

I write the differential equation for membrane dynamics, the differential equation for tensile structure and the energy of a membrane.

The Newton's second law for the element  $dS$  of the membrane is:

$$\rho \frac{\partial^2 z}{\partial t^2} = \rho g + T \Delta z$$

where  $T$  is the tension field norm, and  $g$  is the gravitational acceleration.

In the case of stationary equilibrium:

$$\sqrt{T} \Delta z(x, y) = -\frac{\rho g}{\sqrt{T}}$$

It is possible to obtain the energy density of the membrane, using an analogy with the potential formulation of the electromagnetic field (with unitary permeability)

$$\Delta \mathbf{A}(x, y) = -\mathbf{J}(x, y)$$

where  $\mathbf{A}(x, y) = (x, y, \sqrt{T}z(x, y))$  and  $\mathbf{J} = (0, 0, \rho g/\sqrt{T})$ , so that the energy is:

$$H = \int dxdy \left[ \frac{(\nabla \times \mathbf{A})^2}{2} - \mathbf{J} \cdot \mathbf{A} \right]$$

$$\nabla \times (x, y, Tz(x, y)) = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ x & y & \sqrt{T}z(x, y) \end{pmatrix} = \mathbf{i}\sqrt{T}\partial_y z - \mathbf{j}\sqrt{T}\partial_x z$$

the energy of the membrane is:

$$H = \int dxdy \left[ T \frac{(\partial_x z)^2 + (\partial_y z)^2}{2} + \rho g z(x, y) \right]$$

the second term of the energy is the gravitational potential of the membrane, and the first term is the work done by the tensions to deform the membrane:  $\partial_x z$  and  $\partial_y z$  are angles for little deformations, and the work is done only along the  $z$  axis.

I think that the energy of the electromagnetic field can be seen like an energy of a three-dimensional membrane with constant tension, where the deformation is proportional to the magnetic potential, and the external force is the current.