Massive Gravity Illustrated in the Mandelbrot Set

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Solving problems with giving the graviton mass

Solving the problems arising when we consider that the graviton has mass requires dealing with the unphysical effects that would be seen otherwise. When the graviton is massive; instead of the two modes of the Standard Model or massless graviton, it has 5 polarization modes $(0,\pm 1,\pm 2)$. Because the zero-spin mode would couple to matter nearly as strongly as the spin-2 massless one, and this creates a change in the force and effects of gravity which is discontinuous from what we observe, these effects are labeled the vDVZ discontinuity (for van Dam, Veltman, and Zakharov). When Vainshtein suggested a screening mechanism that would remove zero-mode coupling effects from our local neighborhood, it was found that there would still be (Boulware-Deser) 'ghost' solutions representing unphysical and unobserved effects.

Theories like DGP gravity (for Dvali, Gabadadze, and Porrati) were an attempt to resolve this by using a transition to 5-d at extreme distances to move the discontinuity and ghosts to an arena where they do not affect local observations, and also result in accelerating expansion which we do observe. This was further developed in Cascading DGP (de Rham et al.) where a transition to 6-d is seen even further out. We note that extreme distance also corresponds to an earlier time, and a prior cosmological epoch. There is a literal embodiment of this in theories where a 5-d black hole becomes a white hole and 4-d spacetime bubble, which is our universe (Pourhasan, Afshordi, and Mann). The same transition is clearly represented in the Mandelbrot Set (in higher dimensions) and it is seen to be connected with Cartan's rolling ball analogy for Lie group G_2 symmetries, where the rolling ball's point of contact is at $(-0.75, 0i...)$ and this preserves a 3:1 ratio of radii. This suggests we can use $\mathcal{M}$ to study the 5-d $\rightarrow$ 4-d transition.

Abstract

The long-range transition to 5-d or higher, seen in DGP gravity, cascading DGP, and some other theories, can be interpreted as a prior epoch or precursor universe. This transition has therefore been proposed to be a black hole in 5-d $\rightarrow$ 4-d white hole and a spacetime bubble that is our cosmos. The same transition is represented in the Mandelbrot Set at (-0.75, 0i), where the cardioid meets the circular disc, mimicking Cartan's rolling ball analogy for G_2 symmetries when $\mathcal{M}$ is extended into higher dimensions. We can therefore study this in an archetypical setting, and it suggests a unique exit from a higher-dimensional origin, which conveniently explains the weakness of gravity and accelerating expansion. Other analogies allow us to discuss graviton properties and phenomenology. Notably the Misiurewicz point at about (-1.543689, 0i) represents the quantum critical point in BEC formation, which has been connected to black hole event horizons. A consistent model for gravity with higher-dimensional origins is offered.

Keywords: Massive gravity, Lie group G_2, Mandelbrot Set, higher-d origin, BEC, graviton

Providing answers for cosmological origins

While some theories treat the 5-d $\rightarrow$ 4-d transition from a prior epoch as though any black hole in a 5-d universe could give rise to a cosmos like ours; Mandelbrot Gravity Theory treats it as a unique event -- a universe-ending final singularity. This is in agreement with the Penrose singularity theorems and is illustrated vividly in $\mathcal{M}$. Furthermore; it is seen as an event where the fabric of spacetime is turned inside out or everted. This happens naturally in the context of octonionic inflation, where $\mathcal{M}$ is seen an object arising in octonionic space, because of the action of Lie group G_2 to instate the attribute of interiority/exteriority which strictly defines the associative property in our cosmos. Thus it is seen as a cosmological gauge-fixing event that sets the relative strengths of the fundamental forces during the 5-d $\rightarrow$ 4-d transition, which explains the relative weakness of gravity in the current era.

