

# **On the Ramanujan's Mock $\theta$ -functions of his last letter: mathematical connections with some sectors of Particle Physics, Cosmology, some expressions concerning Monster Group, Black Hole entropies and hypothetical mass of Dark Matter particles.**

**Michele Nardelli<sup>1</sup>, Antonio Nardelli**

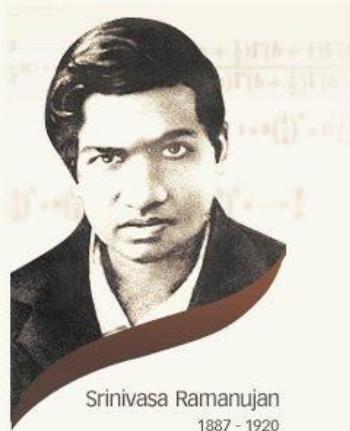
## **Abstract**

*In this research paper we have obtained some interesting mathematical connections between the Mock Theta functions of the Ramanujan's last letter and some sectors of Particle Physics, Cosmology and some expressions concerning the Monster Group, the Black Hole entropies and the hypothetical mass of Dark Matter particles*

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<sup>1</sup> M.Nardelli have studied by Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni “R. Caccioppoli” - Università degli Studi di Napoli “Federico II” – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

$$\sqrt{\frac{\pi e}{2}} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{2}{1 + \cfrac{3}{1 + \cfrac{4}{1 + \cfrac{5}{1 + \cfrac{6}{1 + \cfrac{7}{1 + \cfrac{8}{\ddots}}}}}}}} + \left\{ 1 + \cfrac{1}{1 \cdot 3} + \cfrac{1}{1 \cdot 3 \cdot 5} + \cfrac{1}{1 \cdot 3 \cdot 5 \cdot 7} + \cfrac{1}{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9} + \dots \right\}$$



Srinivasa Ramanujan  
1887 - 1920

Cliff Pickover on | Mathematics | Math, Ap calculus, Mathematics  
Pinterest

From the Ramanujan's last letter:

*"I am extremely sorry for not writing you a single letter up to now . . . I discovered very interesting functions recently which I call "Mock"  $\vartheta$ -functions. Unlike the "False"  $\vartheta$ -functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary  $\vartheta$ -function. I am sending you with this letter some examples . . .*

*If we consider a  $\vartheta$ -function in the transformed Eulerian form e.g...".*

**Analysis of the various mock theta functions and new mathematical connections with some sectors of physics**

For the equation (C) (PAGE 1), we have that:

$$(((\sqrt{0.12/(2\sqrt{5})}) \exp[(\pi^2/(5*0.12)) + 0.12*1/(8\sqrt{5})+0.0144*1/(8\sqrt{5})+0.001728*1/(8\sqrt{5}))])) * (0.257602862254+0.0144) * 10^5$$

Input interpretation:

$$\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{5*0.12} + 0.12 \times \frac{1}{8\sqrt{5}} + 0.0144 \times \frac{1}{8\sqrt{5}} + 0.001728 \times \frac{1}{8\sqrt{5}}\right) \\ (0.257602862254 + 0.0144) \times 10^5$$

Result:

$$6.25273... \times 10^{10}$$

Series representations:

$$\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{5*0.12} + \frac{0.12}{8\sqrt{5}} + \frac{0.0144}{8\sqrt{5}} + \frac{0.001728}{8\sqrt{5}}\right) \\ (0.2576028622540000 + 0.0144) 10^5 = \\ 27200.3 \exp\left(1.66667\pi^2 + \frac{0.017016}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right) \sqrt{z_0} \\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.06}{\sqrt{5}} - z_0\right)^k z_0^{-k}}{k!} \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{5*0.12} + \frac{0.12}{8\sqrt{5}} + \frac{0.0144}{8\sqrt{5}} + \frac{0.001728}{8\sqrt{5}}\right) \\ (0.2576028622540000 + 0.0144) 10^5 = 27200.3 \exp\left(i\pi \left[ \frac{\arg\left(-x + \frac{0.06}{\sqrt{5}}\right)}{2\pi} \right]\right) \\ \exp\left(1.66667\pi^2 + \frac{0.017016}{\exp\left(i\pi \left[ \frac{\arg(5-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}\right) \\ \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(-x + \frac{0.06}{\sqrt{5}}\right)^k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{5 \times 0.12} + \frac{0.12}{8\sqrt{5}} + \frac{0.0144}{8\sqrt{5}} + \frac{0.001728}{8\sqrt{5}}\right)$$

$$(0.2576028622540000 + 0.0144) 10^5 =$$

$$27200.3 \exp\left(1.66667\pi^2 + \frac{0.017016\left(\frac{1}{z_0}\right)^{-1/2[\arg(5-z_0)/(2\pi)]} z_0^{-1/2-1/2[\arg(5-z_0)/(2\pi)]}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right)$$

$$\left(\frac{1}{z_0}\right)^{1/2[\arg(\frac{0.06}{\sqrt{5}}-z_0)/(2\pi)]} z_0^{1/2+1/2[\arg(\frac{0.06}{\sqrt{5}}-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.06}{\sqrt{5}}-z_0\right)^k z_0^{-k}}{k!}$$

$$\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{5 \times 0.12} + \frac{0.12}{8\sqrt{5}} + \frac{0.0144}{8\sqrt{5}} + \frac{0.001728}{8\sqrt{5}}\right)$$

$$(0.2576028622540000 + 0.0144) 10^5 =$$

$$27200.3 \exp\left(1.66667\pi^2 + \frac{0.017016\left(\frac{1}{z_0}\right)^{-1/2[\arg(5-z_0)/(2\pi)]} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right)$$

$$\left(\frac{1}{z_0}\right)^{1/2[\arg(\frac{0.06}{\sqrt{5}}-z_0)/(2\pi)]} z_0^{1/2+1/2[\arg(\frac{0.06}{\sqrt{5}}-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.06}{\sqrt{5}}-z_0\right)^k z_0^{-k}}{k!}$$

Where  $1/(8\sqrt{5}) = 0.0559$ ;  $0.0559 * 0.12 = 0.006708$ ;  $0.001728 / 0.006708 = 0.257602862254$ ;  
 $0.257602862254 + 0.0144$

and:

$$(1+(e^{-0.12})/(1-e^{-0.12})^2+(e^{-0.12})^3/((1-e^{-0.12})^2(1-e^{-0.12^2})^2)+(e^{-0.12})^6/((1-e^{-0.12})^2(1-e^{-0.12^2})^2(1-e^{-0.12^3})^2))$$

Input:

$$1 + \frac{1}{e^{0.12} \left(1 - \frac{1}{e^{0.12}}\right)^2} + \frac{\left(\frac{1}{e^{0.12}}\right)^3}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2} + \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2 \left(1 - e^{-0.12^3}\right)^2}$$

Result:

$$6.24778... \times 10^{10}$$

Alternative representation:

$$\begin{aligned}
& 1 + \frac{1}{e^{0.12} \left(1 - \frac{1}{e^{0.12}}\right)^2} + \frac{\left(\frac{1}{e^{0.12}}\right)^3}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2} + \\
& \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2 \left(1 - e^{-0.12^3}\right)^2} = \\
& 1 + \frac{1}{\exp^{0.12}(z) \left(1 - \frac{1}{\exp^{0.12}(z)}\right)^2} + \frac{\left(\frac{1}{\exp^{0.12}(z)}\right)^3}{\left(1 - \frac{1}{\exp^{0.12}(z)}\right)^2 \left(1 - \exp^{-0.12^2}(z)\right)^2} + \\
& \frac{\left(\frac{1}{\exp^{0.12}(z)}\right)^6}{\left(1 - \frac{1}{\exp^{0.12}(z)}\right)^2 \left(1 - \exp^{-0.12^2}(z)\right)^2 \left(1 - \exp^{-0.12^3}(z)\right)^2} \text{ for } z = 1
\end{aligned}$$

Series representations:

$$\begin{aligned}
& 1 + \frac{1}{e^{0.12} \left(1 - \frac{1}{e^{0.12}}\right)^2} + \frac{\left(\frac{1}{e^{0.12}}\right)^3}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2} + \\
& \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2 \left(1 - e^{-0.12^3}\right)^2} = \\
& 1 + \frac{1}{\left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.12\right)^2\right)^2 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.0144\right)^2\right)^2 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.001728\right)^2\right)^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.72}} + \\
& \frac{1}{\left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.12\right)^2\right)^2 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.0144\right)^2\right)^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.36}} + \\
& \frac{1}{\left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.12\right)^2\right)^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.12}}
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{\left(\frac{1}{e^{0.12}}\right)^3}{e^{0.12} \left(1 - \frac{1}{e^{0.12}}\right)^2} + \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2} + \\
& \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2 \left(1 - e^{-0.12^3}\right)^2} = 1 + \\
& \frac{1.64718}{\left(1 - \frac{1.08673}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.12}}\right)^2 \left(1 - \frac{1.01003}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.0144}}\right)^2 \left(1 - \frac{1.0012}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.001728}}\right)^2 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.72}} \\
& + \frac{1.28343}{\left(1 - \frac{1.08673}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.12}}\right)^2 \left(1 - \frac{1.01003}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.0144}}\right)^2 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.36}} + \\
& \frac{1.08673}{\left(1 - \frac{1.08673}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.12}}\right)^2 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.12}}
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{\left(\frac{1}{e^{0.12}}\right)^3}{e^{0.12} \left(1 - \frac{1}{e^{0.12}}\right)^2} + \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2} + \\
& \frac{\left(\frac{1}{e^{0.12}}\right)^6}{\left(1 - \frac{1}{e^{0.12}}\right)^2 \left(1 - e^{-0.12^2}\right)^2 \left(1 - e^{-0.12^3}\right)^2} = \\
& 1 + 1 / \left( \left(1 - \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.12}}\right)^2 \left(1 - \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.0144}}\right)^2 \right. \\
& \left. \left(1 - \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.001728}}\right)^2 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.72} \right) + \\
& \frac{1}{\left(1 - \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.12}}\right)^2 \left(1 - \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.0144}}\right)^2 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.36}} + \\
& \frac{1}{\left(1 - \frac{1}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.12}}\right)^2 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.12}}
\end{aligned}$$

Note that:

$$27 + 1.08346\pi * (6.24778 \times 10^{10})^{1/4}$$

Input interpretation:

$$27 + 1.08346 \pi \sqrt[4]{6.24778 \times 10^{10}}$$

[Open code](#)

Result:

More digits

1728.74...

This result is very near to the mass of  $f_0(1710)$  candidate glueball

Continued fraction:

Linear form

$$\begin{aligned} 1728 + & \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}} \end{aligned}$$

Possible closed forms:

More

$$-751 + \frac{317}{2\pi} - \frac{564}{\sqrt{\pi}} + 1660\sqrt{\pi} - 62\pi \approx 1728.743839819785700130293$$

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$$\frac{197929 + 175699e - 4931e^2}{136e} \approx 1728.743839819785700114109$$

$$\frac{17390e e! + 12387 + 10307e + 344e^2}{52e} \approx 1728.74383981978570012178461$$

$$1/72 * [27+1.08346\pi * (6.24778 \times 10^{10})^{1/4}]$$

Input interpretation:

$$\frac{1}{72} \left( 27 + 1.08346 \pi \sqrt[4]{6.24778 \times 10^{10}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits

More digits

24.01033110860813472390890291088494296531517821201400679205...

This result 24,01033 is practically equal to 24, are the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

Continued fraction:  
Linear form

$$24 + \cfrac{1}{96 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{17 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{44 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}}$$

Possible closed forms:

More

$$\frac{9712717865 \pi}{1270844744} \approx 24.01033110860813472339648$$

Enlarge Data Customize A Plaintext Interactive

$$\text{root of } 36x^5 - 898x^4 + 829x^3 - 528x^2 + 200x + 899 \text{ near } x = 24.0103 \approx$$

$$24.0103311086081347279547$$

$$\pi \text{ root of } 462x^4 - 4296x^3 + 5639x^2 + 2440x - 6490 \text{ near } x = 7.64273 \approx$$

$$24.01033110860813472375750$$

$$1/(\sqrt{5}) * [27 + 1.08346\pi * (6.24778 \times 10^{10})^{1/4}]$$

Input interpretation:

$$\frac{1}{\sqrt{5}} \left( 27 + 1.08346\pi \sqrt[4]{6.24778 \times 10^{10}} \right)$$

Open code

Result:

More digits

$$773.118\dots$$

This value 773,118 is very near to the rest mass of Charged rho meson

Continued fraction:  
Linear form

$$773 + \cfrac{1}{8 + \cfrac{1}{2 + \cfrac{1}{33 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{148 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \dots}}}}}}}}}}}}}}}}$$

$$1/(137*3) * [27+1.08346\pi * (6.24778 \times 10^{10})^{1/4}]$$

Input interpretation:

$$\frac{1}{137 \times 3} \left( 27 + 1.08346 \pi \sqrt[4]{6.24778 \times 10^{10}} \right)$$

[Open code](#)

[Enlarge](#) [Data](#) [Customize](#) A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

4.206189391289016301998639925994442563266892533491504839483...

This result 4,20618 is in the range of the mass of hypothetical dark matter particles

Continued fraction:

- Linear form

$$4 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{27 + \cfrac{1}{7 + \cfrac{1}{17 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{51 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}$$

$$2\pi * (0.663723)^2 [(6.24778 \times 10^{10})^{1/4}]$$

Input interpretation:

$$2\pi \times 0.663723^2 \sqrt[4]{6.24778 \times 10^{10}}$$

[Open code](#)

Result:

More digits

1383.84...

This result 1383,84 is practically equal to the rest mass of Sigma baryon

1383.8373099004053208878518213417159831225513162043778

Continued fraction:

Linear form

$$1383 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{15 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{30 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

$$\frac{1}{2} (\ln 6.24778 \times 10^{10})$$

Input interpretation:

$$\frac{1}{2} \log(6.24778 \times 10^{10})$$

Result:

$$12.42904\dots$$

This result 12,42904 is very near to the value of black hole entropy (see Tables)

Possible closed forms:

$$-e^{-2+1/e+3/\pi+\pi} \cot(e \pi) \csc(e \pi) \approx 12.429039177$$

$$6 + 2 \sqrt{\frac{31}{3}} \approx 12.42910050$$

$$\frac{724\pi}{183} \approx 12.429033230$$

Continued fraction:  
Linear form

$$12 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{43 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}$$

And

$$1/6 \ln 6.24778 \times 10^{10}$$

Input interpretation:

$$\frac{1}{6} \log(6.24778 \times 10^{10})$$

Result:

$$4.143013\dots$$

This result 4,143013 is in the range of the mass of hypothetical dark matter particles

Continued fraction:

Linear form

$$4 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{130 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{\dots}}}}}}}$$

Note that from the previously expression, if we replaced  $\frac{\pi^2}{5}$  with  $\frac{\pi^2}{6}$  and 27 with 42, we obtain:

$$(((\sqrt{0.12/(2\sqrt{5}))} \exp[(\pi^2/(6*0.12)) + 0.12*1/(8\sqrt{5})+0.0144*1/(8\sqrt{5})+0.001728*1/(8\sqrt{5})])) * 42*10^4$$

Input interpretation:

$$\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{6*0.12} + 0.12 \times \frac{1}{8\sqrt{5}} + 0.0144 \times \frac{1}{8\sqrt{5}} + 0.001728 \times \frac{1}{8\sqrt{5}}\right) \times 42 \times 10^4$$

Result:

$$6.22447... \times 10^{10}$$

$$\frac{((\sqrt{0.12/(2\sqrt{5}))} \exp[(\pi^2/(6*0.12)) + 0.12*1/(8\sqrt{5})+0.0144*1/(8\sqrt{5})+0.001728*1/(8\sqrt{5})])) * 42*10^4}{(37840240076.773331293157064141904)}$$

we can to obtain

$$\frac{\pi^2}{6} = \zeta(2) = 1.644934066848226436472415166646025189218949901206798437735...$$

Input interpretation:

$$\frac{\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{6*0.12} + 0.12 \times \frac{1}{8\sqrt{5}} + 0.0144 \times \frac{1}{8\sqrt{5}} + 0.001728 \times \frac{1}{8\sqrt{5}}\right) \times 42 \times 10^4}{3.7840240076773331293157064141904 \times 10^{10}}$$

Result:

$$1.64493...$$

Result:

- More digits

$$1.644934066848226475081076911228437646701217835062251517803...$$

Or:

$$\frac{((\sqrt{0.12/(2\sqrt{5}))} \exp[(\pi^2/(6*0.12)) + 0.12*1/(8\sqrt{5})+0.0144*1/(8\sqrt{5})+0.001728*1/(8\sqrt{5})])) * 42*10^4}{(196884 - 1728.32026394733275450092328764-576-54)^2}$$

Input interpretation:

$$\frac{\sqrt{\frac{0.12}{2\sqrt{5}}} \exp\left(\frac{\pi^2}{6*0.12} + 0.12 \times \frac{1}{8\sqrt{5}} + 0.0144 \times \frac{1}{8\sqrt{5}} + 0.001728 \times \frac{1}{8\sqrt{5}}\right) \times 42 \times 10^4}{(196884 - 1728.32026394733275450092328764 - 576 - 54)^2}$$

Result:

$$1.64493...$$

Possible closed forms:

$$\frac{\pi^2}{6} \approx 1.644934066848226436472$$

$$\zeta(2) \approx 1.644934066848226436472$$

Considering  $t = 0.14$ ,  $\exp^2$  and  $\frac{\pi^2}{5}$ , we obtain:

Input interpretation:

$$\sqrt{\frac{0.14}{2\sqrt{5}}} \times 0.028 \exp^2 \left( \frac{\pi^2}{5 \times 0.14} + 0.14 \times \frac{1}{8\sqrt{5}} + 0.0196 \times \frac{1}{8\sqrt{5}} + 0.002744 \times \frac{1}{8\sqrt{5}} + 0.00038416 \times \frac{1}{8\sqrt{5}} + 0.0000537824 \times \frac{1}{8\sqrt{5}} \right)$$

