Proof of Riemann hypothesis and other prime conjectures

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Abstract

Riemann hypothesis stands proved in three different ways.To prove Riemann hypothesis from the functional equation concept of Delta function is introduced similar to Gamma and Pi function. Zeta values are renormalised to remove the poles of zeta function. Extending sum to product rule fundamental formula of numbers are defined which then helps proving other prime conjectures namely goldbach conjecture, twin prime conjecture etc.

1 Euler the Grandfather of zeta function

In 1737, Leonard Euler published a paper where he derived a tricky formula that pointed to a wonderful connection between the infinite sum of the reciprocals of all natural integers (zeta function in its simplest form) and all prime numbers.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+...=\frac{2.3.5.7.11....}{1.2.4.6.8....}$$

Now:

$$1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4...=\frac{2}{1}$$

$$1+\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^4...=\frac{3}{2}$$

$\vdots$

Euler product form of zeta function when $s>1$:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\left(1+\frac{1}{p^s}+\frac{1}{p^{2s}}+\frac{1}{p^{3s}}+\frac{1}{p^{4s}}...\right)$$

Equivalent to:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\frac{1}{1-P^{-s}}$$

To carry out the multiplication on the right, we need to pick up exactly one term from every sum that is a factor in the product and, since every integer admits a unique prime factorization, the reciprocal of every integer will be obtained in this manner - each exactly once.

In the year of 1896 - Jacques Hadamard and Charles Jean de la Valle-Poussin independently prove the prime number theorem which essentially says that if there exists a limit to the ratio of primes upto a given number and that numbers natural logarithm, that should be equal to 1. When I started reading about number theory I wondered that if prime number theorem is proved then what is left. The biggest job is done. I questioned myself why zeta function cannot be defined at 1. Calculus has got set of rules for checking convergence of any infinite series, sometime especially when we are enclosing infinities to unity, those rules falls short to check the convergence of infinite series. In spite of that Euler was successful proving sum to product form and calculated zeta values for some numbers by hand only. Leopold Kronecker proved and interpreted Euler's formulas is the outcome of passing to the right-sided limit as $s\to 1^+$. I decided I will stick to Grandpa Eulers approach in attacking the problem.

2 Riemann the father of zeta function

Riemann showed that zeta function have the property of analytic continuation in the whole complex plane except for s=1 where the zeta function has its pole. Riemann Hypothesis is all about non trivial zeros of zeta function. There are trivial zeros which occur at every negative even integer. There are no zeros for $s > 1$. All other zeros lies at a critical strip $0 < s < 1$. In this critical strip he conjectured that all non trivial zeros lies on a critical line of the form of $z = \frac{1}{2} \pm iy$ i.e. the real part of all those complex numbers equals $\frac{1}{2}$.The zeta function satisfies Riemann's functional equation :

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

3 Proof of Riemann Hypothesis

In this section we shall prove Riemann Hypothesis in different ways.

3.1 Proof using Riemanns functional equation

Multiplying both side of Riemanns functional equation by $(s-1)$ we get

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$\zeta(1-s) = \frac{(1-s)\zeta(s)}{2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)}$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$ and $\Gamma(2-1) = \Gamma(1) = 1$

$$\zeta(0) = \frac{-1}{2^1 \pi^0 \sin\left(\frac{\pi}{2}\right)} = -\frac{1}{2}$$

Examining the functional equation we shall observe that the pole of zeta function at $Re(s) = 1$ is solely attributable to the pole of Gamma function. In the critical strip $0 < s < 1$ Delta function (see explanation) holds equally good if not better for factorial function. As zeta function have got the holomorphic property the act of stretching or squeezing preserves the holomorphic character. Using this property we can remove the pole of zeta function. Introducing Delta function for factorial we can remove the poles of Gamma and Pi function and rewrite the functional equation as follows(see explanation below):

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

Now Putting $s = 1$we get:

$$\zeta(1) = 2^1 \pi^{(1-1)} \sin\left(\frac{\pi}{2}\right)\Pi(2-1)\zeta(0) = 1$$

