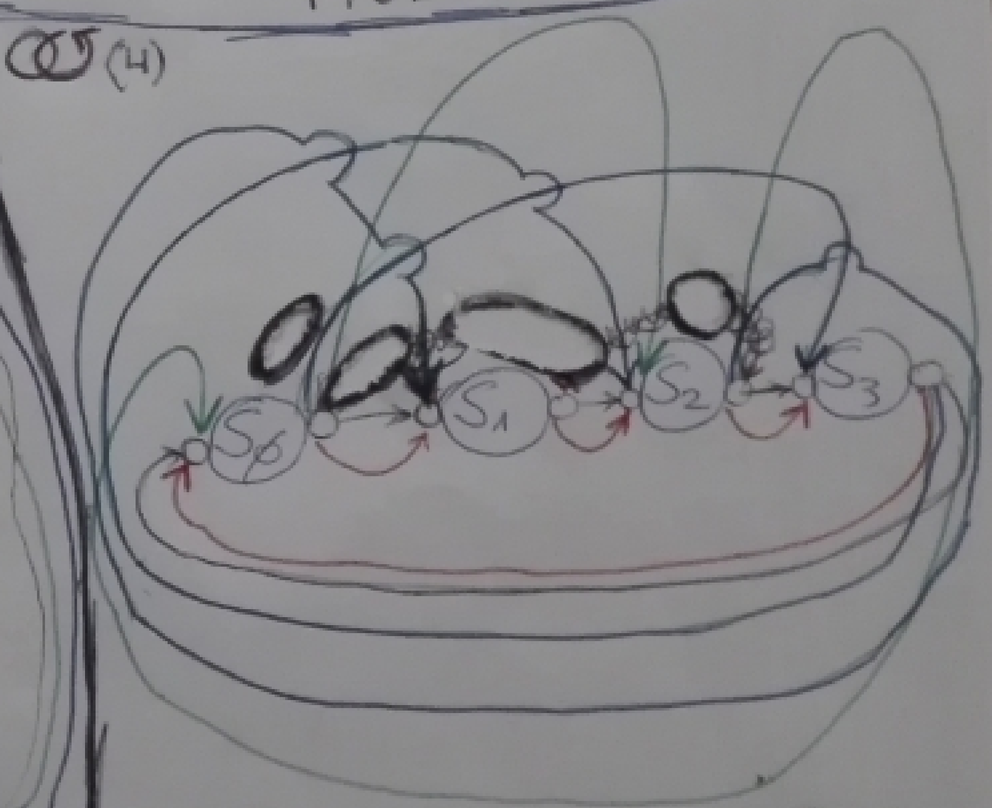
HAMILTONIAN AND CONMUTATIVE

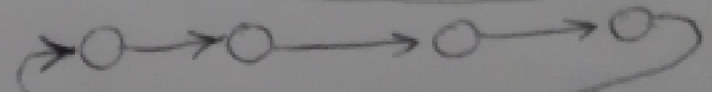
MOD3



$S_\varnothing$	S_1	S_2	S_3
$S_\varnothing$	S_1	S_2	S_3
S_1	S_2	$S_\varnothing$	S_2
S_2	S_3	S_3	$S_\varnothing$
S_3	$S_\varnothing$	S_1	S_1

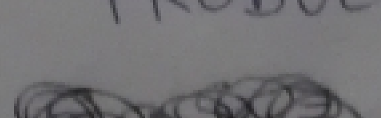
HAMILTONIAN BUT NON-CONMUTATIVE PRODUCT RULE