Result:

- More digits

$$8.90188622348206885089354866855171578336515535828991183... \times 10^9$$

Series representations:

$$\begin{aligned} & \sqrt{\frac{0.14}{2\sqrt{5}}} \quad 0.028 \exp^2 \left( \frac{\pi^2}{5 \times 0.14} + \frac{0.14}{8\sqrt{5}} + \frac{0.0196}{8\sqrt{5}} + \frac{0.002744}{8\sqrt{5}} + \frac{0.00038416}{8\sqrt{5}} + \frac{0.0000537824}{8\sqrt{5}} \right) = \\ & 0.028 \exp^2 \left( 1.42857 \pi^2 + \frac{0.0203477}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (5-z_0)^k z_0^{-k}}{k!}} \right) \sqrt{z_0} \\ & \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k \left(\frac{0.07}{\sqrt{5}} - z_0\right)^k z_0^{-k}}{k!} \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

$$\begin{aligned} & \sqrt{\frac{0.14}{2\sqrt{5}}} \quad 0.028 \exp^2 \left( \frac{\pi^2}{5 \times 0.14} + \frac{0.14}{8\sqrt{5}} + \frac{0.0196}{8\sqrt{5}} + \frac{0.002744}{8\sqrt{5}} + \frac{0.00038416}{8\sqrt{5}} + \frac{0.0000537824}{8\sqrt{5}} \right) = 0.028 \exp \left( i\pi \left[ \frac{\arg(-x + \frac{0.07}{\sqrt{5}})}{2\pi} \right] \right) \\ & \exp^2 \left( 1.42857 \pi^2 + \frac{0.0203477}{\exp(i\pi \left[ \frac{\arg(5-x)}{2\pi} \right]) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} (-\frac{1}{2})_k}{k!}} \right) \\ & \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} (-\frac{1}{2})_k \left(-x + \frac{0.07}{\sqrt{5}}\right)^k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0) \end{aligned}$$

$$\sqrt{\frac{0.14}{2\sqrt{5}}} \quad 0.028$$

$$\exp^2\left(\frac{\pi^2}{5\times 0.14} + \frac{0.14}{8\sqrt{5}} + \frac{0.0196}{8\sqrt{5}} + \frac{0.002744}{8\sqrt{5}} + \frac{0.00038416}{8\sqrt{5}} + \frac{0.0000537824}{8\sqrt{5}}\right) =$$

$$0.028 \exp^2\left(1.42857\pi^2 + \frac{0.0203477\left(\frac{1}{z_0}\right)^{-1/2[\arg(5-z_0)/(2\pi)]} z_0^{-1/2-1/2[\arg(5-z_0)/(2\pi)]}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right)$$

$$\left(\frac{1}{z_0}\right)^{1/2[\arg(\frac{0.07}{\sqrt{5}}-z_0)/(2\pi)]} z_0^{1/2+1/2[\arg(\frac{0.07}{\sqrt{5}}-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.07}{\sqrt{5}}-z_0\right)^k z_0^{-k}}{k!}$$

$$\sqrt{\frac{0.14}{2\sqrt{5}}} \quad 0.028$$

$$\exp^2\left(\frac{\pi^2}{5\times 0.14} + \frac{0.14}{8\sqrt{5}} + \frac{0.0196}{8\sqrt{5}} + \frac{0.002744}{8\sqrt{5}} + \frac{0.00038416}{8\sqrt{5}} + \frac{0.0000537824}{8\sqrt{5}}\right) =$$

$$0.028 \exp^2\left(1.42857\pi^2 + \frac{0.0203477\left(\frac{1}{z_0}\right)^{-1/2[\arg(5-z_0)/(2\pi)]} z_0^{1/2(-1-\arg(5-z_0)/(2\pi))}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right)$$

$$\left(\frac{1}{z_0}\right)^{1/2[\arg(\frac{0.07}{\sqrt{5}}-z_0)/(2\pi)]} z_0^{1/2+1/2[\arg(\frac{0.07}{\sqrt{5}}-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.07}{\sqrt{5}}-z_0\right)^k z_0^{-k}}{k!}$$

Note that:

$$(8.90189 \times 10^9)^{1/9}$$

Input interpretation:

$$\sqrt[9]{8.90189 \times 10^9}$$

Result:

$$12.74964\dots$$

This result 12,74964 is very near to the value of black hole entropy (see Tables)

$$(13*5) \quad (8.90189 \times 10^9)^{1/7}$$

Input interpretation:

$$(13 \times 5) \sqrt[7]{8.90189 \times 10^9}$$

Result:

1715.015...

This result 1715,015 is very near to the mass of  $f_0(1710)$  candidate glueball

$$(53) (8.90189 \times 10^9)^{1/7}$$

Input interpretation:

$$53 \sqrt[7]{8.90189 \times 10^9}$$

Result:

1398.397...

This result 1398,397 is a good approximation to the rest mass of Sigma baryon

$$(29) (8.90189 \times 10^9)^{1/7}$$

Input interpretation:

$$29 \sqrt[7]{8.90189 \times 10^9}$$

Result:

765.1606...

This value 765,1606 is very near to the rest mass of Charged rho meson

$$(8901886223,482)^{1/16} = 4,18641837028$$

$$(8901886223,482)^{1/46} = 1,645481817826 \text{ value very near to the } \frac{\pi^2}{6} = \zeta(2) = 1.644934066848226436472415166646025189218949901206798437735...$$

Obviously, all the results can be represented with infinite continued fractions

And

Input:

$$1 + \frac{1}{e^{0.14} \left(1 - \frac{1}{e^{0.14}}\right)^2} + \frac{\left(\frac{1}{e^{0.14}}\right)^3}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2} + \\ \frac{\left(\frac{1}{e^{0.14}}\right)^6}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2 \left(1 - e^{-0.14^3}\right)^2}$$

Result:

$$8.94217... \times 10^9$$

Alternative representation:

$$1 + \frac{1}{e^{0.14} \left(1 - \frac{1}{e^{0.14}}\right)^2} + \frac{\left(\frac{1}{e^{0.14}}\right)^3}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2} + \\ \frac{\left(\frac{1}{e^{0.14}}\right)^6}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2 \left(1 - e^{-0.14^3}\right)^2} = \\ 1 + \frac{1}{\exp^{0.14}(z) \left(1 - \frac{1}{\exp^{0.14}(z)}\right)^2} + \frac{\left(\frac{1}{\exp^{0.14}(z)}\right)^3}{\left(1 - \frac{1}{\exp^{0.14}(z)}\right)^2 \left(1 - \exp^{-0.14^2}(z)\right)^2} + \\ \frac{\left(\frac{1}{\exp^{0.14}(z)}\right)^6}{\left(1 - \frac{1}{\exp^{0.14}(z)}\right)^2 \left(1 - \exp^{-0.14^2}(z)\right)^2 \left(1 - \exp^{-0.14^3}(z)\right)^2} \text{ for } z = 1$$

Series representations:

$$1 + \frac{1}{e^{0.14} \left(1 - \frac{1}{e^{0.14}}\right)^2} + \frac{\left(\frac{1}{e^{0.14}}\right)^3}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2} + \\ \frac{\left(\frac{1}{e^{0.14}}\right)^6}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2 \left(1 - e^{-0.14^3}\right)^2} = \\ 1 + \frac{1}{\left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.14\right)^2\right)^2 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.0196\right)^2\right)^2 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.002744\right)^2\right)^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.84}} + \\ \frac{1}{\left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.14\right)^2\right)^2 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.0196\right)^2\right)^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.42}} + \\ \frac{1}{\left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!} 0.14\right)^2\right)^2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.14}}$$

$$\begin{aligned}
& 1 + \frac{1}{e^{0.14} \left(1 - \frac{1}{e^{0.14}}\right)^2} + \frac{\left(\frac{1}{e^{0.14}}\right)^3}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2} + \\
& \frac{\left(\frac{1}{e^{0.14}}\right)^6}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2 \left(1 - e^{-0.14^3}\right)^2} = \\
& 1 + 1.79005 / \left( \left( -1.0019 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.002744} \right)^2 \left( -1.01368 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.0196} \right)^2 \right. \\
& \left. \left( -1.10191 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.14} \right)^2 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.515312} \right) + \\
& \frac{1.33793}{\left( -1.10191 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.14} \right)^2} \\
& \frac{\left( -1.01368 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.0196} \right)^2 \left( -1.10191 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.14} \right)^2 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.1008}}{\left( 1.10191 \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.14} \right)} + \\
& \frac{\left( -1.10191 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.14} \right)^2}{\left( -1.10191 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^{0.14} \right)^2}
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{1}{e^{0.14} \left(1 - \frac{1}{e^{0.14}}\right)^2} + \frac{\left(\frac{1}{e^{0.14}}\right)^3}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2} + \\
& \frac{\left(\frac{1}{e^{0.14}}\right)^6}{\left(1 - \frac{1}{e^{0.14}}\right)^2 \left(1 - e^{-0.14^2}\right)^2 \left(1 - e^{-0.14^3}\right)^2} = \\
& 1 + 1 / \left( \left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.14}} \right)^2 \left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.0196}} \right)^2 \right. \\
& \left. \left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.002744}} \right)^2 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.84} \right) + \\
& \frac{1}{\left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.14}} \right)^2 \left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.0196}} \right)^2 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.42}} + \\
& \frac{\left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.14}} \right)^2 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.14}}{\left( 1 - \frac{1}{\left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{0.14}} \right)^2}
\end{aligned}$$

Now, we have, from (A) and (B) (**PAGE 1**):

$$\sqrt{0.12} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)$$

Input:

$$\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)$$

[Open code](#)

Result:

More digits

$$1.23465\dots \times 10^5$$

Series representations:

More

$$\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) = \exp(-0.005 + 1.38889\pi^2) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 + \frac{0.06}{\pi}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

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$$\begin{aligned} \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) &= \\ \exp(-0.005 + 1.38889\pi^2) \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{0.06}{\pi} - z_0\right)^k z_0^{-k}}{k!} \end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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$$\begin{aligned} \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) &= \exp(-0.005 + 1.38889\pi^2) \exp\left(i\pi \left[\frac{\arg\left(\frac{0.06}{\pi} - x\right)}{2\pi}\right]\right) \\ \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{0.06}{\pi} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} &\quad \text{for } (x \in \mathbb{R} \text{ and } x < 0) \end{aligned}$$

Continued fraction:

Linear form

$$123465 + \cfrac{1}{13 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$

$$\exp(\text{Pi}^2/(15*0.72)-0.12/60) /(\sqrt{(5-\sqrt{5})}/2)$$

Input:

$$\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}}$$

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Result:

- Fewer digits  
More digits
- 2.117208961660161995731773296803025758360626158670675011836...

Series representations:

More

$$\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} = \frac{\exp(-0.002 + 0.0925926 \pi^2)}{\sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k (3-\sqrt{5})^k}{k!}}$$

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$$\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} = \frac{\exp(-0.002 + 0.0925926 \pi^2)}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k (5-\sqrt{5}-2z_0)^k z_0^{-k}}{k!}}$$

for not (( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

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$$\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} = -\frac{2 \exp(-0.002 + 0.0925926 \pi^2) \sqrt{\pi}}{\sum_{j=0}^{\infty} \text{Res}_{s=-j} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s}}$$

Continued fraction:

Linear form

$$2 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{63 + \cfrac{1}{14 + \cfrac{1}{11 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{6 + \cfrac{1}{4 + \dots}}}}}}}}}}}}}}}}}}}$$

Now:

$$\ln(((\sqrt((0.12)/(2\pi)) * \exp(\pi^2/0.72-0.12/24) * \exp(\pi^2/(15*0.72)-0.12/60) /(\sqrt((5-\sqrt(5))/2)))))))))))$$

Input:

$$\log\left(\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \times \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}}\right)$$

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- $\log(x)$  is the natural logarithm

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Result:

Fewer digits  
More digits

12.47381228287857112805891354242198544545804851258009386702...

This result 12,4738 is very near to the value of black hole entropy (see Tables)

Alternative representations:

$$\log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$\log_e \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

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$$\log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$\log(a) \log_a \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

Series representation:

$$\log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$\log \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)^{-k}}{k}$$

Integral representations:

$$\log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$\frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\int_1^{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \frac{1}{t} dt}$$

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$$\log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \frac{1}{2i\pi}$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

Continued fraction:  
Linear form

$$12 + \cfrac{1}{2 + \cfrac{1}{9 + \cfrac{1}{21 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{93 + \cfrac{1}{5 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{6 + \cfrac{1}{12 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}}}$$

$$1/3 \ln(((\text{sqrt}(0.12)/(2\pi)) * \exp(\pi^2/0.72 - 0.12/24) * \exp(\pi^2/(15*0.72) - 0.12/60) / (\text{sqrt}(5 - \text{sqrt}(5))/2)))$$

Input:

$$\frac{1}{3} \log \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \times \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

- Fewer digits
- More digits

4.157937427626190376019637847473995148486016170860031289009...

This result 4,1579 is in the range of the mass of hypothetical dark matter particles

Alternative representations:

$$\frac{1}{3} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$
$$\frac{1}{3} \log_e \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

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$$\frac{1}{3} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$
$$\frac{1}{3} \log(a) \log_a \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

Series representation:

$$\begin{aligned} \frac{1}{3} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right) = \\ \frac{1}{3} \log \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right) - \\ \frac{1}{3} \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\exp(-0.002+0.0925926\pi^2)\exp(-0.005+1.38889\pi^2)\sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right)^{-k}}{k} \end{aligned}$$

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Integral representations:

$$\begin{aligned} \frac{1}{3} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right) = \\ \frac{\exp(-0.002+0.0925926\pi^2)\exp(-0.005+1.38889\pi^2)\sqrt{\frac{0.06}{\pi}}}{\frac{1}{3} \int_1^{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \frac{1}{t} dt} \end{aligned}$$

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$$\begin{aligned} \frac{1}{3} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right) = \frac{1}{6i\pi} \\ \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \frac{\exp(-0.002+0.0925926\pi^2)\exp(-0.005+1.38889\pi^2)\sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right)^{-s}}{\Gamma(1-s)} ds \end{aligned}$$

for  $-1 < \gamma < 0$

Continued fraction:  
Linear form

$$4 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{64 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{279 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}$$

$$138 \ln(((\sqrt{(0.12)/(2\pi)}) * \exp(\pi^2/0.72 - 0.12/24) * \exp(\pi^2/(15*0.72) - 0.12/60) / (\sqrt{(5-\sqrt{5})}/2)))$$

Input:

$$138 \log \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \times \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Result:

• Fewer digits  
More digits

$$1721.386095037242815672130068854233991473210694736052953649\dots$$

This result 1721,386 is very near to the mass of  $f_0(1710)$  candidate glueball

Alternative representations:

$$138 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$138 \log_e \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

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$$138 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \\ 138 \log(a) \log_a \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

Series representation:

$$138 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \\ 138 \log \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) - \\ 138 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)^{-k}}{k}$$

Integral representations:

$$138 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \\ \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \\ 138 \int_1^{\frac{\sqrt{\frac{1}{2}(5 - \sqrt{5})}}{\sqrt{\frac{0.06}{\pi}}}} \frac{1}{t} dt$$

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$$138 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \frac{69}{i\pi}$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \frac{\exp(-0.002+0.0925926\pi^2)\exp(-0.005+1.38889\pi^2)\sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

Continued fraction:

Linear form

$$1721 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{10 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$

[Open code](#)

$$62 \ln(((\sqrt{(0.12)/(2\pi)}) * \exp(\pi^2/0.72-0.12/24) * \exp(\pi^2/(15*0.72)-0.12/60) / (\sqrt{(5-\sqrt{5})}/2)))$$

Input:

$$62 \log \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \times \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

[Open code](#)

•  $\log(x)$  is the natural logarithm

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Result:

- More digits  
773.376...

This value 773,376 is very near to the rest mass of Charged rho meson

Alternative representations:

$$62 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$62 \log_e \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

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$$62 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$62 \log(a) \log_a \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right) \sqrt{\frac{0.12}{2\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

[Open code](#)

Series representation:

$$62 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$62 \log \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) -$$

$$62 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)^k}{k}$$

[Open code](#)

Integral representations:

$$62 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) =$$

$$\frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{62 \int_1^{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \frac{1}{t} dt}$$

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$$62 \log \left( \frac{\left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \right) \exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \frac{31}{i\pi}$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \frac{\exp(-0.002 + 0.0925926\pi^2) \exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}}}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

[Open code](#)

(((((ln(((sqrt((0.12)/(2Pi)) \* exp(Pi^2/0.72-0.12/24) \* exp(Pi^2/(15\*0.72)-0.12/60) / (sqrt((5-sqrt(5))/2)))))))^1/5

Input:

$$\sqrt[5]{\log \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \times \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)}$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

1.656532...

1/72 \* (((sqrt((0.12)/(2Pi)) \* exp(Pi^2/0.72-0.12/24) + exp(Pi^2/(15\*0.72)-0.12/60) / (sqrt((5-sqrt(5))/2))))

Input:

$$\frac{1}{72} \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

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Result:

- More digits  
1714.82...