zeta function is now defined on entire $\mathbb{C}$, and as such it becomes an entire function. In complex analysis, Liouville's theorem states that every bounded entire function must be constant. That is, every holomorphic function f for which there exists a positive number M such that $|f(z)| \leq M$ for all z in $\mathbb{C}$ is constant. Being an entire function zeta function is constant as none of the values of zeta function do not exceed $M = \zeta(2) = \frac{\pi^2}{6}$.Maximum modulus principle further requires that non constant holomorphic functions attain maximum modulus on the boundary of the unit circle. Being a constant function zeta function duly complies with maximum modulus principle as it reaches maximum modulus $\frac{\pi^2}{6}$ outside the unit circle i.e. on the boundary of the double unit circle. Gauss's mean value theorem requires that in case a function is bounded in some neighborhood, then its mean value shall occur at the center of the unit circle drawn on the neighborhood. $|\zeta(0)| = \frac{1}{2}$ is the mean modulus of entire zeta function. Inverse of maximum modulus principle implies points on half unit circle give the minimum modulus or zeros of zeta function. Minimum modulus principle requires holomorphic functions having all non zero values shall attain minimum modulus on the boundary of the unit circle. Having lots of zero values holomorphic zeta function do not attain minimum modulus on the boundary of the unit circle rather points on half unit circle gives the minimum modulus or zeros of zeta function. Everything put together it implies that points on the half unit circle will mostly be the zeros of the zeta function which all have $\pm\frac{1}{2}$ as real part as Riemann rightly hypothesized.

Putting $s = \frac{1}{2}$ in $\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$

$$\boxed{\begin{gathered}
\zeta\left(\frac{1}{2}\right) = 2^{\frac{1}{2}} \pi^{(1-\frac{1}{2})} \sin\left(\frac{\pi}{2.2}\right)\Pi\left(\frac{3}{2}\right)\zeta\left(\frac{1}{2}\right) \\
\zeta\left(\frac{1}{2}\right)\left(1 + \frac{3\sqrt{2.\pi.\pi}}{4.\sqrt{2}}\right) = 0 \\
\zeta\left(\frac{1}{2}\right)\left(1 + \frac{3\pi}{4}\right) = 0 \\
\zeta\left(\frac{1}{2}\right) = 0
\end{gathered}}$$

Therefore principal value of $\zeta(\frac{1}{2})$ is zero and Riemann Hypothesis holds good.

Explanation 1 *Euler in the year 1730 proved that the following indefinite integral gives the factorial of x for all real positive numbers,*

$$!x = \Pi(x) = \int_0^\infty t^x e^{-t} dt, x > 1$$

Eulers Pi function satisfies the following recurrence relation for all positive real numbers.

$$\Pi(x+1) = (x+1)\Pi(x), x > 0$$

In 1768, Euler defined Gamma function, $\Gamma(x)$, and extended the concept of factorials to all real negative numbers, except zero and negative integers. $\Gamma(x)$, is an extension of the Pi function, with its argument shifted down by 1 unit.

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

Eulers Gamma function is related to Pi function as follows:

$$\Gamma(x+1) = \Pi(x) = !x$$

Now let us extend factorials of negative integers by way of shifting the argument of Gamma function further down by 1 unit.Let us define Delta function as follows:

$$\Delta(x) = \int_0^\infty t^{x-2} e^{-t} dt$$

The extended Delta function shall have the following recurrence relation.

$$\Delta(x+2) = (x+2)\Delta(x+1) = (x+2)(x+1)\Delta(x) = !x$$

Newly defined Delta function is related to Eulers Gamma function and Pi function as follows:

$$\Delta(x+2) = \Gamma(x+1) = \Pi(x)$$

Plugging into $x = 2$ *above*

$$\Delta(4) = \Gamma(3) = \Pi(2) = 2$$

Plugging into $x = 1$ *above*

$$\Delta(3) = \Gamma(2) = \Pi(1) = 1$$

Plugging into $x = 0$ *above*

$$\Delta(2) = \Gamma(1) = \Pi(0) = 1$$

Plugging into $x = -1$ *above we can remove poles of Gamma and Pi function as follows:*

$$\Delta(1) = \Gamma(0) = \Pi(-1) = 1.\Delta(0) = -1.\Delta(-1) = \int_0^\infty t^{1-1} e^{-t} dt = \Big[-e^{-x} \Big]_0^\infty = \lim_{x \to \infty} -e^{-x} - e^{-0} = 0 + 1 = 1$$

Therefore we can say $\Delta(-1) = -1$. *Similarly plugging into* $x = -2$ *above*

$$\Delta(0) = \Gamma(-1) = \Pi(-2) = -1.\Delta(-1) = -2.\Delta(-2) = \int_0^\infty t^0 e^{-t} dt = \Big[-e^{-x} \Big]_0^\infty = \lim_{x \to \infty} -e^{-x} - e^{-0} = 0 + 1 = 1$$

Therefore we can say $\Delta(-2) = -\frac{1}{2}$. *Continuing further we can remove poles of Gamma and Pi function:*

Plugging into $x = -3$ *above and equating with result found above*

$$\Delta(-1) = \Gamma(-2) = \Pi(-3) = -2. - 1.\Delta(-3) = -1 \implies \Delta(-3) = -\frac{1}{2}$$