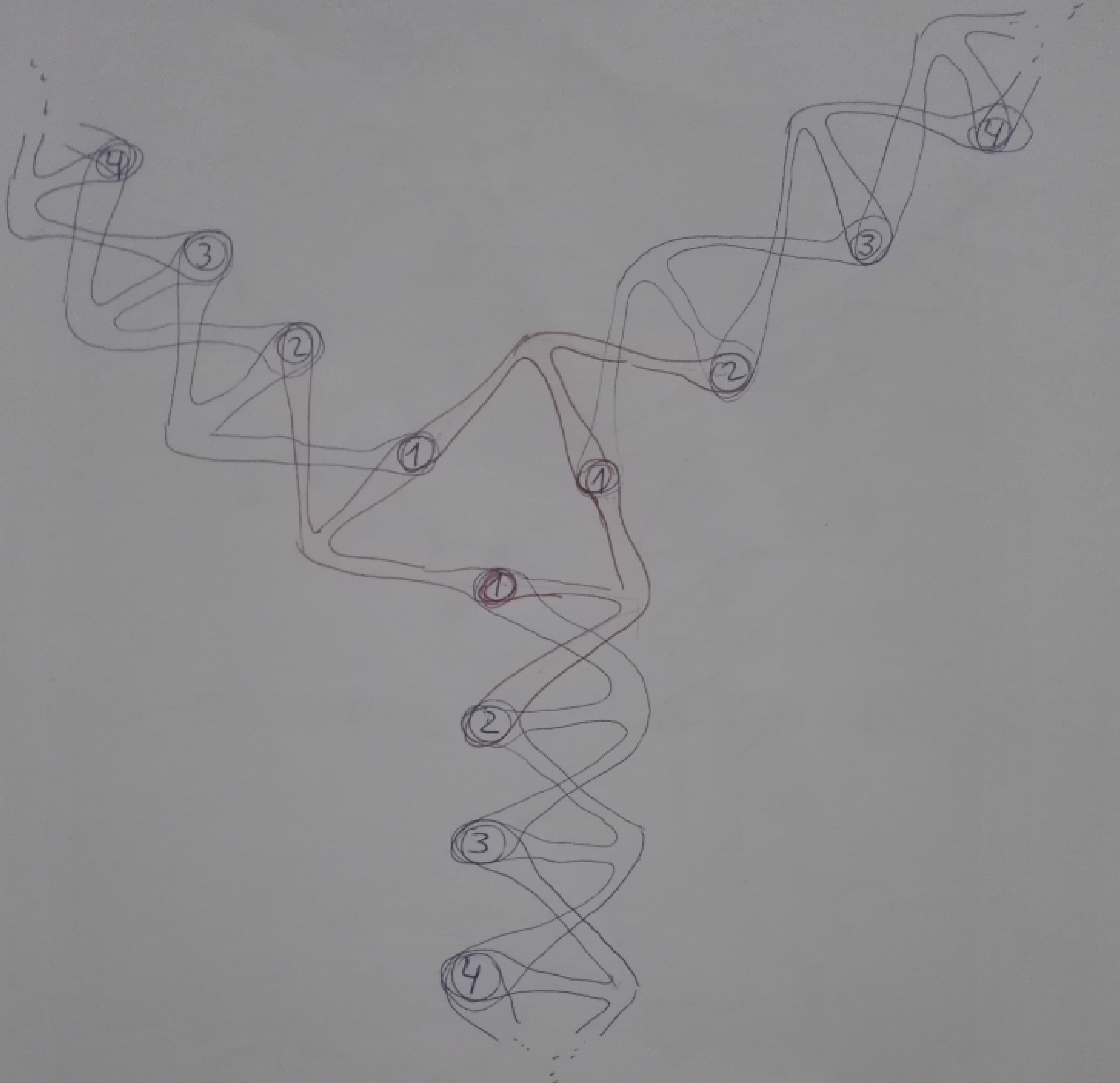
MOD4

CONMUTATIVE BUT NON-HAMILTONIAN PRODUCT RULE

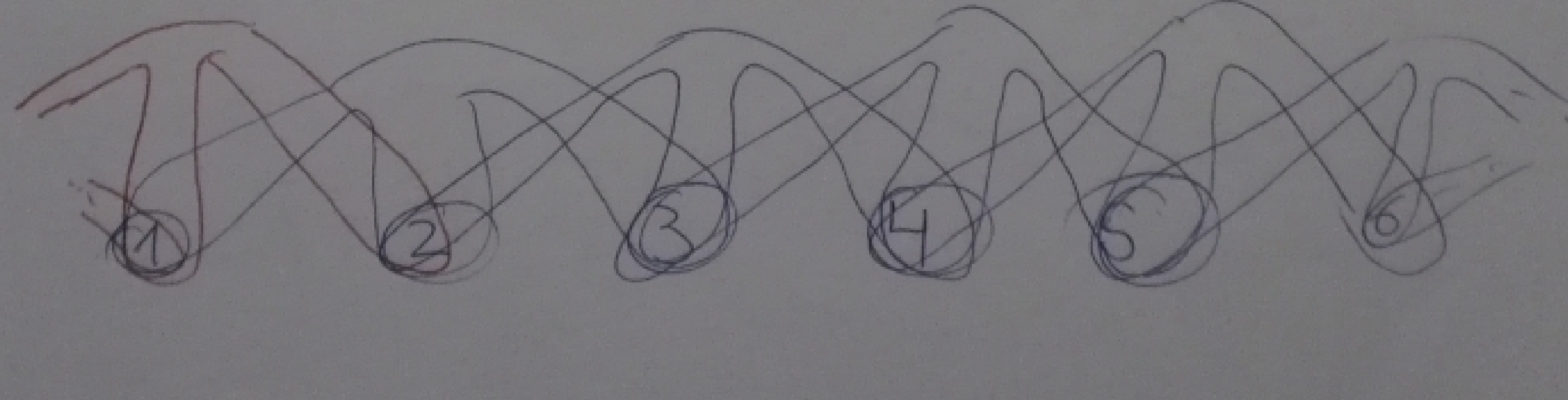


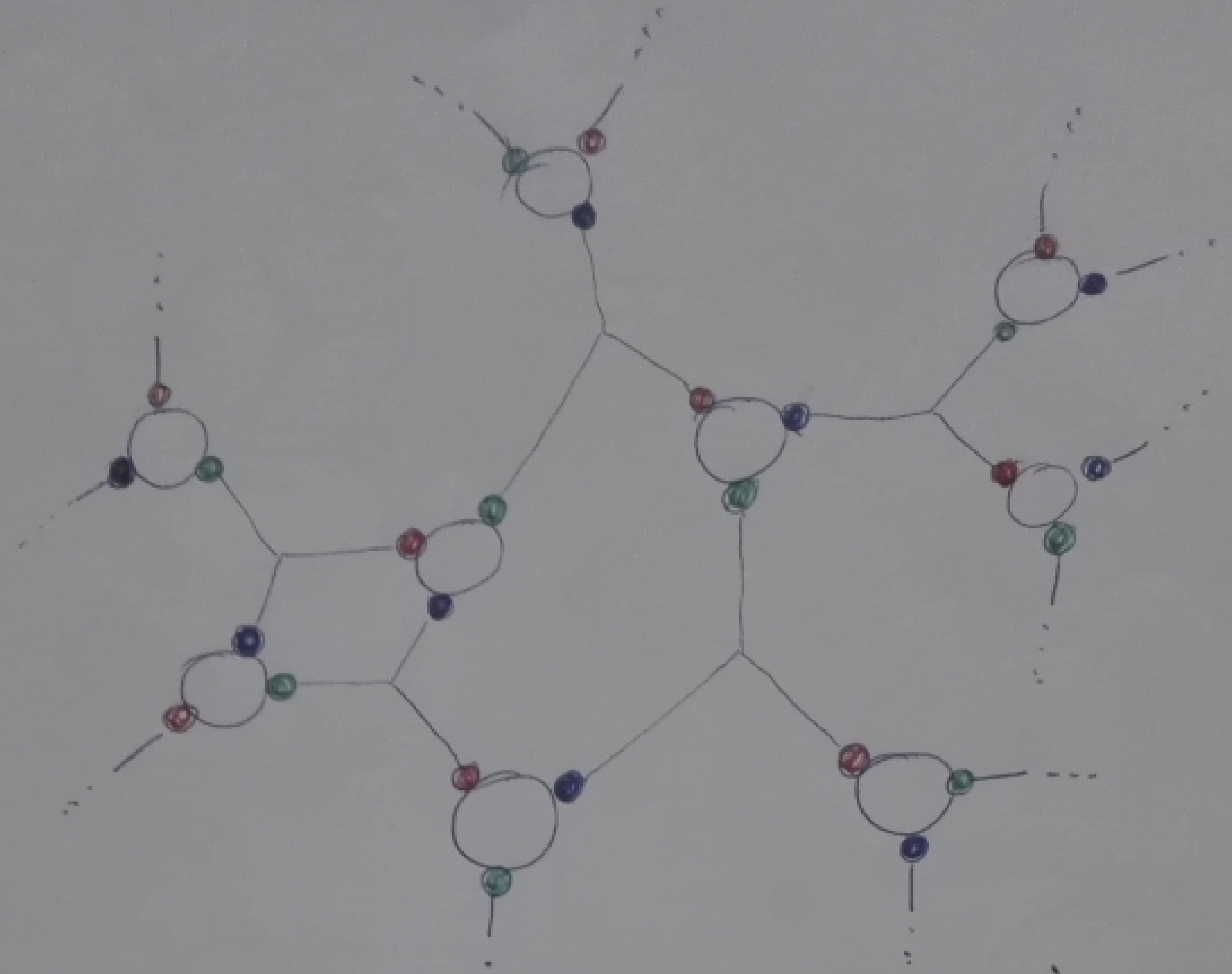
$S_0 \times S_2 = S_2$

$S_2 \times S_2 = S_0$

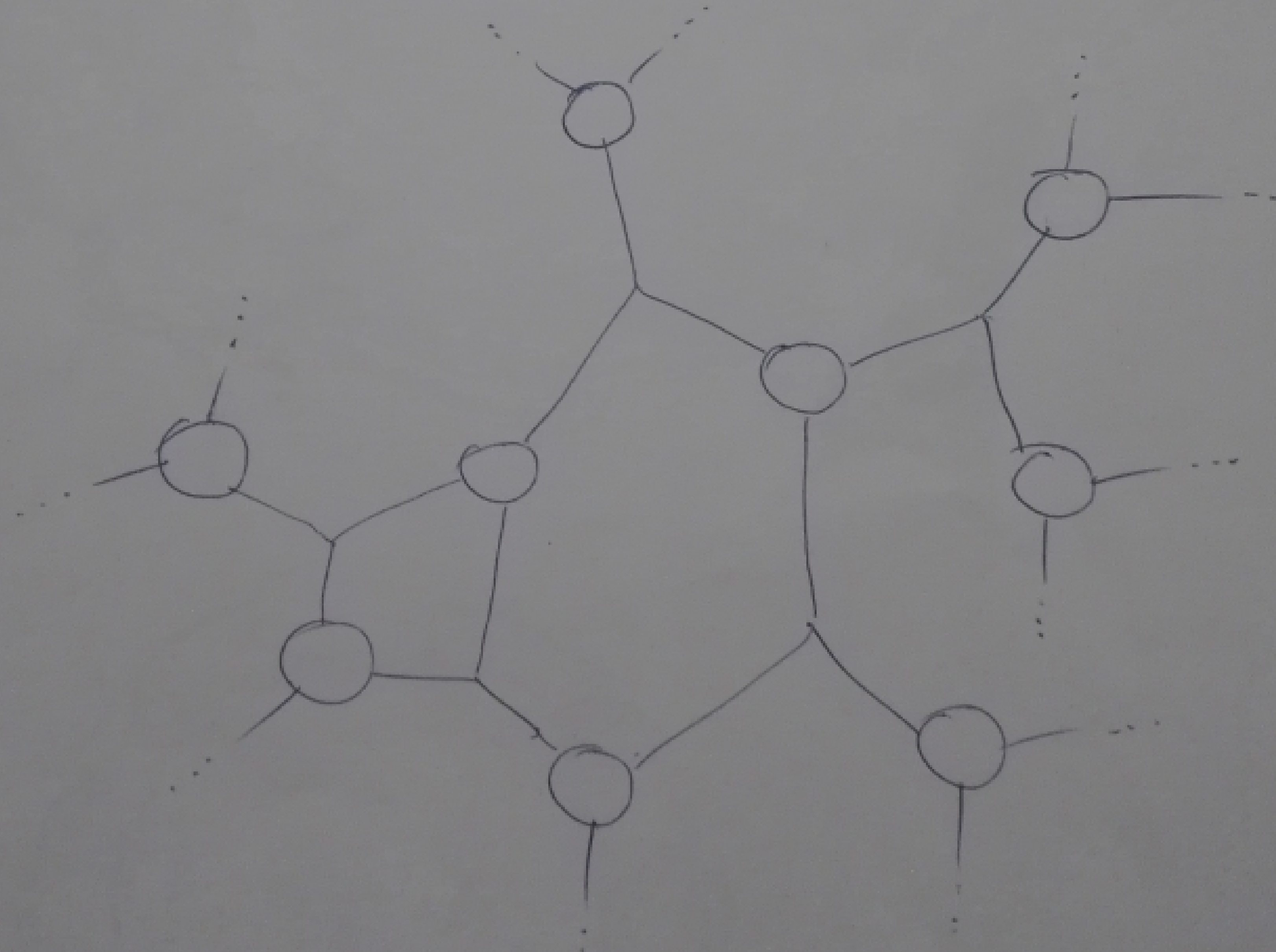


A ESPECIAL KIND OF [redacted] 3-LATTICE

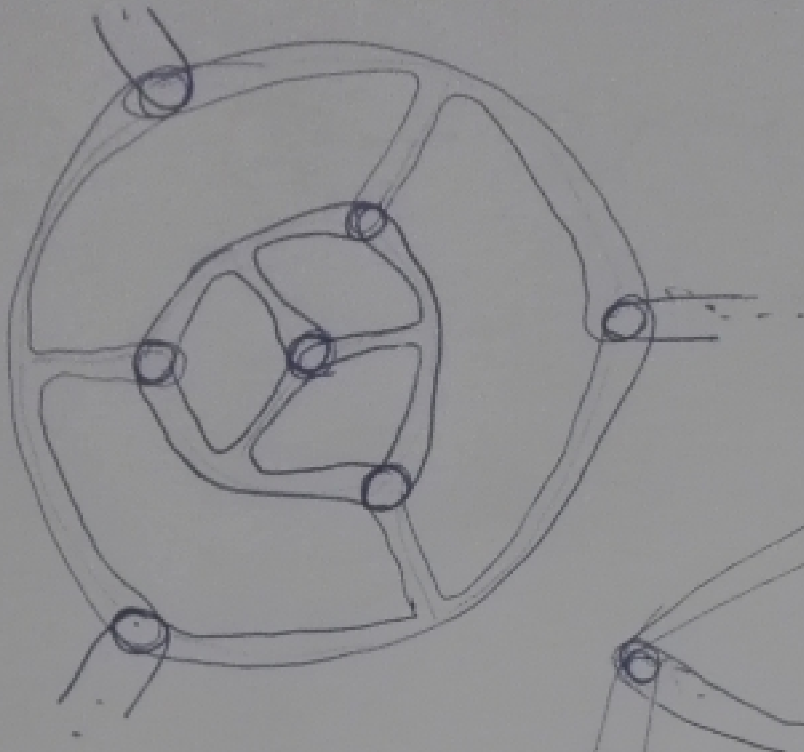


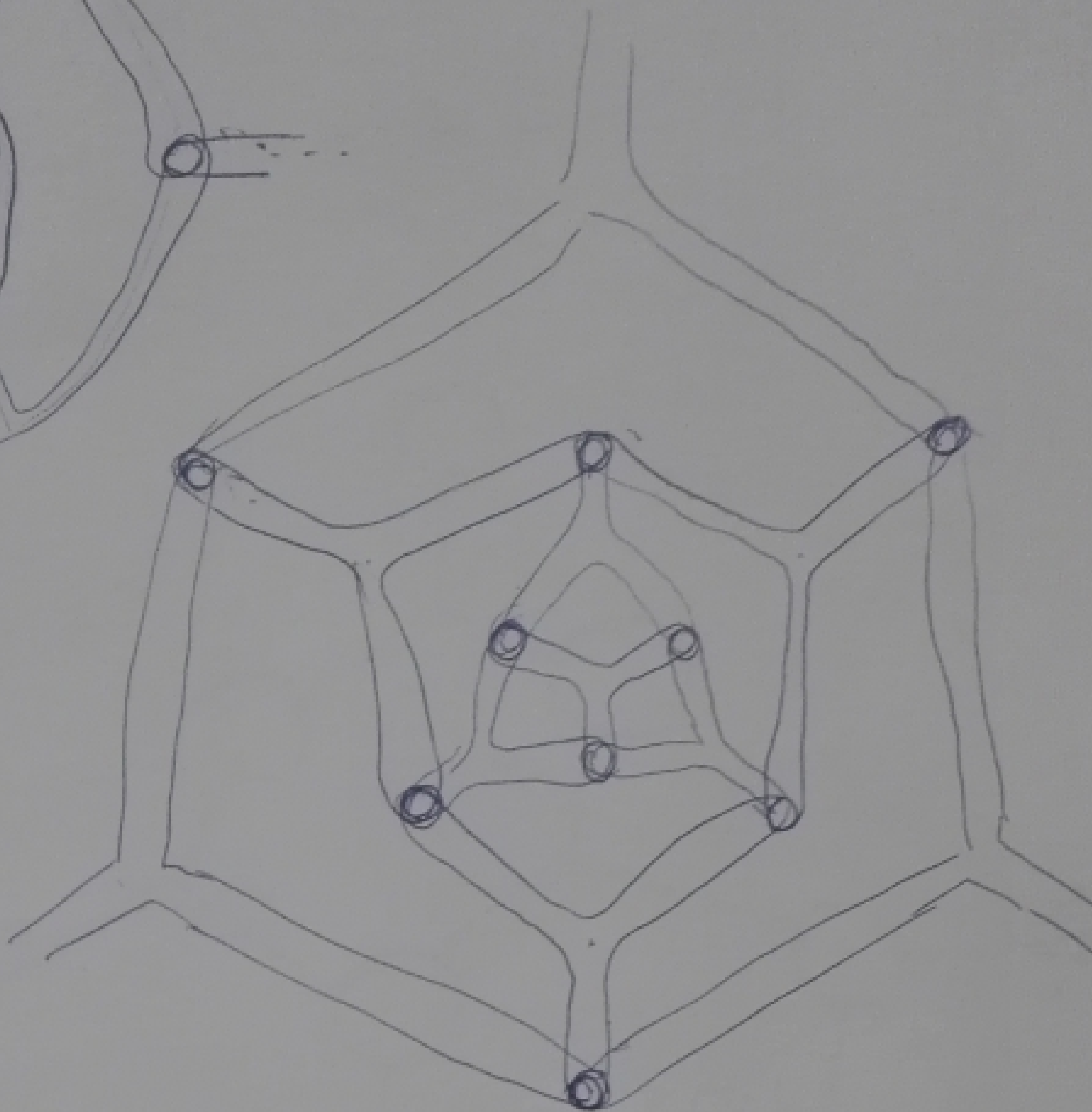


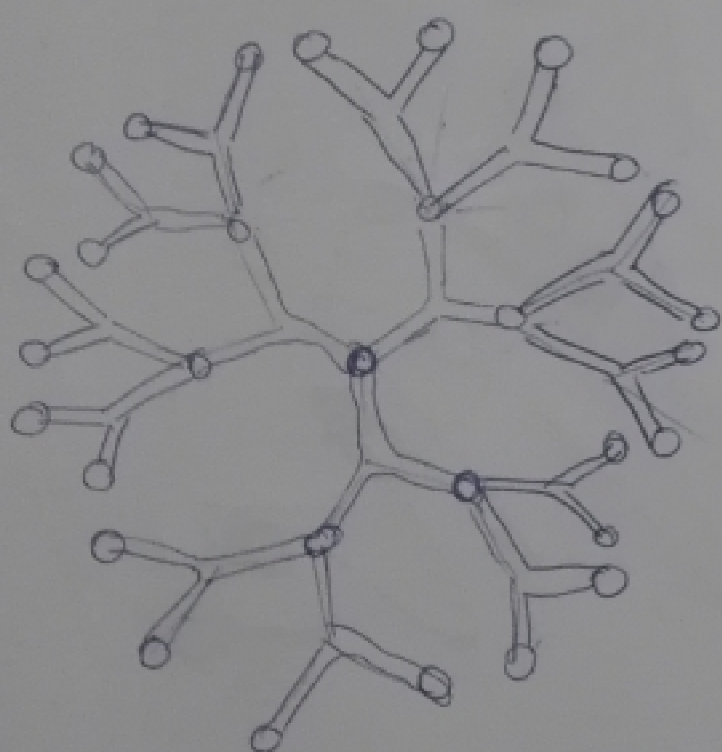
3-ORDERED 3-LATTICE

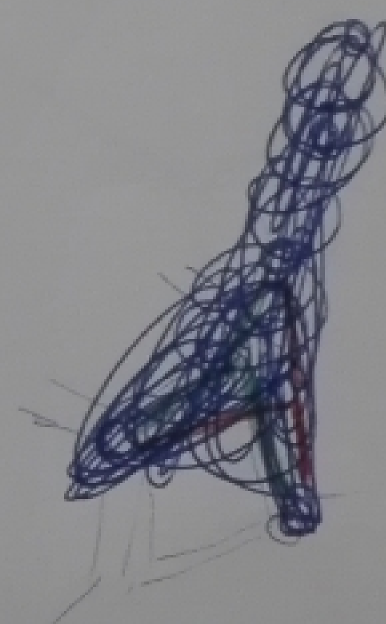


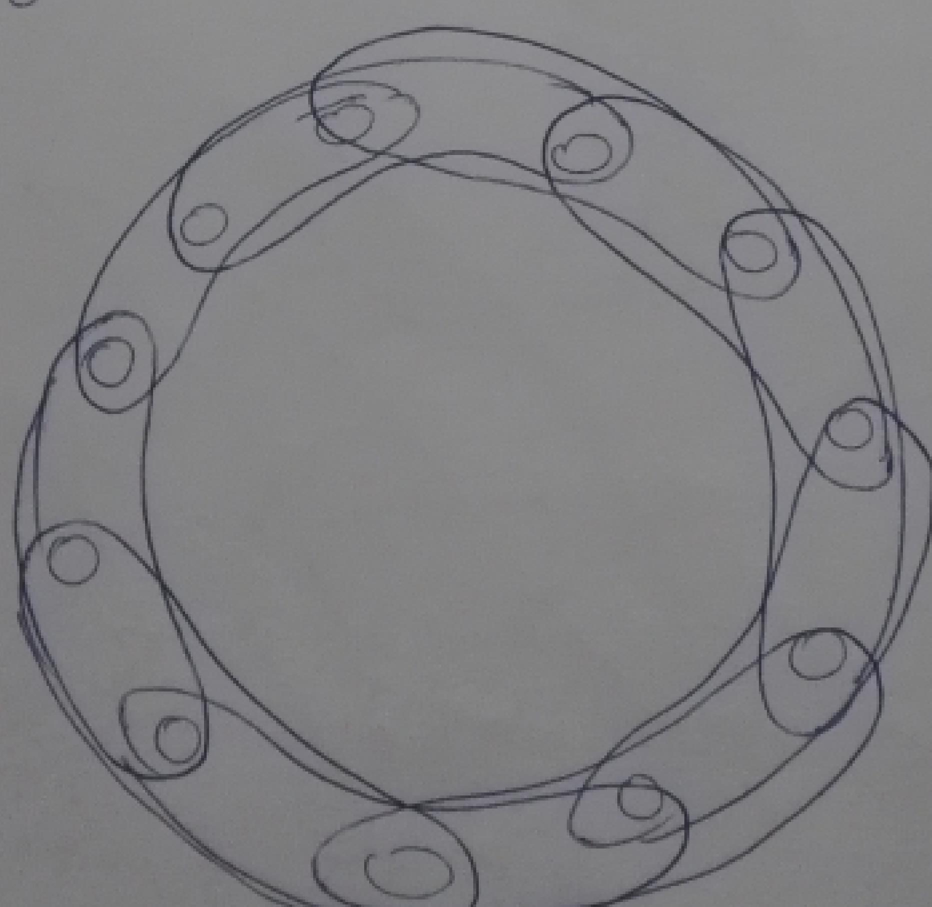
3-LATTICE

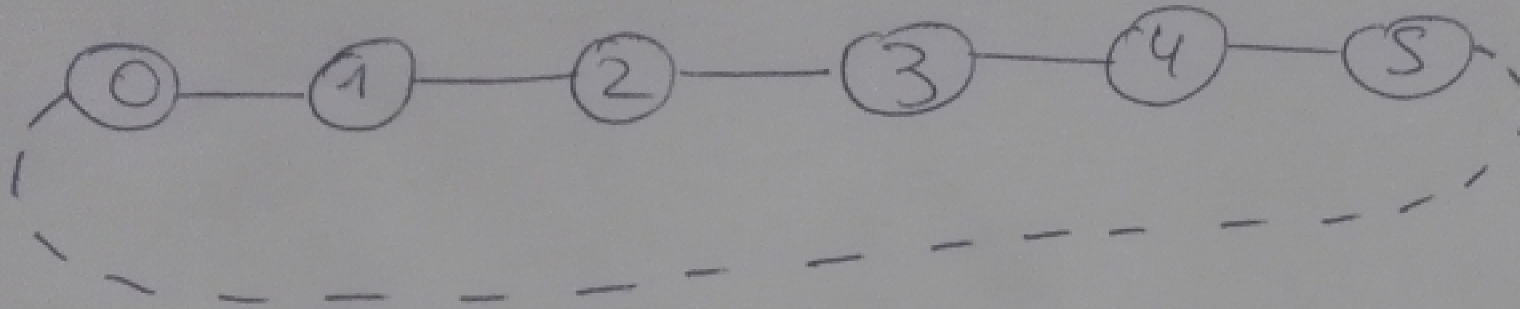


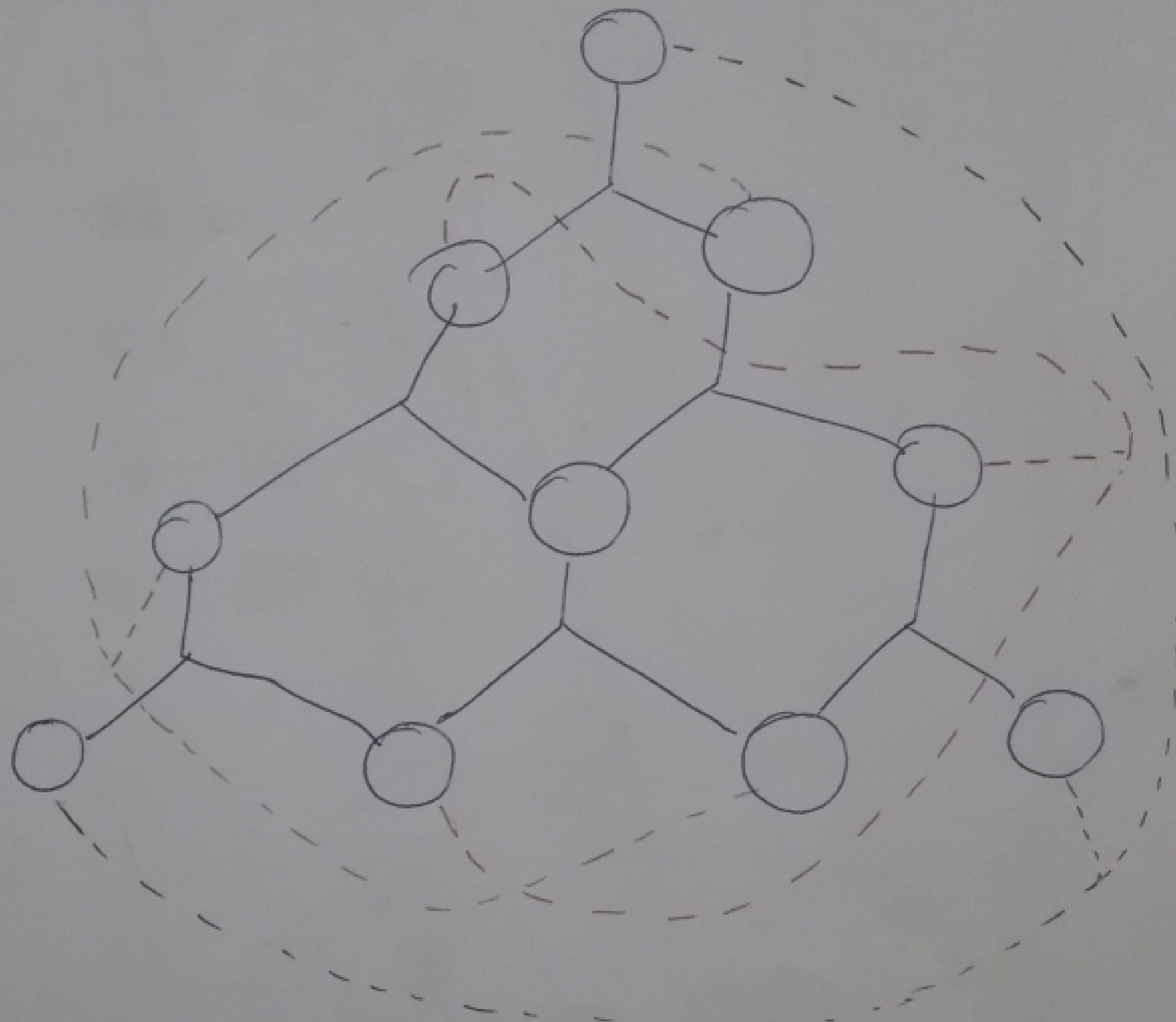


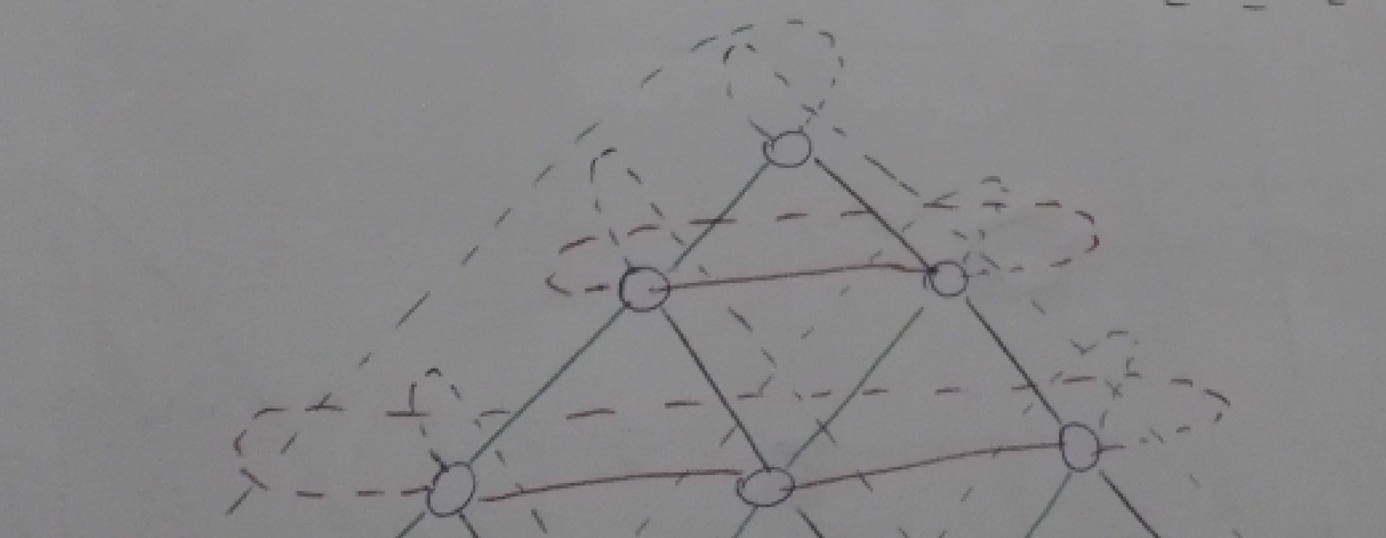


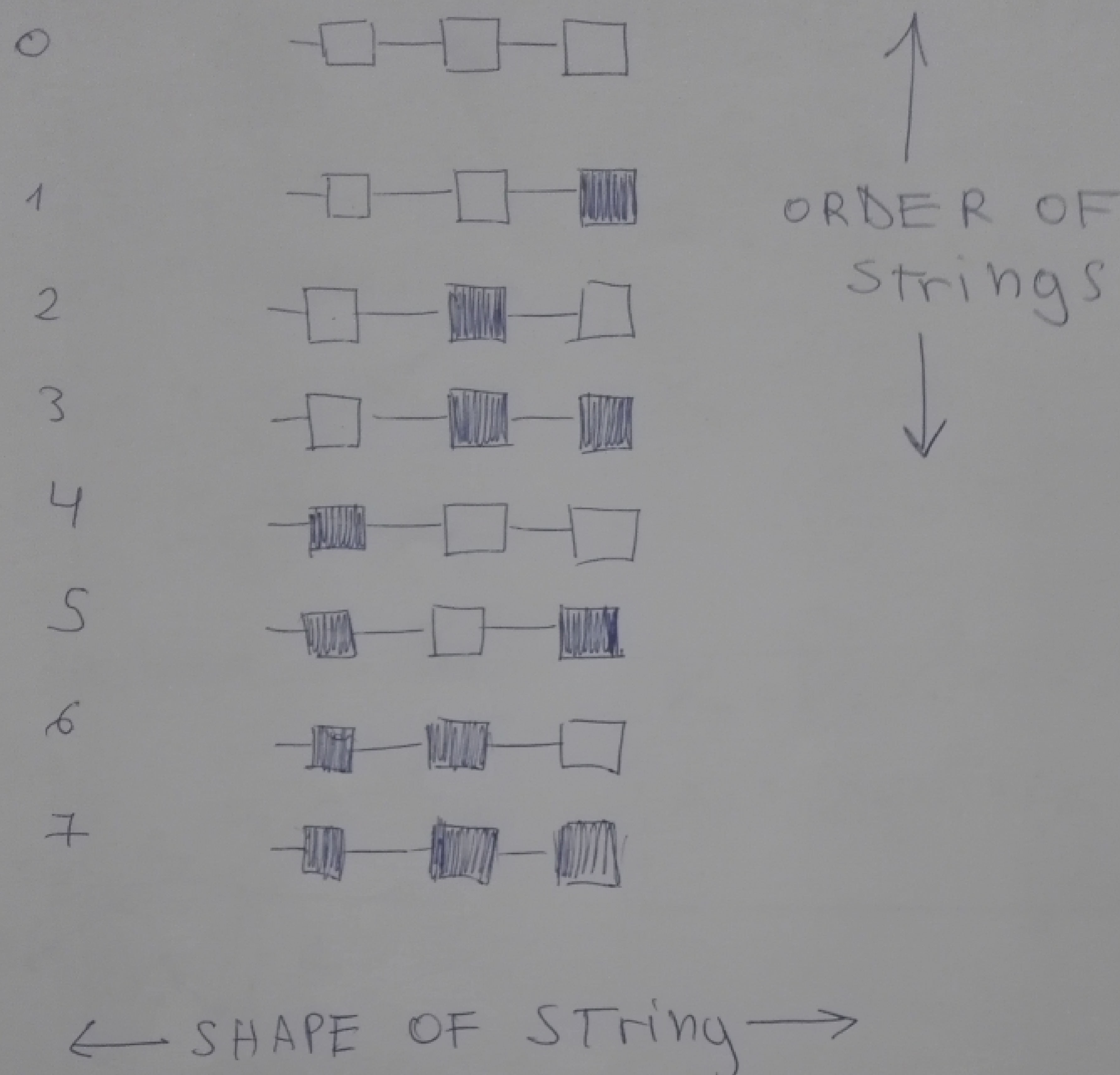


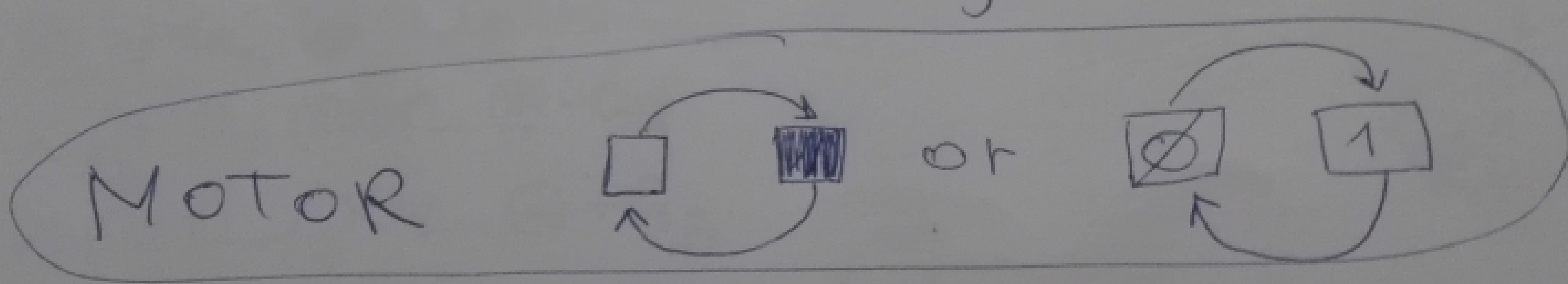


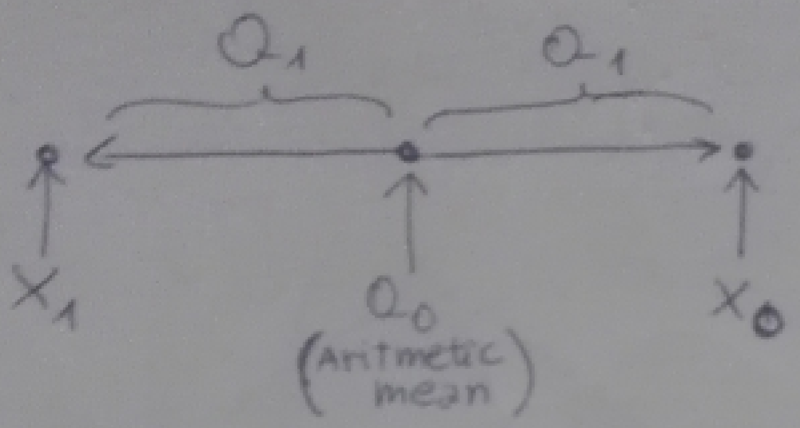

0
1
2
3
4
5






0
1
2
3
4
5
6
7
ORDER OF Strings
SHAPE OF String

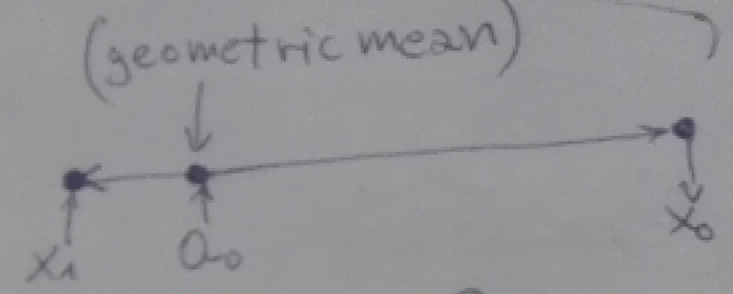

MOTOR
or
Ø
1



$$X_0 = a_0 + a_1$$
$$X_1 = a_0 - a_1$$

$$X_0 = \left(\frac{X_0 + X_1}{2}\right) + \left(\frac{X_0 - X_1}{2}\right)$$
$$X_1 = \left(\frac{X_0 + X_1}{2}\right) - \left(\frac{X_0 - X_1}{2}\right)$$

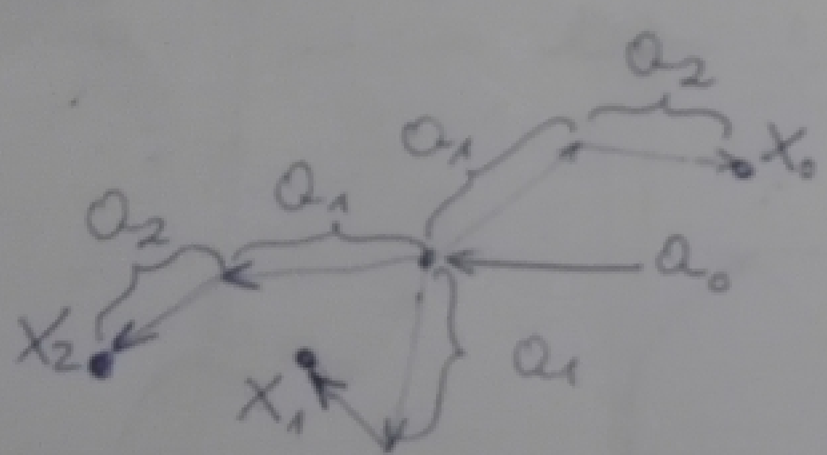
Instead $X_0 - X_1$



$$X_0 = a_0 \times a_1$$
$$X_1 = a_0 \times a_1^{-1}$$

$$X_0 = \left(\sqrt{X_0 \cdot X_1} \times \sqrt{X_0 \cdot X_1^{-1}}\right)$$
$$X_1 = \left(\sqrt{X_0 \cdot X_1} \div \sqrt{X_0 \cdot X_1^{-1}}\right)$$

$$X_0 = (X_0 \cdot X_1)^{\frac{1}{2}} \times (X_0 \cdot X_1^{-1})^{\frac{1}{2}}$$
$$X_1 = (X_0 \cdot X_1)^{\frac{1}{2}} \times (X_0 \cdot X_1^{-1})^{-\frac{1}{2}}$$



$$X_0 = a_0 @ a_1 @ a_2$$
$$X_1 = a_0 @ +a_1 @ -a_2$$
$$X_2 = a_0 @ -a_1 @ +a_2.$$

$$X_0 = \left(\frac{x_0 @ x_1 @ x_2}{3}\right) @ \left(\frac{x_0 @ -x_1 @ +x_2}{3}\right) @ \left(\frac{x_0 @ +x_1 @ -x_2}{3}\right)$$

$$X_1 = \left(\frac{x_0 @ x_1 @ x_2}{3}\right) @ + \left(\frac{x_0 @ -x_1 @ +x_2}{3}\right) @ - \left(\frac{x_0 @ +x_1 @ -x_2}{3}\right).$$

$$X_2 = \left(\frac{x_0 @ x_1 @ x_2}{3}\right) @ - \left(\frac{x_0 @ -x_1 @ +x_2}{3}\right) @ + \left(\frac{x_0 @ +x_1 @ -x_2}{3}\right)$$