This implies a connection between the universe beyond the edge of the observable universe and the realm within subatomic particles – if we assume a higher-dimensional origin for the cosmos. In this theory; black holes early in the post-decoupling universe are predicted to have high spin, and element building in the pre-decoupling universe is assumed to continue to produce both heavy and super-heavy elements – some of which could not pass through the throat of a 5-d $\rightarrow$ 4-d wormhole. While the sub-atomic realm is broadly assumed to contain a higher-dimensional space; this theory asserts it is the same higher-dimensional space which existed before the current era of our cosmos, and is also seen to lie beyond the farthest edge of what is observable. This is because in non-associative spaces interiority/exteriority is relative, such that we can be inside a higher-d space, or riding on its surface, and still find that space within sub-atomic particles.

The Mandelbrot Set is the most complex 2-d form in Mathematics, and it reveals a wonderful interplay between symmetry and asymmetry. Its elegantly simple formula $z \rightarrow z^2 + z$ is maximally complex *and* asymmetrical among such figures, where both ends point toward the extremum of $\mathcal{M}$ at $(-2,0i)$. This allows us to study local symmetry against a background of global asymmetry, which gives us insight into symmetry-breaking in Physics because we can examine it in an archetypical setting, and then apply this.

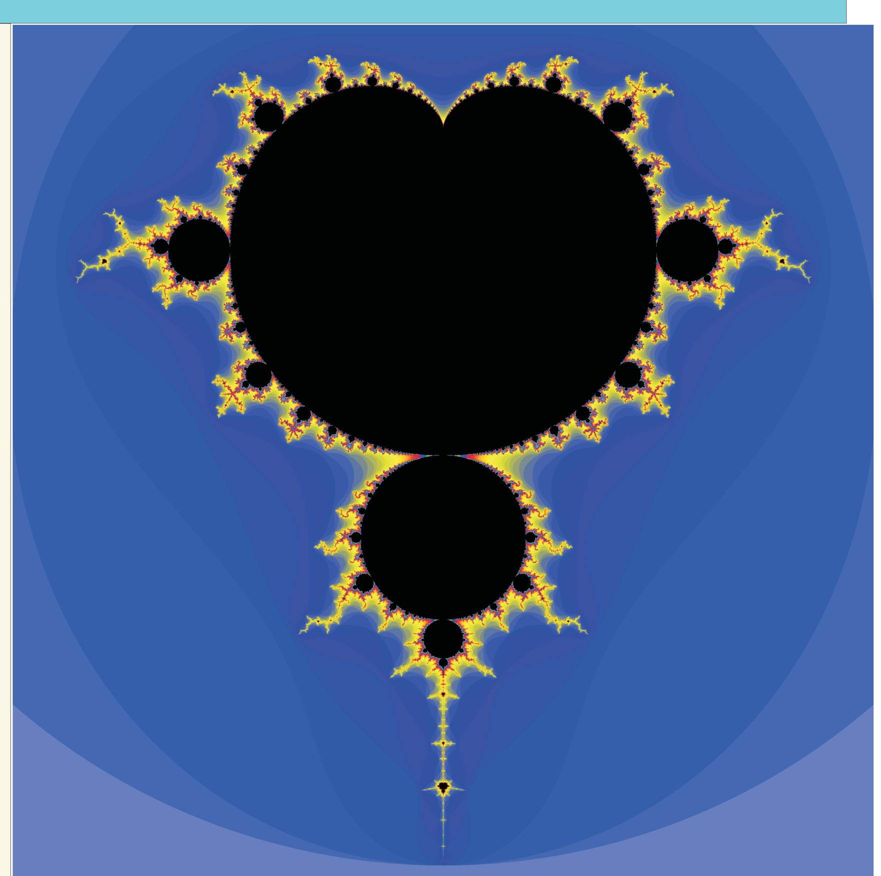

Fig. 1 - The Mandelbrot Set as it is conventionally viewed, in the domain of the complex numbers, is the projection or shadow of a higher-dimensional figure residing in the quaternions and octonions. $\mathcal{M}$ is what remains bounded when $z \rightarrow z^2 + z$ is iterated in the complex or hypercomplex domain.