This result 1714.82 is very near to the mass of  $f_0(1710)$  candidate glueball

Continued fraction:

Linear form

$$1714 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{2 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{...}}}}}}}}}}}}}}}}}}}}$$

[Open code](#)

Series representations:

More

$$\begin{aligned}
& \frac{1}{72} \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15+0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right) = \\
& \left( \exp(-0.002 + 0.0925926\pi^2) + \exp(-0.005 + 1.38889\pi^2) \right. \\
& \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-1 + \frac{0.06}{\pi}\right)^{k_1} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (3-\sqrt{5})^{k_2}}{k_1! k_2!} \right) / \\
& \left( 72 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k (3-\sqrt{5})^k}{k!} \right)
\end{aligned}$$

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$$\begin{aligned}
& \frac{1}{72} \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15+0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \right) = \left( \exp(-0.002 + 0.0925926\pi^2) + \right. \\
& \left. \exp(-0.005 + 1.38889\pi^2) \sqrt{z_0}^{-2} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \right. \\
& \left. \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (5-\sqrt{5}-2z_0)^{k_2} \left(\frac{0.06}{\pi} - z_0\right)^{k_1} z_0^{-k_1-k_2}}{k_1! k_2!} \right) / \\
& \left( 72 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k (5-\sqrt{5}-2z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\begin{aligned}
& \frac{1}{72} \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right) = \\
& - \left[ \left( 4 \exp(-0.002 + 0.0925926\pi^2) \sqrt{\pi}^2 + \right. \right. \\
& \quad \exp(-0.005 + 1.38889\pi^2) \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-j_1} \left( -1 + \frac{0.06}{\pi} \right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right) \\
& \quad \left. \left. \left( \text{Res}_{s=-j_2} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s} \right) \right] / \right. \\
& \left. \left( 144 \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& 1/(1714.82209*3) * (((\text{sqrt}((0.12)/(2\pi)) * \exp(\pi^2/0.72-0.12/24) + \\
& \exp(\pi^2/(15*0.72)-0.12/60) / (\text{sqrt}((5-\text{sqrt}(5))/2)))
\end{aligned}$$

[Input interpretation:](#)

$$\frac{1}{1714.82209 \times 3} \left( \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5 - \sqrt{5})}} \right)$$

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[Result:](#)

- Fewer digits
- More digits

24.0000003759643575672332557545163265237375416470272598385...

This result is practically equal to 24, are the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

[Continued fraction:](#)

[Linear form](#)

$$24 + \cfrac{1}{26598266 + \cfrac{1}{34 + \cfrac{1}{2 + \cfrac{1}{7 + \cfrac{1}{3 + \cfrac{1}{15 + \cfrac{1}{2 + \cfrac{1}{7 + \cfrac{1}{6 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{12 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}$$

[Open code](#)

Series representations:

More

$$\begin{aligned} & \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}}{1714.82 \times 3} = \\ & \left( 0.000194384 \left( \exp(-0.002 + 0.0925926\pi^2) + \exp(-0.005 + 1.38889\pi^2) \right. \right. \\ & \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-1 + \frac{0.06}{\pi}\right)^{k_1} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (3-\sqrt{5})^{k_2}}{k_1! k_2!} \right) \right) / \\ & \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k (3-\sqrt{5})^k}{k!} \right) \end{aligned}$$

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$$\begin{aligned}
& \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}}{1714.82 \times 3} = \\
& \left( 0.000194384 \left( \exp(-0.002 + 0.0925926\pi^2) + \right. \right. \\
& \left. \left. \exp(-0.005 + 1.38889\pi^2) \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! k_2!} (-1)^{k_1+k_2} 2^{-k_2} \right. \right. \\
& \left. \left. \left( -\frac{1}{2} \right)_{k_1} \left( -\frac{1}{2} \right)_{k_2} (5 - \sqrt{5} - 2z_0)^{k_2} \left( \frac{0.06}{\pi} - z_0 \right)^{k_1} z_0^{-k_1-k_2} \right) \right) / \\
& \left( \sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{2} \right)_k^k \left( -\frac{1}{2} \right)_k (5 - \sqrt{5} - 2z_0)^k z_0^{-k}}{k!} \right) \text{ for not} \\
& ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) + \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}}{1714.82 \times 3} = \\
& - \left( \left( 0.0000971918 \left( 4 \exp(-0.002 + 0.0925926\pi^2) \sqrt{\pi}^2 + \exp(-0.005 + 1.38889\pi^2) \right. \right. \right. \\
& \left. \left. \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-j_1} \left( -1 + \frac{0.06}{\pi} \right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right) \right. \right. \\
& \left. \left. \left( \text{Res}_{s=-j_2} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s} \right) \right) \right) / \\
& \left( \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-j} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s} \right)
\end{aligned}$$

1/34 (((sqrt((0.12)/(2Pi)) \* exp(Pi^2/0.72-0.12/24))) / ((exp(Pi^2/(15\*0.72)-0.12/60) /(sqrt((5-sqrt(5))/2))))

Input:

$$\frac{1}{34} \times \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

- Result:  
Fewer digits  
More digits
- 1715.147515379642336365565701143398878468457202683943869237...

This result 1715,1475 is very near to the mass of  $f_0(1710)$  candidate glueball

- Continued fraction:  
Linear form

$$1715 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{15 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

Series representations:

- More

$$\frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)^{34}} =$$

$$\frac{\sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.005 + 1.38889\pi^2) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-1+\frac{0.06}{\pi}\right)^{k_1} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (3-\sqrt{5})^{k_2}}{k_1! k_2!}}$$

$$34 \exp(-0.002 + 0.0925926\pi^2)$$

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$$\frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)^{34}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} = \left( \exp(-0.005 + 1.38889\pi^2) \sqrt{z_0}^2 \right.$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (5-\sqrt{5}-2z_0)^{k_2} \left(\frac{0.06}{\pi} - z_0\right)^{k_1} z_0^{-k_1-k_2}}{k_1! k_2!}$$

$$\left. / (34 \exp(-0.002 + 0.0925926\pi^2)) \right)$$

for not (( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)^{34}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} =$$

$$\left( \exp(-0.005 + 1.38889\pi^2) \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-j_1} \left( -1 + \frac{0.06}{\pi} \right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right. \right.$$

$$\left. \left. \left( \text{Res}_{s=-j_2} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s} \right) \right) / \right.$$

$$\left( 136 \exp(-0.002 + 0.0925926\pi^2) \sqrt{\pi}^2 \right)$$

1/75 (((sqrt((0.12)/(2Pi)) \* exp(Pi^2/0.72-0.12/24))) / ((exp(Pi^2/(15\*0.72)-0.12/60) /(sqrt((5-sqrt(5))/2))))

Input:

$$\frac{1}{75} \times \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)^{34}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}}$$

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Result:

• Fewer digits  
More digits

777.5335403054378591523897845183408249057005985500545540544...

This value 777,533 is very near to the rest mass of Charged rho meson

Continued fraction:

Linear form

$$777 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{20 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{12 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}$$

Series representations:

More

$$\begin{aligned} & \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right) 75}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} = \\ & \frac{\exp(-0.005 + 1.38889\pi^2) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-1+\frac{0.06}{\pi}\right)^{k_1} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (3-\sqrt{5})^{k_2}}{k_1! k_2!}}{75 \exp(-0.002 + 0.0925926\pi^2)} \end{aligned}$$

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$$\frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)^{75}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} = \left( \exp(-0.005 + 1.38889\pi^2) \sqrt{z_0}^2 \right.$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (5-\sqrt{5}-2z_0)^{k_2} \left(\frac{0.06}{\pi} - z_0\right)^{k_1} z_0^{-k_1-k_2}}{k_1! k_2!}$$

$$\left. / (75 \exp(-0.002 + 0.0925926\pi^2)) \right)$$

for not (( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$\frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)^{75}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} =$$

$$\left( \exp(-0.005 + 1.38889\pi^2) \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-j_1} \left( -1 + \frac{0.06}{\pi} \right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right. \right.$$

$$\left. \left. \left( \text{Res}_{s=-j_2} 2^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) (3 - \sqrt{5})^{-s} \right) \right) / \right.$$

$$\left. (300 \exp(-0.002 + 0.0925926\pi^2) \sqrt{\pi}^2) \right)$$

$$1/(2.61803398) \ln (((\text{sqrt}((0.12)/(2\text{Pi})) * \exp(\text{Pi}^2/0.72-0.12/24))) / ((\exp(\text{Pi}^2/(15*0.72)-0.12/60) / (\text{sqrt}((5-\text{sqrt}(5))/2))))$$

Input interpretation:

$$\frac{1}{2.61803398} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)^{75}}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

• Fewer digits  
More digits

4.191547924567337461288822920219450138966980129192222049580...

This result 4,19154 is in the range of the mass of hypothetical dark matter particles

Continued fraction:  
Linear form

$$4 + \cfrac{1}{5 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$

[Open code](#)

Alternative representations:

$$\frac{\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{2.61803} = \frac{\log_e \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \sqrt{\frac{0.12}{2\pi}}}{\frac{\exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{2.61803}$$

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$$\frac{\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{2.61803} = \frac{\log(a) \log_a \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \sqrt{\frac{0.12}{2\pi}}}{\frac{\exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{2.61803}$$

[Open code](#)

Series representation:

$$\log \frac{\left( \begin{array}{c} \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \\ \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \end{array} \right)}{2.61803} =$$

$$0.381966 \log \left( -1 + \frac{\exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}} \sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002 + 0.0925926\pi^2)} \right) -$$

$$0.381966 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\exp(-0.005+1.38889\pi^2) \sqrt{\frac{0.06}{\pi}} \sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002+0.0925926\pi^2)} \right)^k}{k}$$

[Open code](#)

Integral representations:

$$\log \frac{\left( \begin{array}{c} \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \\ \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \end{array} \right)}{2.61803} = 0.381966 \int_1^{\infty} \frac{\exp(-0.005+1.38889\pi^2) \sqrt{\frac{0.06}{\pi}} \sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002+0.0925926\pi^2)} \frac{1}{t} dt$$

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$$\log \frac{\left( \begin{array}{c} \sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right) \\ \frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}} \end{array} \right)}{2.61803} =$$

$$\frac{0.190983}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \frac{\exp(-0.005+1.38889\pi^2) \sqrt{\frac{0.06}{\pi}} \sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002+0.0925926\pi^2)} \right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

[Open code](#)

$$\frac{1}{6.626} \ln \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)$$

Input:

$$\frac{1}{6.626} \log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \cdot 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

Fewer digits  
More digits

1.656144717071501097461677180702890057460198623031999287278...

Continued fraction:

Linear form

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{54 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \dots}}}}}}}}}}}}}}}}}}$$

[Open code](#)

Alternative representations:

$$\frac{\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{6.626} = \frac{\log_e \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \sqrt{\frac{0.12}{2\pi}}}{\frac{\exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{6.626}$$

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$$\frac{\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{6.626} = \frac{\log(a) \log_a \left( \frac{\exp\left(-\frac{0.12}{24} + \frac{\pi^2}{0.72}\right) \sqrt{\frac{0.12}{2\pi}}}{\frac{\exp\left(-\frac{0.12}{60} + \frac{\pi^2}{10.8}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{6.626}$$

[Open code](#)

Series representation:

$$\begin{aligned} & \frac{\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right)}{6.626} = \\ & 0.150921 \log \left( -1 + \frac{\exp(-0.005 + 1.38889\pi^2) \sqrt{\frac{0.06}{\pi}} \sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002 + 0.0925926\pi^2)} \right) - \\ & 0.150921 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\exp(-0.005+1.38889\pi^2) \sqrt{\frac{0.06}{\pi}} \sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002+0.0925926\pi^2)} \right)^k}{k} \end{aligned}$$

[Open code](#)

Integral representations:

$$\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right) = 0.150921 \int_1^{\infty} \frac{\exp(-0.005+1.38889\pi^2)\sqrt{\frac{0.06}{\pi}}\sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002+0.0925926\pi^2)} \frac{1}{t} dt$$

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$$\log \left( \frac{\sqrt{\frac{0.12}{2\pi}} \exp\left(\frac{\pi^2}{0.72} - \frac{0.12}{24}\right)}{\frac{\exp\left(\frac{\pi^2}{15 \times 0.72} - \frac{0.12}{60}\right)}{\sqrt{\frac{1}{2}(5-\sqrt{5})}}} \right) = \frac{0.0754603}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left( -1 + \frac{\exp(-0.005+1.38889\pi^2)\sqrt{\frac{0.06}{\pi}}\sqrt{\frac{1}{2}(5-\sqrt{5})}}{\exp(-0.002+0.0925926\pi^2)} \right)^{-s}}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

[Open code](#)

We have that:

I have proved that if

$$f(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \dots$$

then

$$f(q) + (1-q)(1-q^3)(1-q^5)\dots(1-2q+2q^4-2q^9+\dots) = O(1)$$

at all the points  $q = -1, q^3 = -1, q^5 = -1, q^7 = -1, \dots$ , and at the same time

$$f(q) - (1-q)(1-q^3)(1-q^5)\dots(1-2q+2q^4-\dots) = O(1)$$

at all the points  $q^2 = -1, q^4 = -1, q^6 = -1, \dots$ . Also obviously  $f(q) = O(1)$  at all the points  $q = 1, q^3 = 1, q^5 = 1, \dots$ . And so  $f(q)$  is a Mock  $\vartheta$  function. When  $q = -e^{-t}$  and  $t \rightarrow 0$

$$f(q) + \sqrt{\frac{\pi}{t}} \exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right) \rightarrow 4.$$

From:

$$f(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \dots$$

We obtain, for  $q = -e^{-t}$  for  $t = 0.5$ ;  $q = -0.606530$

$$1 + [((-0.606530)/(1-0.606530)^2)] + [((-0.606530)^4)/[(((1-0.606530)^2)*(1-0.606530^2)^2)]]$$

Input interpretation:

$$1 - \frac{0.606530}{(1 - 0.606530)^2} + \frac{(-0.606530)^4}{(1 - 0.606530)^2 (1 - 0.606530^2)^2}$$

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Result:

More digits

-0.72999480077443047538362776991420567540346048553554829328...

[Open code](#)

and:

$$\sqrt{\frac{\pi}{t}} \exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right)$$

$$\text{sqrt}(\text{Pi}/0.5) \exp((\text{Pi}^2/(24*0.5))-(0.5/24))$$

Input:

$$\sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right)$$

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Result:

More digits

5.58773...

Series representations:

More

$$\sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right) = \\ \exp(-0.0208333 + 0.0833333 \pi^2) \sqrt{-1 + 2\pi} \sum_{k=0}^{\infty} (-1 + 2\pi)^{-k} \binom{\frac{1}{2}}{k}$$

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$$\sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right) = \\ \exp(-0.0208333 + 0.0833333 \pi^2) \sqrt{-1 + 2\pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 2\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$\sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right) = \\ \exp(-0.0208333 + 0.0833333 \pi^2) \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2\pi - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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Thence:

$$-0.7299948 + \sqrt{\frac{\pi}{0.5}} \exp((\pi^2/(24*0.5))-(0.5/24))$$

Input interpretation:

$$-0.7299948 + \sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right)$$

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Result:

More digits

4.85773...

Series representations:

More

$$-0.729995 + \sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right) = \\ -0.729995 + \exp(-0.0208333 + 0.0833333 \pi^2) \sqrt{-1 + 2\pi} \sum_{k=0}^{\infty} (-1 + 2\pi)^{-k} \binom{\frac{1}{2}}{k}$$

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$$-0.729995 + \sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right) = \\ -0.729995 + \exp(-0.0208333 + 0.0833333 \pi^2) \sqrt{-1 + 2 \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 2 \pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$-0.729995 + \sqrt{\frac{\pi}{0.5}} \exp\left(\frac{\pi^2}{24 \times 0.5} - \frac{0.5}{24}\right) = \\ -0.729995 + \exp(-0.0208333 + 0.0833333 \pi^2) \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 \pi - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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For t = 0.6, we obtain:

$$1 + [((-0.548811)/(1-0.548811)^2)] + (((-0.548811)^4)) / [(((1-0.548811)^2) * ((1-0.548811^2)^2))]$$

Input interpretation:

$$1 - \frac{0.548811}{(1 - 0.548811)^2} + \frac{(-0.548811)^4}{(1 - 0.548811)^2 (1 - 0.548811^2)^2}$$

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Result:

More digits

$$-0.78335478262647730587745118425104872042845623277574448731...$$

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$$-0.7833547826264773 + \sqrt{\frac{\pi}{0.6}} \exp\left(\frac{\pi^2}{24 \times 0.6} - \frac{0.6}{24}\right)$$

Input interpretation:

$$-0.7833547826264773 + \sqrt{\frac{\pi}{0.6}} \exp\left(\frac{\pi^2}{24 \times 0.6} - \frac{0.6}{24}\right)$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

$$3.64561...$$

Series representations:

More

- 0.78335478262647730000 +  $\sqrt{\frac{\pi}{0.6}} \exp\left(\frac{\pi^2}{24 \times 0.6} - \frac{0.6}{24}\right) =$   
-0.78335478262647730000 +  
$$\exp(-0.025 + 0.0694444 \pi^2) \sqrt{-1 + 1.66667 \pi} \sum_{k=0}^{\infty} (-1 + 1.66667 \pi)^{-k} \binom{\frac{1}{2}}{k}$$

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- 0.78335478262647730000 +  $\sqrt{\frac{\pi}{0.6}} \exp\left(\frac{\pi^2}{24 \times 0.6} - \frac{0.6}{24}\right) =$   
-0.78335478262647730000 +  
$$\exp(-0.025 + 0.0694444 \pi^2) \sqrt{-1 + 1.66667 \pi} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 1.66667 \pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

- 0.78335478262647730000 +  $\sqrt{\frac{\pi}{0.6}} \exp\left(\frac{\pi^2}{24 \times 0.6} - \frac{0.6}{24}\right) =$   
-0.78335478262647730000 +  
$$\exp(-0.025 + 0.0694444 \pi^2) \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.66667 \pi - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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For t = 0,57721566... (Euler-Mascheroni Constant) we obtain:

-0.81745701519423421359+sqrt(Pi/0.57721566490153286)  
$$\exp((Pi^2/(24*0.57721566490153286))-(0.57721566490153286/24))$$

Input interpretation:

-0.81745701519423421359 +  $\sqrt{\frac{\pi}{0.57721566490153286}}$   
$$\exp\left(\frac{\pi^2}{24 \times 0.57721566490153286} - \frac{0.57721566490153286}{24}\right)$$

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Result:

More digits

3.8263169561696472...

