On Vector Subspaces

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Abstract

The short writing seeks to demonstrate certain lapses in the theory of the linear vector spaces.

Introduction

We set out to bring out a contradiction in the theory of the linear vector spaces.

Basic Calculations

We consider V, a linear vector space[1]. W is a proper subspace[2]: $W \subset V$. We take $e \in V - W$ and N vectors $y_i \in W; i = 1,2,3 \ldots N \gg n = \dim V$. All y_i are not linearly independent. We consider $k = \dim W$ of the $y_i \in W$ as linearly independent. They form a basis of W. The rest of the y_i are linear combinations of the basic vectors of W.

We form the sums

$$\alpha_i = e + y_i; i = 1,2,3 \ldots N$$

$$\alpha_i \in V - W$$

Next we consider the equation

$$\sum_i c_i \alpha_i = 0 \ (1)$$

$$\sum_i c_i (e + y_i) = 0$$

$$\Rightarrow e \sum_i c_i = \sum_i c_i y_i \quad (2)$$

The right side of (11)belongs to Wwhile the left side belongs to$V - W$

This is not possible unless each side of …is equal to zero. We cannot have all $c_i = 0$ since that will produce N linearly independent vectors with N>>n=dim V.

Equations:

$$\sum c_i = 0; i = 1,2, \dots N \quad (3.1)$$

$$\sum_i c_i y_i = 0; i = 1,2,3 \dots N \ (3.2)$$

Equation $\sum c_i = 0$ implies $c_N = -c_1 - c_2 - \cdots - c_{N-1}$

Using the above result in (3.2) we obtain,

$$y_N = \frac{c_1}{c_1 + c_2 + \cdots + c_{N-1}} y_1 + \frac{c_2}{c_1 + c_2 + \cdots + c_{N-1}} y_2 + \cdots . \frac{c_{n-1}}{c_1 + c_2 + \cdots + c_{N-1}} y_{n-1} \quad (13)$$

$$y_N = a_1 y_1 + a_2 y_2 + a_2 y_3 + \cdots . a_{N-1} y_{N-1} \quad (4)$$

where $a_i = \frac{c_i}{c_1 + c_2 + \cdots + c_{N-1}}; i = 1,2,3 \dots N - 1$

It is important to take note of the fact that with (4)

$$a_1 + a_2 + \cdots + a_{N-1} = 1 \ (5)$$

But y_N could be an arbitrary superposition of the rest of y_i,according to our choice. Constraint given by (5) is quite unwarranted. We may exert our choice in order to have y_N with

$$a_1 + a_2 + \cdots + a_{N-1} \neq 1 \ (6)$$

Conclusions

As claimed at the outset, there are contradictions in the theory of the linear vector spaces. A restructuring of the subject could be necessary

References

1. Dym H., Linear Algebra in Action, American Mathematical Society, Vol 78, Second Indian Reprint 2014, page 2

2. Hoffman K,Kunze R;Linear algebra,Second Edition, PHI Learning Private Limited, New Delhi,2014[India Reprint], Chapter 6=2:Section 2.2, Subspaces[De