Algebra behind Equality

$a+b=c \iff (a+b)+-(c)=0$

$a\times b=c \iff (a\times b)^{+1}\times(c)^{-1}=1$

with abstract operations

$a\odot b=c \iff (a\odot b)^{e_+}\odot(c)^{e_-}=e_0$

where e_0: neutral element under $\odot$

e_-: opposite element under $\odot$

$e_+\equiv e_0$

1 5 3 6 :, 45

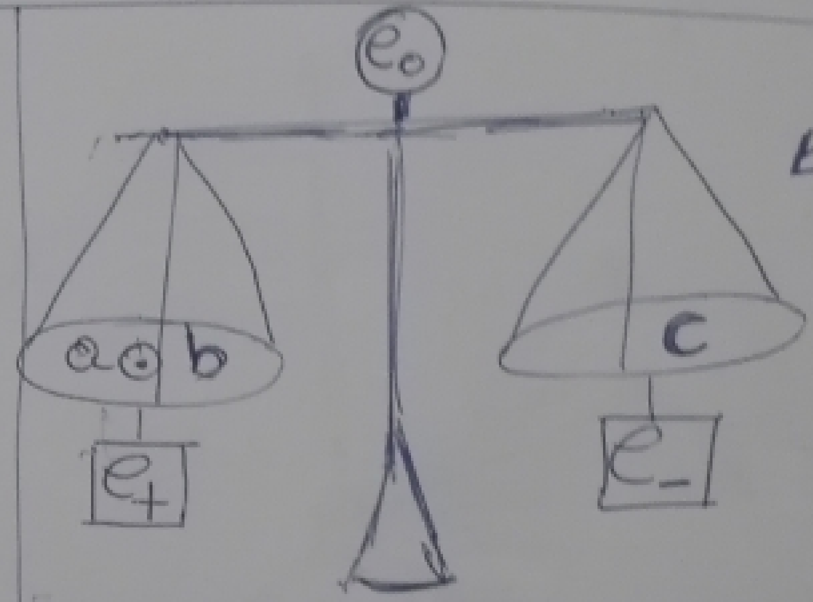
4 0 8

8 3 7 5

1 9 8 1 0

2 3 3 0

0 5 6



P_3

$-a @+a@ a=0$

$-a @+a=\bar{a}$

PD_3

$a \curlyvee a = a^{@1}\cdot a^{-1}\cdot a^{+1}=1$

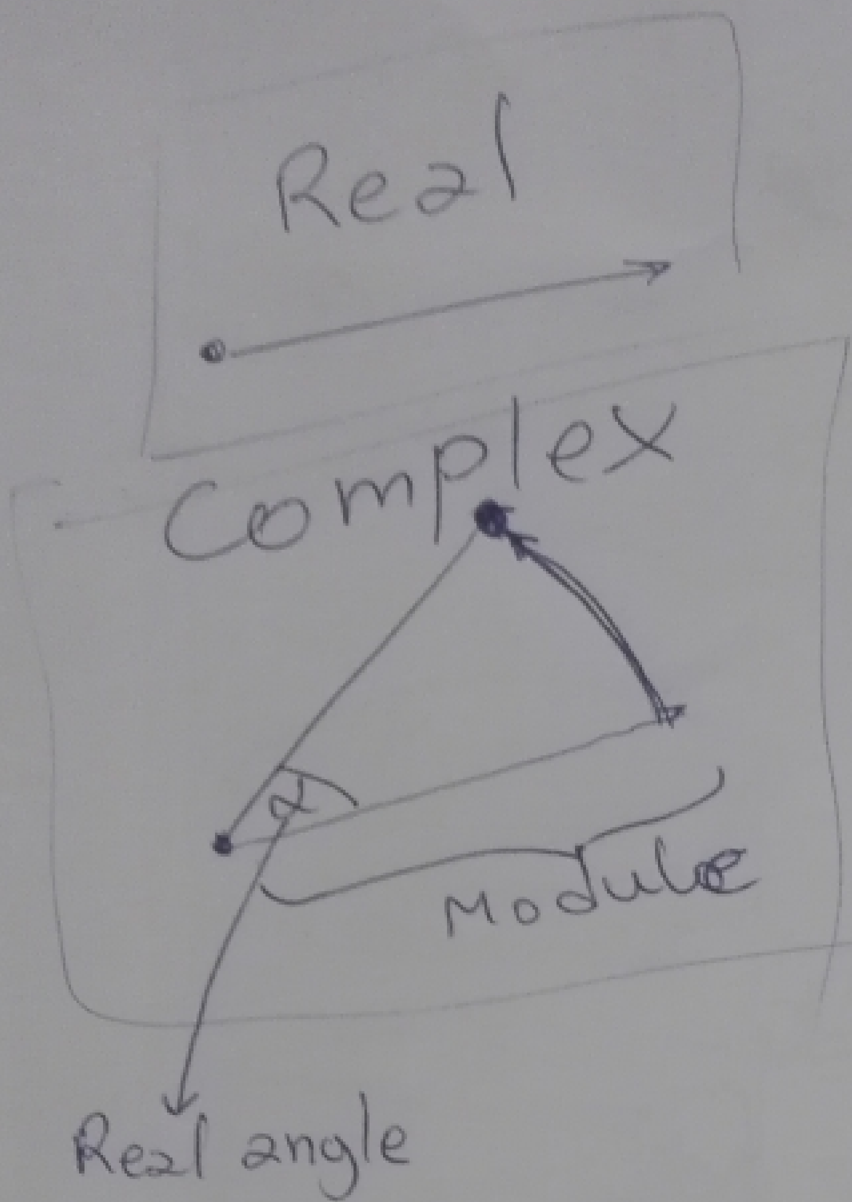
$\frac{1}{a \curlyvee a}=a^{+1}\cdot a^{-1}=a^{+1@-1}=a^{\bar{1}}=\frac{1}{a}$

Directed Division

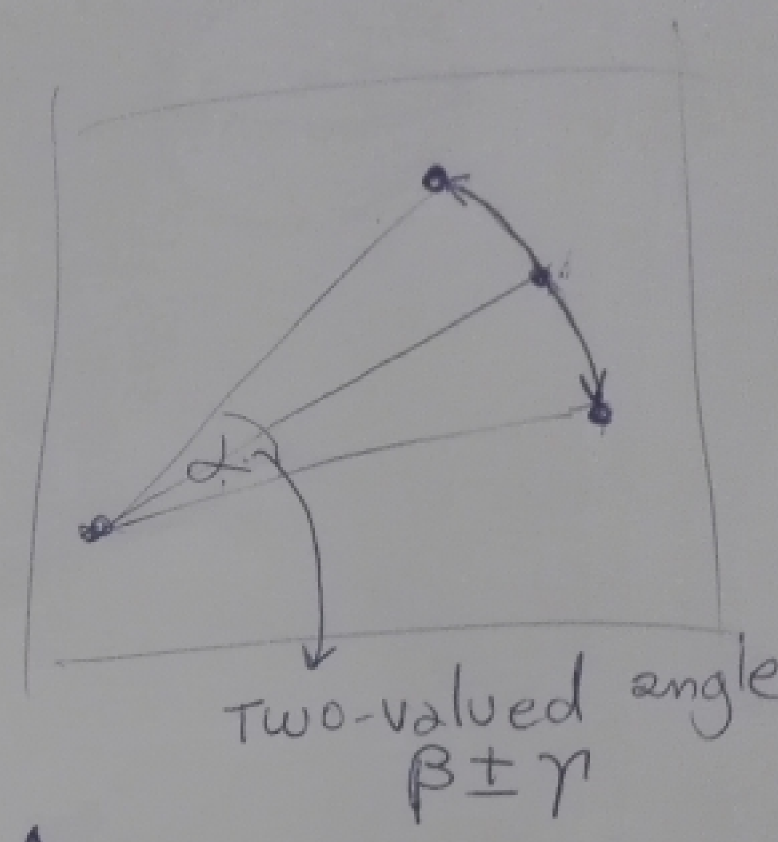
ABCD $\div$ FGH = 0, IJK.....

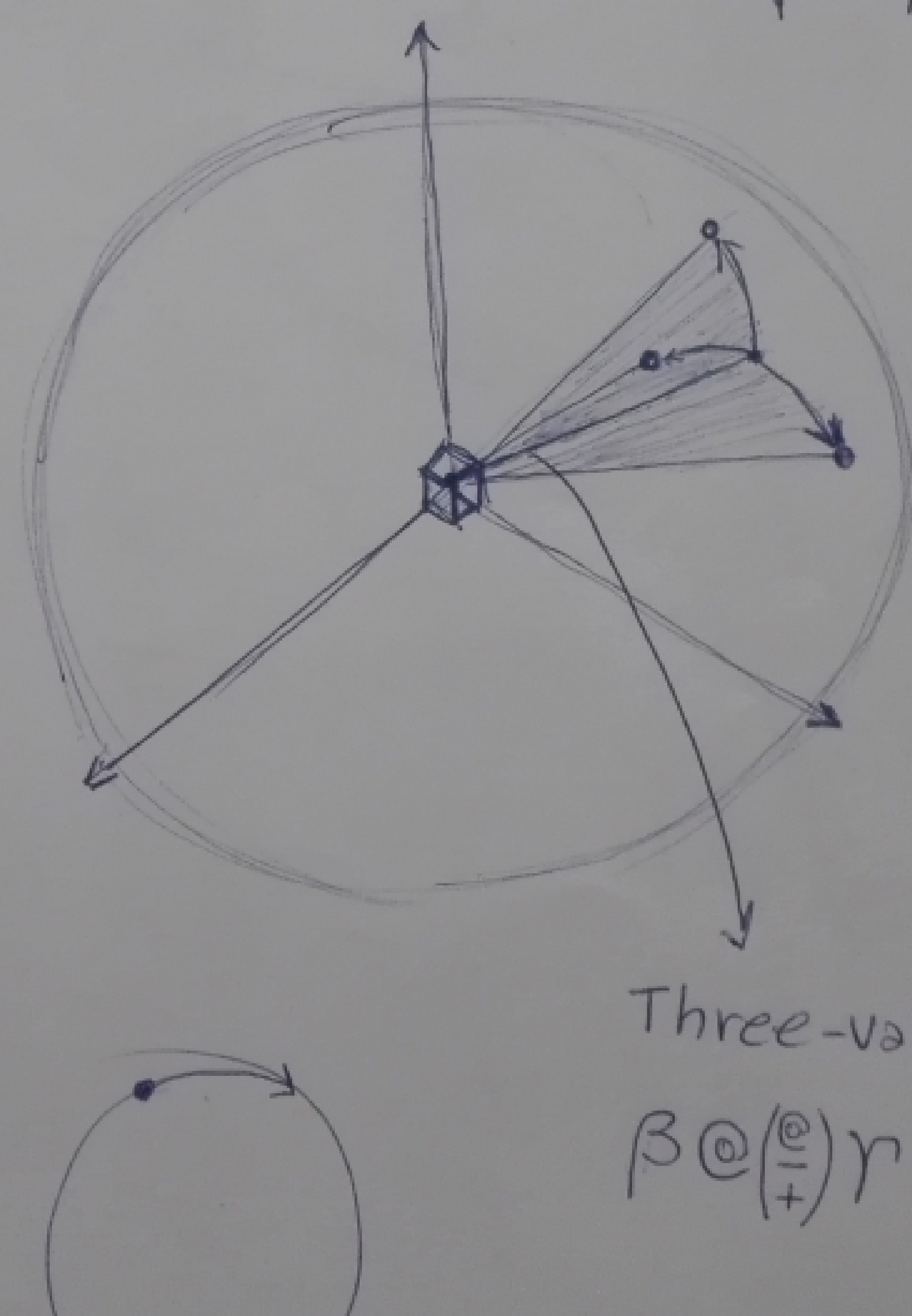
ABCD $\frac{\ominus}{\ominus}$ FGH = 0, I J K M :

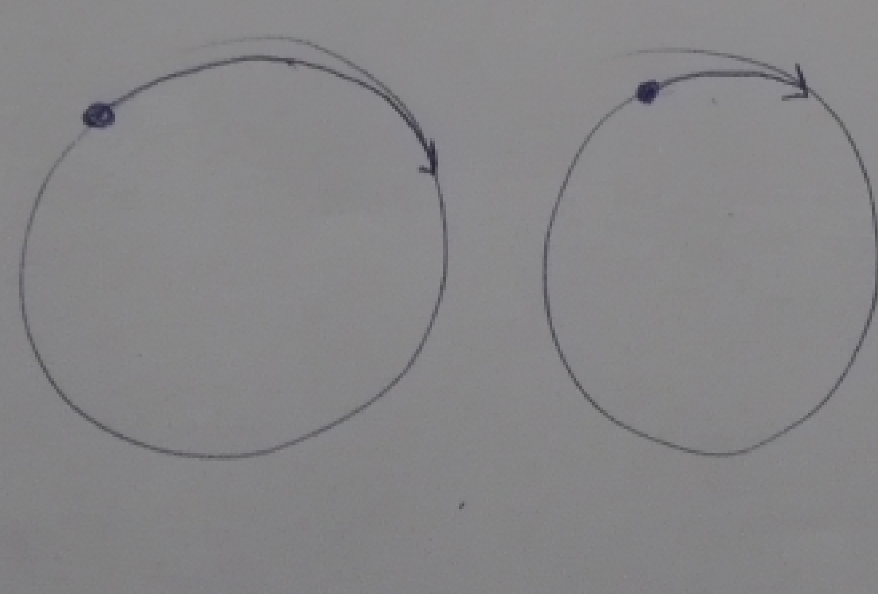
ABCD $\frac{\oplus}{\oplus}$ FGH = 0, I J K

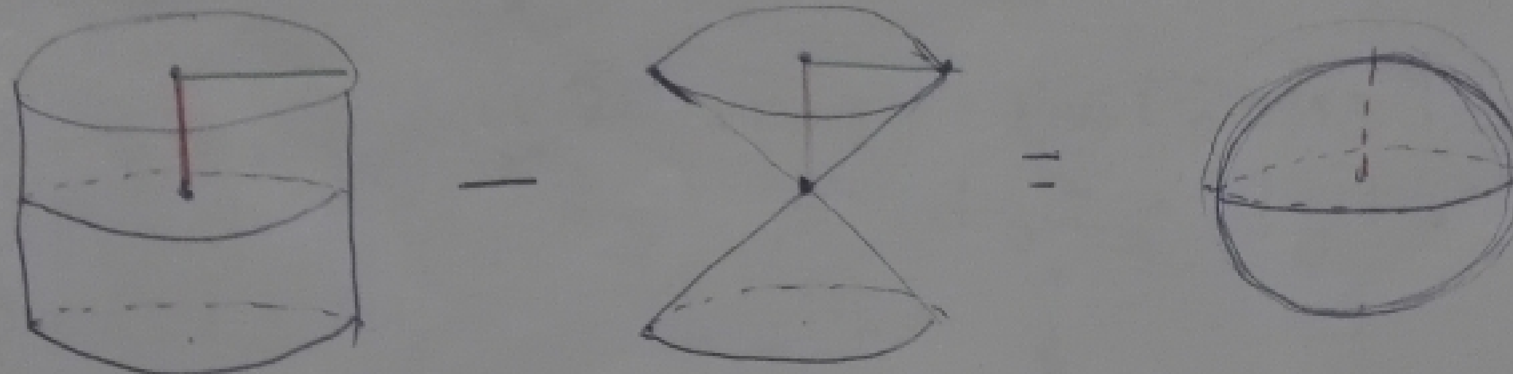

Real
Complex
α
Module
Real angle

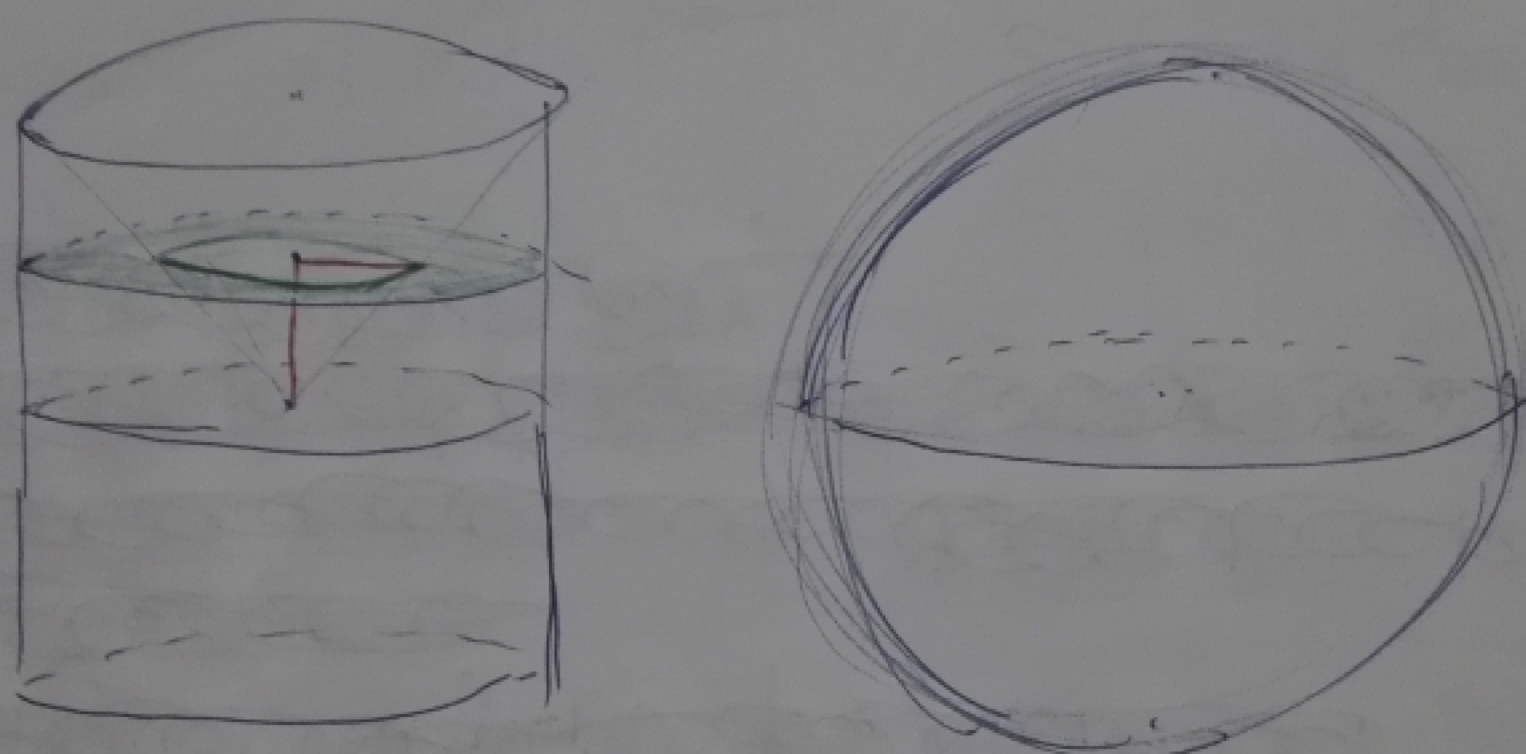
⟺


α
Two-valued angle
β±γ


Three-valued angle
β@(@/+)γ







To be able to compare trajectories of $R \times C$ vs P_4.

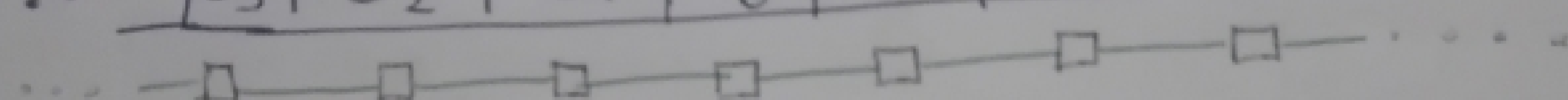
How To represent Positive $\mathbb{P}_2$ Rationals.

Example in Quaternary positional.

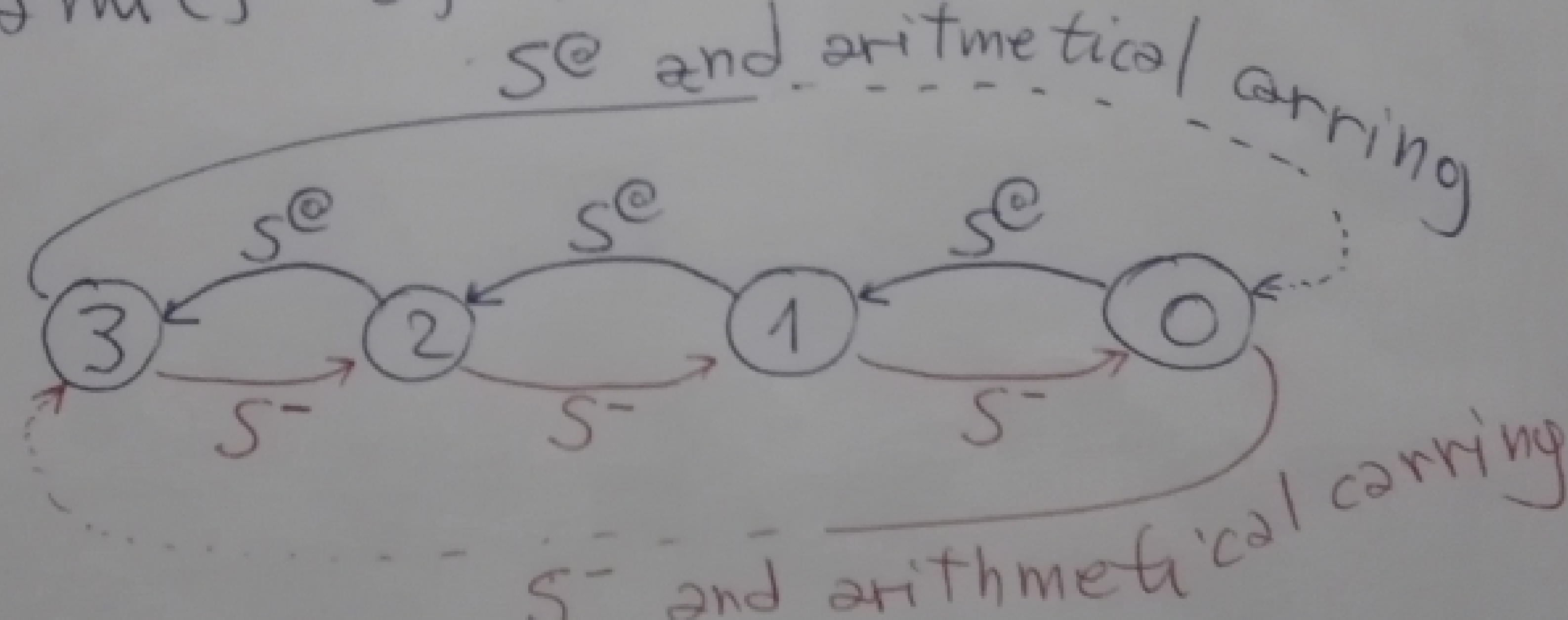
	digit "∅"	digit "•"	digit "••"	digit "∴"
- Digits "d_i"	↕	↕	↕	↕
$i \in \{\varnothing, \cdot, :, \therefore\}$	0	1	2	3.

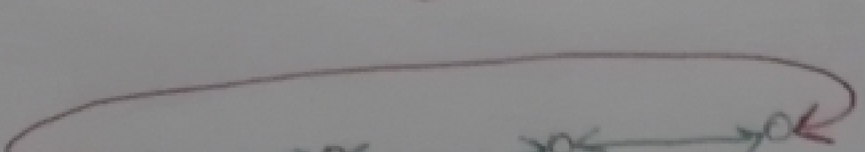
- Successor of a digit "d_i": $S^{@}(d_i) = d_{i@1}$
- Predecessor of a digit "d_i": $S^{-}(d_i) = d_{i@-1}$
- Shape of a String: $\sum_{0}^{+\infty,-\infty} C_i \times (\therefore)^i$ with $C_i \in \{d_i\}$

... | C_3 | C_2 | C_1 | C_0 | C_{-1} | C_{-2} | C_{-3} | ...