Continued fraction:

Linear form

$$3 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{22 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}$$

Series representations:

More

- 0.817457015194234213590000 +  $\sqrt{\frac{\pi}{0.577215664901532860000}}$   
 $\exp\left(\frac{\pi^2}{24 \times 0.577215664901532860000} - \frac{0.577215664901532860000}{24}\right) =$
- 0.817457015194234213590000 +  
 $\exp(-0.0240506527042305358333 + 0.0721856131083597281418 \pi^2)$   
 $\sqrt{-1 + 1.73245471460063347540 \pi}$   
 $\sum_{k=0}^{\infty} (-1 + 1.73245471460063347540 \pi)^{-k} \binom{\frac{1}{2}}{k}$

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- 0.817457015194234213590000 +  $\sqrt{\frac{\pi}{0.577215664901532860000}}$   
 $\exp\left(\frac{\pi^2}{24 \times 0.577215664901532860000} - \frac{0.577215664901532860000}{24}\right) =$
- 0.817457015194234213590000 +  
 $\exp(-0.0240506527042305358333 + 0.0721856131083597281418 \pi^2)$   
 $\sqrt{-1 + 1.73245471460063347540 \pi}$   
 $\sum_{k=0}^{\infty} \frac{(-1)^k (-1 + 1.73245471460063347540 \pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}$

[Open code](#)

$$\begin{aligned}
& -0.817457015194234213590000 + \sqrt{\frac{\pi}{0.577215664901532860000}} \\
& \exp\left(\frac{\pi^2}{24 \times 0.577215664901532860000} - \frac{0.577215664901532860000}{24}\right) = \\
& -0.817457015194234213590000 + \\
& \exp(-0.0240506527042305358333 + 0.0721856131083597281418 \pi^2) \\
& \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1.73245471460063347540 \pi - z_0)^k z_0^{-k}}{k!}
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

[Open code](#)

$$3.8263169561696472\dots + 3.64561\dots + 4.85773\dots / 3 =$$

= Input interpretation:

$$\frac{1}{3} (3.8263169561696472 + 3.64561 + 4.85773)$$

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Result:

More digits

$$4.1098856520565490666\dots$$

[Open code](#)

This result 4,1098 is the mean between 0,5 - 0,57721566... - 0,6 and can be considered the best range so that:

$$f(q) + \sqrt{\frac{\pi}{t}} \exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right) \rightarrow 4.$$

Furthermore, this result 4,1098 is also in the range of the mass of hypothetical dark matter particles

From the first fundamental expression of PAGE 4, i.e.

$$(-1)^{n-1} \frac{\exp\left(\pi \sqrt{\frac{n}{6} - \frac{1}{144}}\right)}{2 \sqrt{n - \frac{1}{24}}} + O\left(\frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{\sqrt{n - \frac{1}{24}}}\right)$$

we have, for  $n = 64$ , if we replaced  $\frac{\pi}{6}$  with  $\frac{\pi^2}{6}$ , a similar result. Indeed:

$$[-(\exp(\text{Pi}*\text{sqrt}(64/6-1/144)) / (2\text{sqrt}(64-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}/6-1/144)) / (\text{sqrt}(64-1/24))]$$

Input:

$$-\left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{\sqrt{64 - \frac{1}{24}}} \right)$$

[Open code](#)

Exact result:

$$-\sqrt{\frac{6}{1535}} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/2 \sqrt{\pi/6-1/144} \pi}$$

Decimal approximation:

More digits

•  $-1781.14584997706528471576132644218868840329191415810390947\dots$

This result  $-1781,1458$  is in the range of the mass of  $f_0(1710)$  candidate glueball, with minus sign

Alternate forms:

$$-\sqrt{\frac{6}{1535}} \left( e^{(\sqrt{1535} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \right)$$

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$$-\sqrt{\frac{6}{1535}} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/24 \pi \sqrt{24 \pi - 1}}$$

Continued fraction:

Linear form

$$-1781 + \cfrac{1}{-6 + \cfrac{1}{-1 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-25 + \cfrac{1}{-4 + \cfrac{1}{-2 + \cfrac{1}{-10 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-8 + \cfrac{1}{-1 + \cfrac{1}{\dots}}}}}}}}}}}}}}$$

Series representations:

$$\begin{aligned} & -\left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2\sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)\pi}{\sqrt{64 - \frac{1}{24}}} \right) = \\ & -\left( \left[ \left( \exp\left(\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \right. \right. \\ & \quad \left. \left. \left. 2 \exp\left(\frac{1}{2}\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right] / \\ & \quad \left( 2\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

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$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{64 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{1535}{144} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. \\
& \quad \left. + 2 \exp\left(\frac{1}{2} \pi \exp\left(i \pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi}{6} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \left( 2 \exp\left(i \pi \left[ \frac{\arg\left(\frac{1535}{24} - x\right)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{64 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2} [\arg\left(\frac{1535}{144} - z_0\right)/(2\pi)] z_0^{1/2 (1 + [\arg\left(\frac{1535}{144} - z_0\right)/(2\pi)])} \right. \right. \right. \\
& \quad \left. \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)] z_0^{1/2 (1 + [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)])} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\
& \left( \frac{1}{z_0} \right)^{-1/2 [\arg\left(\frac{1535}{24} - z_0\right)/(2\pi)]} z_0^{-1/2 - 1/2 [\arg\left(\frac{1535}{24} - z_0\right)/(2\pi)]} \Bigg) / \\
& \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$[-(\exp(\text{Pi})*\sqrt{64/6-1/144}) / (2\sqrt{64-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\pi^2/6-1/144}) / (\sqrt{64-1/24})]$$

Input:

$$-\frac{\exp\left(\pi \sqrt{\frac{64}{6}-\frac{1}{144}}\right)}{2 \sqrt{64-\frac{1}{24}}}+\frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6}-\frac{1}{144}}\right)}{\sqrt{64-\frac{1}{24}}}$$

[Open code](#)

Exact result:

$$-\sqrt{\frac{6}{1535}} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}}$$

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[Decimal approximation:](#)

More digits

•  $-1781.69268916956715292273155970926095696562359453667315382\dots$

[Alternate forms:](#)

This result -1781,692 is in the range of the mass of  $f_0(1710)$  candidate glueball, with minus sign

$$-\sqrt{\frac{6}{1535}} \left( e^{(\sqrt{1535} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \right)$$

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$$-\sqrt{\frac{6}{1535}} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/24 \pi \sqrt{24 \pi^2 - 1}}$$

[Continued fraction:](#)

Linear form

$$\begin{aligned} & -1781 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-14 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-7 + \cfrac{1}{-154 + \cfrac{1}{-3 + \cfrac{1}{-17 + \cfrac{1}{-3 + \cfrac{1}{\dots}}}}}}}}}}}}}} \end{aligned}$$

[Series representations:](#)

$$\begin{aligned}
& - \left\{ \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \pi\right)}{\sqrt{64 - \frac{1}{24}}} \right\} = \\
& - \left\{ \left[ \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{144} - z_0\right)^k z_0^{-k}}{k!}\right) + \right. \right. \\
& \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!}\right) \right] / \\
& \quad \left. \left( 2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \right\} \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

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$$\begin{aligned}
& - \left\{ \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \pi\right)}{\sqrt{64 - \frac{1}{24}}} \right\} = \\
& - \left\{ \left[ \exp\left(\pi \exp\left(i \pi \left| \frac{\arg\left(\frac{1535}{144} - x\right)}{2\pi} \right| \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \exp\left(i \pi \left| \frac{\arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)}{2\pi} \right| \right) \right] \sqrt{x} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right] \right\} / \\
& \quad \left. \left( 2 \exp\left(i \pi \left| \frac{\arg\left(\frac{1535}{24} - x\right)}{2\pi} \right| \right) \right] \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right) + \exp\left(\frac{1}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \pi\right)}{2 \sqrt{64 - \frac{1}{24}}} \right) = \\
& - \left( \left( \left( \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2} [\arg(\frac{1535}{144} - z_0)/(2\pi)]\right) z_0^{1/2 (1 + [\arg(\frac{1535}{144} - z_0)/(2\pi)])} \right. \right. \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \\
& \quad 2 \exp\left(\frac{1}{2} \pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(-\frac{1}{144} + \frac{\pi^2}{6} - z_0)/(2\pi)]} z_0^{1/2 (1 + [\arg(-\frac{1}{144} + \frac{\pi^2}{6} - z_0)/(2\pi)])} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \\
& \quad \left( \frac{1}{z_0} \right)^{-1/2 [\arg(\frac{1535}{24} - z_0)/(2\pi)]} z_0^{-1/2 - 1/2 [\arg(\frac{1535}{24} - z_0)/(2\pi)]} \Bigg) / \\
& \quad \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

Thence, results practically equals: -1781.1458 and -1781.6926 with the important difference that  $\frac{\pi^2}{6}$  is equal to

- [Decimal approximation:](#)  
More digits

1.644934066848226436472415166646025189218949901206798437735...

[Open code](#)

Property:

$\frac{\pi^2}{6}$  is a transcendental number

And represent zeta of 2

[Alternative representations:](#)

More

$\frac{\pi^2}{6} = \frac{1}{6} (180^\circ)^2$

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$$\frac{\pi^2}{6} = \zeta(2)$$

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$$\frac{\pi^2}{6} = \frac{1}{6} (-i \log(-1))^2$$

[Series representations:](#)

More

$$\frac{\pi^2}{6} = \sum_{k=1}^{\infty} \frac{1}{k^2}$$

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$$\frac{\pi^2}{6} = -2 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

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$$\frac{\pi^2}{6} = \frac{4}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

[Integral representations:](#)

More

$$\frac{\pi^2}{6} = \frac{8}{3} \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

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$$\frac{\pi^2}{6} = \frac{2}{3} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

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$$\frac{\pi^2}{6} = \frac{2}{3} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

We note that:

$$54 + [-(\exp(\text{Pi}*\text{sqrt}(64/6-1/144)) / (2\text{sqrt}(64-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}^2/6-1/144)) / (\text{sqrt}(64-1/24)))]$$

Input:

$$54 - \left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2\sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{64 - \frac{1}{24}}} \right)$$

[Open code](#)

Exact result:

$$54 - \sqrt{\frac{6}{1535}} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/2\pi \sqrt{\pi^2/6-1/144}}$$

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Decimal approximation:

More digits

-1727.69268916956715292273155970926095696562359453667315382...

This result -1727,692 is very near to the mass of  $f_0(1710)$  candidate glueball, with minus sign

Alternate forms:

$$54 - \sqrt{\frac{6}{1535}} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/24\pi \sqrt{24\pi^2-1}}$$

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$$\frac{82890 - \sqrt{9210} e^{(\sqrt{1535} \pi)/12} - 2 \sqrt{9210} e^{1/24\pi \sqrt{24\pi^2-1}}}{1535}$$

Continued fraction:

Linear form

$$\begin{aligned} -1727 + & \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-14 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-7 + \cfrac{1}{-154 + \cfrac{1}{-3 + \cfrac{1}{-17 + \cfrac{1}{-3 + \cfrac{1}{\dots}}}}}}}}}}}}}} \end{aligned}$$

Series representations:

$$\begin{aligned}
54 - & \left( \frac{\exp\left(\pi\sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2\sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\pi\right)}{\sqrt{64 - \frac{1}{24}}} \right) = \\
& - \left( \left[ \exp\left(\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{144} - z_0\right)^k z_0^{-k}}{k!}\right) + \right. \right. \\
& \quad \left. \left. 2 \exp\left(\frac{1}{2}\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!}\right) - \right. \\
& \quad \left. \left. 108\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right] / \right. \\
& \quad \left. \left. \left( 2\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \right)
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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$$\begin{aligned}
54 - & \left( \frac{\exp\left(\pi\sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2\sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\pi\right)}{\sqrt{64 - \frac{1}{24}}} \right) = \\
& - \left( -\exp\left(\pi \exp\left(i\pi \left[ \frac{\arg\left(\frac{1535}{144} - x\right)}{2\pi} \right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \\
& \quad \left. 2 \exp\left(\frac{1}{2}\pi \exp\left(i\pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)}{2\pi} \right]\right)\right) \right. \\
& \quad \left. \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& \quad \left. 108 \exp\left(i\pi \left[ \frac{\arg\left(\frac{1535}{24} - x\right)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right] / \\
& \quad \left. \left( 2 \exp\left(i\pi \left[ \frac{\arg\left(\frac{1535}{24} - x\right)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1535}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)
\end{aligned}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& 54 - \left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \pi\right)}{\sqrt{64 - \frac{1}{24}}} \right) = \\
& \left( \left( \frac{1}{z_0} \right)^{-1/2} \left[ \arg\left(\frac{1535}{24} - z_0\right)/(2\pi) \right] z_0^{-1/2} - \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(\frac{1535}{144} - z_0\right)/(2\pi) \right]\right) \right. \\
& \quad \left. - \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{144} - z_0\right)^k z_0^{-k}}{k!} \right) - \\
& 2 \exp\left(\frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right]\right) z_0^{1/2} \left( 1 + \left[ \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right] \right) \\
& \quad \left. + \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!} \right) + \\
& 108 \left( \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(\frac{1535}{24} - z_0\right)/(2\pi) \right] z_0^{1/2+1/2} \left[ \arg\left(\frac{1535}{24} - z_0\right)/(2\pi) \right] \right. \\
& \quad \left. \left. + \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1535}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$1/\sqrt{2e} * [-(\exp(Pi*\sqrt{64/6-1/144})) / (2\sqrt{64-1/24}) + (\exp(Pi/2*\sqrt{Pi^2/6-1/144})) / (\sqrt{64-1/24})]$$

Input:

$$\frac{1}{\sqrt{2e}} \left( \left( \frac{\exp\left(\pi \sqrt{\frac{64}{6} - \frac{1}{144}}\right)}{2 \sqrt{64 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \pi\right)}{\sqrt{64 - \frac{1}{24}}} \right) \right)$$

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Exact result:

$$\frac{-\sqrt{\frac{6}{1535}} e^{\left(\sqrt{1535} \pi\right)/12} - 2 \sqrt{\frac{6}{1535}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}}}{\sqrt{2e}}$$

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Decimal approximation:

More digits

-764.135821434088465707046654009089493504812748262303831508...

This value -764,1358 is very near to the rest mass of Charged rho meson, with minus sign

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Alternate forms:

More

$$-\sqrt{\frac{3}{1535 e}} \left( e^{\left(\sqrt{1535} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \right)$$

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$$\frac{-\sqrt{\frac{3}{1535}} e^{\left(\sqrt{1535} \pi\right)/12} - 2 \sqrt{\frac{3}{1535}} e^{1/24 \pi \sqrt{24 \pi^2 - 1}}}{\sqrt{e}}$$

$$-\sqrt{\frac{3}{1535}} e^{\left(\sqrt{1535} \pi\right)/12 - 1/2} - 2 \sqrt{\frac{3}{1535}} e^{1/24 \pi \sqrt{24 \pi^2 - 1} - 1/2}$$

Continued fraction:

Linear form

$$\begin{aligned} & -764 + \cfrac{1}{-7 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{-7 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-7 + \cfrac{1}{-1 + \cfrac{1}{-16 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}}}}} \end{aligned}$$

Now:

$$[-(\exp(\text{Pi}*\text{sqrt}(2/6-1/144)) / (2\text{sqrt}(2-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}/6-1/144)) / (\text{sqrt}(2-1/24))]$$

Input:

$$-\left( \frac{\exp\left(\pi \sqrt{\frac{2}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{2 \sqrt{2 - \frac{1}{24}}} \right)$$

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Exact result:

$$-\sqrt{\frac{6}{47}} e^{(\sqrt{47} \pi)/12} - 2 \sqrt{\frac{6}{47}} e^{1/2 \sqrt{\pi/6-1/144} \pi}$$

Decimal approximation:

More digits

-4.36037234447371942763505615372938415285081428954656404865...

Alternate forms:

$$-\sqrt{\frac{6}{47}} \left( e^{(\sqrt{47} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \right)$$

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## Enlarge Data Customize A Plaintext Interactive

$$-\sqrt{\frac{6}{47}} e^{(\sqrt{47} \pi)/12} - 2 \sqrt{\frac{6}{47}} e^{1/24 \pi \sqrt{24 \pi - 1}}$$

Continued fraction:

Linear form

$$\begin{aligned} -4 + & \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{-2 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-435 + \cfrac{1}{-1 + \cfrac{1}{-31 + \cfrac{1}{-20 + \cfrac{1}{-1 + \cfrac{1}{-8 + \cfrac{1}{\dots}}}}}}}}}}}}}} \end{aligned}$$

Series representations:

More

$$\begin{aligned} & \left[ \frac{\exp\left(\pi \sqrt{\frac{2}{6} - \frac{1}{144}}\right) + \exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{2 \sqrt{2 - \frac{1}{24}}} \right] = \\ & - \frac{\exp\left(\pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{97}{144}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + 2 \exp\left(\frac{1}{2} \pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{145}{144} + \frac{\pi}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}{2 \sum_{k=0}^{\infty} \frac{\left(-\frac{23}{24}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \end{aligned}$$

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$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{2}{6} - \frac{1}{144}}\right) + \exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{2 \sqrt{2 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{47}{144} - z_0\right)^k z_0^{-k}}{k!}\right) + \right. \right. \\
& \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!}\right)\right) \right) / \\
& \left( 2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{47}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{2}{6} - \frac{1}{144}}\right) + \exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{2 \sqrt{2 - \frac{1}{24}}} \right) = \\
& \left( \left( \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} \left(-\frac{97}{144}\right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{2 \sqrt{\pi}}\right) + \right. \right. \\
& \quad \left. \left. 2 \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} \left(-\frac{145}{144} + \frac{\pi}{6}\right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{4 \sqrt{\pi}}\right)\right) \right. \\
& \left. \sqrt{\pi} \right) / \left( \sum_{j=0}^{\infty} \text{Res}_{s=-j} \left(\frac{24}{23}\right)^s \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right)
\end{aligned}$$

$$[-(\exp(\text{Pi})*\sqrt{8/6-1/144}) / (2\sqrt{8-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{8-1/24})]$$

Input:

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{8}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{2 \sqrt{8 - \frac{1}{24}}} \right)
\end{aligned}$$

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Exact result:

$$-\sqrt{\frac{6}{191}} e^{(\sqrt{191} \pi)/12} - 2 \sqrt{\frac{6}{191}} e^{1/2 \sqrt{\pi/6-1/144} \pi}$$

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Decimal approximation:

More digits

-7.70168576232505364644057076339729663723484731410883992966...