Plugging into $x = -4$ *above and equating with result found above*

$$\Delta(-2) = \Gamma(-3) = \Pi(-4) = -3. - 2.\Delta(-4) = -\frac{1}{2} \implies \Delta(-4) = -\frac{1}{12}$$

Plugging into $x = -5$ *above and equating with result found above*

$$\Delta(-3) = \Gamma(-4) = \Pi(-5) = -4. - 3.\Delta(-5) = -\frac{1}{2} \implies \Delta(-5) = -\frac{1}{24}$$

Plugging into $x = -6$ *above and equating with result found above*

$$\Delta(-4) = \Gamma(-5) = \Pi(-6) = -5. - 4.\Delta(-6) = -\frac{1}{12} \implies \Delta(-6) = -\frac{1}{240}$$

$\vdots$

And the pattern continues upto infinity.

Explanation 2 *Multiplying both side of Riemanns functional equation by* $(s-1)$ *we get*

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ *we get:*

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s)$$

$s \to 1$*we get:* $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$

$$\boxed{2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s) = -1}$$

Similarly multiplying both numerator and denominator right hand side of Riemanns functional equation by $(1-s)(2-s)$ *before applying any limit we get :*

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{(1-s)(2-s)\Gamma(1-s)\zeta(1-s)}{(1-s)(2-s)}$$

Putting $(1-s)(2-s)\Gamma(1-s) = \Gamma(3-s)$ *we get:*

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(1-s)(2-s)}$$

Multiplying both side of the above equation by $(s-1)$ *we get*

$$(s-1)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(s-2)}$$

$s \to 1$*we get:* $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$

$$-1 = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(s-2)}$$

Multiplying both side of the above equation further by $(s-2)$ *we get:*

$$(2-s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Multiplying both side of the above equation by $\zeta(s-1)$ *we get*

$$(2-s)\zeta(s-1) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1)$$

$s \to 1$*we get:* $\because \lim_{s\to 1}(s-2)\zeta(s-2) = 1 \therefore (2-s)\zeta(s-1) = -1$

$$2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1) = -1$$

If we can set $\zeta(s-1) = 1$ *then we can write*

$$\boxed{2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = -1}$$

To manually define zeta function such a way that it takes value 1, Euler's induction approach was applied and it was observed that zeta function have the potential unit value as demonstrated in the section (4.1).Both the above boxed forms are numerically equivalent to Riemann's original functional equation therefore for positive unit argument functional equation can be analytically continued as:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(2-s)\zeta(1-s)$$

Justification of the definition we set for $\zeta(1) = 1$ and consistency of the above forms of functional equation have been cross checked in the main proof and also it was found that the proposition complies with all the theorems used in complex analysis.Having showed that the zeta function can take unit value, multiplying both side any of the boxed equations by -1 we can analytically continue the functional equation applicable for negative unit argument as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(2-s)\zeta(1-s)$$

Justification of the definition we set for $\zeta(-1) = \frac{1}{2}$ and consistency of the above forms of functional equation have been cross checked in the in the section 4.2. $\zeta(-1) = \frac{1}{2}$ must be the second solution to $\zeta(-1)$ apart from the known Ramanujan's proof $\zeta(-1) = \frac{-1}{12}$. One has to accept that following the zeta functions recurrence pattern, at-least one zeta value can have multiple solutions.

3.2 Proof using Eulers original product form

Eulers Product form of zeta Function in Eulers exponential form of complex numbers is as follows:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(1 + re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ...\right)$$

Now any such factor $\left(1 + re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ...\right)$ will be zero if

$$\left(re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ...\right) = -1 = e^{i\pi}$$

Comparing both side of the equation and equating left side to right side on the unit circle we can say: *

$$\boxed{\theta + 2\theta + 3\theta + 4\theta ... = \pi}$$

$$\boxed{r + r^2 + r^3 + r^4 .... = 1}$$

We can solve θ and r as follows:

$$\boxed{\begin{aligned} \theta + 2\theta + 3\theta + 4\theta ... &= \pi \\ \theta(1+2+3+4...) &= \pi \\ \theta.\zeta(-1) &= \pi \\ \theta.\frac{-1}{12} &= \pi \\ \theta &= -12\pi \end{aligned} \qquad \begin{aligned} r + r^2 + r^3 + r^4 .... &= 1 \\ r(1 + r + r^2 + r^3 + r^4 ....) &= 1 \\ r\frac{1}{1-r} &= 1 \\ r &= 1-r \\ r &= \frac{1}{2} \end{aligned}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = \frac{1}{2}\cos(-12\pi) = \frac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, hence Riemann Hypothesis is proved.

