Energy-Momentum Density's Conservation Law of Electromagnetic Field in Rindler Space-time

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ABSTRACT

We find the energy-momentum density of electromagnetic field by energy-momentum tensor of electromagnetic field in Rindler space-time. We find the energy-momentum density's conservation law of electromagnetic field in Rindler spacetime.

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1. Introduction

Our article's aim is that we find the energy-momentum density of electromagnetic field by energy-momentum tensor of electromagnetic field in Rindler space-time. We find the energy-momentum density's conservation law of electromagnetic field in Rindler space-time.

In inertial frame, the energy-momentum tensor $T^{\mu\upsilon}$ of the electromagnetic field is

$$T^{\mu\upsilon} = \frac{1}{4\pi c}(F^{\mu}{}_{\rho}F^{\upsilon\rho} - \frac{1}{4}\eta^{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \tag{1}$$

In this time, in inertial frame, Faraday tensors $F^{\mu\upsilon}, F_{\mu\upsilon}$ are

$$F^{\mu\upsilon} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}, F_{\mu\upsilon} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \tag{2}$$

Hence, the energy density $p_f{}^0$ and the momentum density $\vec{p}_f$ of electromagnetic field are

$$T^{00} = p_f{}^0 = \frac{E^2 + B^2}{8\pi c}, \quad T^{0\upsilon} = \vec{p}_f = \frac{\vec{E} \times \vec{B}}{4\pi c}$$

$$\left|\vec{E}\right| = E, \left|\vec{B}\right| = B \tag{3}$$

In inertial frame, the energy-momentum conservation law of electromagnetic field is by Noether theorem,

$$T^{\mu\upsilon}{}_{,\upsilon} = T^{00}{}_{,0} + T^{0i}{}_{,i} \ , \ i = 1,2,3$$

$$= \frac{1}{c}\frac{\partial}{\partial t}(\frac{E^2 + B^2}{8\pi c}) + \vec{\nabla} \cdot (\frac{\vec{E} \times \vec{B}}{4\pi c}) = 0 \tag{4}$$

2. Energy-Momentum Density's Conservation Electromagnetic Field in Rindler Spacetime .

Rindler space-time is

$$d\tau^2 = (1 + \frac{a_0\xi^1}{c^2})(d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] = g_{\mu\upsilon}d\xi^{\mu}d\xi^{\upsilon} \tag{5}$$

In Rindler space-time, the energy-momentum tensor $T^{\mu\upsilon}$ of the electromagnetic field is

$$T^{\mu\upsilon} = \frac{1}{4\pi c}(F^{\mu}{}_{\rho}F^{\upsilon\rho} - \frac{1}{4}g^{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma}) \tag{6}$$

In this time, in Rindler space-time, Faraday tensors $F_{\xi}{}^{\mu\upsilon}$ is[2]

$$F_{\xi}^{\ \mu\upsilon} = \begin{pmatrix} 0 & E_{\xi^1} & E_{\xi^2} & E_{\xi^3} \\ -E_{\xi^1} & 0 & (1+\frac{a_0\xi^1}{c^2})B_{\xi^3} & -(1+\frac{a_0\xi^1}{c^2})B_{\xi^2} \\ -E_{\xi^2} & -(1+\frac{a_0\xi^1}{c^2})B_{\xi^3} & 0 & (1+\frac{a_0\xi^1}{c^2})B_{\xi^1} \\ -E_{\xi^3} & (1+\frac{a_0\xi^1}{c^2})B_{\xi^2} & -(1+\frac{a_0\xi^1}{c^2})B_{\xi^1} & 0 \end{pmatrix} \quad (7)$$