Alternate forms:

$$-\sqrt{\frac{6}{191}} \left( e^{\left(\sqrt{191} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \right)$$

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$$-\sqrt{\frac{6}{191}} e^{\left(\sqrt{191} \pi\right)/12} - 2 \sqrt{\frac{6}{191}} e^{1/24 \pi \sqrt{24 \pi - 1}}$$

Continued fraction:

Linear form

$$\begin{aligned} & -7 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-5 + \cfrac{1}{-4 + \cfrac{1}{-3 + \cfrac{1}{-4 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-7 + \cfrac{1}{-4 + \cfrac{1}{\dots}}}}}}}}}}}}}}}} \end{aligned}$$

Series representations:

$$\begin{aligned} & \left( \frac{\exp\left(\pi \sqrt{\frac{8}{6} - \frac{1}{144}}\right)}{2 \sqrt{8 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{8 - \frac{1}{24}}} \right) = \\ & - \left( \left( \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{191}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \right. \\ & \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right) / \\ & \quad \left( 2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{191}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

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$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{8}{6} - \frac{1}{144}}\right)}{2 \sqrt{8 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{8 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{191}{144} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{191}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. + \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \exp\left(i \pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi}{6} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \quad \left( 2 \exp\left(i \pi \left[ \frac{\arg\left(\frac{191}{24} - x\right)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{191}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{8}{6} - \frac{1}{144}}\right)}{2 \sqrt{8 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{8 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2} [\arg\left(\frac{191}{144} - z_0\right)/(2\pi)] \right) z_0^{1/2 [1 + [\arg\left(\frac{191}{144} - z_0\right)/(2\pi)]]} \right. \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{191}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)] \right) z_0^{1/2 [1 + [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)]]} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\
& \quad \left( \frac{1}{z_0} \right)^{-1/2 [\arg\left(\frac{191}{24} - z_0\right)/(2\pi)]} z_0^{-1/2 - 1/2 [\arg\left(\frac{191}{24} - z_0\right)/(2\pi)]} \Bigg) / \\
& \quad \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{191}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$[-(\exp(\text{Pi}*\text{sqrt}(16/6-1/144)) / (2\text{sqrt}(16-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}/6-1/144)) / (\text{sqrt}(16-1/24))]$$

Input:

$$-\left( \frac{\exp\left(\pi\sqrt{\frac{16}{6} - \frac{1}{144}}\right)}{2\sqrt{16 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{\sqrt{16 - \frac{1}{24}}} \right)$$

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Exact result:

$$-\sqrt{\frac{6}{383}} e^{(\sqrt{383}\pi)/12} - 2\sqrt{\frac{6}{383}} e^{1/2\sqrt{\pi/6-1/144}\pi}$$

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Decimal approximation:

More digits

- 21.7921604566254747127459424621662443480967531405723267207...

This result -21,79216 is very near to the value of black hole entropy (see Tables), with minus sign

Alternate forms:

$$-\sqrt{\frac{6}{383}} \left( e^{(\sqrt{383}\pi)/12} + 2e^{1/24\pi\sqrt{24\pi-1}} \right)$$

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$$-\sqrt{\frac{6}{383}} e^{(\sqrt{383}\pi)/12} - 2\sqrt{\frac{6}{383}} e^{1/24\pi\sqrt{24\pi-1}}$$

Continued fraction:

Linear form

$$\begin{array}{r} 1 \\ -21 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-3 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-42 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}}}}}}$$

Series representations:

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{16}{6} - \frac{1}{144}}\right)}{2 \sqrt{16 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{16 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{383}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \right. \\
& \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\
& \quad \left. \left( 2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{383}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

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$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{16}{6} - \frac{1}{144}}\right)}{2 \sqrt{16 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{16 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{383}{144} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{383}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \exp\left(i \pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi}{6} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \\
& \quad \left. \left( 2 \exp\left(i \pi \left[ \frac{\arg\left(\frac{383}{24} - x\right)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{383}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{16}{6} - \frac{1}{144}}\right)}{2 \sqrt{16 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{16 - \frac{1}{24}}} \right) = \\
& - \left( \left( \left( \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2} [\arg(\frac{383}{144} - z_0)/(2\pi)]\right) z_0^{1/2(1+\lfloor \arg(\frac{383}{144} - z_0)/(2\pi) \rfloor)} \right. \right. \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{383}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \\
& \quad 2 \exp\left(\frac{1}{2} \pi \left(\frac{1}{z_0}\right)^{1/2} [\arg(-\frac{1}{144} + \frac{\pi}{6} - z_0)/(2\pi)]\right) z_0^{1/2(1+\lfloor \arg(-\frac{1}{144} + \frac{\pi}{6} - z_0)/(2\pi) \rfloor)} \\
& \quad \left. \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right) \\
& \quad \left( \frac{1}{z_0} \right)^{-1/2[\arg(\frac{383}{24} - z_0)/(2\pi)]} z_0^{-1/2-1/2[\arg(\frac{383}{24} - z_0)/(2\pi)]} \Bigg) / \\
& \quad \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{383}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$[-(\exp(\text{Pi})*\sqrt{24/6-1/144}) / (2\sqrt{24-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{24-1/24})]$$

Input:

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{24}{6} - \frac{1}{144}}\right)}{2 \sqrt{24 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{24 - \frac{1}{24}}} \right)
\end{aligned}$$

[Open code](#)

Exact result:

$$-\frac{1}{5} \sqrt{\frac{6}{23}} e^{(5\sqrt{23}\pi)/12} - \frac{2}{5} \sqrt{\frac{6}{23}} e^{1/2 \sqrt{\pi/6-1/144} \pi}$$

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Decimal approximation:](#)

More digits

-55.0350864316374396091524112838137802449821710998154612076...

[Alternate forms:](#)

$$-\frac{1}{5} \sqrt{\frac{6}{23}} \left( e^{(5\sqrt{23}\pi)/12} + 2 e^{1/24\pi\sqrt{24\pi-1}} \right)$$

[Open code](#)

[Enlarge](#) [Data](#) [Customize](#) A Plaintext [Interactive](#)

$$-\frac{1}{5} \sqrt{\frac{6}{23}} e^{\left(5\sqrt{23}\pi\right)/12} - \frac{2}{5} \sqrt{\frac{6}{23}} e^{1/24\pi\sqrt{24\pi-1}}$$

Continued fraction:

Linear form

$$\begin{aligned} & -55 + \cfrac{1}{-28 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-238 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-12 + \cfrac{1}{-2 + \cfrac{1}{-22 + \cfrac{1}{-2 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{\dots}}}}}}}}}}}}}} \end{aligned}$$

Series representations:

$$\begin{aligned} & \left( \frac{\exp\left(\pi\sqrt{\frac{24}{6} - \frac{1}{144}}\right)}{2\sqrt{24 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\sqrt{\frac{\pi}{6} - \frac{1}{144}}\pi\right)}{\sqrt{24 - \frac{1}{24}}} \right) = \\ & - \left( \left( \exp\left(\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{575}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \right. \\ & \quad \left. \left. 2 \exp\left(\frac{1}{2}\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right) / \\ & \left( 2\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{575}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

[Enlarge](#) [Data](#) [Customize](#) A Plaintext [Interactive](#)

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{24}{6} - \frac{1}{144}}\right)}{2 \sqrt{24 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{24 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{575}{144} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{575}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. + \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \exp\left(i \pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi}{6} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \quad \left( 2 \exp\left(i \pi \left[ \frac{\arg\left(\frac{575}{24} - x\right)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{575}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{24}{6} - \frac{1}{144}}\right)}{2 \sqrt{24 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{24 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2} [\arg\left(\frac{575}{144} - z_0\right)/(2\pi)] \right) z_0^{1/2 [1 + [\arg\left(\frac{575}{144} - z_0\right)/(2\pi)]]} \right. \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{575}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)] \right) z_0^{1/2 [1 + [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)]]} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \Bigg) / \\
& \quad \left( \frac{1}{z_0} \right)^{-1/2 [\arg\left(\frac{575}{24} - z_0\right)/(2\pi)]} z_0^{-1/2 - 1/2 [\arg\left(\frac{575}{24} - z_0\right)/(2\pi)]} \Bigg) / \\
& \quad \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{575}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$[-(\exp(\text{Pi})*\text{sqrt}(48/6-1/144)) / (2\text{sqrt}(48-1/24)) + (\exp(\text{Pi}/2)*\text{sqrt}(\text{Pi}/6-1/144)) / (\text{sqrt}(48-1/24))]$$

Exact result:

$$-\sqrt{\frac{6}{1151}} e^{\left(\sqrt{1151} \pi\right)/12} - 2 \sqrt{\frac{6}{1151}} e^{1/2 \sqrt{\pi/6-1/144} \pi}$$

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Decimal approximation:](#)

More digits

-520.324977175415226219045375149602966830562620617104898006...

A good approximation to the rest mass of Eta meson

[Alternate forms:](#)

$$-\sqrt{\frac{6}{1151}} \left( e^{\left(\sqrt{1151} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \right)$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

$$-\sqrt{\frac{6}{1151}} e^{\left(\sqrt{1151} \pi\right)/12} - 2 \sqrt{\frac{6}{1151}} e^{1/24 \pi \sqrt{24 \pi - 1}}$$

[Continued fraction:](#)

Linear form

$$\begin{aligned} -520 + & \cfrac{1}{-3 + \cfrac{1}{-12 + \cfrac{1}{-1 + \cfrac{1}{-26 + \cfrac{1}{-2 + \cfrac{1}{-5 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}}}}} \end{aligned}$$

[Series representations:](#)

$$\begin{aligned} & \left( \frac{\exp\left(\pi \sqrt{\frac{48}{6} - \frac{1}{144}}\right)}{2 \sqrt{48 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{48 - \frac{1}{24}}} \right) = \\ & - \left( \left( \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1151}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \right. \\ & \quad \left. \left. 2 \exp\left(\frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) \right) / \\ & \left( 2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1151}{24} - z_0\right)^k z_0^{-k}}{k!} \right) \text{ for } \text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

Enlarge Data Customize A Plaintext Interactive

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{48}{6} - \frac{1}{144}}\right)}{2 \sqrt{48 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{48 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{1151}{144} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1151}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right. + \right. \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \exp\left(i \pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi}{6} - x\right)}{2\pi} \right] \right)\right) \sqrt{x} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \quad \left( 2 \exp\left(i \pi \left[ \frac{\arg\left(\frac{1151}{24} - x\right)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1151}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \\
& \quad \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& - \left( \frac{\exp\left(\pi \sqrt{\frac{48}{6} - \frac{1}{144}}\right)}{2 \sqrt{48 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}} \pi\right)}{\sqrt{48 - \frac{1}{24}}} \right) = \\
& - \left( \left( \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(\frac{1151}{144} - z_0\right)/(2\pi) \right] \right) z_0^{1/2 \left( 1 + [\arg\left(\frac{1151}{144} - z_0\right)/(2\pi)] \right)} \right. \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1151}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \\
& \quad \left. 2 \exp\left(\frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi) \right] \right) z_0^{1/2 \left( 1 + [\arg\left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)/(2\pi)] \right)} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \Bigg) / \\
& \quad \left( \frac{1}{z_0} \right)^{-1/2 \left[ \arg\left(\frac{1151}{24} - z_0\right)/(2\pi) \right]} z_0^{-1/2 - 1/2 \left[ \arg\left(\frac{1151}{24} - z_0\right)/(2\pi) \right]} \Bigg) / \\
& \quad \left( 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1151}{24} - z_0\right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

$$[-(\exp(\text{Pi})*\sqrt{54/6-1/144}) / (2\sqrt{54-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{54-1/24})]$$

Input:

$$-\frac{\exp\left(\pi \sqrt{\frac{54}{6} - \frac{1}{144}}\right)}{2 \sqrt{54 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{54 - \frac{1}{24}}}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Exact result:

$$-\sqrt{\frac{6}{1295}} e^{(\sqrt{1295} \pi)/12} - 2 \sqrt{\frac{6}{1295}} e^{1/2 \sqrt{\pi/6-1/144} \pi}$$

Decimal approximation:

More digits

-840.829384784908297995410706810926169820451153617261985029...

Alternate forms:

$$-\sqrt{\frac{6}{1295}} \left( e^{(\sqrt{1295} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \right)$$

[Open code](#)

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$$-\sqrt{\frac{6}{1295}} e^{(\sqrt{1295} \pi)/12} - 2 \sqrt{\frac{6}{1295}} e^{1/24 \pi \sqrt{24 \pi - 1}}$$

We note that:

$$\text{Pi} [-(\exp(\text{Pi})*\sqrt{54/6-1/144}) / (2\sqrt{54-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{54-1/24})] \text{Pi} [-(\exp(\text{Pi})*\sqrt{54/6-1/144}) / (2\sqrt{54-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{54-1/24})]$$

Input:

$$\pi \left( -\frac{\exp\left(\pi \sqrt{\frac{54}{6} - \frac{1}{144}}\right)}{2 \sqrt{54 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{54 - \frac{1}{24}}} \right)$$

[Open code](#)

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Exact result:

$$\left( -\sqrt{\frac{6}{1295}} e^{\left(\sqrt{1295} \pi\right)/12} - 2\sqrt{\frac{6}{1295}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}} \right) \pi$$

Decimal approximation:

More digits

-2643.41379369142944342937563614270970030474695509126014466...

This result is very near to the rest mass of charmed Xi baryon

Alternate forms:

$$-\sqrt{\frac{6}{1295}} \left( e^{\left(\sqrt{1295} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \right) \pi$$

[Open code](#)

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$$-\sqrt{\frac{6}{1295}} e^{\left(\sqrt{1295} \pi\right)/12} \pi - 2\sqrt{\frac{6}{1295}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}} \pi$$

Continued fraction:

Linear form

$$\begin{aligned} -2643 + & \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-2021 + \cfrac{1}{-1 + \cfrac{1}{-6 + \cfrac{1}{-3 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}}}}} \end{aligned}$$

and that the result -2643,4137... is practically equal to the rest mass of charmed Xi baryon  $2645.9 \pm 0.5$  with minus sign.

$$[-(\exp(\text{Pi}*\text{sqrt}(256/6-1/144)) / (2\text{sqrt}(256-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}/6-1/144)) / (\text{sqrt}(256-1/24))]$$

Input:

$$-\frac{\exp\left(\pi \sqrt{\frac{256}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{2 \sqrt{256 - \frac{1}{24}} + \sqrt{256 - \frac{1}{24}}}$$

[Open code](#)

Exact result:

$$-\sqrt{\frac{6}{6143}} e^{(\sqrt{6143} \pi)/12} - 2\sqrt{\frac{6}{6143}} e^{1/2\sqrt{\pi/6-1/144} \pi}$$

Decimal approximation:

More digits

$$-2.5481741128894845511527240984310489334247038391341796... \times 10^7$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Alternate forms:

$$-\sqrt{\frac{6}{6143}} \left( e^{(\sqrt{6143} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \right)$$

[Open code](#)

$$-\sqrt{\frac{6}{6143}} e^{(\sqrt{6143} \pi)/12} - 2\sqrt{\frac{6}{6143}} e^{1/24 \pi \sqrt{24 \pi - 1}}$$

Continued fraction:

Linear form

$$\begin{aligned} & -25481741 + \cfrac{1}{-7 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{-7 + \cfrac{1}{-3 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-38 + \cfrac{1}{...}}}}}}}}}}}}}} \end{aligned}$$

$$((-(\exp(\text{Pi})*\sqrt{256/6-1/144})) / (2\sqrt{256-1/24}) + (\exp(\text{Pi}/2)*\sqrt{256/6-1/144})) / (\sqrt{256-1/24}))])^1/3 * 2\text{Pi}$$

Input:

$$-\sqrt[3]{\frac{\exp\left(\pi \sqrt{\frac{256}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi}{6} - \frac{1}{144}}\right) \times 2\pi}{2\sqrt{256 - \frac{1}{24}} + \sqrt{256 - \frac{1}{24}}}}$$

[Open code](#)

Exact result:

• [Units »](#)

$$2 \sqrt[3]{\sqrt{\frac{6}{6143}} e^{\left(\sqrt{6143} \pi\right)/12} + 2 \sqrt{\frac{6}{6143}} e^{1/2 \sqrt{\pi/6-1/144} \pi} \pi}$$

• Units »

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Decimal approximation:](#)

More digits

1848.940355032163723639557516290408915880979834826482144597...

[Open code](#)

Alternate form:

$$2 \sqrt[6]{\frac{6}{6143}} \sqrt[3]{e^{\left(\sqrt{6143} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi - 1}} \pi}$$

[Continued fraction:](#)

Linear form

$$1848 + \cfrac{1}{1 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{21 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \dots}}}}}}}}}}}}}}}}}}$$

The result 1848.94 is a good approximation to the rest mass of D meson

1864.84±0.17

$$[-(\exp(\text{Pi} * \sqrt{496/6-1/144})) / (2\sqrt{496-1/24}) + (\exp(\text{Pi}/2 * \sqrt{\text{Pi}^2/6-1/144})) / (\sqrt{496-1/24})]$$

Input:

$$-\left( \frac{\exp\left(\pi \sqrt{\frac{496}{6} - \frac{1}{144}}\right)}{2 \sqrt{496 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{496 - \frac{1}{24}}} \right)$$

[Open code](#)

Exact result:

$$-\sqrt{\frac{6}{11903}} e^{(\sqrt{11903} \pi)/12} - 2\sqrt{\frac{6}{11903}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}}$$

[Decimal approximation:](#)

[More digits](#)

- 5.699036373166463553624780102376593290362262007576723...  $\times 10^{10}$

[Open code](#)

[Alternate forms:](#)

$$-\sqrt{\frac{6}{11903}} \left( e^{(\sqrt{11903} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \right)$$

[Open code](#)

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$$-\sqrt{\frac{6}{11903}} e^{(\sqrt{11903} \pi)/12} - 2\sqrt{\frac{6}{11903}} e^{1/24 \pi \sqrt{24 \pi^2 - 1}}$$

$$(((27 - [-(\exp(\text{Pi} * \text{sqrt}(496/6 - 1/144)) / (2\sqrt{496 - 1/24})) + (\exp(\text{Pi}/2 * \text{sqrt}(\text{Pi}^2/6 - 1/144)) / (\sqrt{496 - 1/24}))]))^{1/4} * 2\text{Pi}$$

[Input:](#)

$$27 - \frac{\exp\left(\pi \sqrt{\frac{496}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{2 \sqrt{496 - \frac{1}{24}}} \times 2\pi$$

[Open code](#)

[Exact result:](#)

$$27 + 2\sqrt[4]{\sqrt{\frac{6}{11903}} e^{(\sqrt{11903} \pi)/12} + 2\sqrt{\frac{6}{11903}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}} \pi}$$

[Decimal approximation:](#)

[More digits](#)

- 3096.942389275415119196141396729212278886160029049826229679...