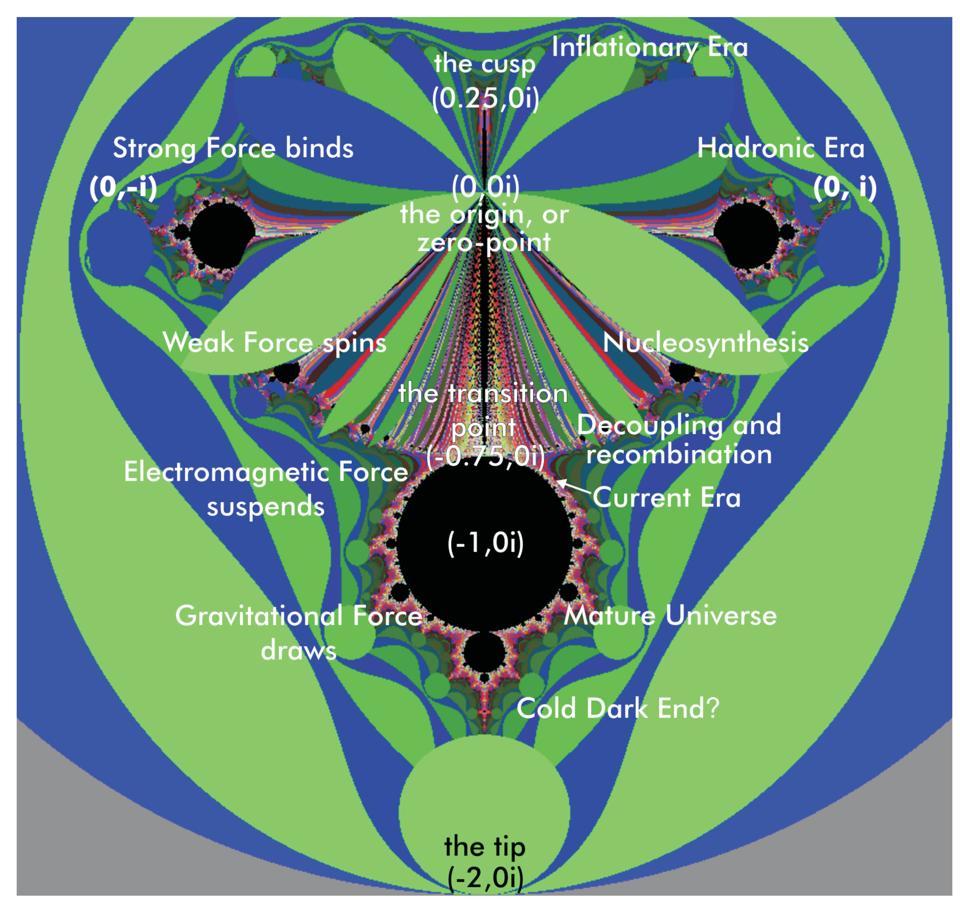

Fig. 2 - The Mandelbrot Butterfly is annotated here to show the analogy of $\mathcal{M}$ to the fundamental forces and cosmological epochs.