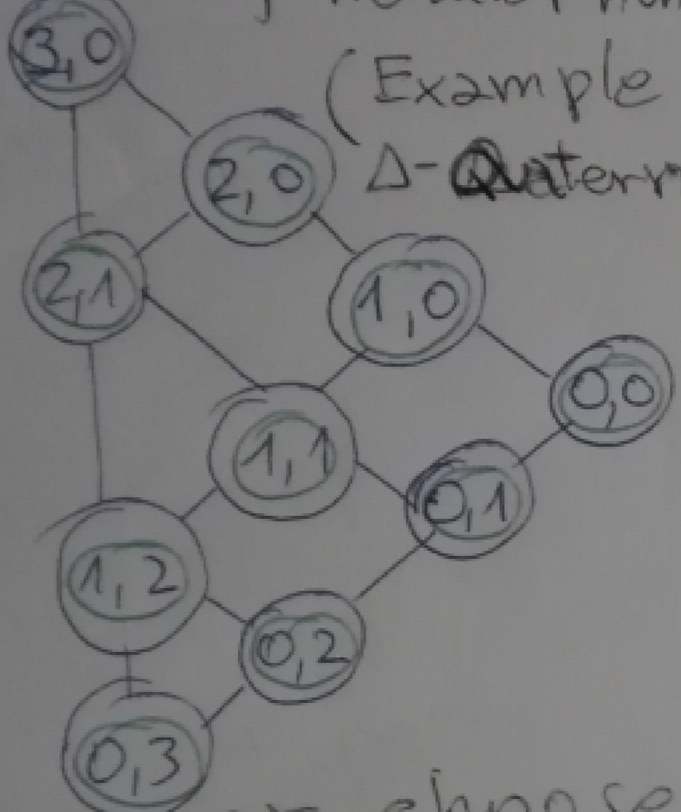
- Dynamics of THe Motor



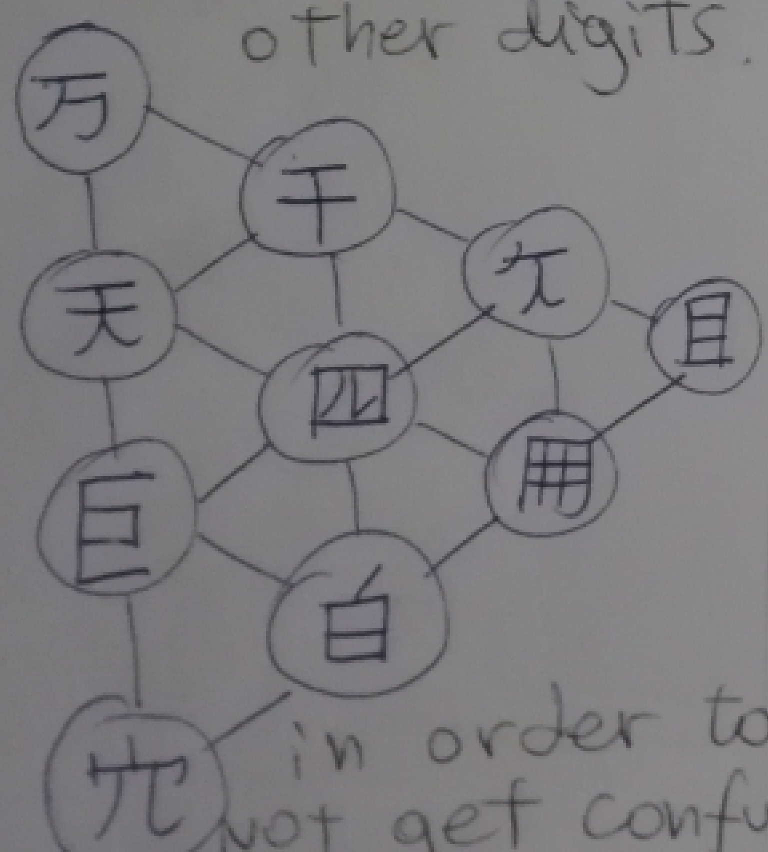


- Restriction with The digit "0" in The left.

Digit require a pair of natural numbers (Example in Δ-Quaternary)

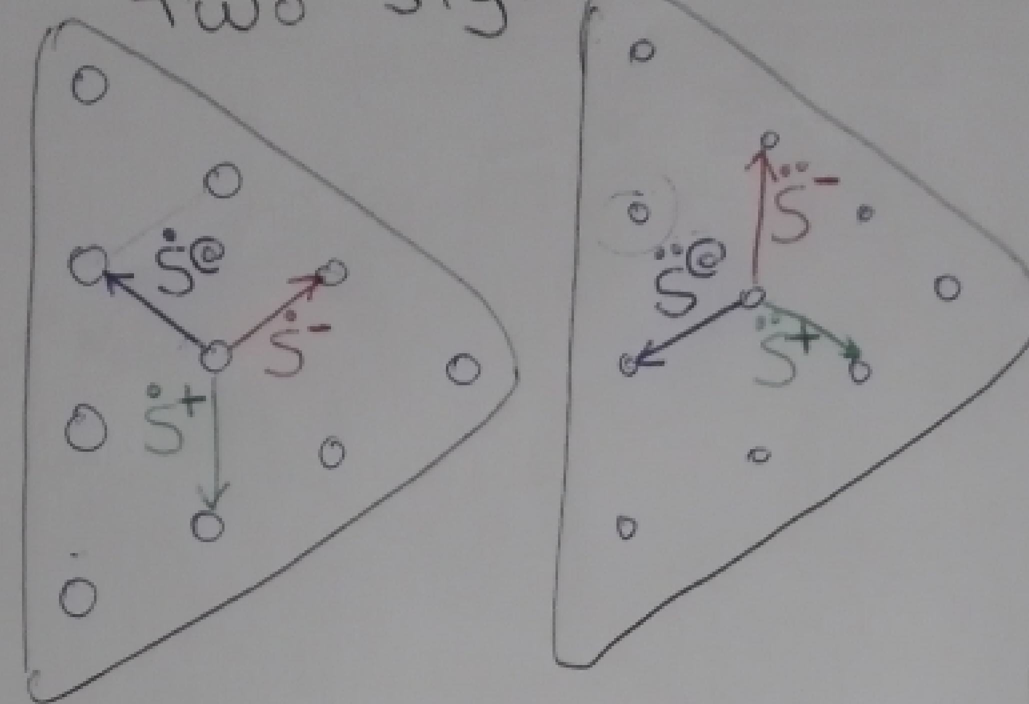


or choose other digits.

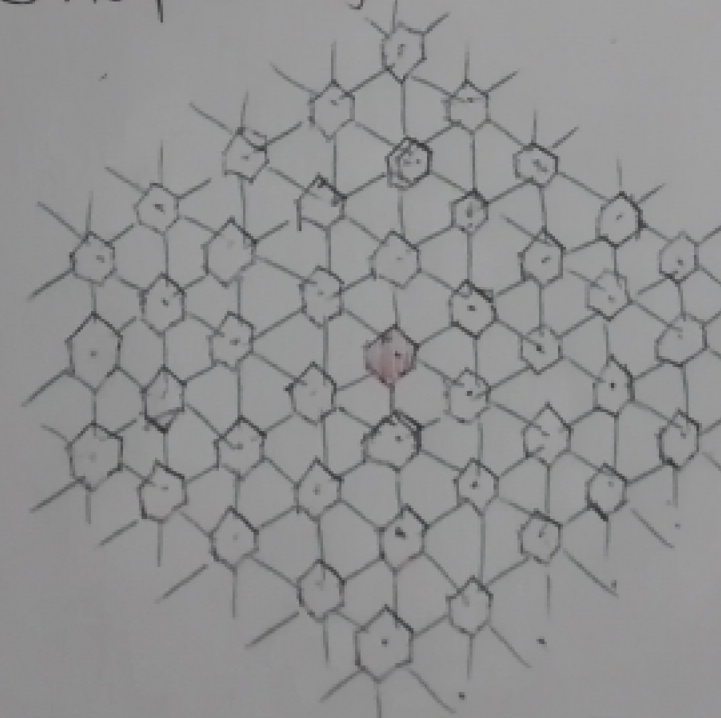


in order to not get confused.

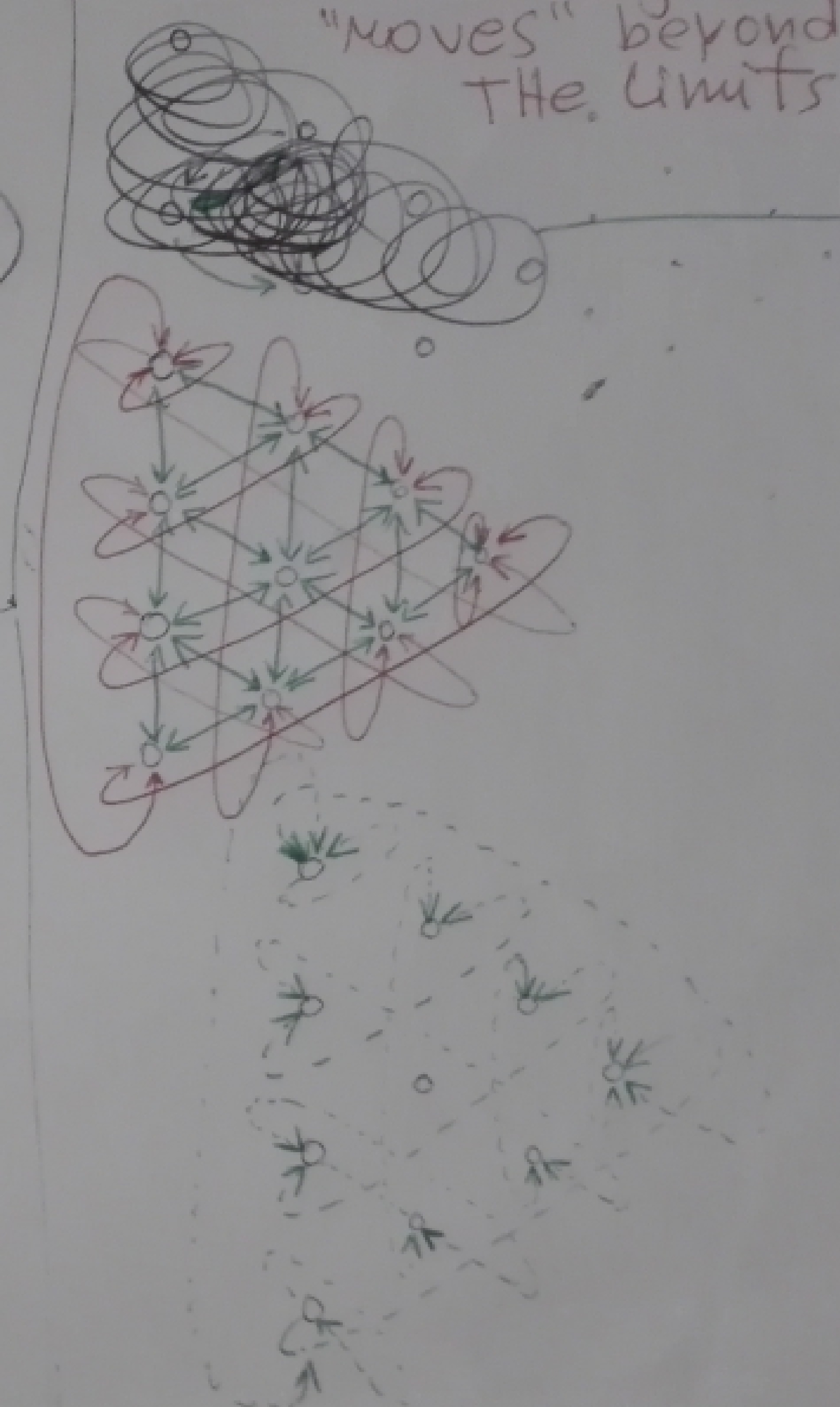
every digit Has two "signed" Succesor



Shape of PD3 string.

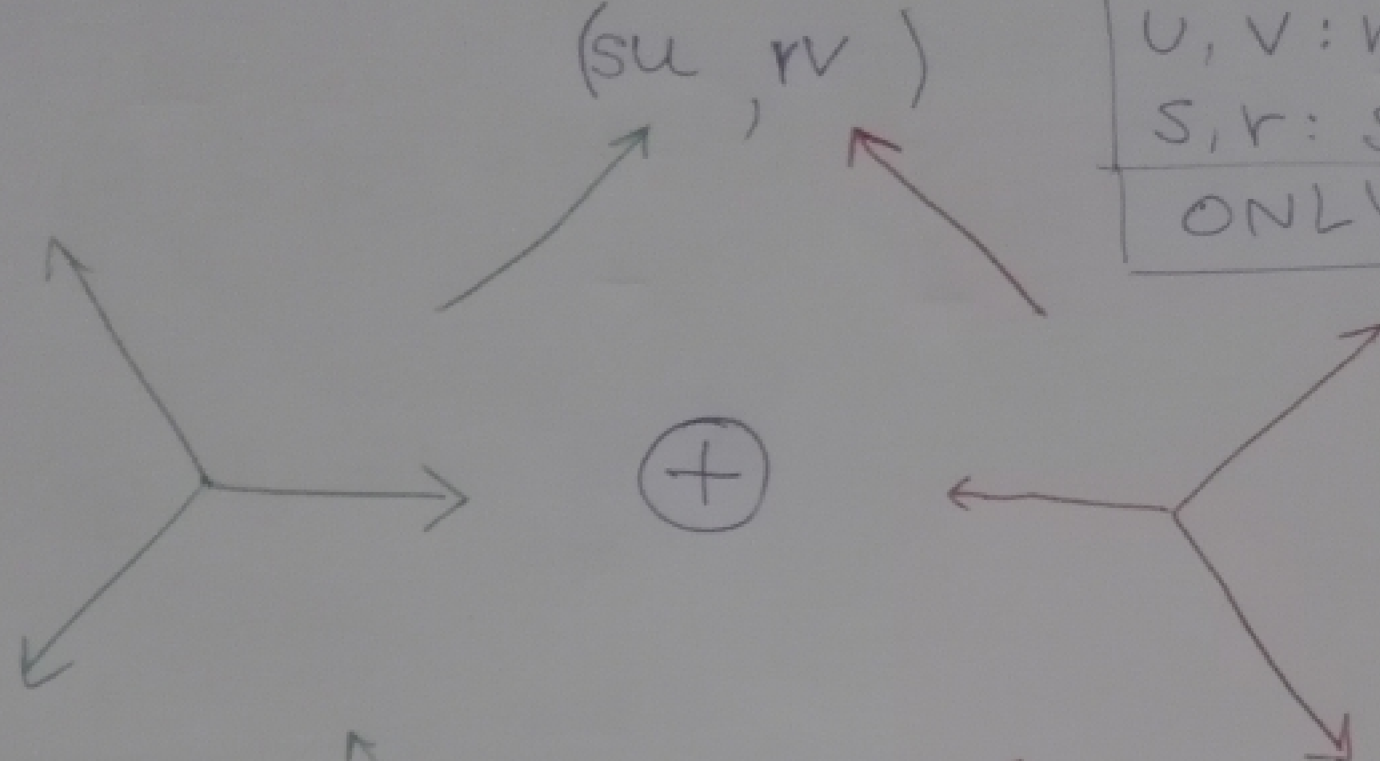


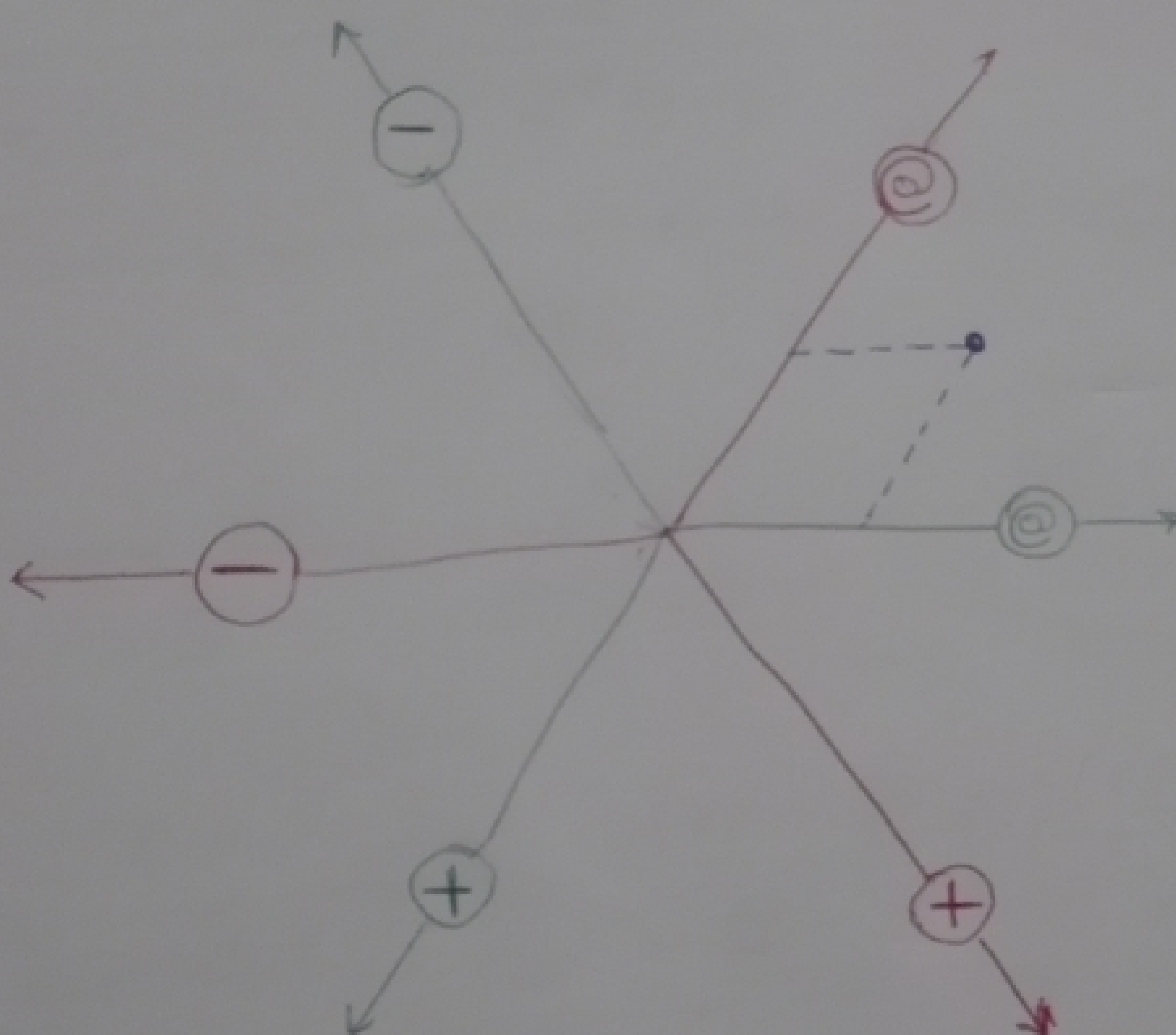
ArithMetical carring when the digit "moves" beyond the limits



P3-iZED Coordinates

U, V: magnitudes
S, r: signs
ONLY RAYS!!





$$(@A, 0) + (0, -A) = (0, 0)$$
$$(-A, 0) + (0, +A) = (0, 0)$$
$$(+A, 0) + (0, @A) = (0, 0)$$
$$(@A, 0) + (0, @B) = (@A, @B)$$
$$(@A, 0) + (0, +B) = (@A, +B)$$

For P_2

$$(a @ -b) = (a @ -b) \cdot \frac{(a @ b)}{(a @ b)}$$ ← Multiplied by one

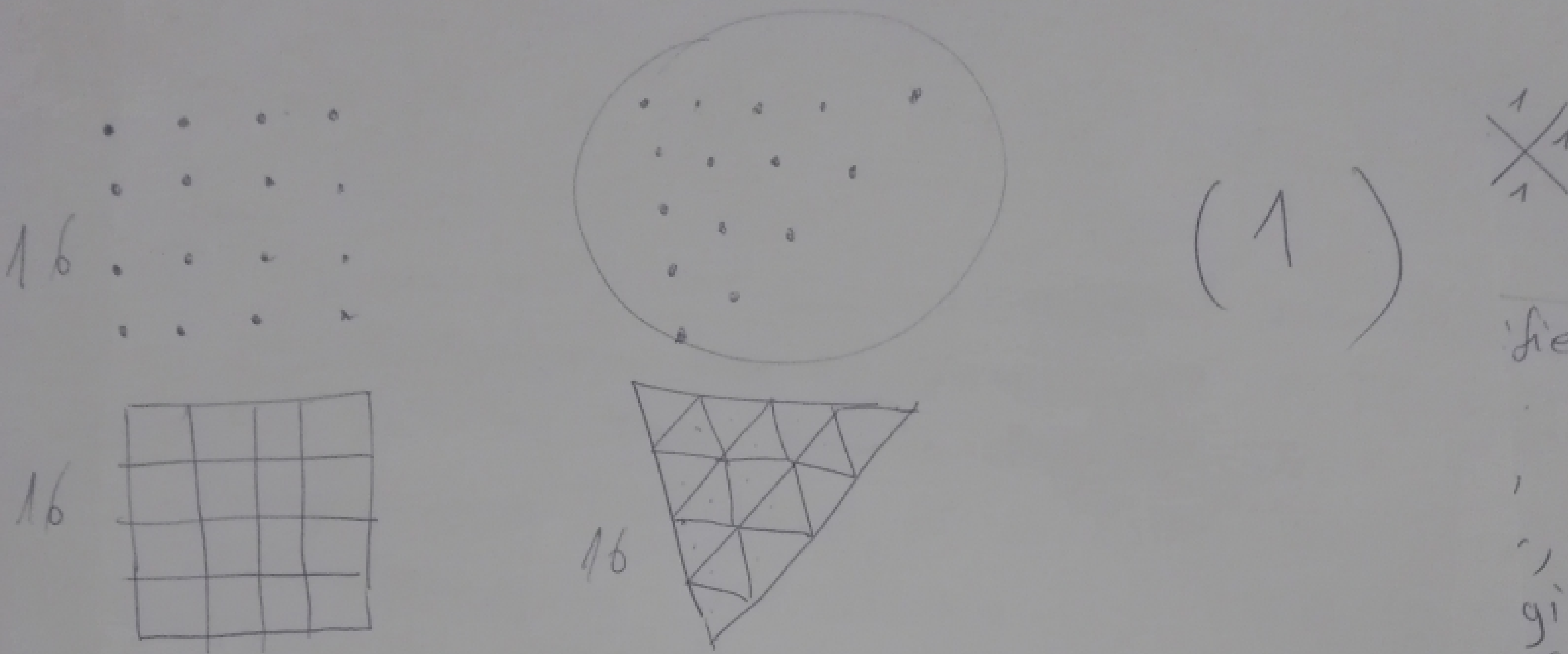
$$a @ -b = \frac{a^2 @ -b^2}{a @ b}$$

$$\frac{1}{a @ -b} = \frac{a @ b}{a^2 @ -b^2}$$

For P_3

$$\frac{1}{a @ -b @ +c} = \frac{1}{a @ -b @ +c} \cdot \frac{(a @ +b @ -c)(a @ b @ c)}{(a @ +b @ -c)(a @ b @ c)}$$ → multiplied by one

$$\frac{1}{a @ -b @ +c} = \frac{(a @ +b @ -c)(a @ b @ c)}{a^3 @ b^3 @ c^3 @ 3\overline{abc}}$$



(1)

→ RADIUS OF THE CIRCUNSCRIBED SPHERE (INSCRIBED?)

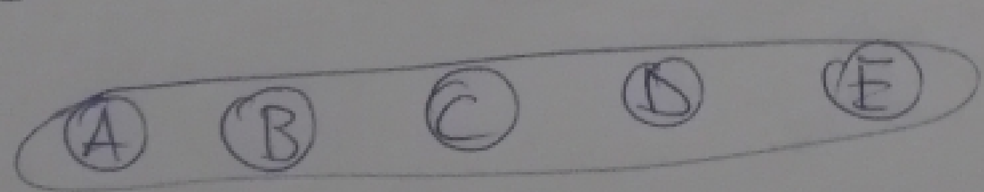
$$\frac{R_u}{N} = \frac{\text{HyperVOLUMEN}}{\text{HypersurFACE (TOTAL)}}$$

↓ NUMBER OF CARTESIAN DIMENSIONS

FOR Regular
n-Simplex,
n-cubes
n-spheres.

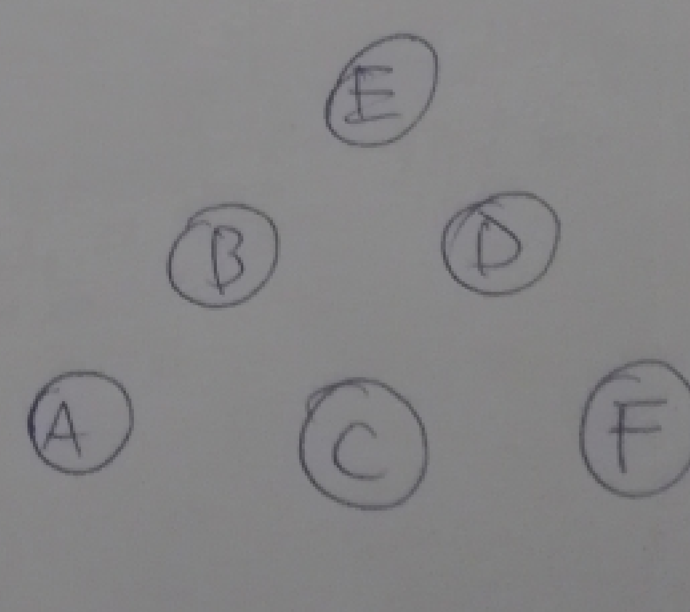
FACTORIAL

LINEAR ARRANGEMENT

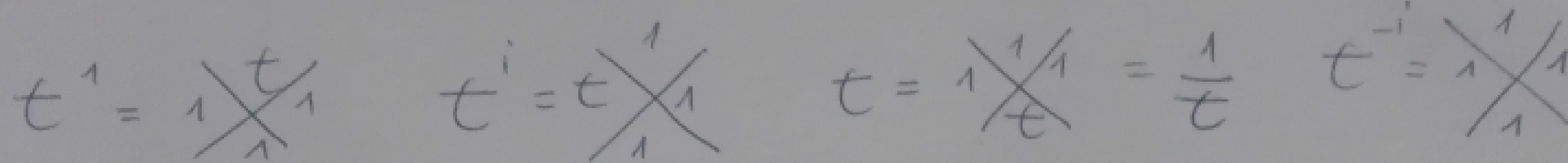


↓

5!