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Alternate forms:](#)

$$27 + 2\sqrt[8]{\frac{6}{11903}} \sqrt[4]{e^{(\sqrt{11903} \pi)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \pi}$$

[Open code](#)

$$\frac{321381 + 2\sqrt[8]{6} \cdot 11903^{7/8} \sqrt[4]{e^{(\sqrt{11903}\pi)/12} + 2e^{1/24\pi}\sqrt{24\pi^2-1}}}{11903} \pi$$

Continued fraction:

Linear form

$$3096 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{7 + \cfrac{1}{1991 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{30 + \cfrac{1}{1 + \cfrac{1}{12 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$

The result 3096,94 is practically equal to the rest mass of the J/Psi meson  
 $3096.916 \pm 0.011$

$$[-(\exp(\text{Pi})*\sqrt{1024/6-1/144}) / (2\sqrt{1024-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{1024-1/24})]$$

Input:

$$-\left( \frac{\exp\left(\pi\sqrt{\frac{1024}{6}-\frac{1}{144}}\right)}{2\sqrt{1024-\frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6}-\frac{1}{144}}\right)}{\sqrt{1024-\frac{1}{24}}} \right)$$

[Open code](#)

Exact result:

$$-\frac{1}{5}\sqrt{\frac{6}{983}}e^{(5\sqrt{983}\pi)/12} - \frac{2}{5}\sqrt{\frac{6}{983}}e^{1/2\sqrt{\pi/6-1/144}\pi}$$

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Decimal approximation:

More digits

$$-1.041368108406279399413155957904606754806783037095726... \times 10^{16}$$

[Open code](#)

Alternate forms:

$$-\frac{1}{5} \sqrt{\frac{6}{983}} \left( e^{\left(5\sqrt{983}\pi\right)/12} + 2e^{1/24\pi\sqrt{24\pi-1}} \right)$$

[Open code](#)

$$-\frac{1}{5} \sqrt{\frac{6}{983}} e^{\left(5\sqrt{983}\pi\right)/12} - \frac{2}{5} \sqrt{\frac{6}{983}} e^{1/24\pi\sqrt{24\pi-1}}$$

Continued fraction:

Linear form

$$\bullet -10413681084062793 + \cfrac{1}{-1 + \cfrac{1}{-169 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-12 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}$$

$$((([-(\exp(\text{Pi})*\sqrt{1024/6-1/144})) / (2\sqrt{1024-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}/6-1/144})) / (\sqrt{1024-1/24})))^{1/6} * 2\text{Pi}$$

Input:

$$-\left( -\frac{\exp\left(\pi\sqrt{\frac{1024}{6}-\frac{1}{144}}\right)}{2\sqrt{1024-\frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6}-\frac{1}{144}}\right)}{\sqrt{1024-\frac{1}{24}}} \times 2\pi \right)$$

[Open code](#)

Exact result:

$$2\sqrt[6]{\frac{1}{5}\sqrt{\frac{6}{983}}e^{\left(5\sqrt{983}\pi\right)/12} + \frac{2}{5}\sqrt{\frac{6}{983}}e^{1/2\sqrt{\pi/6-1/144}\pi}\pi}$$

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Decimal approximation:

More digits

2936.165832636106007583223363922232337304567574612003522605...

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Alternate form:

$$\frac{2^{12} \sqrt{\frac{6}{983}} \pi}{\sqrt[6]{e^{\left(5 \sqrt{983} \pi\right)/12} + 2 e^{1/24} \pi \sqrt{24 \pi - 1}}}$$

Continued fraction:

Linear form

$$2936 + \cfrac{1}{6 + \cfrac{1}{33 + \cfrac{1}{7 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{25 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{15 + \cfrac{1}{71 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}$$

The result 2936,1658 is a good approximation to the rest mass of the Charmed eta meson  $2980.3 \pm 1.2$

In conclusion, we have:

$$[-(\exp(\text{Pi})*\sqrt{4096/6-1/144}) / (2\sqrt{4096-1/24}) + (\exp(\text{Pi}/2)*\sqrt{\text{Pi}^2/6-1/144}) / (\sqrt{4096-1/24})]$$

Input:

$$-\frac{\exp\left(\pi \sqrt{\frac{4096}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{2 \sqrt{4096 - \frac{1}{24}}} \sqrt{4096 - \frac{1}{24}}$$

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Exact result:

$$-\sqrt{\frac{6}{98303}} e^{\left(5 \sqrt{98303} \pi\right)/12} - 2 \sqrt{\frac{6}{98303}} e^{1/2 \pi \sqrt{\pi^2/6-1/144}}$$

Decimal approximation:

More digits

$$-3.474457619702701019494461189742470223474370730063589 \dots \times 10^{33}$$

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Alternate forms:

$$-\sqrt{\frac{6}{98303}} \left( e^{\left(\sqrt{98303} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \right)$$

[Open code](#)

$$-\sqrt{\frac{6}{98303}} e^{\left(\sqrt{98303} \pi\right)/12} - 2 \sqrt{\frac{6}{98303}} e^{1/24 \pi \sqrt{24 \pi^2 - 1}}$$

Continued fraction:

Linear form

$$\bullet \quad -3474457619702701019494461189742470 + \cfrac{1}{-4 + \cfrac{1}{-2 + \cfrac{1}{-9 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{...}}}}}}$$

$$2\pi * \ln((( -[-(\exp(\pi * \sqrt{4096/6-1/144})) / (2\sqrt{4096-1/24})) + (\exp(\pi/2 * \sqrt{4096/6-1/144})) / (\sqrt{4096-1/24}))))$$

Input:

$$2\pi \log \left( - \left( - \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right) + \exp \left( \frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} \right) \right) \right)$$

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•  $\log(x)$  is the natural logarithm

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Exact result:

$$2\pi \log \left( \sqrt{\frac{6}{98303}} e^{\left(\sqrt{98303} \pi\right)/12} + 2 \sqrt{\frac{6}{98303}} e^{1/24 \pi \sqrt{\pi^2/6-1/144}} \right)$$

Decimal approximation:

More digits

$$\bullet \quad 485.2550913884254054893620260853642044016470740645878905363...$$

Alternate forms:

More

$$\bullet \quad \log \left( \left( \frac{6}{98303} \right)^\pi \left( e^{\left(\sqrt{98303} \pi\right)/12} + 2 e^{1/24 \pi \sqrt{24 \pi^2 - 1}} \right)^{2\pi} \right)$$

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$$2\pi \log \left( \sqrt{\frac{6}{98303}} \left( e^{\left(\sqrt{98303}\right)\pi/12} + 2e^{1/24\pi\sqrt{24\pi^2-1}} \right) \right)$$

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$$2\pi \log \left( e^{\left(\sqrt{98303}\right)\pi/12} + 2e^{1/24\pi\sqrt{24\pi^2-1}} \right) - \pi \log \left( \frac{98303}{6} \right)$$

Continued fraction:

Linear form

$$\bullet \quad 485 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{14 + \cfrac{1}{48 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}}}$$

Alternative representations:

$$2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right) + \exp \left( \frac{1}{2} \pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} \right) \right) =$$

$$2\pi \log_e \left( \frac{\exp \left( \frac{1}{2} \pi \sqrt{-\frac{1}{144} + \frac{\pi^2}{6}} \right) + \exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}} + 2 \sqrt{4096 - \frac{1}{24}}} \right)$$

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$$2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}}} \right) \right) =$$

$$2\pi \log(a) \log_a \left( \frac{\exp \left( \frac{1}{2}\pi \sqrt{-\frac{1}{144} + \frac{\pi^2}{6}} \right)}{\sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} \right)$$

Series representations:

More

$$2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}}} \right) \right) =$$

$$2\pi \log \left( -1 + \sqrt{\frac{6}{98303}} e^{(\sqrt{98303}\pi)/12} + 2\sqrt{\frac{6}{98303}} e^{1/24\pi\sqrt{-1+24\pi^2}} \right) -$$

$$2\pi \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{-\sqrt{\frac{6}{98303}} e^{(\sqrt{98303}\pi)/12} + 2\sqrt{\frac{6}{98303}} e^{1/24\pi\sqrt{-1+24\pi^2}}} \right)^k}{k}$$

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$$2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}}} \right) \right) =$$

$$4i\pi^2 \left[ \frac{\pi - \arg \left( \frac{1}{z_0} \right) - \arg(z_0)}{2\pi} \right] + 2\pi \log(z_0) -$$

$$2\pi \sum_{k=1}^{\infty} \frac{(-1)^k \left( \sqrt{\frac{6}{98303}} e^{(\sqrt{98303}\pi)/12} + 2\sqrt{\frac{6}{98303}} e^{1/24\pi\sqrt{-1+24\pi^2}} - z_0 \right)^k z_0^{-k}}{k}$$

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$$\begin{aligned}
& 2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}}} \right) \right) = \\
& 2\pi \log \left( -1 + \sqrt{\frac{6}{98303}} e^{(\sqrt{98303}\pi)/12} + 2 \sqrt{\frac{6}{98303}} e^{1/2\pi \sqrt{-1/144+\pi^2/6}} \right) - \\
& 2\pi \sum_{k=1}^{\infty} \frac{\left( \frac{1}{-1+\sqrt{\frac{6}{98303}} e^{(\sqrt{98303}\pi)/12} + 2\sqrt{\frac{6}{98303}} e^{1/24\pi \sqrt{-1+24\pi^2}}} \right)^k}{k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& 2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}}} \right) \right) = \\
& 2\pi \int_1^{\sqrt{\frac{6}{98303}} \left( e^{(\sqrt{98303}\pi)/12} + 2e^{1/24\pi \sqrt{-1+24\pi^2}} \right)} \frac{1}{t} dt
\end{aligned}$$

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$$\begin{aligned}
& 2\pi \log \left( -(-1) \left( \frac{\exp \left( \pi \sqrt{\frac{4096}{6} - \frac{1}{144}} \right)}{2 \sqrt{4096 - \frac{1}{24}}} + \frac{\exp \left( \frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}} \right)}{\sqrt{4096 - \frac{1}{24}}} \right) \right) = -i \int_{-i\infty+\gamma}^{i\infty+\gamma} \\
& \frac{\left( -1 + \sqrt{\frac{6}{98303}} e^{(\sqrt{98303}\pi)/12} + 2 \sqrt{\frac{6}{98303}} e^{1/24\pi \sqrt{-1+24\pi^2}} \right)^{-s}}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) \\
& ds \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

The result 485,25 is very near to the value of the rest mass of the Kaon meson  
 $493.677 \pm 0.016$

From:

## Black Hole Entropy and Long Strings

Andreas Blommaert

Academic year 2015-2016

Faculty of Engineering and Architecture

Chair: Prof. dr. Dirk Ryckbosch

Vakgroep Fysica en Sterrenkunde

Master's dissertation submitted in order to obtain the academic degree of Master of Science in  
Engineering Physics

Counsellor: Prof. dr. Henri Verschelde

Supervisor: Prof. dr. Henri Verschelde

Furthermore, we have that:

The only remaining problem is thus the summation over  $\alpha$ . To solve this, it is important to realize that the value  $M^2$  only actually depends on the value of  $N^\perp$  of a certain state. The question is thus now how many string states there are in the full open bosonic string spectrum for each value of  $N^\perp$ . Following the literature we call this number  $P_{24}(N^\perp)$ . The number 24 refers to the number of transverse coordinates the oscillators of which contribute to the mass of a certain string state. As explained e.g. in section 2.4, for bosonic open strings in uncompactified 26 dimensional spacetime this number equals 24.  $P_{24}(N^\perp)$  is called the number of partitions of  $N^\perp$  (we will not mention the number of transverse coordinates explicitly when talking of the number of partitions in what follows). The single open string partition function is now reduced to

$$Z_1 = \sum_{N^\perp} P_{24}(N^\perp) Z_{part}(N^\perp). \quad (3.5)$$

Since we are interested only in the high energy behavior of the single string partition function, we are interested in finding a asymptotic formula for  $P_{24}(N^\perp)$  for high values of  $N^\perp$ . Although this might not seem difficult for the reader, the problem is tougher than it looks. Luckily for us, it was solved before by Hardy and Ramanujan [22] who found

$$P_{24}(N^\perp) = \frac{1}{\sqrt{2}} N^{\perp - \frac{27}{4}} e^{4\pi\sqrt{N^\perp}} \quad (3.6)$$

or for a more general number of transverse oscillators

$$P_b(N) = \frac{1}{\sqrt{2}} \left( \frac{b}{24} \right)^{\frac{b+1}{4}} N^{-\frac{b+3}{4}} e^{2\pi\sqrt{\frac{N}{b}}}. \quad (3.7)$$

And from:

## Chapter 16 String thermodynamics and black holes

The result (16.2.29) is only the leading term of the celebrated Hardy-Ramanujan asymptotic expansion of  $p(N)$ :

$$p(N) \simeq \frac{1}{4N\sqrt{3}} \exp\left(2\pi\sqrt{\frac{N}{6}}\right). \quad (16.2.31)$$

This is not an exact formula either, but is an accurate estimate of  $p(N)$ , as opposed to our accurate estimate of the logarithm of  $p(N)$ . We will not give here a derivation of the Hardy-Ramanujan result. It is fun, however, to test the accuracy of the Hardy-Ramanujan expansion. In Table 16.2 we compare the values of  $p(N)$ , as calculated exactly, with the estimate  $p_{\text{est}}(N)$  provided by (16.2.31). The estimate gives an error of about one-half of a percent for  $N = 10000$ .

We now need a minor generalization of (16.2.31). Assume the string can vibrate in  $d$  transverse directions. Then, for each frequency  $\ell\omega_0$ , we must have  $d$  harmonic oscillators representing the possible polarizations of the motion. Furthermore, the associated occupation numbers need a superscript labelling the  $d$  polarizations:

$$\begin{matrix} n_1^{(1)} & n_1^{(2)} & \dots & n_1^{(d)} \\ n_2^{(1)} & n_2^{(2)} & \dots & n_2^{(d)} \\ \dots & \dots & \dots & \dots \\ n_l^{(1)} & n_l^{(2)} & \dots & n_l^{(d)} \\ \dots & \dots & \dots & \dots \end{matrix} \quad (16.2.32)$$

In order to sum over all possible states in the new partition function  $Z_d$ , we must sum over all possible values of the occupation numbers  $n_k^{(q)}$ , where

$N$	$p(N)$	$p(N)_{\text{est}}$	$p(N)/p_{\text{est}}(N)$
5	7	8.94	0.7829
10	42	48.10	0.8731
100	190569292	199281893.25	0.9563
1000	$2.406 \times 10^{31}$	$2.440 \times 10^{31}$	0.9860
10000	$3.617 \times 10^{106}$	$3.633 \times 10^{106}$	0.9956

Table 16.2: Comparing the exact values of  $p(N)$  with the estimate  $p(N)_{\text{est}}$  provided by the Hardy-Ramanujan formula.

$k = 1, 2, \dots, \infty$ , and  $q = 1, 2, \dots, d$ . This gives

$$Z_d = \sum_{n_k^{(1)}, \dots, n_k^{(d)}} \exp \left[ -\frac{\hbar\omega_0}{kT} \sum_{\ell=0}^{\infty} \sum_{q=1}^d \ell n_{\ell}^{(q)} \right]. \quad (16.2.33)$$

The sums over the various  $n^{(q)}$  factorize, so,

$$Z_d = \sum_{n_k^{(1)}} \exp \left[ -\frac{\hbar\omega_0}{kT} \sum_{\ell=0}^{\infty} \ell n_{\ell}^{(1)} \right] \dots \sum_{n_k^{(d)}} \exp \left[ -\frac{\hbar\omega_0}{kT} \sum_{\ell=0}^{\infty} \ell n_{\ell}^{(d)} \right]. \quad (16.2.34)$$

Each factor here is equal to the previously calculated partition function  $Z$ . We therefore have

$$Z_d = (Z)^d. \quad (16.2.35)$$

The new free energy  $F_d$  is also easy to calculate:

$$F_d = -kT \ln Z_d = -kT d \ln Z = F d. \quad (16.2.36)$$

The entropy, obtained by differentiation of the free energy, also acquires a multiplicative factor of  $d$ :

$$S_d = S d. \quad (16.2.37)$$

For the energy  $E_d$ , the same multiplicative factor exists on account of (16.1.6). We also note that  $E_d$  is equal to  $\hbar\omega_0 N$ , where  $N$  is now the total occupation

number

$$E_d = E d = \hbar\omega_0 N, \quad N = \sum_{\ell,q} \ell n_\ell^{(q)}. \quad (16.2.38)$$

Using our earlier result for  $S$  in (16.2.28) we now have

$$S_d = d(k 2\pi) \sqrt{\frac{1}{6} \frac{E}{\hbar\omega_0}} = k 2\pi \sqrt{\frac{d}{6} \frac{E d}{\hbar\omega_0}} = k 2\pi \sqrt{\frac{Nd}{6}}, \quad (16.2.39)$$

where we made use of (16.2.38).