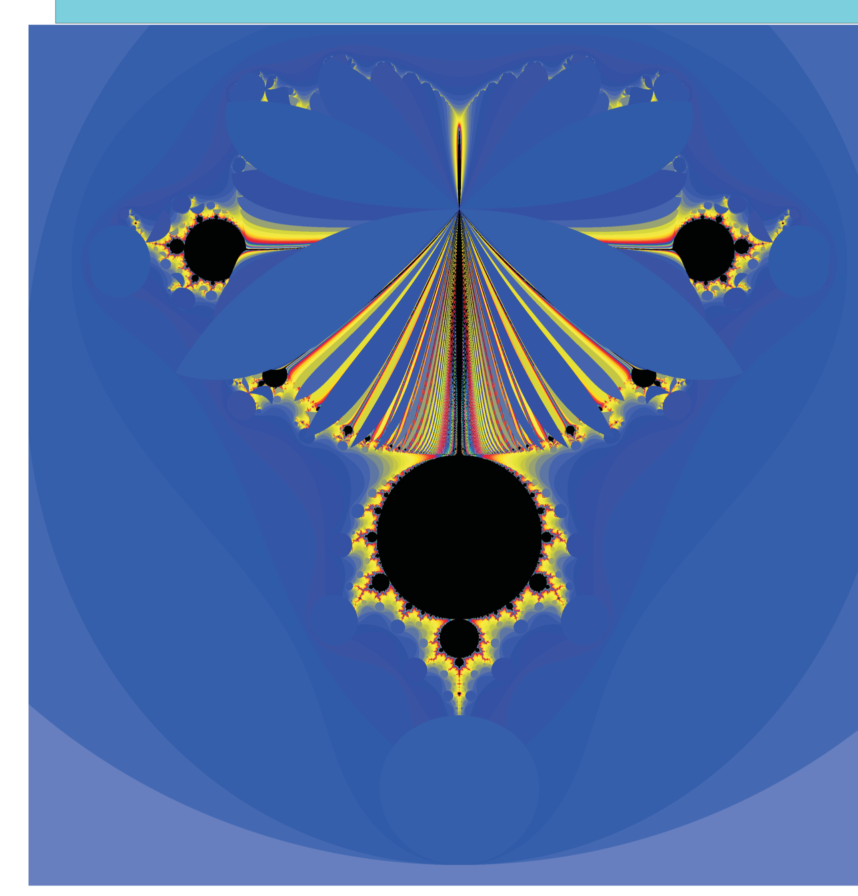

Fig. 3 - The Mandelbrot Butterfly that appears when one colors in points where the iterand magnitude decreases monotonically (over 3 iterations) reveals the Mandelbrot Set's connections to cosmology and particle Physics.

The Butterfly figure was discovered by the author 32 years ago, when trying to find a calculational shortcut for $\mathcal{M}$. It reveals features like the basins of attraction around the mini-$\mathcal{M}$s that lie past branching Misiurewicz points. But it suggests many analogies to physical processes, and a progression that appears to represent the stages or epochs of Cosmology. As the formula $z \rightarrow z^2 + z$ is iterated, prior values of the iterand magnitude are retained, and points are colored in where it steadily diminishes over 3 calculations.

While the familiar Lie groups SO(n), SU(n), and Sp(n) come from the reals, complex, and quaternions respectively; the exceptional Lie groups E_8, E_7, E_6, F_4, and G_2, all spring from the octonions! If we designate the octonion imaginaries (i,j,k,l,J,K,L) the action of G_2 is seen to split the basis elements such that (l,J,K,L) remain in the 5-d precursor or parent universe, while the (i,j,k) axes are the rotations in our 4-d spacetime.

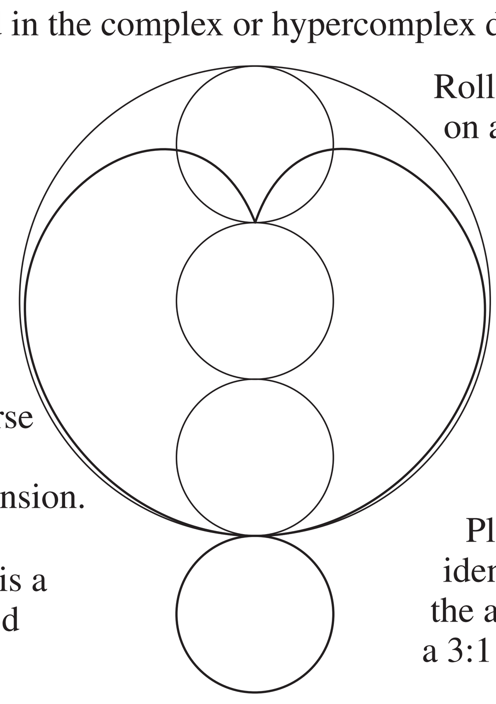

Fig. 4 - The major geometry of the Mandelbrot Set can be created by rolling a circle on a circle, and this recreates Cartan's rolling ball analogy for G_2 symmetries, when $\mathcal{M}$ is regarded as a higher-dimensional figure. Cartan's analogy works best in 5-d which is the maximal (hyper-) volume for a ball, defined as the space contained by a sphere or hypersphere of a given dimension.