① complexified fractions.



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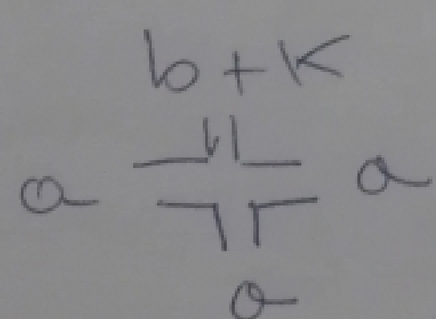
② Opcionally, it can be added a complexified positional number system, but instead of taking the Knuth Route, that is, complexifying the base, it's taken another, complexifying the shape of string of digits, This is, instead of an linear arrangement of digits using integers, use a planar "string" of digits, gaussian Integers.

---

③ $a = b + k$ $\Updownarrow$ $1(a) + -1(b+k) = 0$

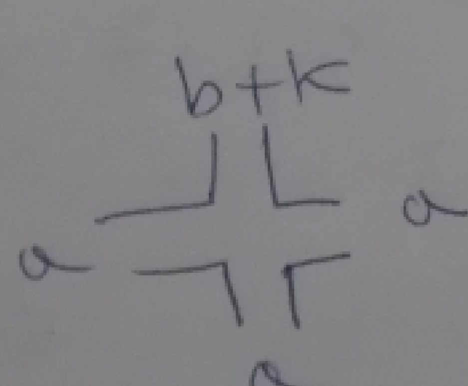
$a = b k$ $\Updownarrow$ $(a)^1 (bk)^{-1} = 1.$

complexified version:



$\Leftrightarrow 1(a) + i(b+k) + -1(a) + -i(a) = 0$

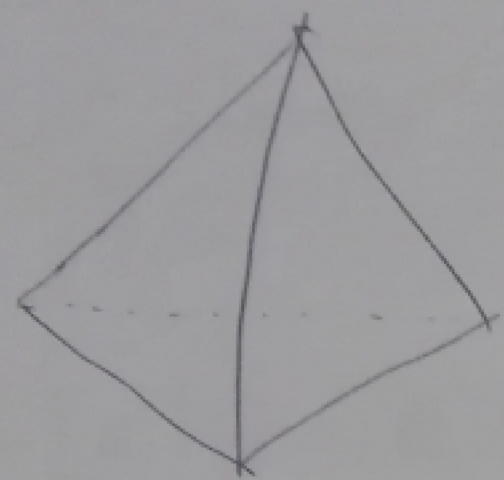
with $a = b+k$.

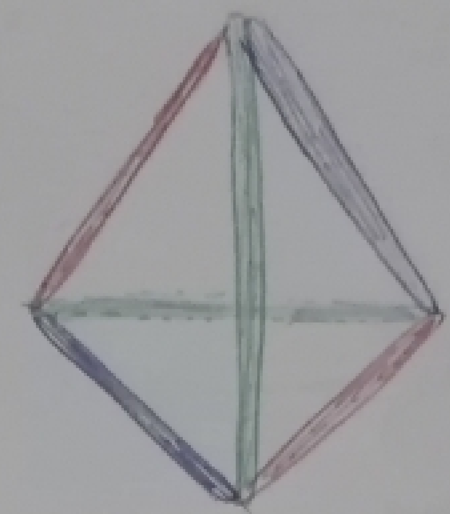


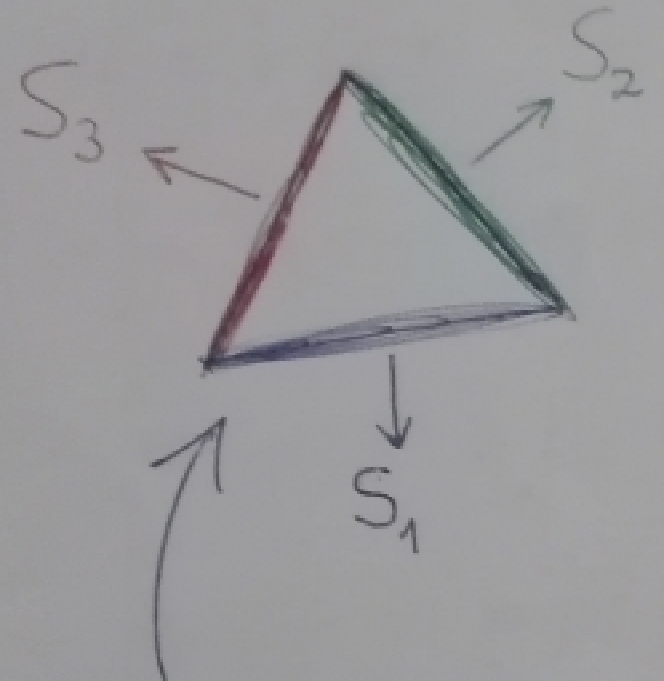
$\Leftrightarrow (a)^1 \cdot (bk)^i \cdot (a)^{-1} \cdot (a)^{-i} = 1.$

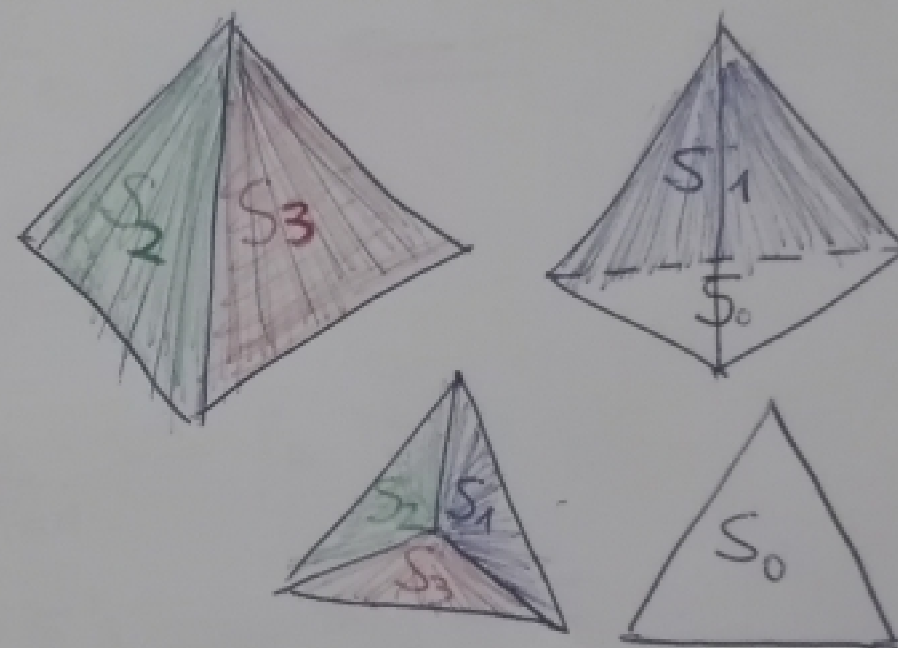
with $a = b \cdot k$.

OTher P_4 Product Rule, generated by a tetrahedron.



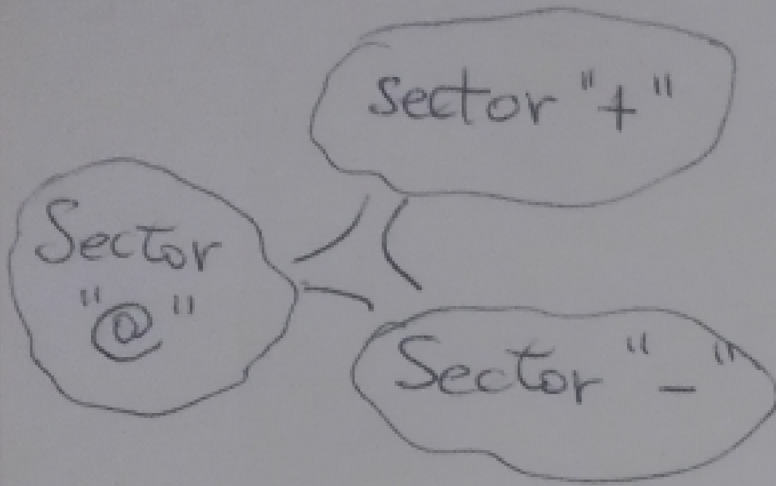


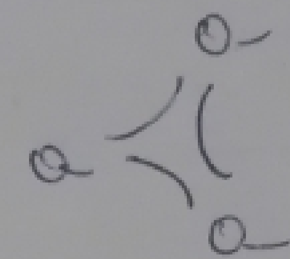




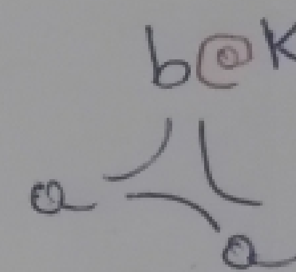
Product Rule.

- The tetrahedron is , Liying on one face, when the sign. is multiplied, it can change the side, Turning it respect some edge, or Remaining equal (multiplied by S_0)
- The current sign is indicated with The face Touching The ground.
- Each face correpond one sign.



$-[a]@+[a]@@[a]=0 \Longleftrightarrow$ 

$a=b@k$

$-[a]@+[b@k]@@[a]=0 \Longleftrightarrow$ 

$-[a]@+[b]@(+k)@@[a]=0$

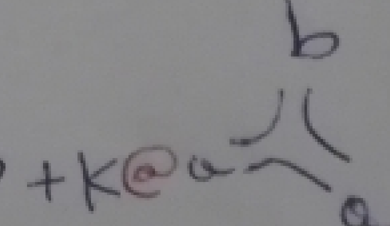
$-[a]@+[b]@\left(+k\cdot\left(\frac{-1}{-1}\right)\right)@@[a]=0$

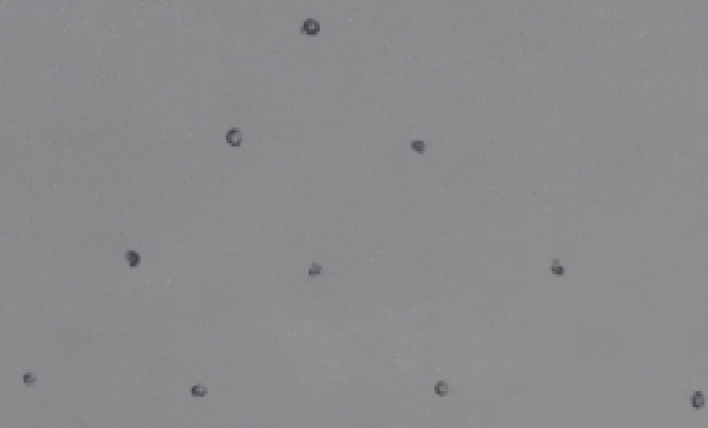
$-[a]@+[b]@\left(\frac{@k}{-1}\right)@@[a]=0$

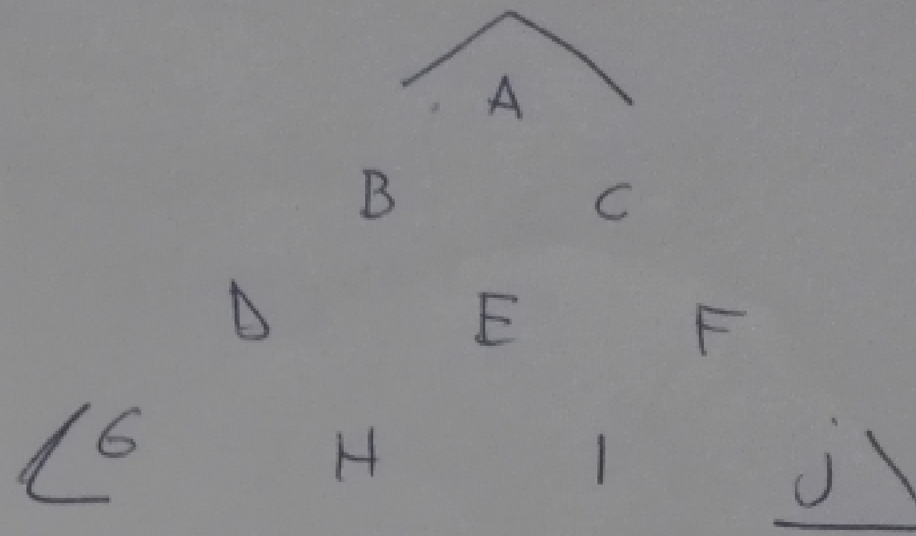
$-[a]@+[b]@@\left[\frac{k}{-1}@a\right]=0$

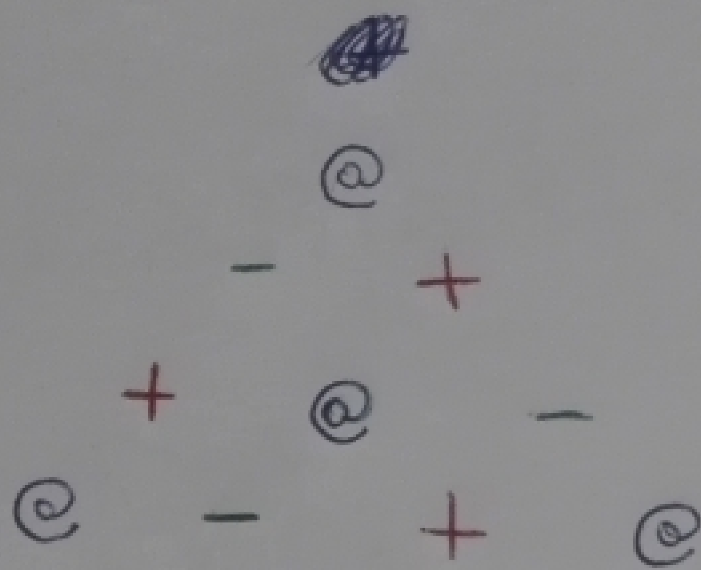
$-[a]@+[b]@@\left[\left(\frac{k}{-1}\right)\left(\frac{+1}{+1}\right)@a\right]=0$

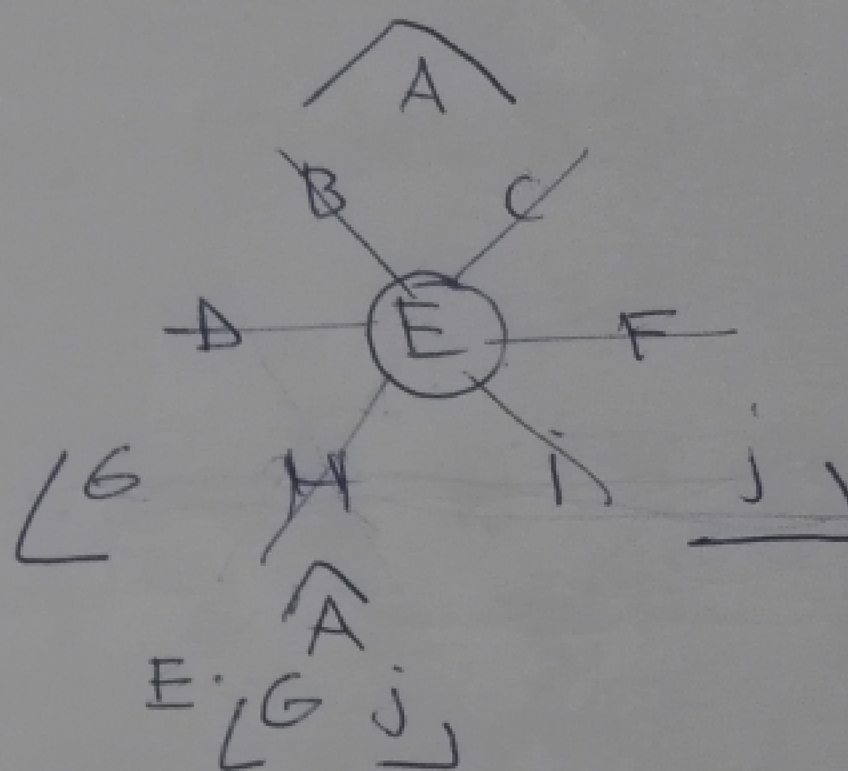
$-[a]@+[b]@@\left[\frac{+k}{1}@a\right]=0$

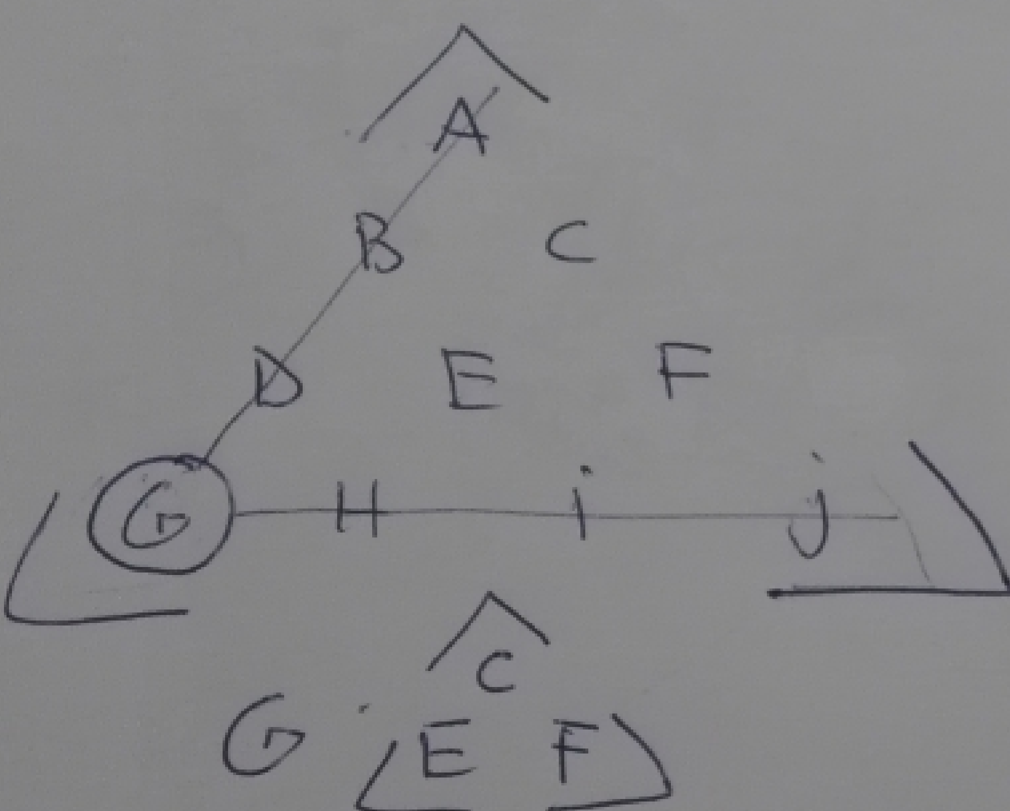
$-[a]@+[b]@@[+k@a]=0 \Longleftrightarrow$ 

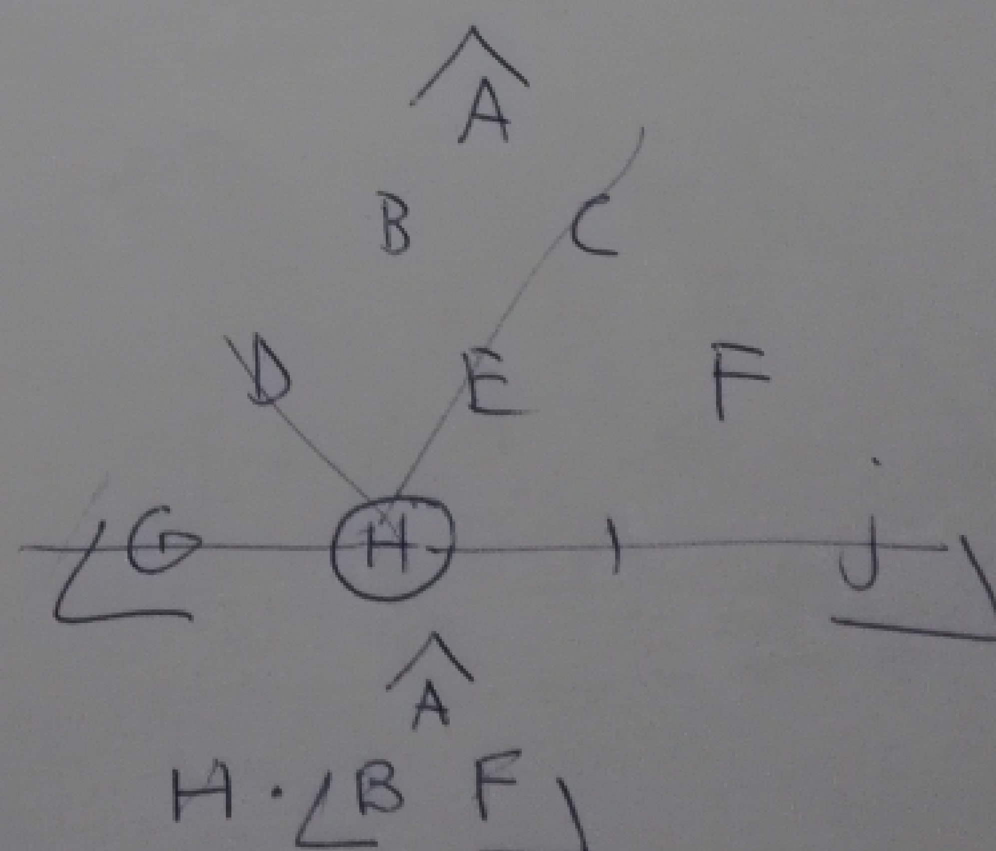



A
B C
D E F
G H I J


@
− +
+ @ −
@ − + @

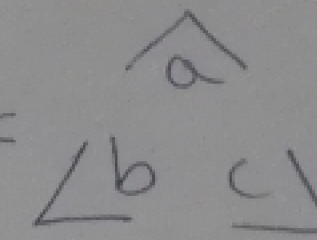

A
B C
D E F
G H I J
A
E · G J


A
B C
D E F
G H i J
C
G · E F


A
B C
D E F
G H I J
A
H · B F

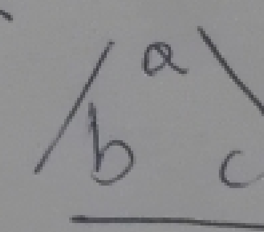
SQUARE MATRIX A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Triangular Matrix =