Explanation 3 ** We can try back the trigonometric form then the algebraic form of complex numbers do the summation algebraically and then come back to exponential form as follows:*

$$\begin{aligned} & re^{i\theta} + r^2 e^{i2\theta} + r^3 e^{i3\theta} ... \\ &= (r\cos\theta + ir\sin\theta) + (r^2\cos 2\theta + ir^2 \sin 2\theta) + (r^3\cos 3\theta + ir^3\sin 3\theta) + (r^4\cos 4\theta + ir^4\sin 4\theta).... \\ &= (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) + (x_4 + iy_4) + (x_5 + iy_5).... \\ &= (x_1 + x_2 + x_3 + x_4 + x_5 + ...) + i(y_1 + y_2 + y_3 + y_4 + y_5 + ...) \\ &= R\cos\Theta + iR\sin\Theta \end{aligned}$$

Explanation 4 *One may attempt to show that $(re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...) = -1$ actually results $\frac{re^{i\theta}}{1-re^{i\theta}}$ which implies in $0 = -1$. Correct way to evaluate $\frac{re^{i\theta}}{1-re^{i\theta}}$ is to apply the complex conjugate of denominator before reaching any conclusion. $\frac{re^{i\theta}(1+re^{i\theta})}{(1-re^{i\theta})(1+re^{i\theta})}$ then shall result to $re^{i\theta} = -1$ which points towards the unit circle. In the present proof we need to go deeper into the d-unit circle and come up with the interpretation which can explain the Riemann Hypothesis.*

Explanation 5 *One may attempt to show inequality of the reverse calculation $\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}... = 1 \neq -1$. $re^{i\pi} = -1$ need to be interpreted as the exponent which then satisfies $1^{-1} = 1$ or $2.2^{-1} = 1$ on the unit or d-unit circle. There is nothing called t-unit circle satisfying $3.3^{-1} = 1$.*

3.3 Proof using alternate product form

Eulers alternate Product form of zeta Function in Eulers exponential form of complex numbers is as follows:

$$\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\left(\frac{1}{1-\frac{1}{re^{i\theta}}}\right)=\prod_p\left(\frac{re^{i\theta}}{re^{i\theta}-1}\right)$$

Multiplying both numerator and denominator by $re^{i\theta}+1$we get:

$$\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\left(\frac{re^{i\theta}(re^{i\theta}+1)}{(re^{i\theta}-1)(re^{i\theta}+1)}\right)$$

Now any such factor $\left(\frac{re^{i\theta}(re^{i\theta}+1)}{(r^2e^{i2\theta}-1)}\right)$ will be zero if $re^{i\theta}(re^{i\theta}+1)$ is zero:

$$\boxed{\begin{aligned} re^{i\theta}(re^{i\theta}+1) &= 0\\ re^{i\theta}(re^{i\theta}-e^{i\pi}) &= 0\\ r^2e^{i2\theta}-re^{i(\pi-\theta)*} &= 0\\ r^2e^{i2\theta} &= re^{i(\pi-\theta)}\end{aligned}}$$

We can solve θ and r as follows:

$$\boxed{\begin{array}{rrrr} 2\theta = & (\pi-\theta) & r^2 = & r\\ 3\theta = & \pi & \frac{r^2}{r} = & \frac{r}{r}\\ \theta = & \frac{\pi}{3} & r = & 1\end{array}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = 1.\cos(\tfrac{\pi}{3})=\tfrac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, and Riemann Hypothesis is proved.

Explanation 6 ** $e^{i(-\theta)}$ is arrived as follows:*

$$e^{i\theta} = \left(e^{i\theta}\right)^{1} = \left(e^{i\theta}\right)^{1^{-1}} = \left(e^{i\theta}\right)^{-1^{1}} = \left(\left(e^{i\theta}\right)^{i^2}\right)^{1} = \left(e^{i\theta}\right)^{i^2} = e^{i^3(\theta)} = e^{-i\theta}$$

Explanation 7 *Essentially proving $\log_2(\frac{1}{2})=-1$ in a complex turned simple way is equivalent of saying $\log(1)=0$ in real way. Primes other than 2 satisfy $\log_p(\frac{1}{2})=e^{i\theta}$ also in a pure complex way.*

4 Infinite product or sum of zeta values

4.1 Infinite product of positive zeta values converges

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}... = \left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2}...\right)...$$

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}... = \left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4}...\right)...$$

$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}... = \left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6}...\right)...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)...\\
&= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right)\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}...\right)\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}...\right)...\\
&= 1 + \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right) + \left(\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right) + \left(\frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3}...\right)...\\
&= 1 + 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} + \frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1} + \frac{1}{9^1}...\\
&= 1 + \zeta(1)
\end{aligned}$$

$\vdots$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)... =\\
&\left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...\\
&\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...\\
&\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...
\end{aligned}$$

$\vdots$

$$\begin{aligned}
&= \left(1 + 1\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...\\
&\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...\\
&\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...
\end{aligned}$$

$\vdots$

continued to next page....

continued from last page....

