Prime Gap near a Primorial Number

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Abstract

In this article, we use method of sieve of Eratosthenes to prove that there is a larger prime gap near any primorial number.

A primorial number is defined as the product of the first m primes:

$p_m\sharp = \prod_{i=1,...,m} p_i$,

here p_i is the i^{th} prime number.

By using method of sieve of Eratosthenes up to the first m primes, p_i, i=1,...m, there are total $K = \prod_{i=1,...,m}(1-1/p_i)p_m\sharp$ numbers of the remaining numbers smaller than $p_m\sharp$. Obviously 1 and $p_m\sharp$ -1 are remainging numbers.

here $K = p_m\sharp \prod_{i=1,...,m}((p_i - 1)/p_i) = \prod_{i=1,...,m}(p_i - 1)$.

These remaining numbers are symmetric to the number $\frac{p_m\sharp}{2}$, so they can be paired up as (x, $p_m\sharp$-x), here x and $p_m\sharp$-x are remaining numbers.

Let p_{m+1} be the smallest remaining number, obviously it is a prime number.

There are no remaining numbers in (1, p_{m+1}), nor in ($p_m\sharp$-p_{m+1}, $p_m\sharp$-1).

So there is a prime gap larger than, or equal to $p_{m+1} - 1$ near the primorial number $p_m\sharp$

Equal happens when both numbers, $p_m\sharp$-p_{m+1} and $p_m\sharp$-1 are prime numbers.