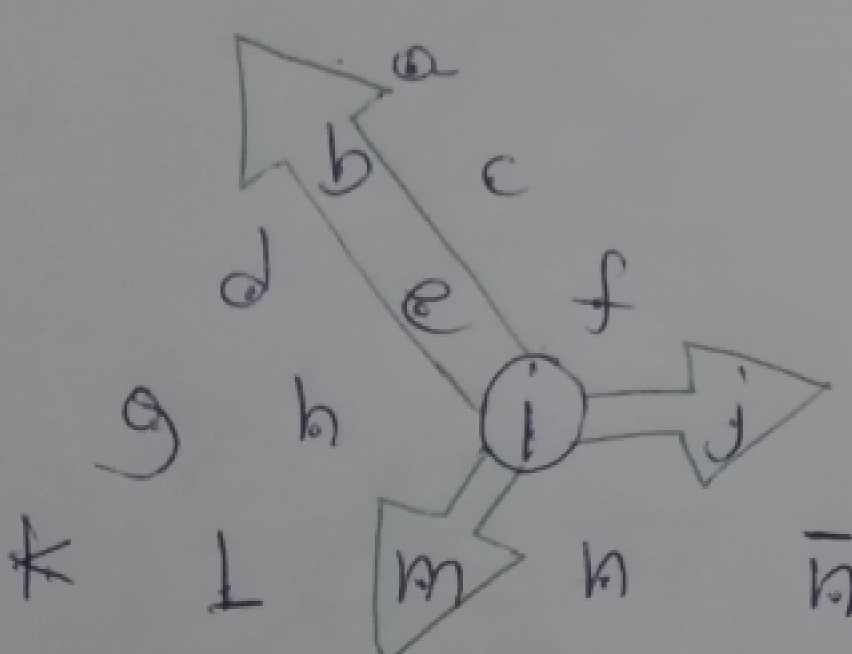


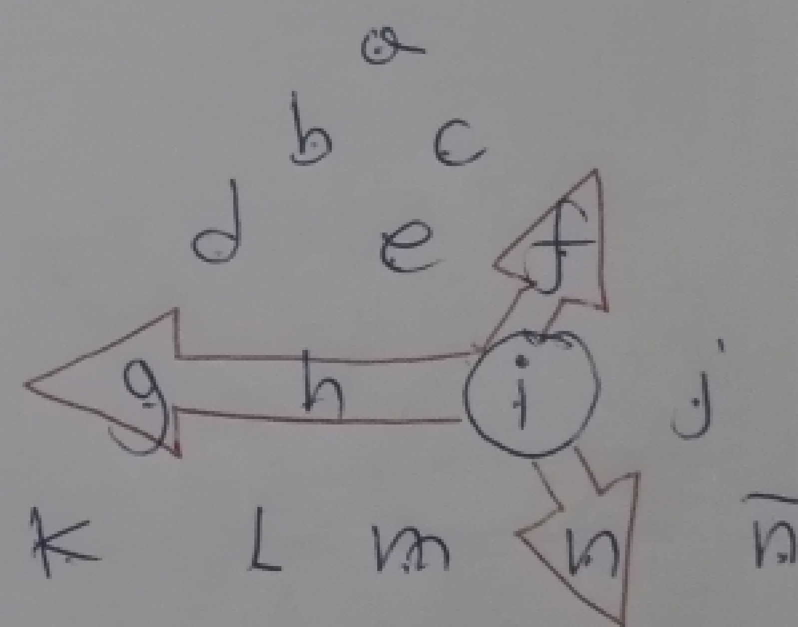
Determinant of a SQUARE MATRIX = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinant triangular =

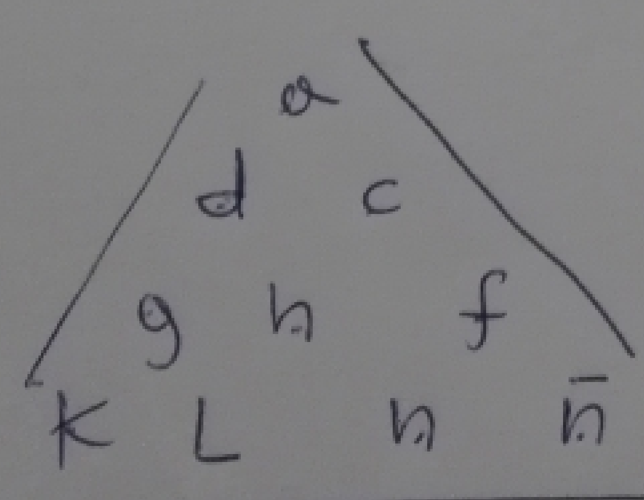


EXPANSION IN MINORS "??

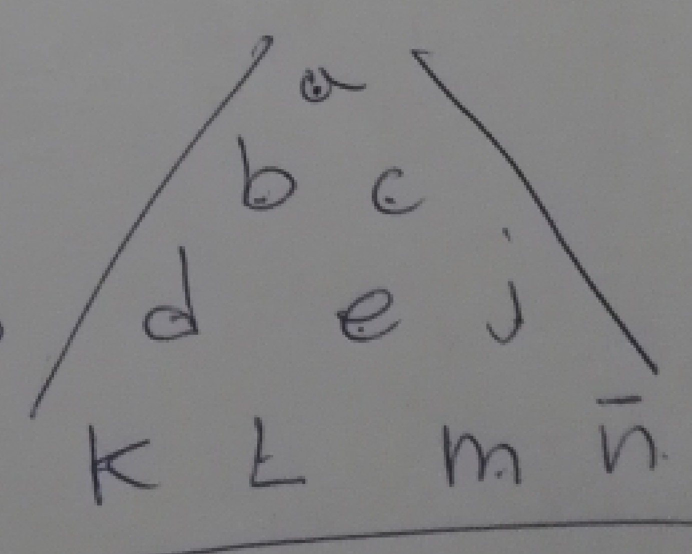




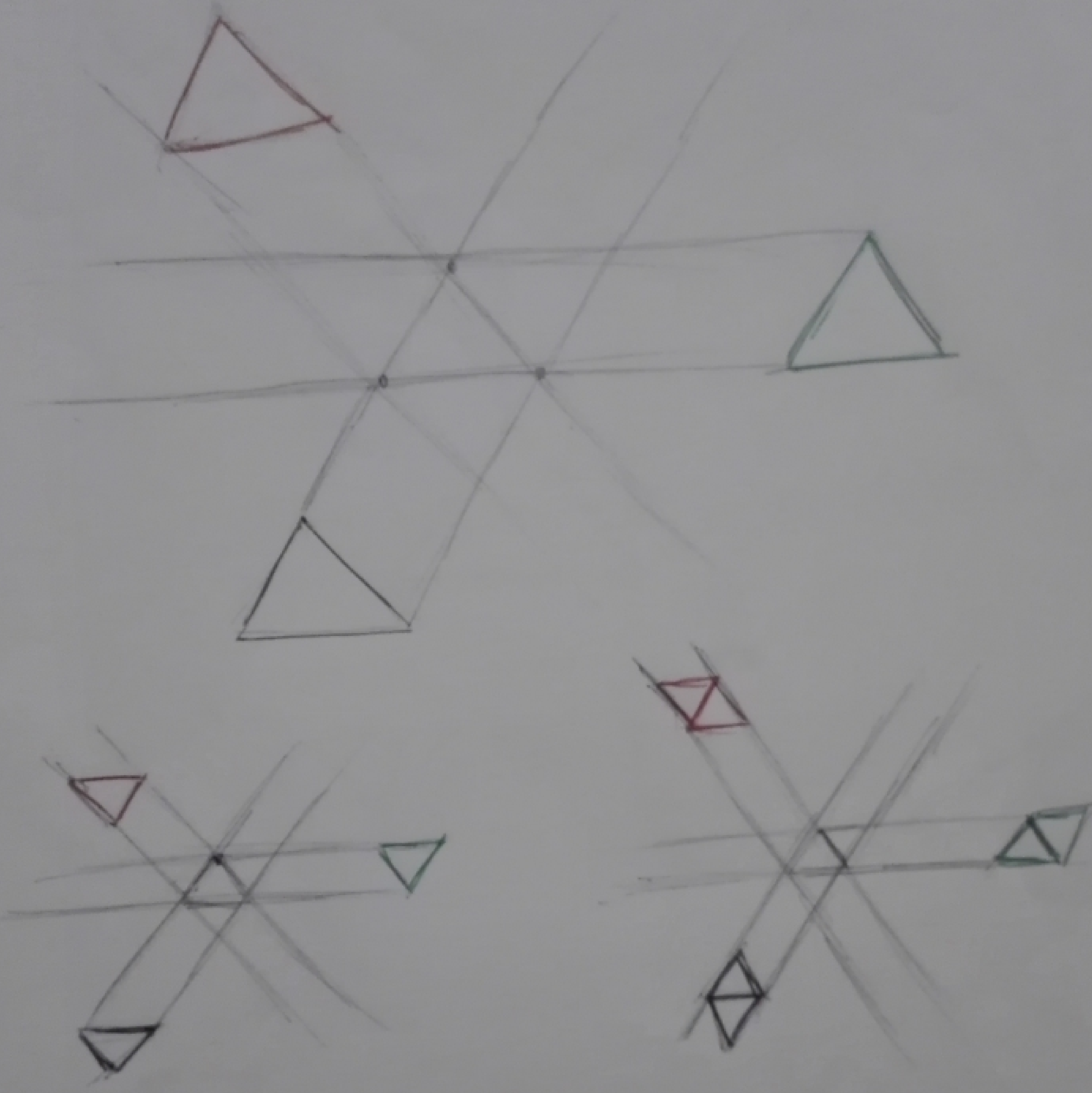
i ·



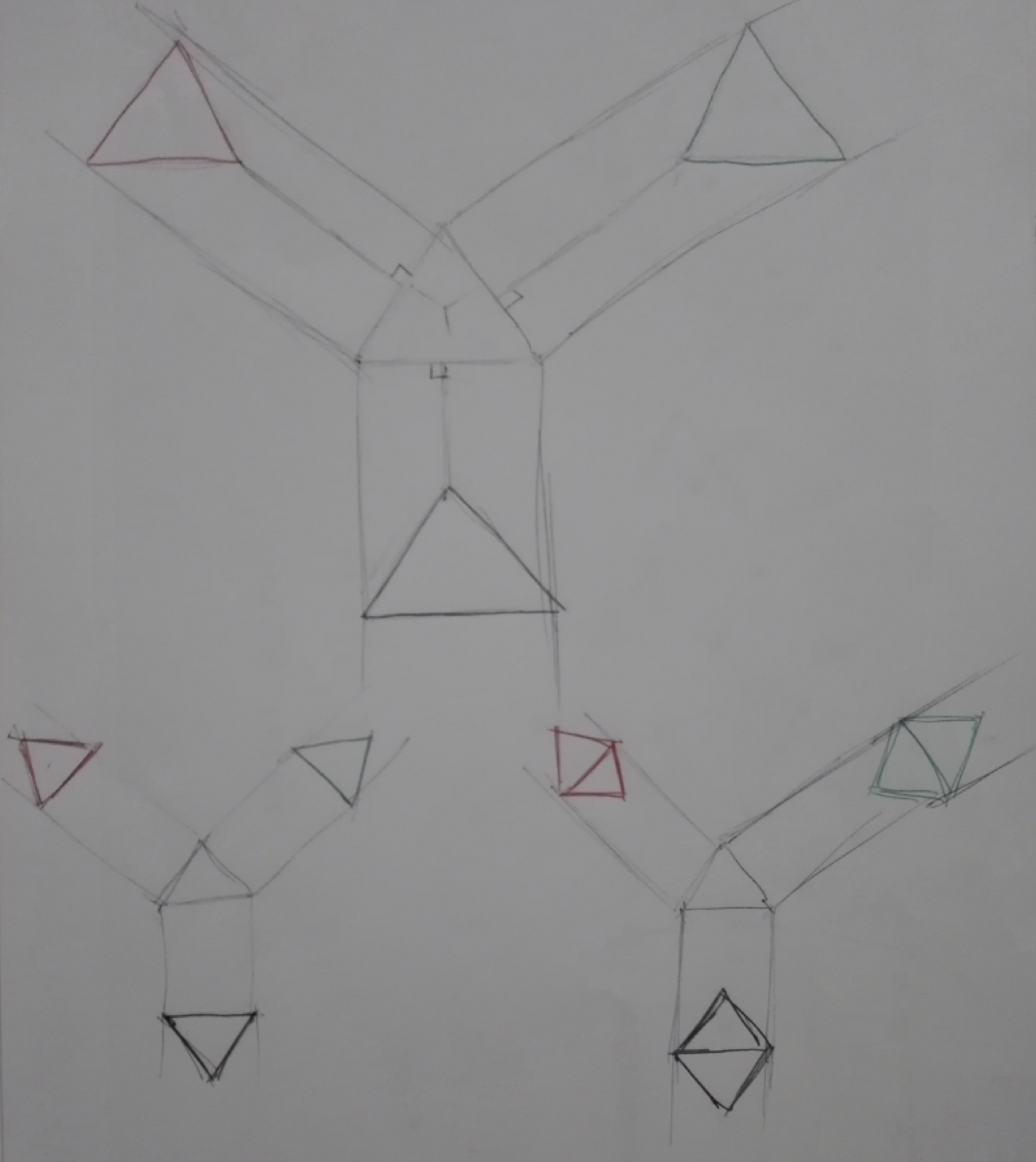
i ·



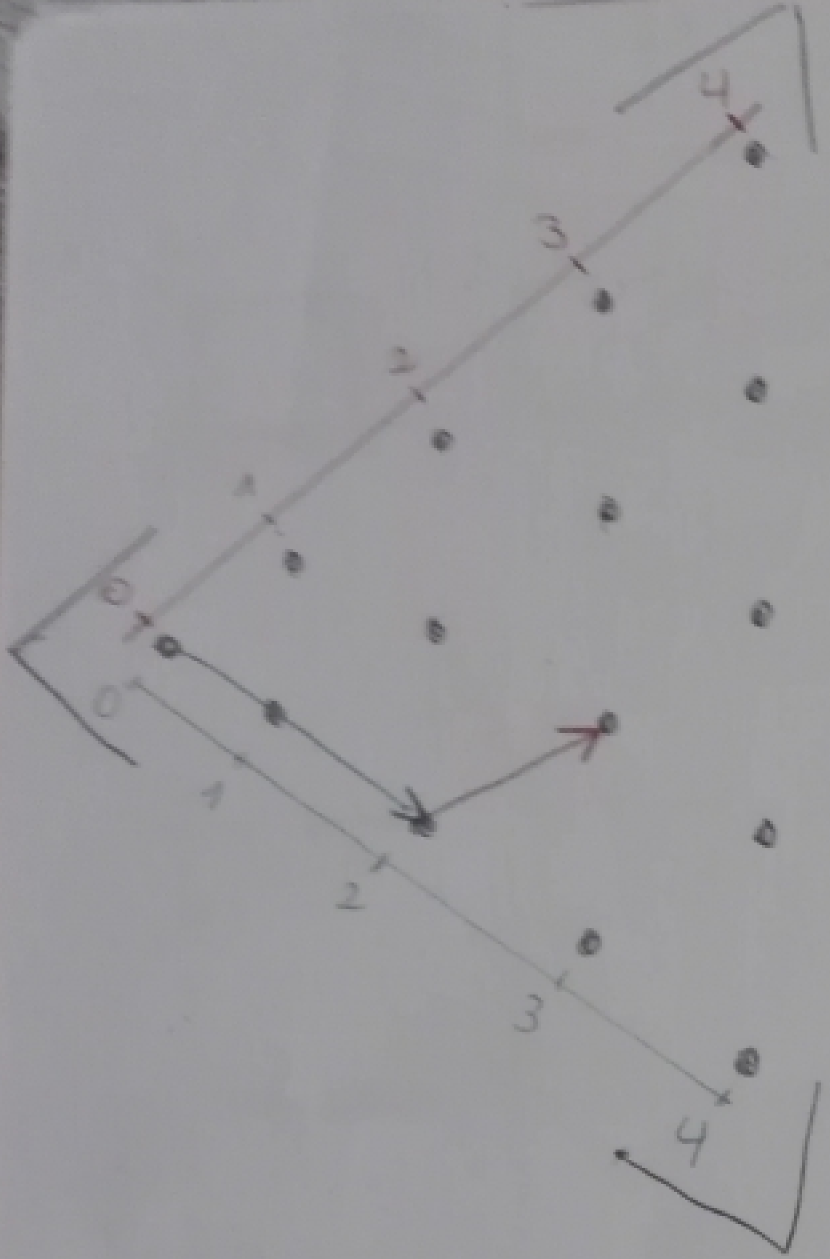
¿2 versions?