Simultaneously halfing and doubling each factor and writing it sum of two equivalent forms

$$= 2\left(\frac{1}{2}\left(1+\frac{\frac{1}{3}}{1-\frac{1}{3}}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{5}}{1-\frac{1}{5}}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{4}}{1-\frac{1}{4}}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{9}}{1-\frac{1}{9}}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)..\right)$$

$$\left(\frac{1}{2}\left(1+\frac{\frac{1}{8}}{1-\frac{1}{8}}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{\frac{1}{27}}{1-\frac{1}{27}}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(\frac{1}{2}\left(1+\frac{1}{2}+1+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1+\frac{1}{4}+1+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{1}{3}+1+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\left(\frac{1}{2}\left(1+\frac{1}{8}+1+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1+\frac{1}{7}+1+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1+\frac{1}{26}+1+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(1+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3^1}+\frac{1}{3^2}+\frac{1}{3^3}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{4}+\frac{1}{5^1}+\frac{1}{5^2}+\frac{1}{5^3}...\right)\right)...$$

$$\left(1+\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{8}+\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}...\right)\right)...$$

$$\left(1+\frac{1}{2}\left(\frac{1}{7}+\frac{1}{2^3}+\frac{1}{2^6}+\frac{1}{2^9}...\right)\right)\left(1+\frac{1}{2}\left(\frac{1}{26}+\frac{1}{3^3}+\frac{1}{3^6}+\frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(1+\frac{1}{2}\left(\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...+\frac{1}{2^1}+\frac{1}{3^1}+\frac{1}{4^1}...\right)\right)$$

$$= 2\left(1+\frac{1}{2}\left(2\zeta(1)-2\right)\right)$$

$$= 2(1-1+\zeta(1))$$

$$= 2\zeta(1)$$

Now comparing two identities:

$$\boxed{1+\zeta(1) = 2\zeta(1))}$$

$$\boxed{\zeta(1) = 1}$$

Hence Infinite product of positive zeta values converges to 2

4.2 Infinite product of negative zeta values converges

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1 ... = \Bigg(1 + 2 + 2^2 + 2^3 ...\Bigg)\Bigg(1 + 3 + 3^2 + 3^3 ...\Bigg)\Bigg(1 + 5 + 5^2 + 5^3 ...\Bigg)...$$

$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2 ... = \Bigg(1 + 2^2 + 2^4 + 2^6 ...\Bigg)\Bigg(1 + 3^2 + 3^4 + 3^6 ...\Bigg)\Bigg(1 + 5^2 + 5^4 + 5^6 ...\Bigg)...$$

$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3 ... = \Bigg(1 + 2^3 + 2^6 + 2^9 ...\Bigg)\Bigg(1 + 3^3 + 3^6 + 3^9 ...\Bigg)\Bigg(1 + 5^3 + 5^6 + 5^9 ...\Bigg)...$$

$$\vdots$$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)...\\
&= \Bigg(1 + 2^1 + 3^1 + 4^1 + 5^1 ...\Bigg)\Bigg(1 + 2^2 + 3^2 + 4^2 + 5^2 ...\Bigg)\Bigg(1 + 2^3 + 3^3 + 4^3 + 5^3 ...\Bigg)...\\
&= 1 + \Bigg(2 + 2^2 + 2^3 ...\Bigg) + \Bigg(3 + 3^2 + 3^3 ...\Bigg) + \Bigg(4 + 4^2 + 4^3 ...\Bigg)...\\
&= 1 + \Bigg(1 + 2 + 2^2 + 2^3 ... - 1\Bigg) + \Bigg(1 + 3 + 3^2 + 3^3 ... - 1\Bigg) + \Bigg(1 + 4 + 4^2 + 4^3 ... - 1\Bigg)...\\
&= 1 + \Bigg(-\frac{1}{1} - 1\Bigg) + \Bigg(-\frac{1}{2} - 1\Bigg) + \Bigg(-\frac{1}{3} - 1\Bigg) + \Bigg(-\frac{1}{4} - 1\Bigg)...\\
&= 1 - \Bigg(\Bigg(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} ...\Bigg) + 1 + 1 + 1 + 1 ...\Bigg)\\
&= 1 - \Bigg(\zeta(1) + \zeta(0)\Bigg)\\
&= 1 - \Bigg(1 - \frac{1}{2}\Bigg)\\
&= \frac{1}{2}
\end{aligned}$$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)... =\\
&\Bigg(1 + 2 + 2^2 + 2^3 ...\Bigg)\Bigg(1 + 3 + 3^2 + 3^3 ...\Bigg)\Bigg(1 + 5 + 5^2 + 5^3 ...\Bigg)...\\
&\Bigg(1 + 2^2 + 2^4 + 2^6 ...\Bigg)\Bigg(1 + 3^2 + 3^4 + 3^6 ...\Bigg)\Bigg(1 + 5^2 + 5^4 + 5^6 ...\Bigg)...\\
&\Bigg(1 + 2^3 + 2^6 + 2^9 ...\Bigg)\Bigg(1 + 3^3 + 3^6 + 3^9 ...\Bigg)\Bigg(1 + 5^3 + 5^6 + 5^9 ...\Bigg)...\\
&\vdots\\
&= 1 + 2^1 + 3^1 + 4^1 + 5^1 ...\\
&= \zeta(-1)
\end{aligned}$$