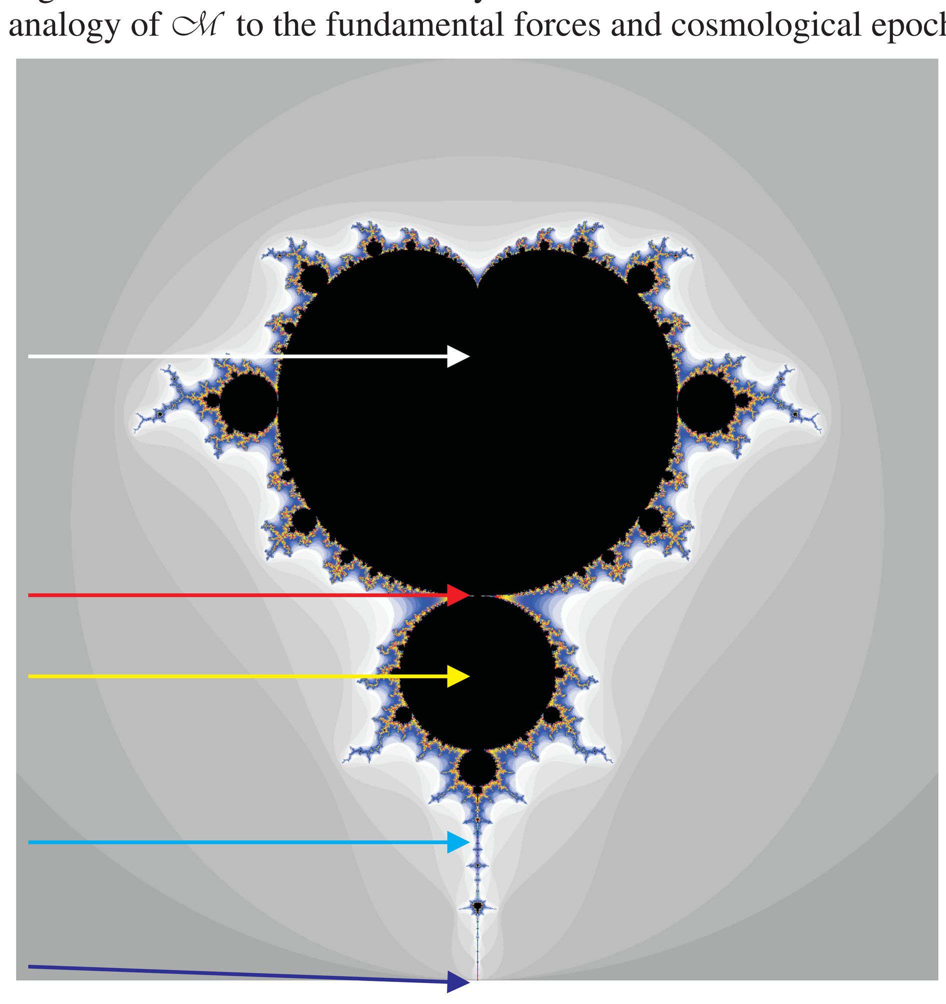

Fig. 5 - The Mandelbrot Set is annotated here to show features of massive gravity theory, and to illustrate how $\mathcal{M}$ represents both the different flavors of graviton and various aspects of the corresponding cosmology.