PRODUCT PARALLEL

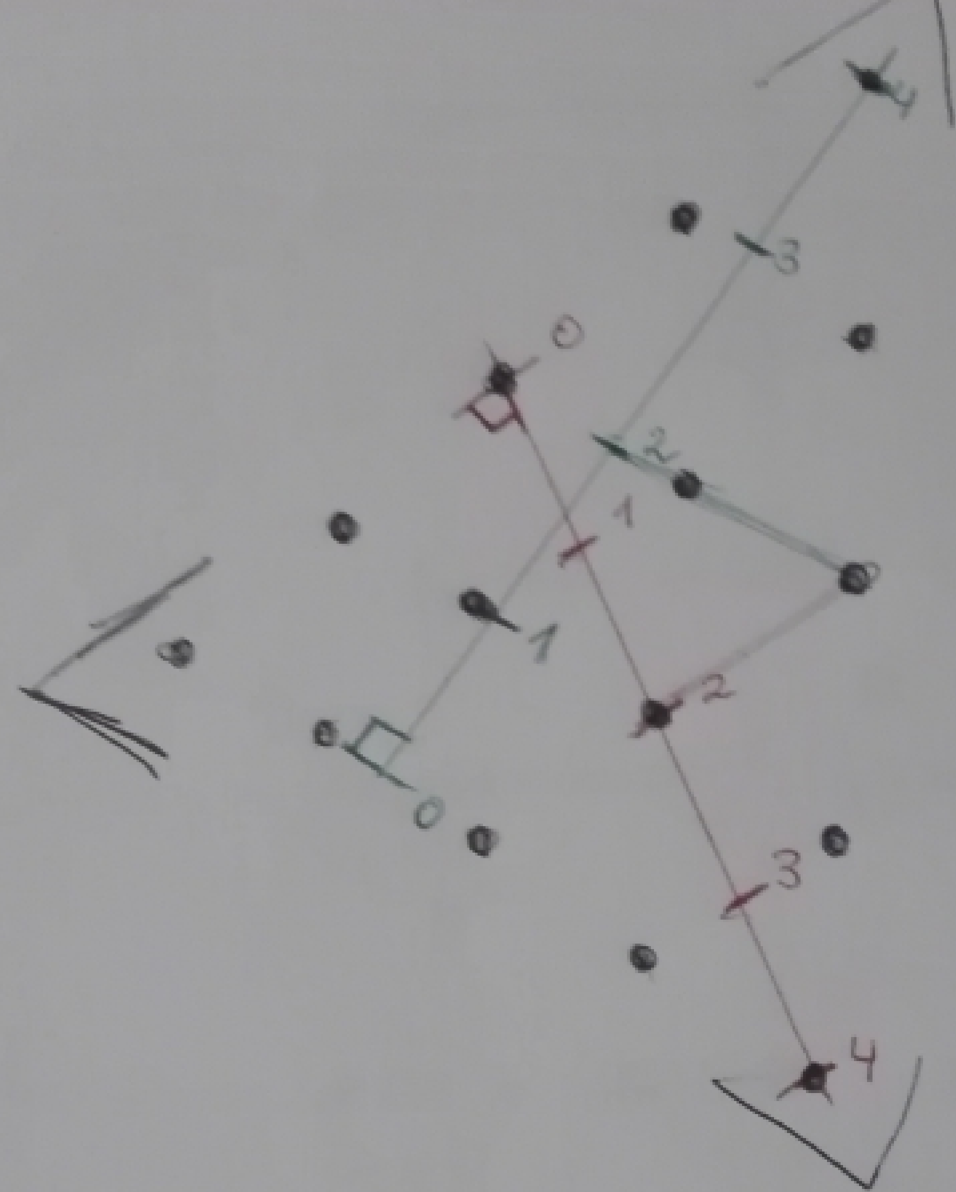


PRODUCTO PERPENDICULAR



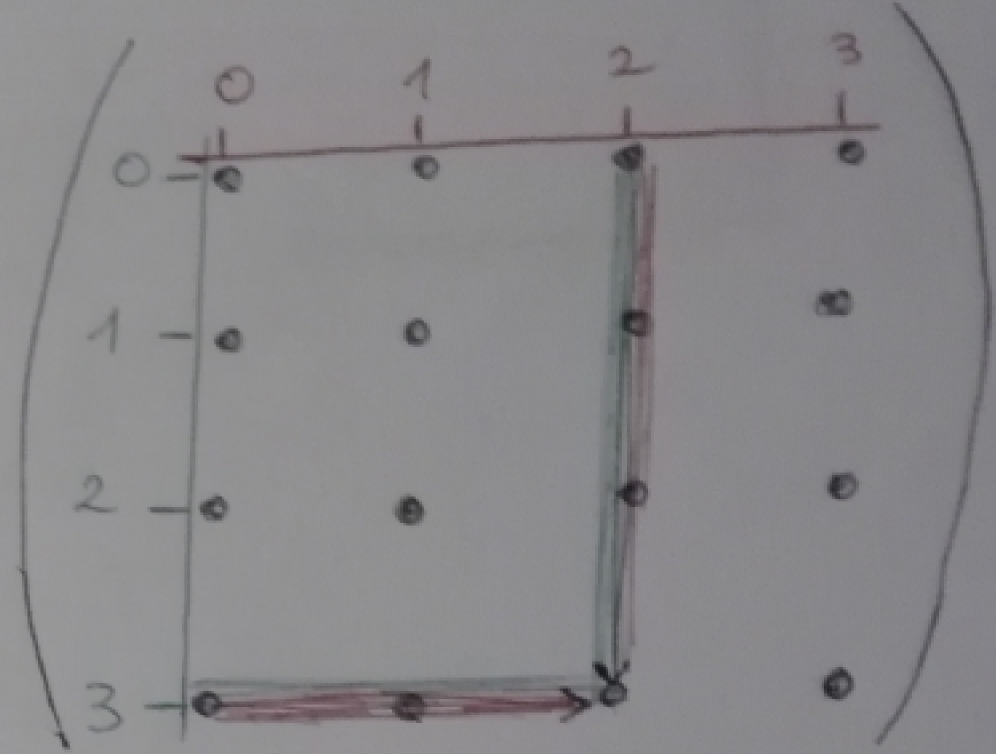
$A = [a_{ij}]_5$

WITH $i+j \leq 4$



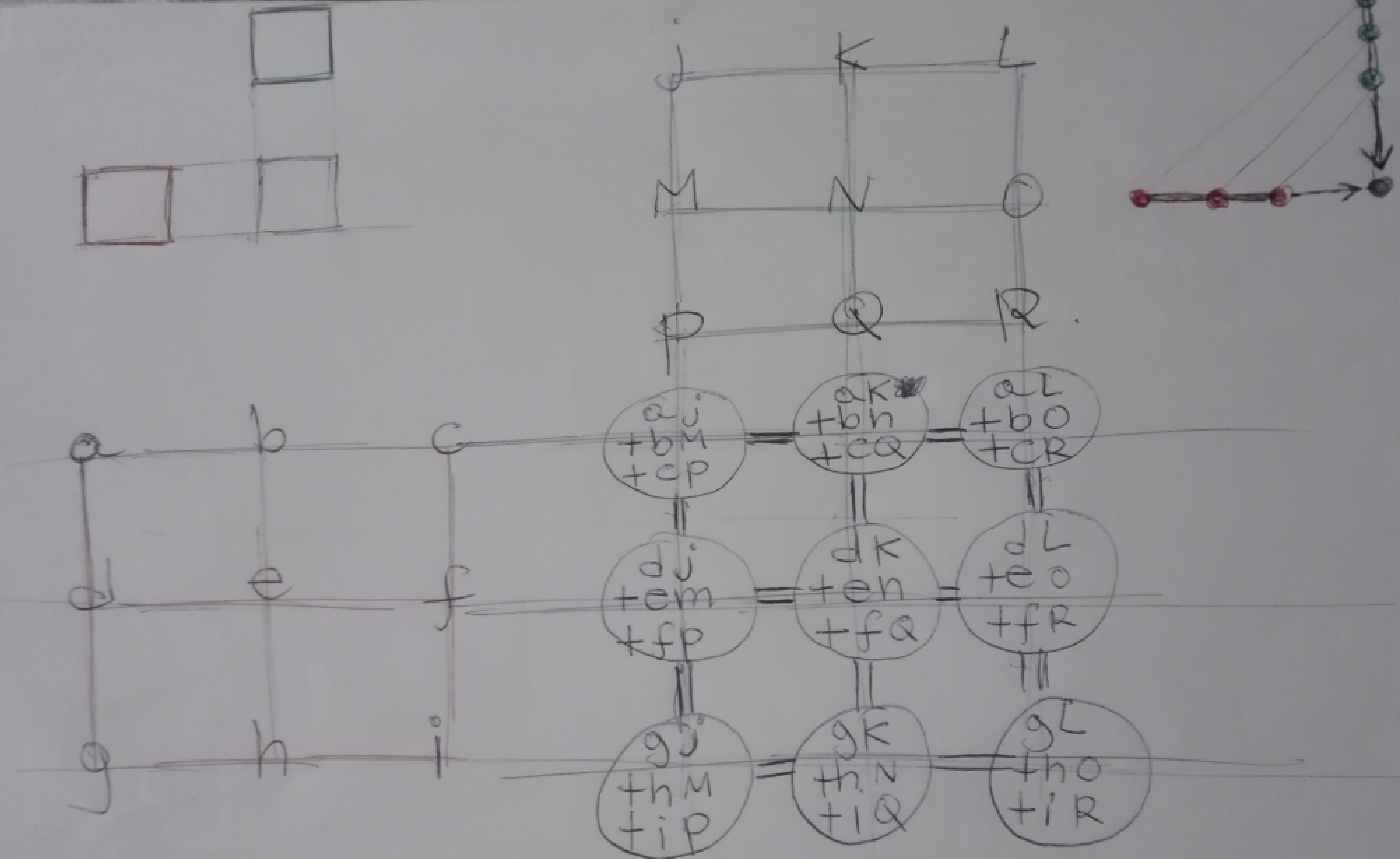
$A = [a_{ij}]_5$

WITH $i+j \leq 4$

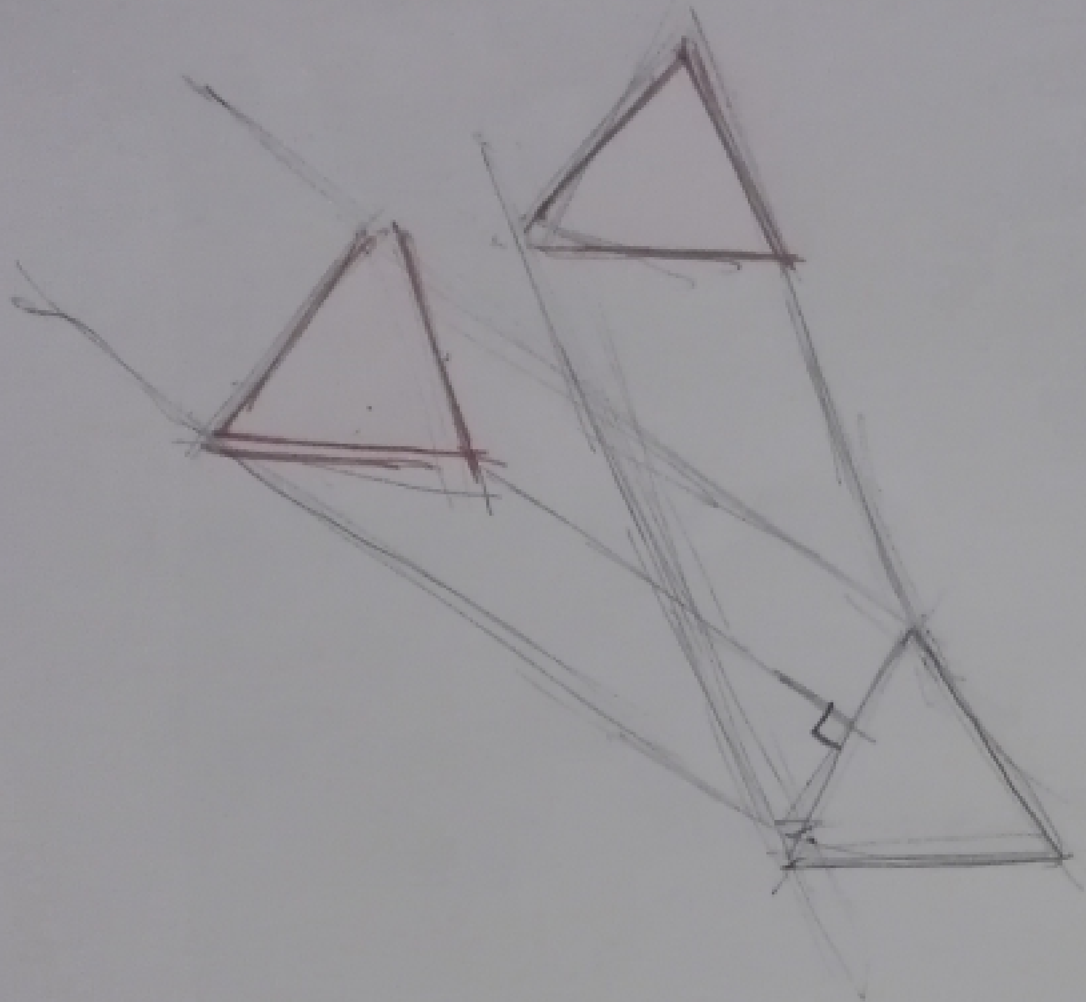


$A = [a_{i,j}]_{4,4}$

WITH $i \leq 3, j \leq 3$



VISUALIZATION OF SQUARE PRODUC



HYBRID PRODUCT.

Powers of the number "1,1"

$(1,1)^0 = 1$

$(1,1)^1 = 1,1$

$(1,1)^2 = 1,21$

$(1,1)^3 = 1,331$

$(1,1)^4 = 1,4641.$

As long as The overflow does not interfere, choose a suficient large base.

Aproximation of constant "e" by The left.

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{\substack{x \to 10 \\ b \to \infty}} \left(1_b + \frac{1_b}{x_b}\right)^{x_b} = \lim_{b \to \infty} \left(1_b + \frac{1_b}{10_b}\right)^{10_b}$$

b : base

$$= \lim_{b \to \infty} \left(1_b + 0,1_b\right)^{10_b}$$

$$= \lim_{b \to \infty} \left(1,1_b\right)^{10_b}$$

$$\lim_{x \to \infty} \left(1 + {}^{\;1}_{x \curlyvee 1} + {}^{\;1}_{1 \curlyvee x}\right)^x = e_{<}$$

$$x^2 + y^2 = 1 \Longrightarrow x^2 = 1 - y^2$$

$$x^3 + y^3 + z^3 = 1 \longrightarrow x^3 = 1 - y^3 - z^3$$

plot a surface in space

$\downarrow$

$$+(x^3 + y^3 + z^3) + -(1) = 0 \qquad (P_2)$$

Reinterpret (-1) by $\bar{1}$ in P_3.

P_2 Aditive inverse

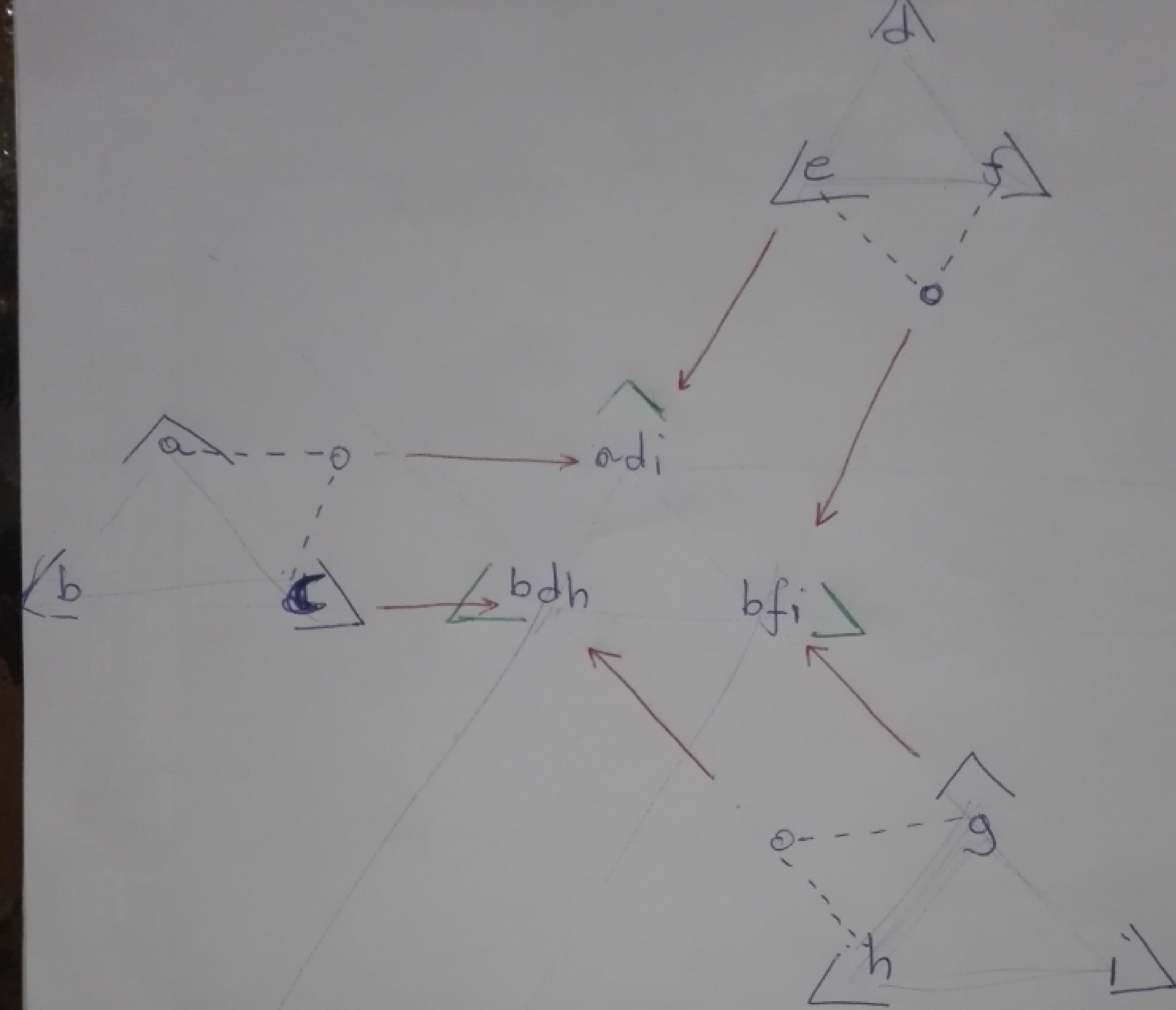
$$(x^3 @ y^3 @ z^3) @ (\bar{1}) = 0$$

$$(x^3 @ y^3 @ z^3) @ -(1) @ +(1) = 0$$

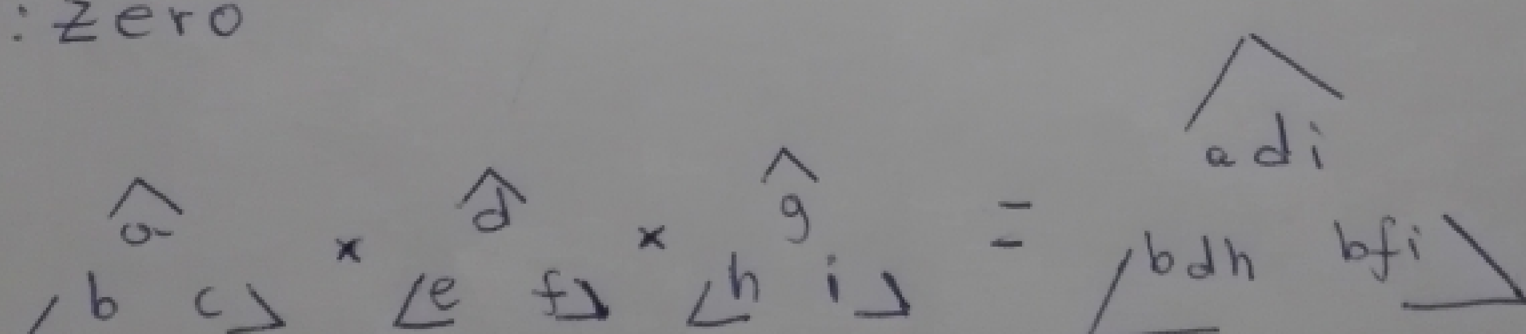
$$(x^3 @ y^3 @ z^3) \prec \begin{matrix} 1 \\ 1 \end{matrix}$$

$$x^3 \prec \begin{matrix} 1 @ -y^3 \\ 1 @ +z^3 \end{matrix}$$

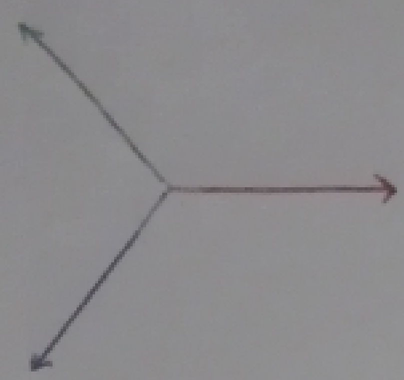
plot a curve in space



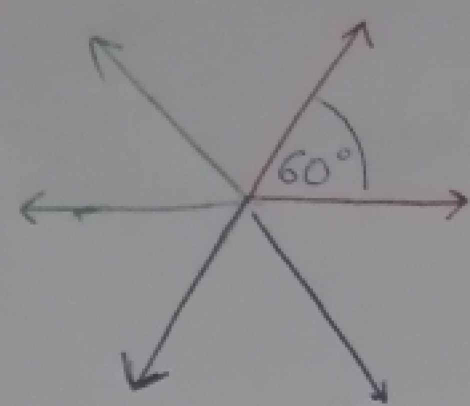
o : zero



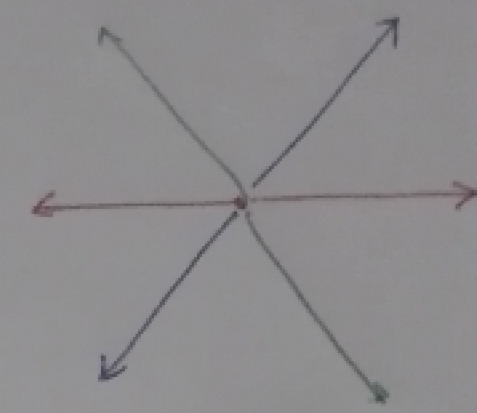
in a P2 arrangement



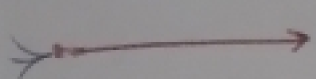
(a, b, c)
P_3



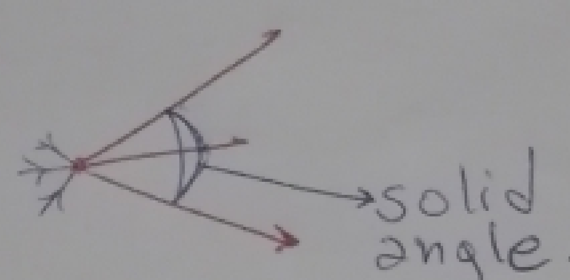
(a, b)
P_3-ized V1



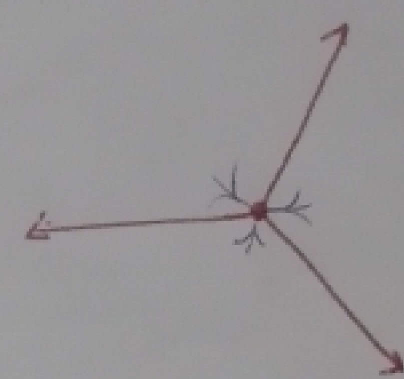
(a, b)
P_3-ized V2.



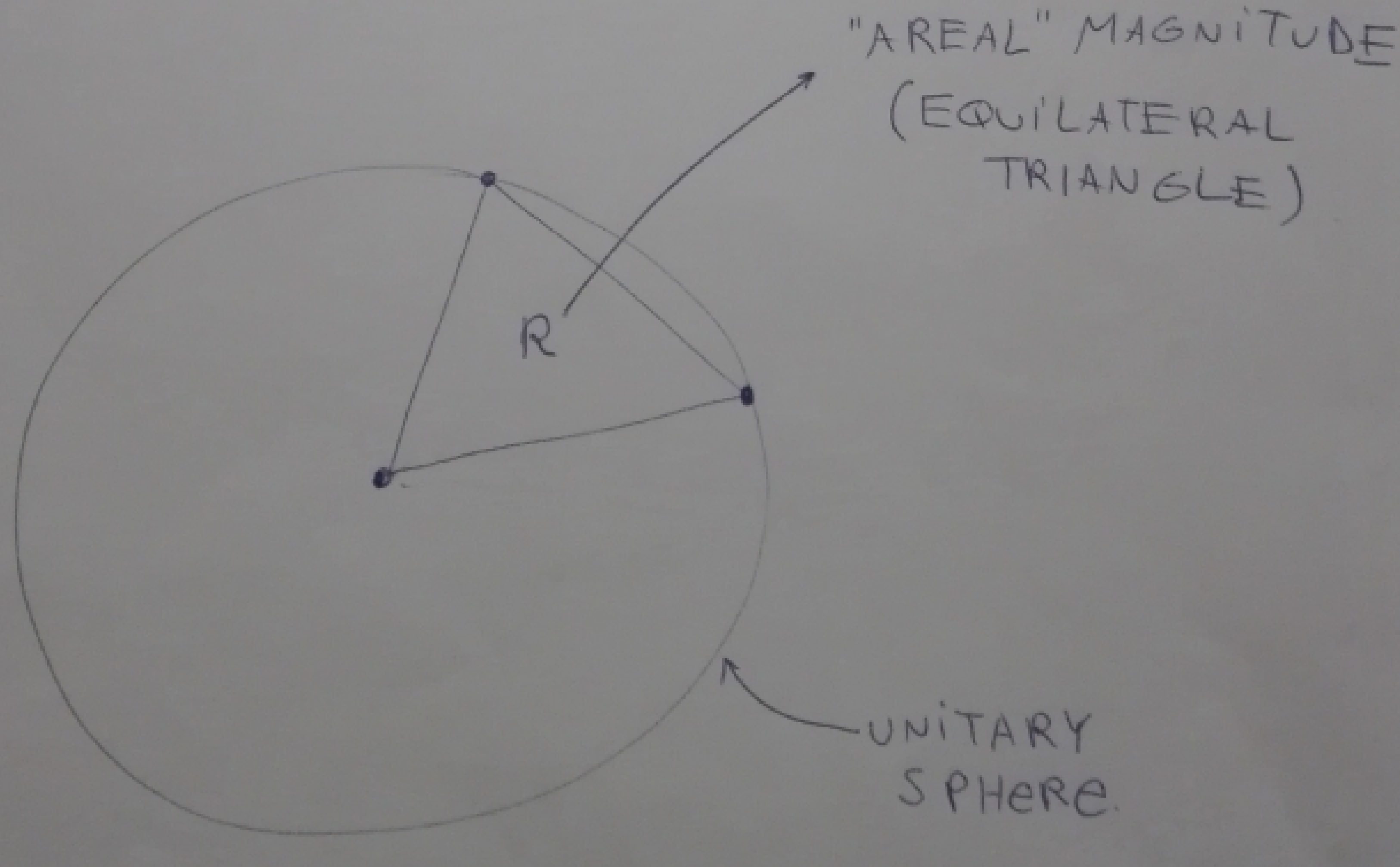
(a, b, c, d)
P_4

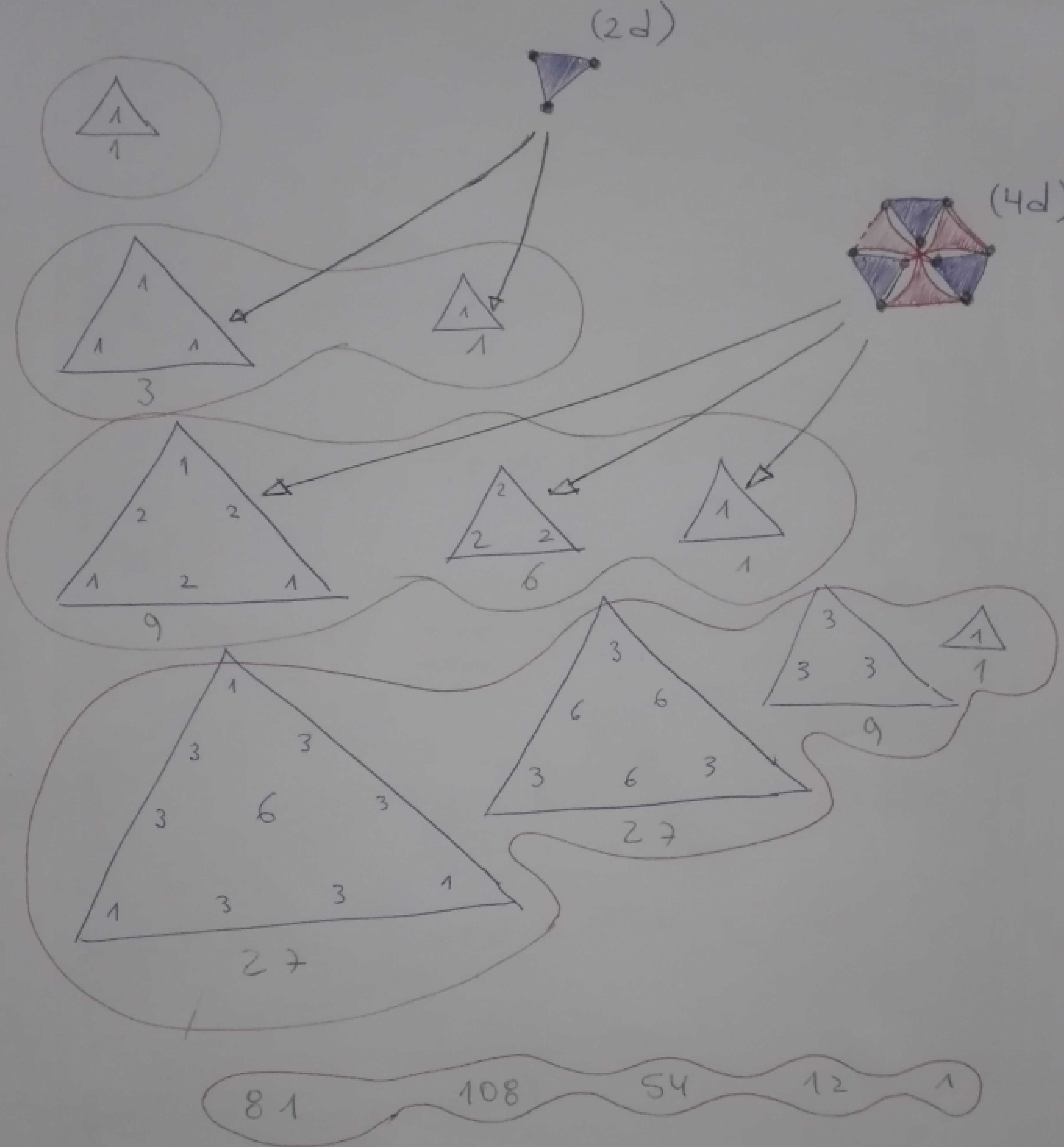


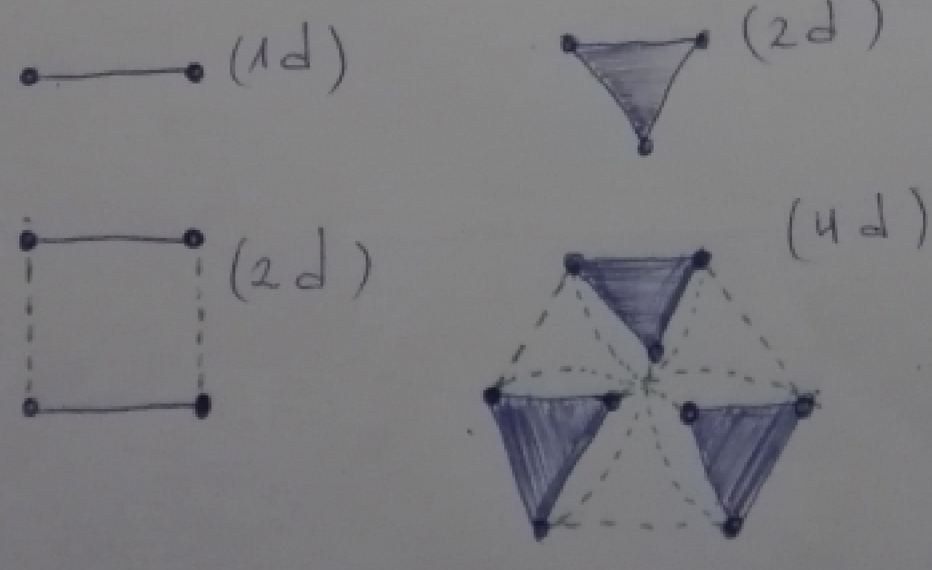
(a, b, c)
P_4-ized V1



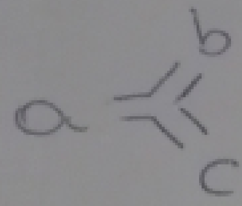
(a, b, c)
P_4-ized V2.
"positive R4 Rays
in a P_3 arrangement"