Therefore $\boxed{\zeta(-1) = \frac{1}{2}}$ must be the second solution of $\zeta(-1)$ apart from the known one $\zeta(-1) = \frac{-1}{12}$.

Using Delta function instead of Gamma function on the d-unit circle we can rewrite the functional equation applicable for negative argument as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(2-s)\zeta(1-s)$$

Putting $s = -1$we get:

$$\zeta(-1) = -2^{-1} \pi^{(-1-1)} \sin\left(\frac{-\pi}{2}\right) \Gamma(3-s)\zeta(2) = \frac{1}{2}$$

To proof Ramanujans Way

$$\sigma = \boxed{1+2+3+4+5+6+7+8+9.....}$$

$$2\sigma = \boxed{0+1+2+3+4+5+6+7+8+9...} + 1+1+1+1+1+1+1...*$$

Subtracting the bottom from the top one we get:

$$-\sigma = 0+1+1+1+1+1+1+1+1... +1+1+1+1+1+1+1...$$

$$\sigma = -(1+1+1+1+1+1+1+1+1+1........)$$

$$\sigma = \frac{1}{2}$$

*The second part is calculated subtracting bottom from the top before doubling.

4.3 Counter proof on Nicole Oresme's proof of divergent harmonic series

Nicole Oresme in around 1350 proved divergence of harmonic series by comparing the harmonic series with another divergent series. He replaced each denominator with the next-largest power of two.

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}...$$
$$> 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}...$$
$$> 1+\left(\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+...$$
$$> 1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}...$$

He then concluded that the harmonic series must diverge as the above series diverges.

Now in my finding it was too quick to conclude as we can go ahead and show:

$$> 1+\frac{1}{2}\left(1+1+1+1+1+1+1+1+1+1+1+1+1+1+...\right)$$
$$> 1+\frac{1}{2}.\frac{-1}{2}$$
$$> 1-\frac{1}{4}$$

If we replace $\zeta(1)$ by 1 as found in previous section then also it passes the comparison test.

$$\Rightarrow 1 > 1-\frac{1}{4}$$

Therefore We need to come out of the belief that harmonic series diverges.

$$> 1-\left(1-2+3-4+5-6+7-8+9...\right)$$
$$> 1-\left(1+2+3+4+5+6+7+8+9...\right)-\left(1+2+4+6+8+...\right)+1$$
$$= 1-\frac{1}{2}-\frac{1}{2}+1$$
$$= 1-0$$
$$\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}... = 1$$

4.4 Infinite product of All zeta values converges

$$\boxed{\zeta(-1)\zeta(-2)\zeta(-3)...\zeta(1)\zeta(2)\zeta(3)... = \zeta(-1).2.\zeta(1) = \frac{1}{2}.2.1 = 1}$$

4.5 Infinite sum of Positive zeta values converges

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...$$
$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}...$$
$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}...$$
$$\vdots$$
$$\zeta(1) + \zeta(2) + \zeta(3)...$$
$$= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right) + \left(1 + 1 + 1 + 1 + ...\right)$$
$$= \zeta(1) + \zeta(0) = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\boxed{\zeta(1) + \zeta(2) + \zeta(3)... = \frac{1}{2}}$$

4.6 Infinite sum of Negative zeta values converges

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1...$$
$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2...$$
$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3...$$
$$\vdots$$
$$\zeta(-1) + \zeta(-2) + \zeta(-3)...$$
$$= \left(1 + 2^1 + 3^1 + 4^1 + 5^1...\right) + \left(1 + 1 + 1 + 1 + ...\right)$$
$$= \zeta(-1) + \zeta(0) = \frac{1}{2} - \frac{1}{2} = 0$$
$$\boxed{\zeta(-1) + \zeta(-2) + \zeta(-3)... = \zeta(-1) + \zeta(0) = 0}$$

4.7 Infinite sum of All zeta values converges

$$\boxed{\zeta(-1) + \zeta(-2) + \zeta(-3)... + \zeta(1) + \zeta(2) + \zeta(3)... = 0 + \frac{1}{2} = \frac{1}{2}}$$

4.8 Primes product = 2.Sum of numbers

We know :

$\zeta(-1) = \zeta(1) + \zeta(0)$

$$or\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}...\right)+\left(1+1+1+1+...\right)=\frac{1}{2}$$

$$or\left(1+1\right)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{4}\right)+...=\frac{1}{2}$$

$$or\left(\frac{2}{1}+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5}...\right)=\frac{1}{2}$$

LCM of the denominators can be shown to equal the square root of primes product.