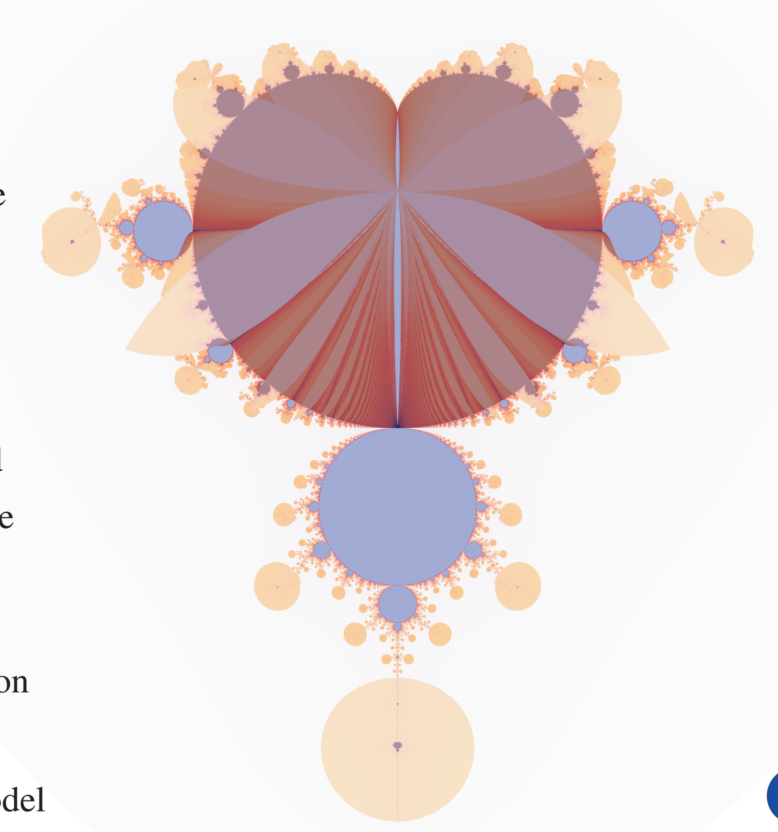

Fig. 6 - This image of the Mandelbrot Butterfly by Paul Bourke shows how it resembles a cosmic thermometer, where the cusp is the Cosmos' origin and the far tip at (-2,0i) is its final end.

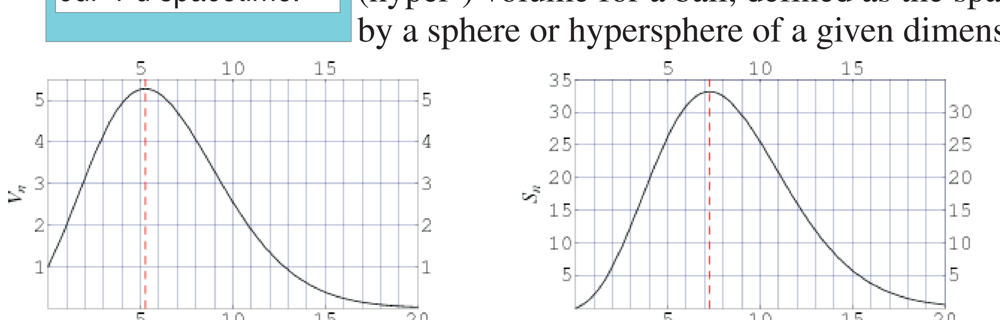

Fig. 7 - Graphs of (hyper-) volume and (hyper-) surface area for the n-ball (on left) and the n-sphere (on right). Note that as the number of dimensions is increased, both volume and surface area rise to a maximal value and decrease thereafter.

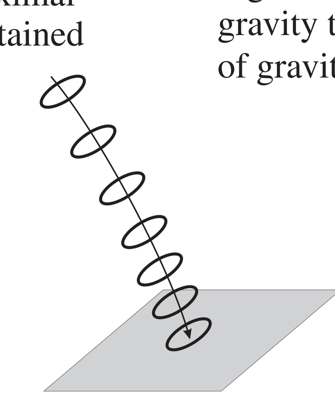

Fig. 8 - Gravitons settle and lie flat on a condensing surface or horizon, following gravitational field lines.