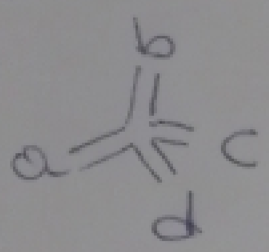

(2d)
(4d)
1
3
1
9
6
1
27
9
1
27
81 108 54 12 1


(1d)
(2d)
(2d)
(4d)

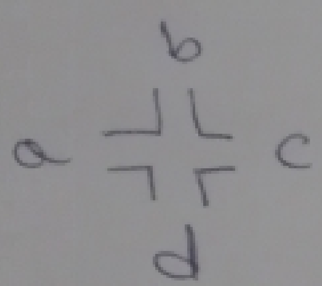
$a = b \quad \Longleftrightarrow \quad (a)@-(b) = 0$



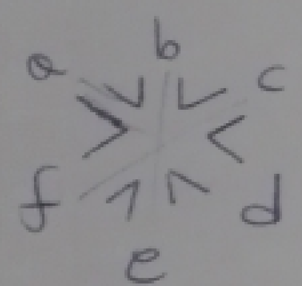
$\Longleftrightarrow \quad (a)@-(b)@+(c)=0$



$\Longleftrightarrow \quad (a)@-(b)@+(c)@*(d)=0$



$\Longleftrightarrow \quad (a)@i(b)@-(c)@-i(d)=0$



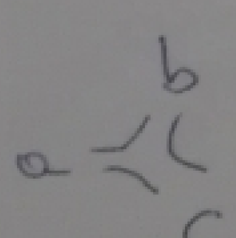
$\Longleftrightarrow \quad (a)@-\text{ϟ}(b)@\Delta(c)@-(d)@\text{ϟ}(e)@-\Delta(f)=0$

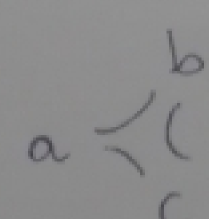
$a = b$

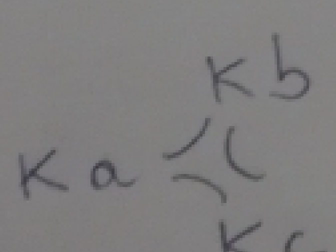
$ka = kb$

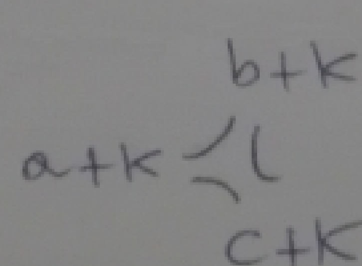
$a = b$

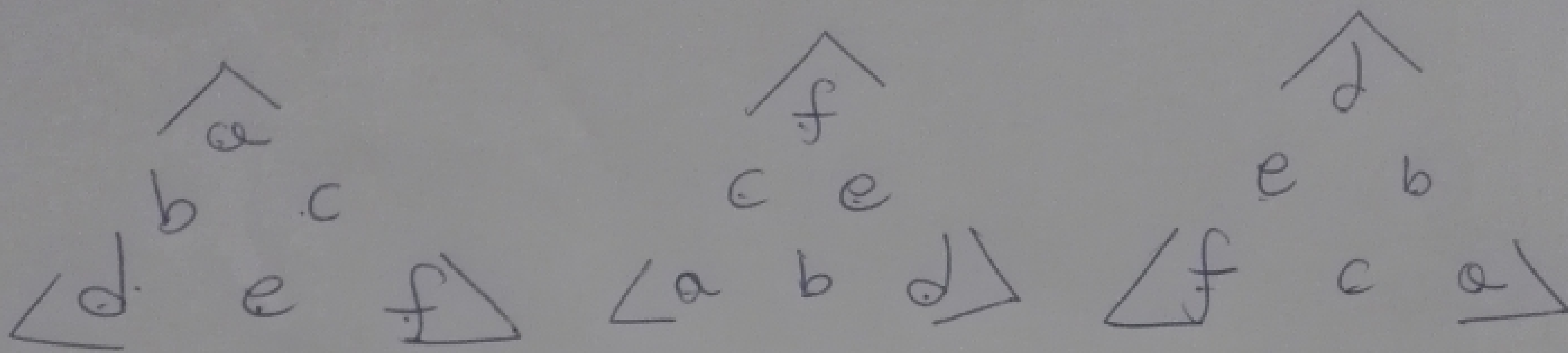
$a+k = b+k$

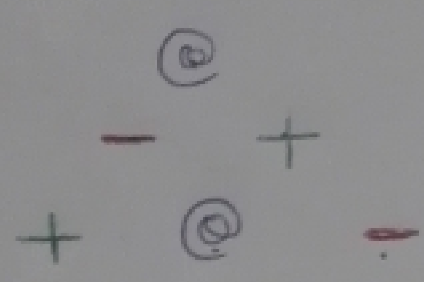




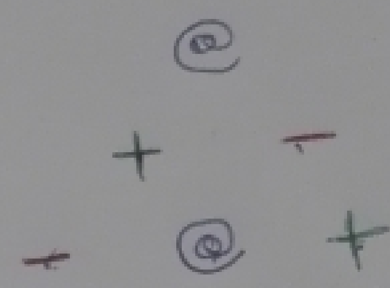




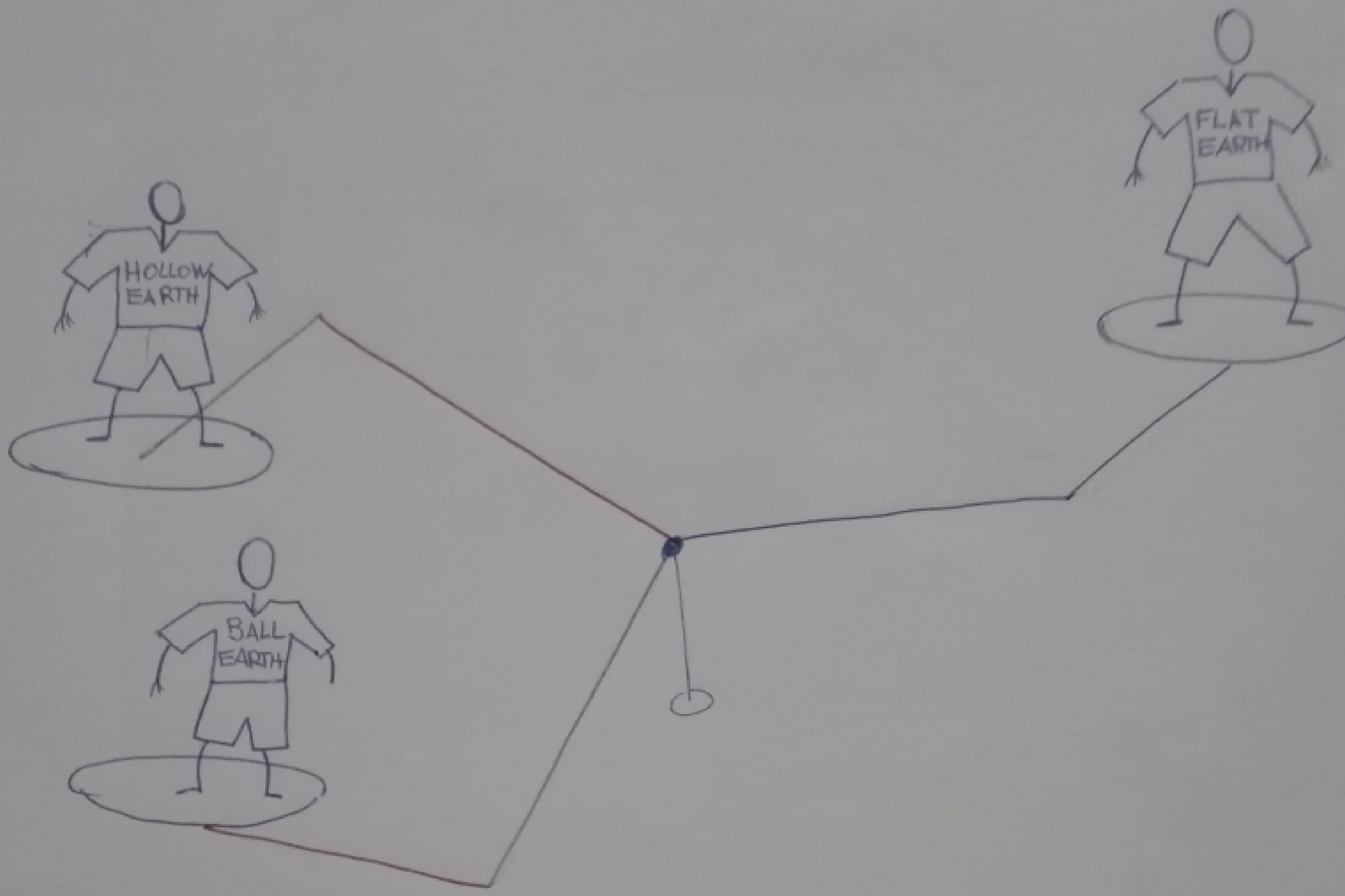


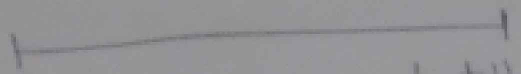


or

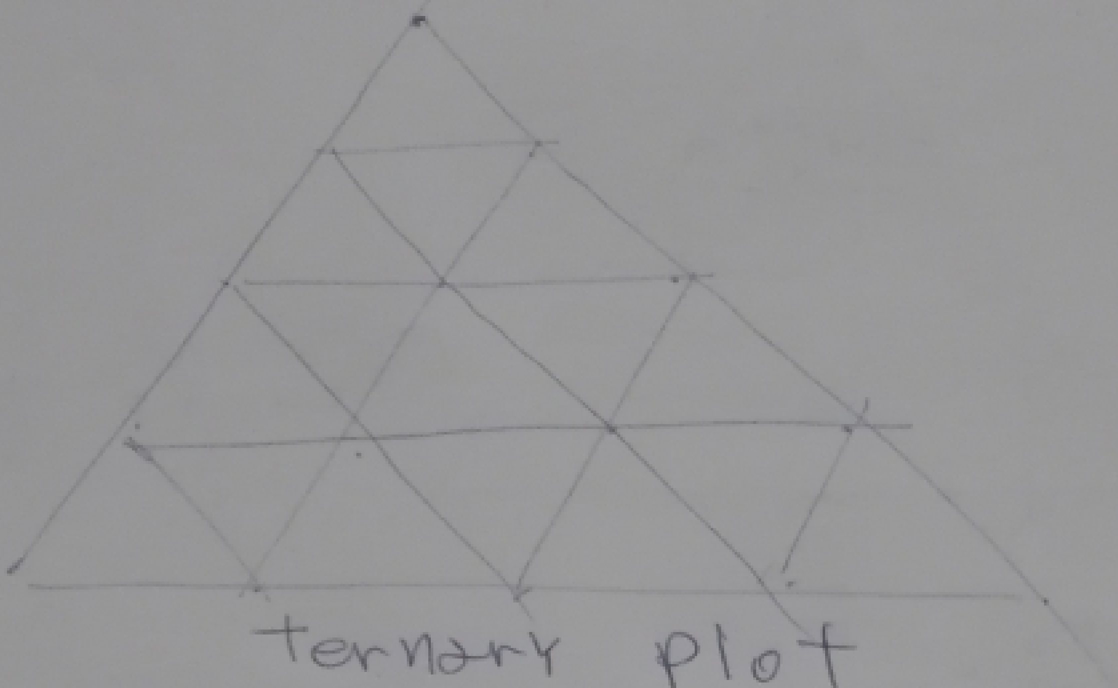


Rotation Rather trasposition ?


HOLLOW EARTH
BALL EARTH
FLAT EARTH

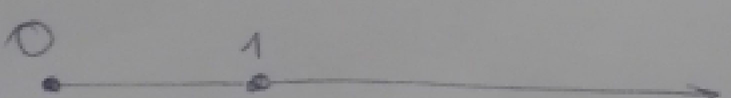


"binary plot".

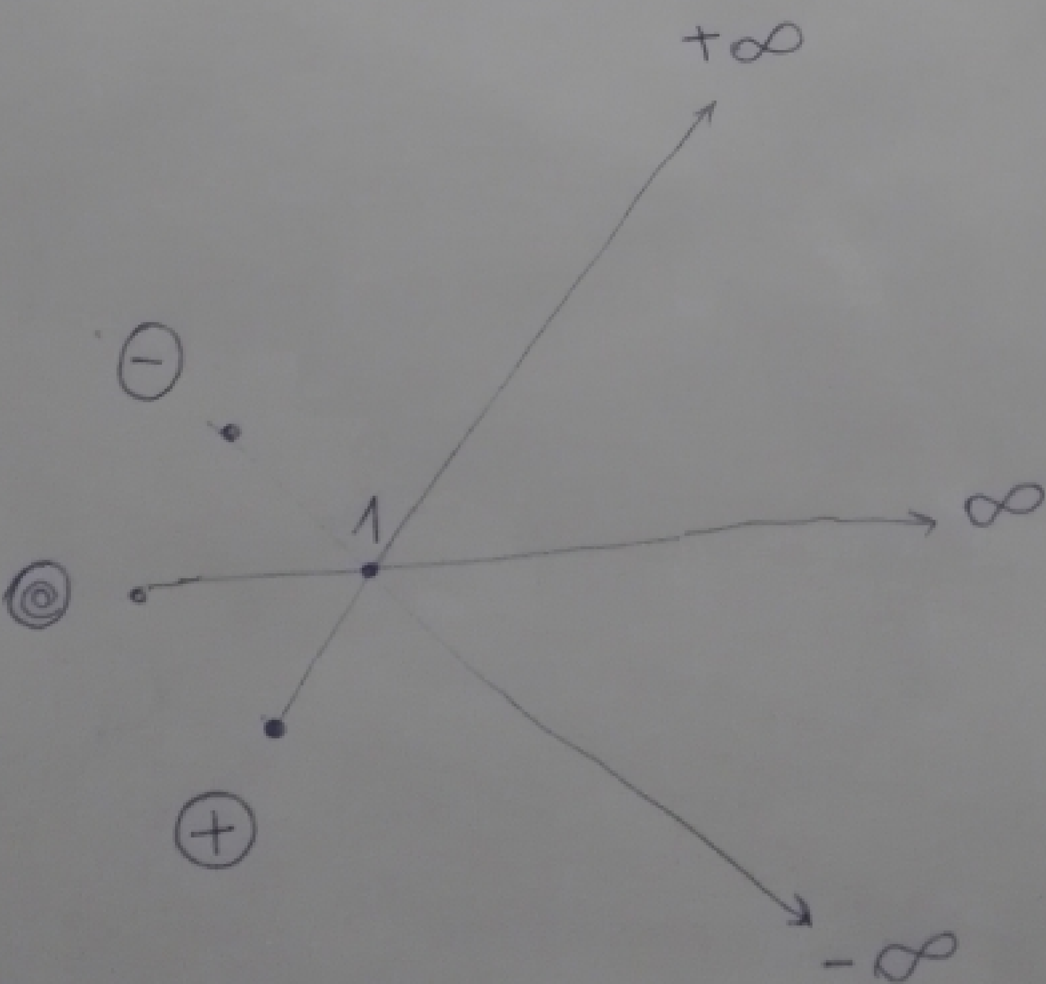


ternary plot

VS

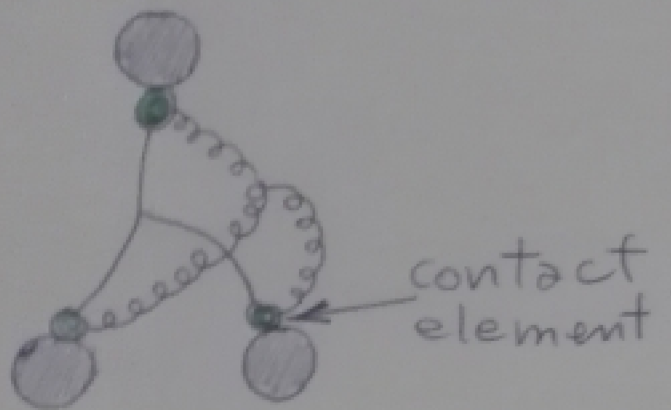


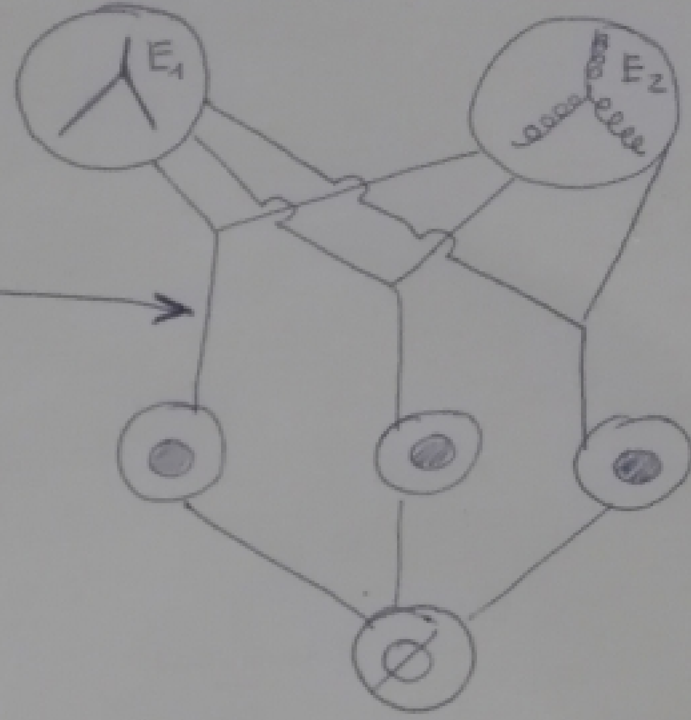
PD_2



PD_3

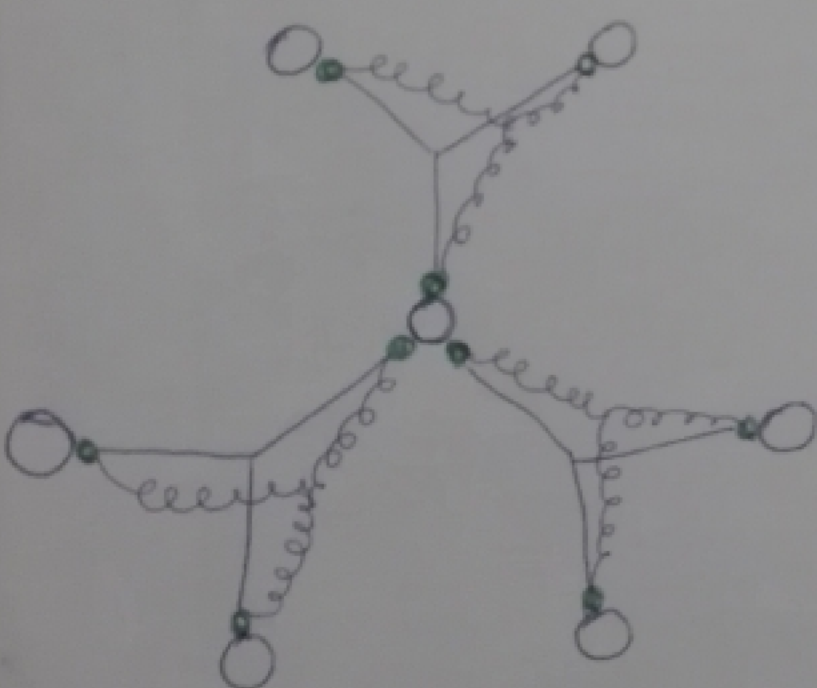
geometric constructor.

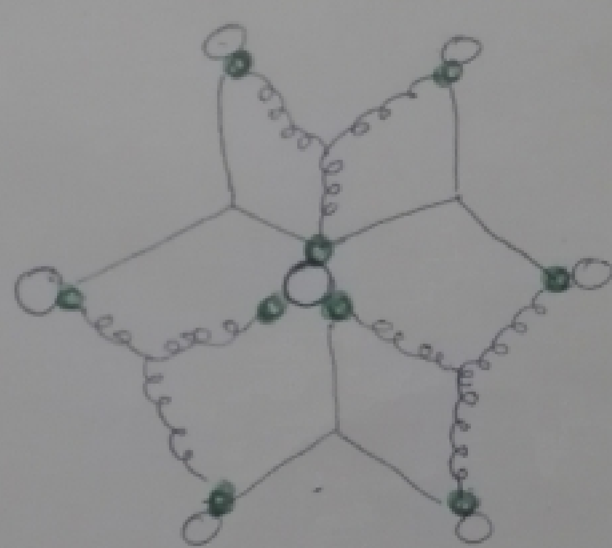


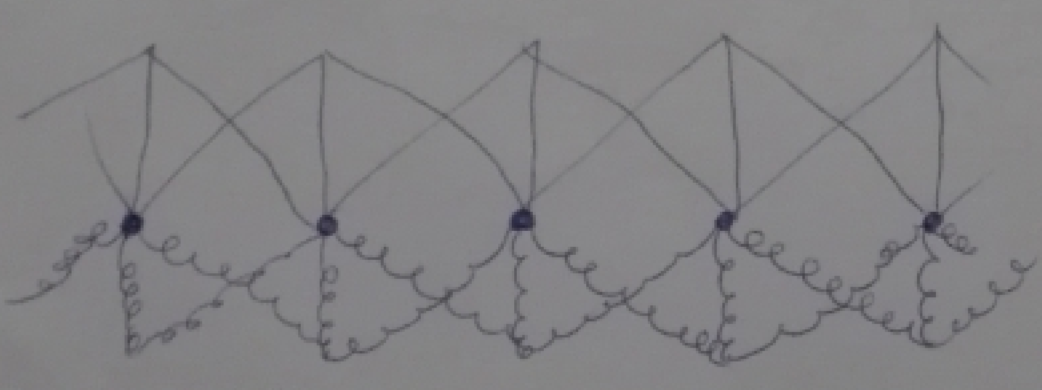


Hasse diagram
+
Contact graph.

N° of vertexes = N° of E_1 = N° of E_2.







Commutative arithmetics
Product n-ary (case 2 and 3)

$(+)(+) = (+)$
$(-)(-) = (+)$

N° mode = 2

$(-)\cdot(+) = (-)$

N° Mode = 1

$(-)(-)(-) = (*)$
$(+)(+)(+) = (*)$
$(*)(*)(*) = (*)$

N° mode = 3

$(-)(+)(+) = (+)$
$(-)(*)(*) = (+)$
$(+)(-)(-) = (+)$
$(+)(*)(*) = (+)$
$(*)(-)(-) = (+)$
$(*)(+)(+) = (+)$

N° mode = 2

$(-)(+)(*) = (-)$

N° mode = 1

---

other possible way to produce alternative product rule in P_n is

(1) Reduced Residue Systems
(modular product

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