Reversing the numerator sequence can shown to equal the sum of integers.

$$or\left(\frac{1+2+3+4+5+6+7...*}{2.3.5.7.11...**}\right)=\frac{1}{2}$$

$$or 2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$$

*Series of terms written in reverse order.

**Product of All numbers can be written as 2 series of infinite product of all prime powers

**One arises from individual numbers and another from the number series.Then

$$LCM = \prod_{i=1}^{\infty} P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{(1+2+3+4+5+6+7...)+(1+2+3+4+5+6+7...)}...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{\frac{1}{2}+\frac{1}{2}}...$$

$$LCM = 2.3.5.7.11...$$

4.9 Fundamental formula of integers

Primes product = 2.Sum of numbers can be generalized to all even numbers as zeta function all the poles being removed become bijectively holomorphic and as such become absolutely analytic or literally an entire function. $2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$ is open ended and self replicating. Similarly $\sum_{N=1}^{\infty} N = \prod_{i\neq 1}^{\infty} P_i$ is self sufficient. We can pick partial series, truncate series to get even and odd numbers.

- $2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$ can be regarded as Fundamental formula of all even numbers.
- $\sum_{N=1}^{\infty} N = \prod_{i\neq 1}^{\infty} P_i$ can be regarded as Fundamental formula of all odd numbers excluding primes.
- $\sum_{N=1}^{\infty} N = P_i$ can be regarded as Fundamental formula of all primes.

5 zeta results confirms Cantors theory

We have seen both sum and product of positive zeta values are greater than sum and product of negative zeta values respectively. This actually proves a different flavor of Cantors theory numerically. If negative zeta values are associated with the set of rational numbers and positive zeta values are associated with the set of natural numbers then the numerical inequality between sum and product of both proves that there are more ordinal numbers in the form of rational numbers than cardinal numbers in the form of natural numbers in spite of having one to one correlation among them. This actually happens because of dual nature of reality. Every unit fractions can be written in two different ways i.e. one upon the integer or two upon the double of the integer as they are equivalent. But the number of integer representation being unique will always fall short of the former. Even if we bring into products,factors,sum,partitions etc. then also the result remain same. So there are more rational numbers than natural numbers. Stepping down the ladder we can say there are more ordinal numbers than cardinal numbers.

6 Proof of other unsolved problems

In the light of identities derived most of the unsolved prime conjectures turns obvious as follows:

6.1 Goldbach Binary/Even Conjecture

If we take two odd prime in the left hand side of fundamental formula of even numbers then retaining the fundamental pattern both the side have a highest common factor of 2. That means all the even numbers can be expressed as sum of at least 1 pair of primes i.e. 2 primes and if we multiply both side by 2 then some even numbers can be expressed as sum of 2 pair of primes i.e. 4 primes. Overall an even number can be expressed as sum of maximum 2+2+2=6 primes one pair each in half unit, unit and d-unit circle. However immediately after 3 pairs of prime one pi rotation completes and the prime partition sequence breaks, i.e. beyond d-unit circle it starts over and over again cyclically along the number line. Ramanujans derived value $2\zeta(-1) = -\frac{2}{12} = -\frac{1}{6}$ actually indicates that limit .

$$2(p_1 + p_2) = 2.p_3...$$
$$4(p_1 + p_2 + p_3 + p_4) = 2.2.p_5.p_6...$$
$$8(p_1 + p_2 + p_3 + p_4 + p_5 + p_6) = 2.2.2.p_7.p_8.p_9...$$

6.2 Goldbach Tarnary/Odd Conjecture

In case of odd number also we can have combination of 3 and 6 primes. Beyond that no more prime partition is possible.

$$(p_1 + p_2 + p_3) = p_4.p_5...$$
$$(p_1 + p_2 + p_3 + p_4 + p_5 + p_6) = p_7.p_8.p_9...$$

6.3 Polignac prime conjectures

6.3.1 Twin prime conjecture

Lets test whether prime gap of 2 preserves the fundamental formula of numbers.