The graviton in String Theory is a simple loop with no extent, but here we assume it has a minimal extent. Its phenomenology is determined in part by how it relates to the geometrical transitions and boundaries it encounters. The Mandelbrot Set gives insights into both matter and the fabric of space which inform us about how gravity emerges from the unified force. Examining the form of $\mathcal{M}$ and the Butterfly near (-2,0i), (-1,0i), and (0,0i); we see how gravitons couple to both.

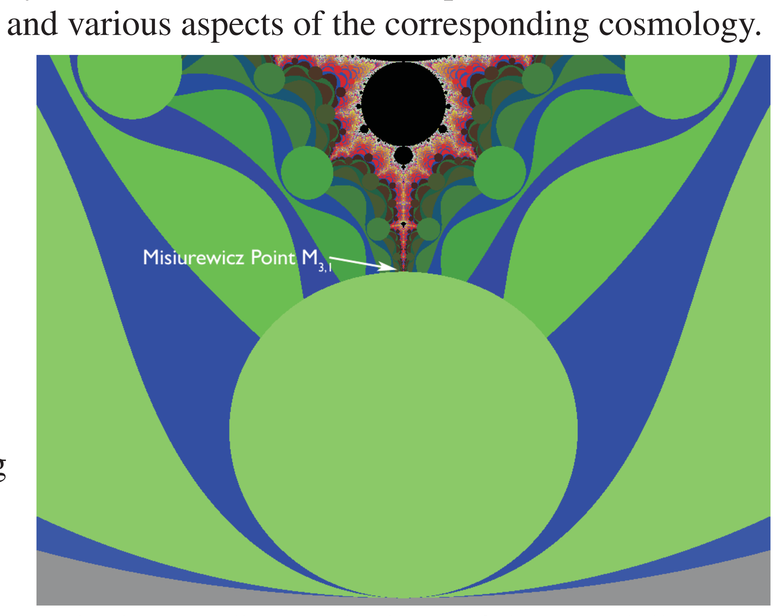

Fig. 9 - The largest disc, at the base of the Mandelbrot Butterfly, represents a BEC or Schwarzschild black hole – where Misiurewicz point $M_{3,1}$ is the quantum critical point or event horizon, and $(-2,0i)$ is the BH antipode at 0°K.

$$\mathbb{O} \supset \mathbb{H} \supset \mathbb{C} \supset \mathbb{R}$$

Eqn. 1 - Normed division algebras have a hierarchy where the octonions are the most general and contain the quaternions, which contain the complex numbers, that in turn hold the reals.

$$\textit{Smooth} \supset \textit{Top} \supset \textit{Meas}$$

Eqn. 2 - There is also a hierarchy of the geometric forms and spaces, where the *Smooth* forms contain the *Topological* forms, which contain the *Measurable* forms, and likewise with the geometric spaces.

$$\textit{Gases} \supseteq \textit{Liquids} \supseteq \textit{Solids}$$

Eqn. 3 - A partial analogy with this exists, if we consider the phases of matter.

Note - the relations above show a pattern in pure Mathematics that carries over into Physics, where definite forms are a condensate of more variable and general ones that precede them.

This document combines and extends the material presented on my poster for GR21 in NYC, in my talk at FFP15 in Orihuela, and detailed in recent publications in Prespacetime Journal. It includes insights from the author's finalist essays in recent FQXi contests, and findings from his paper submitted to this year's Gravity Research Foundation awards competition. The accompanying paper, with the same title as this poster, should appear within a month or so of this event, detailing some of the most relevant findings that are sketched on this poster.