Let us call  $p_d(N)$  the number of partitions of  $N$  when we have a  $d$ -fold degeneracy. This means, for example, that the partition  $\{3, 2, 1\}$  of 6 now gives rise to many partitions written like  $\{3_{p_1}, 2_{p_2}, 1_{p_3}\}$ , where we include subscripts  $p_i$  that can take all possible values from one to  $d$ . A partition with different subscripts is considered a different partition. We now see that, for a given energy, with associated number  $N$ , the number of states is  $p_d(N)$ . Therefore  $S_d = k \ln p_d(N)$ , and comparing with (16.2.39) we conclude that for large  $N$

$$\ln p_d(N) \simeq 2\pi \sqrt{\frac{Nd}{6}}. \quad (16.2.40)$$

The more accurate version of this result can be shown to be

$$p_d(N) \simeq \frac{1}{\sqrt{2}} \left(\frac{d}{24}\right)^{(d+1)/4} N^{-(d+3)/4} \exp\left(2\pi\sqrt{\frac{Nd}{6}}\right). \quad (16.2.41)$$

You can see that for  $d = 1$  this reduces to  $p(N)$ , as given in (16.2.31). For  $d = 24$ , the number of transverse light-cone directions in the bosonic string, the expression simplifies a little:

$$p_{24}(N) \simeq \frac{1}{\sqrt{2}} N^{-27/4} \exp\left(4\pi\sqrt{N}\right). \quad (16.2.42)$$

Note that, from the Table 16.2, and utilizing the Ramanujan equation, we obtain:

$$0.57516 ((((([-(\exp(\text{Pi}*\text{sqrt}(10000/6-1/144))) / (2\text{sqrt}(10000-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}^2/6-1/144)) / (\text{sqrt}(10000-1/24))))])))^2$$

Input:

$$0.57516 \left( - \left( - \frac{\left( \exp\left(\pi \sqrt{\frac{10000}{6} - \frac{1}{144}}\right) + \exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right) \right)^2}{2 \sqrt{10000 - \frac{1}{24}}} \right) \right)$$

[Open code](#)

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Result:

More digits

$3.61711\dots \times 10^{106}$

Series representations:

$$\begin{aligned}
& 0.57516(-1) \left( -\frac{\exp\left(\pi \sqrt{\frac{10000}{6} - \frac{1}{144}}\right)}{2\sqrt{10000 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{10000 - \frac{1}{24}}} \right)^2 = \\
& 0.14379 \left( \exp^2\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{239999}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad \left. 4 \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{239999}{144} - z_0\right)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \exp\left(\frac{1}{2}\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad \left. 4 \exp^2\left(\frac{1}{2}\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\
& \left( \sqrt{z_0}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{239999}{24} - z_0\right)^k z_0^{-k}}{k!} \right)^2 \right)
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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$$\begin{aligned}
& 0.57516(-1) \left( - \left( \frac{\exp\left(\pi \sqrt{\frac{10000}{6} - \frac{1}{144}}\right)}{2 \sqrt{10000 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{10000 - \frac{1}{24}}} \right)^2 \right) = \\
& 0.14379 \left( \exp^2 \left( \pi \exp \left( i\pi \left[ \frac{\arg\left(\frac{239999}{144} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{239999}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& 4 \exp \left( \pi \exp \left( i\pi \left[ \frac{\arg\left(\frac{239999}{144} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{239999}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& \left. \exp \left( \frac{1}{2}\pi \exp \left( i\pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& 4 \exp^2 \left( \frac{1}{2}\pi \exp \left( i\pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \Bigg) / \\
& \left( \exp^2 \left( i\pi \left[ \frac{\arg\left(\frac{239999}{24} - x\right)}{2\pi} \right] \right) \sqrt{x}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{239999}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^2 \right)
\end{aligned}$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )

$$\begin{aligned}
& 0.57516(-1) \left( - \left( \frac{\exp\left(\pi \sqrt{\frac{10000}{6} - \frac{1}{144}}\right)}{2 \sqrt{10000 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{10000 - \frac{1}{24}}} \right)^2 \right) = \\
& \left( 0.14379 \left( \exp^2 \left( \pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(\frac{239999}{144} - z_0\right)/(2\pi) \right] \right. \right. \right. \\
& \quad \left. \left. \left. z_0^{1/2 \left( 1 + \left[ \arg\left(\frac{239999}{144} - z_0\right)/(2\pi) \right] \right)} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{239999}{144} - z_0 \right)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad \left. 4 \exp \left( \pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(\frac{239999}{144} - z_0\right)/(2\pi) \right] z_0^{1/2 \left( 1 + \left[ \arg\left(\frac{239999}{144} - z_0\right)/(2\pi) \right] \right)} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{239999}{144} - z_0 \right)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \exp \left( \frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right] z_0^{1/2 \left( 1 + \left[ \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right] \right)} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( -\frac{1}{144} + \frac{\pi^2}{6} - z_0 \right)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad \left. 4 \exp^2 \left( \frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} \left[ \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right] z_0^{1/2 \left( 1 + \left[ \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right] \right)} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( -\frac{1}{144} + \frac{\pi^2}{6} - z_0 \right)^k z_0^{-k}}{k!} \right) \right) \right. \\
& \quad \left. \left( \frac{1}{z_0} \right)^{-\left[ \arg\left(\frac{239999}{24} - z_0\right)/(2\pi) \right]} z_0^{-1-\left[ \arg\left(\frac{239999}{24} - z_0\right)/(2\pi) \right]} \right) / \\
& \quad \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{239999}{24} - z_0 \right)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

Result practically equal to the value as in the Table, i.e. for  $N = 10000$   $p(N) = 3,617 * 10^{106}$ . Also for  $N = 5$ ,  $N = 10$ ,  $N = 100$  and  $N = 1000$ , we obtain similar results.

$$0.1329 ((((([-(\exp(\text{Pi}*\text{sqrt}(5/6-1/144)) / (2\text{sqrt}(5-1/24))) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}^2/6-1/144)) / (\text{sqrt}(5-1/24))))]))^2$$

Input:

$$0.1329 \left( -\left( -\frac{\exp\left(\pi \sqrt{\frac{5}{6} - \frac{1}{144}}\right)}{2 \sqrt{5 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{5 - \frac{1}{24}}}\right)^2 \right)$$

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[Result:](#)

• Fewer digits  
More digits

7.000489715690419184227742861162617611770809936686792152198...

Series representations:

$$\begin{aligned} 0.1329(-1) & \left( -\left( \frac{\exp\left(\pi \sqrt{\frac{5}{6} - \frac{1}{144}}\right)}{2 \sqrt{5 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2} \pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{5 - \frac{1}{24}}} \right)^2 \right) = \\ & 0.033225 \left( \exp^2 \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{119}{144} - z_0\right)^k z_0^{-k}}{k!} \right) + \right. \\ & 4 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{119}{144} - z_0\right)^k z_0^{-k}}{k!} \right) \\ & \exp \left( \frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!} \right) + \\ & \left. 4 \exp^2 \left( \frac{1}{2} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)^k z_0^{-k}}{k!} \right) \right) / \\ & \left( \sqrt{z_0}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{119}{24} - z_0\right)^k z_0^{-k}}{k!} \right)^2 \right) \end{aligned}$$

for  $\text{not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

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$$\begin{aligned}
& 0.1329(-1) \left( - \frac{\exp\left(\pi \sqrt{\frac{5}{6} - \frac{1}{144}}\right)}{2 \sqrt{5 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{5 - \frac{1}{24}}} \right)^2 = \\
& 0.033225 \left( \exp^2 \left( \pi \exp \left( i\pi \left[ \frac{\arg\left(\frac{119}{144} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{119}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& 4 \exp \left( \pi \exp \left( i\pi \left[ \frac{\arg\left(\frac{119}{144} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{119}{144} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \\
& \left. \exp \left( \frac{1}{2}\pi \exp \left( i\pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) + \\
& 4 \exp^2 \left( \frac{1}{2}\pi \exp \left( i\pi \left[ \frac{\arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)}{2\pi} \right] \right) \right) \sqrt{x} \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{144} + \frac{\pi^2}{6} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \Bigg) / \\
& \left( \exp^2 \left( i\pi \left[ \frac{\arg\left(\frac{119}{24} - x\right)}{2\pi} \right] \right) \sqrt{x}^2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{119}{24} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)^2 \right)
\end{aligned}$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )

$$\begin{aligned}
& 0.1329(-1) \left( - \left[ \frac{\exp\left(\pi \sqrt{\frac{5}{6} - \frac{1}{144}}\right)}{2 \sqrt{5 - \frac{1}{24}}} + \frac{\exp\left(\frac{1}{2}\pi \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{5 - \frac{1}{24}}} \right]^2 \right) = \\
& \left( 0.033225 \left( \exp^2 \left[ \pi \left( \frac{1}{z_0} \right)^{1/2} \left| \arg\left(\frac{119}{144} - z_0\right)/(2\pi) \right| \right] z_0^{1/2} \left( 1 + \left| \arg\left(\frac{119}{144} - z_0\right)/(2\pi) \right| \right) \right. \right. \\
& \quad \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{119}{144} - z_0 \right)^k z_0^{-k}}{k!} \left. \right) + 4 \exp \left( \pi \left( \frac{1}{z_0} \right)^{1/2} \left| \arg\left(\frac{119}{144} - z_0\right)/(2\pi) \right| \right. \\
& \quad \left. \left. z_0^{1/2} \left( 1 + \left| \arg\left(\frac{119}{144} - z_0\right)/(2\pi) \right| \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{119}{144} - z_0 \right)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \exp \left( \frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} \left| \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right| \right) z_0^{1/2} \left( 1 + \left| \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right| \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( -\frac{1}{144} + \frac{\pi^2}{6} - z_0 \right)^k z_0^{-k}}{k!} \right) + \\
& \quad \left. 4 \exp^2 \left( \frac{1}{2} \pi \left( \frac{1}{z_0} \right)^{1/2} \left| \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right| \right) z_0^{1/2} \left( 1 + \left| \arg\left(-\frac{1}{144} + \frac{\pi^2}{6} - z_0\right)/(2\pi) \right| \right) \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( -\frac{1}{144} + \frac{\pi^2}{6} - z_0 \right)^k z_0^{-k}}{k!} \right) \Bigg) \\
& \quad \left( \frac{1}{z_0} \right)^{-\left| \arg\left(\frac{119}{24} - z_0\right)/(2\pi) \right|} z_0^{-1-\left| \arg\left(\frac{119}{24} - z_0\right)/(2\pi) \right|} \Bigg) / \\
& \quad \left( \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k \left( \frac{119}{24} - z_0 \right)^k z_0^{-k}}{k!} \right)^2
\end{aligned}$$

$$0.321158 (((((-[-(\exp(\text{Pi}*\text{sqrt}(10/6-1/144)) / (2\text{sqrt}(10-1/24)) + (\exp(\text{Pi}/2*\text{sqrt}(\text{Pi}^2/6-1/144)) / (\text{sqrt}(10-1/24))))]))))^2$$

Input interpretation:

$$0.321158 \left( - \left( - \left[ \frac{\exp\left(\pi \sqrt{\frac{10}{6} - \frac{1}{144}}\right)}{2 \sqrt{10 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{10 - \frac{1}{24}}} \right]^2 \right) \right)$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

42.00078768262369264059794486211334619364672129732630395117...

$$0.554792 ((((((-[\exp(\text{Pi}*\sqrt{100/6-1/144})) / (2\sqrt{100-1/24})) + (\exp(\text{Pi}/2*\sqrt{\text{Pi}^2/6-1/144}) / (\sqrt{100-1/24}))))))^2$$

Input interpretation:

$$0.554792 \left( - \left( - \frac{\exp\left(\pi \sqrt{\frac{100}{6} - \frac{1}{144}}\right)}{2 \sqrt{100 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{100 - \frac{1}{24}}} \right)^2 \right)$$

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Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$1.90569... \times 10^8$

$$0.57032 ((((((-[\exp(\text{Pi}*\sqrt{1000/6-1/144})) / (2\sqrt{1000-1/24})) + (\exp(\text{Pi}/2*\sqrt{\text{Pi}^2/6-1/144}) / (\sqrt{1000-1/24}))))))^2$$

Input:

$$0.57032 \left( - \left( - \frac{\exp\left(\pi \sqrt{\frac{1000}{6} - \frac{1}{144}}\right)}{2 \sqrt{1000 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2} \sqrt{\frac{\pi^2}{6} - \frac{1}{144}}\right)}{\sqrt{1000 - \frac{1}{24}}} \right)^2 \right)$$

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Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$2.40652... \times 10^{31}$

Now, we take the expression

$$[-(\exp(\text{Pi}*\sqrt{16/6-1/144})) / (2\sqrt{16-1/24}) + (\exp(\text{Pi}/2*\sqrt{\text{Pi}/6-1/144}) / (\sqrt{16-1/24}))]$$

Input:

$$-\left( \frac{\exp\left(\pi\sqrt{\frac{16}{6}-\frac{1}{144}}\right)}{2\sqrt{16-\frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6}-\frac{1}{144}}\right)}{\sqrt{16-\frac{1}{24}}}\right)$$

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Exact result:

$$-\sqrt{\frac{6}{383}} e^{(\sqrt{383}\pi)/12} - 2\sqrt{\frac{6}{383}} e^{1/2\sqrt{\pi/6-1/144}\pi}$$

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Decimal approximation:

More digits

-21.7921604566254747127459424621662443480967531405723267207...

The result is about -21,79216. This is the coefficient of  $q^n$  in the next mathematical expressions.

We have that:

When  $q = -e^{-t}$  and  $t \rightarrow 0$

$$f(q) + \sqrt{\frac{\pi}{t}} \exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right) \rightarrow 4.$$

The coefficient of  $q^n$  in  $f(q)$  is

$$(-1)^{n-1} \frac{\exp\left(\pi\sqrt{\frac{n}{6}-\frac{1}{144}}\right)}{2\sqrt{n-\frac{1}{24}}} + O\left(\frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6}-\frac{1}{144}}\right)}{\sqrt{n-\frac{1}{24}}}\right)$$

Now, we take  $n = 16$  and obtain, considering the following develop of the above formula:

$$-\left( \frac{\exp\left(\pi\sqrt{\frac{16}{6}-\frac{1}{144}}\right)}{2\sqrt{16-\frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6}-\frac{1}{144}}\right)}{\sqrt{16-\frac{1}{24}}}\right)$$

the value -21.79216 (the coefficient).

Thence  $q = \text{coefficient} * -e^{-t}$ ; for  $t = 0.5$ ,  $q = (-e^{-0.5}) - 21.79216$  for each  $q$ .

For example:  $q^5 = ((-e^{-0.5}) * -21.79216)^5$  and so on.

Now:

The coefficient of  $q^n$  in  $f(q)$  is

$$(-1)^{n-1} \frac{\exp\left(\pi\sqrt{\frac{n}{6} - \frac{1}{144}}\right)}{2\sqrt{n - \frac{1}{24}}} + O\left(\frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{n}{6} - \frac{1}{144}}\right)}{\sqrt{n - \frac{1}{24}}}\right)$$

It is inconceivable that a single  $\vartheta$  function could be found to cut out the singularities of  $f(q)$ .

Mock  $\vartheta$ -functions

$$\begin{aligned}\phi(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots \\ \psi(q) &= \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots \\ \chi(q) &= 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \dots\end{aligned}$$

These are related to  $f(q)$  as shown below.

$$2\phi(-q) - f(q) = f(q) + 4\psi(-q)$$

$$\begin{aligned}&= \frac{1 - 2q + 2q^4 - 2q^9 + \dots}{(1+q)(1+q^2)(1+q^3)\dots} \\ 4\chi(q) - f(q) &= \frac{(1 - 2q^3 + 2q^{12} - \dots)^2}{(1-q)(1-q^2)(1-q^3)\dots}\end{aligned}$$

These are of the 3rd order.

---

We have that, for  $t = 2$ :

$$((((1 - 2((-e^{-2} * (-21.79216)))) + 2((-e^{-2} * (-21.79216))^4) - 2((-e^{-2} * (-21.79216))^9))) / (((((1 + ((-e^{-2} * (-21.79216)))))(((1 + ((-e^{-2} * (-21.79216))^2)))))(((1 + ((-e^{-2} * (-21.79216))^3)))))$$

Input interpretation:

$$\frac{1 - \frac{2(-21.79216)}{e^2} + 2\left(\left(-\frac{21.79216}{e^2}\right)^4 - 2\left(-\frac{21.79216}{e^2}\right)^9\right)}{\left(1 - \frac{21.79216}{e^2}\right)\left(1 + \left(-\frac{21.79216}{e^2}\right)^2\right)\left(1 + \left(-\frac{21.79216}{e^2}\right)^3\right)}$$

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Result:

- Fewer digits
- More digits

-66.0051133800458917738884796007671739802453535254896602167...