$p^2 + 2p = p(p + 2)$

adding 2 both side will turn both side into prime as $p^2 + 2p + 2$ cannot be factorised really.

$p^2 + 2p + 2 = p(p + 2) + 2$

$p^2 + 2p + 2 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of twin primes.

6.3.2 Cousin prime conjecture

Lets test whether prime gap of 4 preserves the fundamental formula of numbers.

$p^2 + 4p = p(p + 4)$

adding 1 both side will turn both side into prime as $p^2 + 4p + 1$ cannot be factorised really.

$p^2 + 4p + 1 = p(p + 4) + 1$

$p^2 + 4p + 1 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of cousin primes.

6.3.3 Sexy prime conjecture

Lets test whether prime gap of 6 preserves the fundamental formula of numbers.

$p^2 + 6p = p(p + 6)$

adding 1 both side will turn both side into prime as $p^2 + 6p + 1$ cannot be factorised really.

$p^2 + 6p + 1 = p(p + 6) + 1$

$p^2 + 6p + 1 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of cousin primes.

6.3.4 Other Polignac prime conjectures

Similarly all other polignac primes of the form of p+2n shall be there infinitely.

6.4 Sophie Germain prime conjecture

Lets test whether prime gap of 2p preserves the fundamental formula of numbers which will generate sophie germain prime pairs.

$2p^2 = p(2p)$

adding 1 both side will turn both side into prime as $2p^2 + 1$ cannot be factorised really.

$2p^2 + 1 = p(2p) + 1$

$2p^2 + 1 = p_1.p_2$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be infinite number of Sophie Germain primes.

6.5 Landau's prime conjecture

We need to check whether there shall always be infinite number of $N^2 + 1$ primes.

$N^2 + 1 = N^2 + 1$

Adding N and multiplying both side by 2 will turn both side into an even number.

$2(N^2 + N + 1) = 2.(N^2 + N + 1)$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised really.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall always be infinite number of $N^2 + 1$ primes.

6.6 Legendre's prime conjecture

Lets take sum of two successive numbers square and test whether they conform to the fundamental formula of numbers.

$N^2 + N^2 + 2N + 1 = N^2 + (N + 1)^2$

adding 1 both side will turn both side into an even number.

$2(N^2 + N + 1) = 2.P$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised really.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall always be a prime between two successive numbers square.

6.7 Brocard's prime conjecture

As square of a prime and square of its successor both have identical powers they shall have a highest common factor of 4 in $4(p_1 + p_2 + p_3 + p_4...) = 2.2.p_5.p_6...$, and there shall be at least four primes between them as Brocard conjectured.

6.8 Opperman's prime conjecture

Lets test whether gap of N between $N(N-1)$ and N^2 preserves the fundamental formula of numbers which will give us the count of primes between the pairs.

$N^2 - N + N^2 = N(N-1) + N^2$

adding 3N+1 both side will turn both side into an even number.

$2(N^2 + N + 1) = 2.(N^2 + N + 1)$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised really.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be atleast one prime between $N(N-1)$ and N^2 as Opperman conjectured.

Lets test whether gap of N between $N(N+1)$ and N^2 preserves the fundamental formula of numbers which will give us the count of primes between the pairs.

$N^2 + N + N^2 = N(N+1) + N^2$

adding N+2 both side will turn both side into an even number.

$2(N^2 + N + 1) = 2.(N^2 + N + 1)$

dividing by 2 both side will turn it into prime as $N^2 + N + 1$ cannot be factorised really.

$(N^2 + N + 1) = P$

And this has happened without violating fundamental formula of prime numbers.

As the form is preserved there shall be atleast one prime between N^2 and $N(N+1)$ as Opperman conjectured.

6.9 Collatz conjecture

As fundamental formula of numbers is proved to be continuous, Collatz conjectured operations on any number (i.e. halving the even numbers or simultaneously tripling and adding 1 to odd numbers) may either blow up to infinity or come down to singularity. We have seen that among the odd numbers odd primes are descendants of sole even prime 2. This small bias turns the game of equal probability into one sided game i.e Collatz conjecture cannot blow upto infinity, it ends with 2 and one last step before the final whistle bring it down to singularity 1 as Collatz conjectured. Hence Collatz conjecture is proved to be trivial.

7 Conclusion

Nature is dual and infinite by nature. We human created other numbers to put a limit to the concept of infinity; we divided numbers according to its divisibility into three types i.e. odd, even and primes. But following the legacy of number 2 all these 3 types of numbers carries the same dual and infinite nature.

a As shown mathematically in the section (3), Riemann hypothesis stands proved.

b As shown mathematically in the section (6), other prime conjectures like Goldbach conjecture, twin prime conjecture etc. stands proved.

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8 Bibilography

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