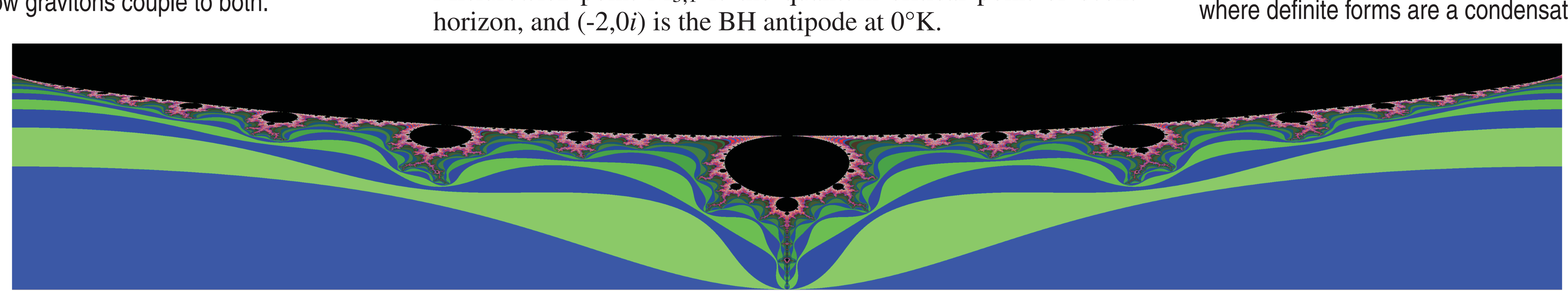

Fig. 10 - The Mandelbrot Set, where concentric circles around $(0,0i)$ are displayed as rows of pixels, shows a resemblance to anti de Sitter space

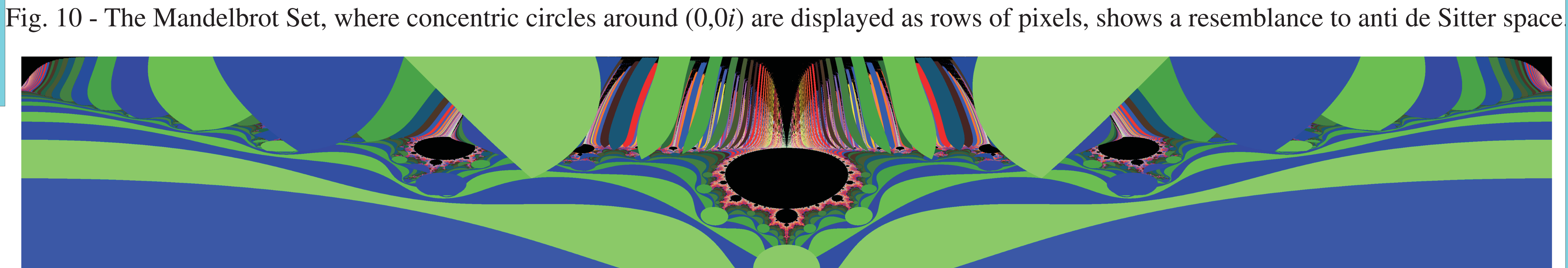

Fig. 11 - The Mandelbrot Butterfly displayed in the same manner shows space in the current era as the open bulb at the center, where the bands above are creation/annihilation operators in the early or parent universe pre-decoupling.

Condensation in Massive Gravity

Condensation appears to be an overarching theme in Math that carries over into Physics, where the appearance of form requires that mathematical properties be strictly or unambiguously defined, before material forms with consistent properties can exist. This is of particular interest in the study of gravity, because it not only assists in defining gravity's action, but it is also part of the natural mechanism that allows massive particles to be formed in the early universe. The modeling of this process that occurs at all of the primary branching Misiurewicz points is augmented by the use of the Mandelbrot Butterfly algorithm to reveal the associated basins of attraction. This helps us to grasp how nature uses processes of condensation to define the natural forces and characterize their action. We see the progression of their appearance, because this is graphically depicted in $\mathcal{M}$.

Poster for GR22/Amaldi13
Valencia Spain, July 2019
Session A3 - Modified theories of gravity (theoretical aspects)