In Rindler space-time, Faraday tensors $F_{\xi\,\mu\upsilon}$ is[2]

$$F_{\xi\,\mu\upsilon} = \begin{pmatrix} 0 & -(1+\frac{a_0\xi^1}{c^2})E_{\xi^1} & -(1+\frac{a_0\xi^1}{c^2})E_{\xi^2} & -(1+\frac{a_0\xi^1}{c^2})E_{\xi^3} \\ (1+\frac{a_0\xi^1}{c^2})E_{\xi^1} & 0 & B_{\xi^3} & -B_{\xi^2} \\ (1+\frac{a_0\xi^1}{c^2})E_{\xi^2} & -B_{\xi^3} & 0 & B_{\xi^1} \\ (1+\frac{a_0\xi^1}{c^2})E_{\xi^3} & B_{\xi^2} & -B_{\xi^1} & 0 \end{pmatrix} \quad (8)$$

Hence, the energy density $p_{\xi_f}^{\ 0}$ and the momentum density $\vec{p}_{\xi_f}$ of electromagnetic field are in Rindler space-time.

$$T^{00} = p_{\xi_f}^{\ 0} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{E_{\xi}^{\ 2}+B_{\xi}^{\ 2}}{8\pi c} \quad (9)$$

$$T^{0\upsilon} = \vec{p}_{\xi_f} = \frac{\vec{E}_{\xi}\times\vec{B}_{\xi}}{4\pi c} \quad (10)$$

$$\left|\vec{E}_{\xi}\right| = E_{\xi}, \left|\vec{B}_{\xi}\right| = B_{\xi} \quad (11)$$

In Rindler space-time, the energy-momentum conservation law of electromagnetic field is by Noether theorem,

$$\begin{aligned} T^{\mu\upsilon}{}_{;\upsilon} &= T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{0\upsilon}{}_{;\upsilon} \ , \quad i = 1,2,3 \\ &= \frac{\partial T^{0\upsilon}}{\partial x^{\upsilon}} + \Gamma^{0}{}_{\sigma\upsilon}T^{\sigma\upsilon} + \Gamma^{\upsilon}{}_{\sigma\upsilon}T^{0\sigma} \end{aligned} \quad (12)$$

In this time, the affine connection is in Rindler space-time

$$\Gamma^1{}_{00} = -(1+\frac{a_0}{c^2}\xi^1)\frac{a_0}{c^2}, \quad \Gamma^0{}_{10} = \Gamma^0{}_{01} = \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2} \tag{13}$$

Hence, in Rindler space-time, the energy-momentum conservation law of electromagnetic field is

$$\begin{aligned} T^{\mu\upsilon}{}_{;\upsilon} &= T^{00}{}_{;0} + T^{0i}{}_{;i} = T^{0\upsilon}{}_{;\upsilon} \\ &= \frac{\partial T^{0\upsilon}}{\partial x^\upsilon} + \Gamma^0{}_{\sigma\upsilon}T^{\sigma\upsilon} + \Gamma^\upsilon{}_{\sigma\upsilon}T^{0\sigma} \\ &= \frac{\partial T^{0\upsilon}}{\partial x^\upsilon} + 3\Gamma^0{}_{01}T^{01} \end{aligned}$$

$$= \frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{1}{c}\frac{\partial}{\partial\xi^0}(\frac{E_\xi{}^2 + B_\xi{}^2}{8\pi c}) + \vec{\nabla}_\xi\cdot(\frac{\vec{E}_\xi\times\vec{B}_\xi}{4\pi c}) + 3\frac{1}{(1+\frac{a_0\xi^1}{c^2})}\frac{a_0}{c^2}\frac{1}{4\pi c}(E_{\xi^3}B_{\xi^2} - E_{\xi^2}B_{\xi^3})$$

$$= 0 \qquad \vec{\nabla}_\xi = (\frac{\partial}{\partial\xi^1}, \frac{\partial}{\partial\xi^2}, \frac{\partial}{\partial\xi^3})$$

$$|\vec{E}_\xi| = E_\xi, |\vec{B}_\xi| = B_\xi \tag{14}$$

3. Conclusion

We find the energy-momentum density's conservation law of electromagnetic field in Rindler space-time.

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