Alternative representation:

$$\frac{1 - \frac{2(-1)(-21.7922)}{e^2} + 2\left(\left(-\frac{-21.7922}{e^2}\right)^4 - 2\left(-\frac{-21.7922}{e^2}\right)^9\right)}{\left(1 - \frac{21.7922}{e^2}\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^3\right)} =$$

$$\frac{1 - \frac{2(-1)(-21.7922)}{\exp^2(z)} + 2\left(\left(-\frac{-21.7922}{\exp^2(z)}\right)^4 - 2\left(-\frac{-21.7922}{\exp^2(z)}\right)^9\right)}{\left(1 - \frac{21.7922}{\exp^2(z)}\right)\left(1 + \left(-\frac{-21.7922}{\exp^2(z)}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{\exp^2(z)}\right)^3\right)} \text{ for } z = 1$$

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Series representations:

More

$$\frac{1 - \frac{2(-1)(-21.7922)}{e^2} + 2\left(\left(-\frac{-21.7922}{e^2}\right)^4 - 2\left(-\frac{-21.7922}{e^2}\right)^9\right)}{\left(1 - \frac{21.7922}{e^2}\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^3\right)} =$$

$$\frac{9.33681 \times 10^{-9} \left(1 + \sum_{k=0}^{\infty} \frac{-4.43366 \times 10^{12} (-18)^k + 451057. (-8)^k - 43.5843 (-2)^k}{k!}\right)}{\left(0.0000966271 + \sum_{k=0}^{\infty} \frac{(-6)^k}{k!}\right)\left(0.00210571 + \sum_{k=0}^{\infty} \frac{(-4)^k}{k!}\right)\left(0.0458881 + \sum_{k=0}^{\infty} \frac{(-2)^k}{k!}\right)}$$

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$$\begin{aligned}
& \frac{1 - \frac{2(-1)(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} = \\
& - \left[ \left( 41396.2 \left( z^{18} - 1.01735 \times 10^{-7} z^8 \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^{10} + 9.83032 \times 10^{-12} z^2 \right. \right. \right. \\
& \quad \left. \left. \left. \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^{16} - 2.25547 \times 10^{-13} \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^{18} \right) \right] / \\
& \quad \left( \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^6 \left( z^2 + 0.0458881 \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^2 \right) \right) \left( z^4 + \right. \\
& \quad \left. \left. \left. 0.00210571 \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^4 \right) \left( z^6 + 0.0000966271 \left( \sum_{k=0}^{\infty} \frac{-1+k+z}{k!} \right)^6 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1 - \frac{2(-1)(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} = \\
& \left( 7.10543 \times 10^{-15} \left( -6.23982 \times 10^{26} + 6.34806 \times 10^{19} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{10} - \right. \right. \\
& \quad \left. \left. 6.13395 \times 10^{15} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{16} + 1.40737 \times 10^{14} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{18} \right) \right) / \\
& \left( \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6 \left( 21.7922 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right)^2 \left( 21.7922 - 8.08557 \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \right. \\
& \quad \left( 21.7922 - 6.60184 \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \left( 21.7922 + 6.60184 \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \\
& \quad \left. \left( 21.7922 + 8.08557 \sum_{k=0}^{\infty} \frac{1}{k!} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^2 \right) \right)
\end{aligned}$$

Continued fraction:

Linear form

$$\begin{aligned}
& -66 + \cfrac{1}{-195 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-3 + \cfrac{1}{-3 + \cfrac{1}{-13 + \cfrac{1}{-7 + \cfrac{1}{-1 + \cfrac{1}{-14 + \cfrac{1}{-4 + \cfrac{1}{-21 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}}}}}
\end{aligned}$$

[Open code](#)

$$((((1 - 2((-e^{-2} * (-21.79216))^3)) + 2((-e^{-2} * (-21.79216))^12)))^2 / (((((1 - ((-e^{-2} * (-21.79216))))^2) * (((1 + ((-e^{-2} * (-21.79216))^3))))$$

Input interpretation:

$$\frac{\left(1 - 2\left(-\frac{-21.79216}{e^2}\right)^3\right) + 2\left(-\frac{-21.79216}{e^2}\right)^{12})^2}{\left(1 - \left(-\frac{-21.79216}{e^2}\right)\right)\left(1 - \left(-\frac{-21.79216}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.79216}{e^2}\right)^3\right)}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
  - More digits
- 1.87538473673149311183011011168097647494683359215598068...  $\times 10^9$

Alternative representation:

$$\frac{\left(1 - 2\left(-\frac{-21.7922}{e^2}\right)^3\right) + 2\left(-\frac{-21.7922}{e^2}\right)^{12})^2}{\left(1 - \left(-\frac{-21.7922}{e^2}\right)\right)\left(1 - \left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^3\right)} =$$

$$\frac{\left(1 - 2\left(-\frac{-21.7922}{\exp^2(z)}\right)^3\right) + 2\left(-\frac{-21.7922}{\exp^2(z)}\right)^{12})^2}{\left(1 - \left(-\frac{-21.7922}{\exp^2(z)}\right)\right)\left(1 - \left(-\frac{-21.7922}{\exp^2(z)}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{\exp^2(z)}\right)^3\right)} \text{ for } z = 1$$

[Open code](#)

Series representations:

More

$$\frac{\left(1 - 2\left(-\frac{-21.7922}{e^2}\right)^3\right) + 2\left(-\frac{-21.7922}{e^2}\right)^{12})^2}{\left(1 - \left(-\frac{-21.7922}{e^2}\right)\right)\left(1 - \left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^3\right)} =$$

$$9.33681 \times 10^{-9} \left( 1 + 2 \sum_{k=0}^{\infty} \frac{2.29421 \times 10^{16} (-24)^k - 20698.1 (-6)^k}{k!} + \right.$$

$$\left. \left( \sum_{k=0}^{\infty} \frac{2.29421 \times 10^{16} (-24)^k - 20698.1 (-6)^k}{k!} \right)^2 \right) /$$

$$\left( \left( 0.0000966271 + \sum_{k=0}^{\infty} \frac{(-6)^k}{k!} \right) \left( -0.00210571 + \sum_{k=0}^{\infty} \frac{(-4)^k}{k!} \right) \left( -0.0458881 + \sum_{k=0}^{\infty} \frac{(-2)^k}{k!} \right) \right)$$

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$$\begin{aligned}
& \frac{\left(1 - 2\left(-\frac{-21.7922}{e^2}\right)^3 + 2\left(-\frac{-21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - -\frac{-21.7922}{e^2}\right)\left(1 - \left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^3\right)} = \\
& \left(4.91434 \times 10^{24} z^{48} - 8.86732 \times 10^{12} z^{30} \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^{18} + \right. \\
& \quad 4.28412 \times 10^8 z^{24} \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^{24} + 4z^{12} \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^{36} - \\
& \quad \left. 0.000386509 z^6 \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^{42} + 9.33681 \times 10^{-9} \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^{48}\right) / \\
& \left(\left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^{36} \left(z^2 - 0.0458881 \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^2\right)\right. \\
& \quad \left.\left(z^4 - 0.00210571 \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^4\right) \left(z^6 + 0.0000966271 \left(\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}\right)^6\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(1 - 2\left(-\frac{-21.7922}{e^2}\right)^3 + 2\left(-\frac{-21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - -\frac{-21.7922}{e^2}\right)\left(1 - \left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^2}\right)^3\right)} = \\
& \left(5.2634 \times 10^{32} - 9.49717 \times 10^{20} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{18} + 4.58842 \times 10^{16} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} + \right. \\
& \quad 4.28412 \times 10^8 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{36} - 41396.2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{42} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{48}\Big) / \\
& \left(\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{36} \left(-4.66821 + \sum_{k=0}^{\infty} \frac{1}{k!}\right)^2 \left(4.66821 + \sum_{k=0}^{\infty} \frac{1}{k!}\right)^2 \left(21.7922 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^2\right)^2 \right. \\
& \quad \left.\left(21.7922 - 8.08557 \sum_{k=0}^{\infty} \frac{1}{k!} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^2\right) \left(21.7922 + 8.08557 \sum_{k=0}^{\infty} \frac{1}{k!} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^2\right)\right)
\end{aligned}$$

$\ln [[[(((1 - 2((-e^2 * (-21.79216))^3)) + 2((-e^2 * (-21.79216))^12)))^2 / (((1 - ((-e^2 * (-21.79216))))((1 - ((-e^2 * (-21.79216))^2))))(((1 + ((-e^2 * (-21.79216))^3))))]]]$

Input interpretation:

$$\log \left( \frac{\left(1 - 2\left(-\frac{-21.79216}{e^2}\right)^3 + 2\left(-\frac{-21.79216}{e^2}\right)^{12}\right)^2}{\left(1 - -\frac{-21.79216}{e^2}\right)\left(1 - \left(-\frac{-21.79216}{e^2}\right)^2\right)\left(1 + \left(-\frac{-21.79216}{e^2}\right)^3\right)} \right)$$

Open code

- $\log(x)$  is the natural logarithm

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Result:

• Fewer digits  
More digits

21.35207966824305974769486763276860398561069308852414622005...

This result 21,352 is very near to the value of black hole entropy (see Tables)

Alternative representations:

More

$$\log \left( \frac{\left(1 - 2 \left(-\frac{21.7922}{e^2}\right)^3 + 2 \left(-\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(-\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(-\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(-\frac{21.7922}{e^2}\right)^3\right)} \right) =$$

$$\log_e \left( \frac{\left(1 - 2 \left(\frac{21.7922}{e^2}\right)^3 + 2 \left(\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(\frac{21.7922}{e^2}\right)^3\right)} \right)$$

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$$\log \left( \frac{\left(1 - 2 \left(-\frac{21.7922}{e^2}\right)^3 + 2 \left(-\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(-\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(-\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(-\frac{21.7922}{e^2}\right)^3\right)} \right) =$$

$$\log(a) \log_a \left( \frac{\left(1 - 2 \left(\frac{21.7922}{e^2}\right)^3 + 2 \left(\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(\frac{21.7922}{e^2}\right)^3\right)} \right)$$

[Open code](#)

$$\log \left( \frac{\left(1 - 2 \left(-\frac{21.7922}{e^2}\right)^3 + 2 \left(-\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(-\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(-\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(-\frac{21.7922}{e^2}\right)^3\right)} \right) =$$

$$-\text{Li}_1 \left( 1 - \frac{\left(1 - 2 \left(\frac{21.7922}{e^2}\right)^3 + 2 \left(\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(\frac{21.7922}{e^2}\right)^3\right)} \right)$$

[Open code](#)

Series representations:

More

$$\log \left( \frac{\left(1 - 2 \left(-\frac{21.7922}{e^2}\right)^3 + 2 \left(-\frac{21.7922}{e^2}\right)^{12}\right)^2}{\left(1 - \left(-\frac{21.7922}{e^2}\right)^3\right) \left(1 - \left(-\frac{21.7922}{e^2}\right)^2\right) \left(1 + \left(-\frac{21.7922}{e^2}\right)^3\right)} \right) =$$

$$\log \left( -1 + \frac{\left(1 + \frac{2.29421 \times 10^{16}}{e^{24}} - \frac{20.698.1}{e^6}\right)^2}{\left(1 + \frac{10.349.1}{e^6}\right) \left(1 - \frac{474.898}{e^4}\right) \left(1 - \frac{21.7922}{e^2}\right)} \right) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{(2.29421 \times 10^{16} - 20.698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10.349.1 + e^6)} \right)^{-k}}{k}$$

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$$\log \left( \frac{\left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$2 i \pi \left[ - \frac{-\pi + \arg \left( \frac{(2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6) z_0} \right) + \arg(z_0)}{2 \pi} \right] +$$

$$\log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{(2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6)} - z_0 \right)^k z_0^{-k}}{k}$$

$$\log \left( \frac{\left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$2 i \pi \left[ \arg \left( \frac{\left( 1 + \frac{2.29421 \times 10^{16}}{e^{24}} - \frac{20698.1}{e^6} \right)^2}{\left( 1 + \frac{10349.1}{e^6} \right) \left( 1 - \frac{474.898}{e^4} \right) \left( 1 - \frac{-21.7922}{e^2} \right)} - x \right) \right] + \log(x) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{(2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6)} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0$$

Integral representations:

$$\log \left( \frac{\left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\int_1^{\infty} \frac{\frac{(2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6)}}{t} dt$$

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$$\log \left( \frac{\left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\frac{1}{2 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\left( -1 + \frac{(2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6)} \right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

[Open code](#)

$$\ln [[[(-1 - 2((-e^{-2} * (-21.79216))) + 2((-e^{-2} * (-21.79216))^4 - 2((-e^{-2} * (-21.79216))^{16})) / (((1 + ((-e^{-2} * (-21.79216))))((1 + ((-e^{-2} * (-21.79216))^{16}))(((1 + ((-e^{-2} * (-21.79216))^{16})^3))))]]]$$

Input interpretation:

$$\log \left( -\frac{1 - \frac{2(-21.79216)}{e^2} + 2 \left( \left( -\frac{21.79216}{e^2} \right)^4 - 2 \left( -\frac{21.79216}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.79216}{e^2} \right) \left( 1 + \left( -\frac{21.79216}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.79216}{e^2} \right)^3 \right)} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

- Fewer digits
- More digits

4.189732214480598252020155238278436967247158861726685147661...

This result 4,1897 is in the range of the mass of hypothetical dark matter particles

Alternative representations:

More

$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\log_e \left( \frac{-1 + \frac{43.5843}{e^2} - 2 \left( \left( \frac{21.7922}{e^2} \right)^4 - 2 \left( \frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 + \frac{21.7922}{e^2} \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)$$

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$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\log(a) \log_a \left( \frac{-1 + \frac{43.5843}{e^2} - 2 \left( \left( \frac{21.7922}{e^2} \right)^4 - 2 \left( \frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 + \frac{21.7922}{e^2} \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)$$

[Open code](#)

$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$-\text{Li}_1 \left( 1 - \frac{-1 + \frac{43.5843}{e^2} - 2 \left( \left( \frac{21.7922}{e^2} \right)^4 - 2 \left( \frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 + \frac{21.7922}{e^2} \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)$$

[Open code](#)

Series representations:

More

$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\log \left( -1 + \frac{-1 - 2 \left( -\frac{2.21683 \times 10^{12}}{e^{18}} + \frac{225528}{e^8} \right) + \frac{43.5843}{e^2}}{\left( 1 + \frac{10349.1}{e^6} \right) \left( 1 + \frac{474.898}{e^4} \right) \left( 1 + \frac{21.7922}{e^2} \right)} \right) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} \right)^{-k}}{k}$$

[Open code](#)

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$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$2i\pi \left[ -\frac{-\pi + \arg \left( \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} z_0 \right) + \arg(z_0)}{2\pi} \right] +$$

$$\log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} - z_0 \right)^k z_0^{-k}}{k}$$

$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) =$$

$$2i\pi \left[ \frac{\arg \left( \frac{-1 - 2 \left( -\frac{2.21683 \times 10^{12}}{e^{18}} + \frac{225528}{e^8} \right) + \frac{43.5843}{e^2}}{\left( 1 + \frac{10349.1}{e^6} \right) \left( 1 + \frac{474.898}{e^4} \right) \left( 1 + \frac{21.7922}{e^2} \right)} - x \right)}{2\pi} \right] + \log(x) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0$$

Integral representations:

$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\int_1^{\frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)}} \frac{1}{t} dt$$

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$$\log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\frac{1}{2 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\left( -1 + \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} \right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

[Open code](#)

Continued fraction:

Linear form

$$4 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{69 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{12 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$

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$$3 \ln [[[(((1 - 2((-e^{-2} * (-21.79216))) + 2((-e^{-2} * (-21.79216))^4 - 2((-e^{-2} * (-21.79216))^9)))) / (((((1 + ((-e^{-2} * (-21.79216)))) * (((1 + ((-e^{-2} * (-21.79216))^2)))) * (((((1 + ((-e^{-2} * (-21.79216))^3))))]]]$$

Input interpretation:

$$3 \log \left( -\frac{1 - \frac{2(-21.79216)}{e^2} + 2 \left( \left( -\frac{21.79216}{e^2} \right)^4 - 2 \left( -\frac{21.79216}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.79216}{e^2} \right) \left( 1 + \left( -\frac{21.79216}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.79216}{e^2} \right)^3 \right)} \right)$$

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- $\log(x)$  is the natural logarithm

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Result:

- Fewer digits
- More digits

12.56919664344179475606046571483531090174147658518005544298...

This result 12,5691 is practically equal to the value of black hole entropy (see Tables)

Alternative representations:

More

$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$3 \log_e \left( \frac{-1 + \frac{43.5843}{e^2} - 2 \left( \left( \frac{21.7922}{e^2} \right)^4 - 2 \left( \frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 + \frac{21.7922}{e^2} \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)$$

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$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$3 \log(a) \log_a \left( \frac{-1 + \frac{43.5843}{e^2} - 2 \left( \left( \frac{21.7922}{e^2} \right)^4 - 2 \left( \frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 + \frac{21.7922}{e^2} \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)$$

[Open code](#)

$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$-3 \text{Li}_1 \left( 1 - \frac{-1 + \frac{43.5843}{e^2} - 2 \left( \left( \frac{21.7922}{e^2} \right)^4 - 2 \left( \frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 + \frac{21.7922}{e^2} \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)$$

[Open code](#)

Series representations:

More

$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) = \\ 3 \log \left( -1 + \frac{-1 - 2 \left( -\frac{2.21683 \times 10^{12}}{e^{18}} + \frac{225528.}{e^8} \right) + \frac{43.5843}{e^2}}{\left( 1 + \frac{10349.1}{e^6} \right) \left( 1 + \frac{474.898}{e^4} \right) \left( 1 + \frac{21.7922}{e^2} \right)} \right) - \\ 3 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} \right)^k}{k}$$

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$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) = \\ 6 i \pi \left[ -\frac{-\pi + \arg \left( \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6) z_0} \right) + \arg(z_0)}{2 \pi} \right] + \\ 3 \log(z_0) - 3 \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} - z_0 \right)^k}{k} z_0^{-k}$$

$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) = \\ 6 i \pi \left[ \frac{\arg \left( \frac{-1 - 2 \left( -\frac{2.21683 \times 10^{12}}{e^{18}} + \frac{225528.}{e^8} \right) + \frac{43.5843}{e^2}}{\left( 1 + \frac{10349.1}{e^6} \right) \left( 1 + \frac{474.898}{e^4} \right) \left( 1 + \frac{21.7922}{e^2} \right)} - x \right)}{2 \pi} \right] + 3 \log(x) - \\ 3 \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)} - x \right)^k}{k} x^{-k} \quad \text{for } x < 0$$

Integral representations:

$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{-21.7922}{e^2} \right)^4 - 2 \left( -\frac{-21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{-21.7922}{e^2} \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) = \\ 3 \int_1^{\frac{4.43366 \times 10^{12} - 451057. e^{10} + 43.5843 e^{16} - e^{18}}{e^6 (21.7922 + e^2) (474.898 + e^4) (10349.1 + e^6)}} \frac{1}{t} dt$$

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$$3 \log \left( -\frac{1 - \frac{2(-21.7922)}{e^2} + 2 \left( \left( -\frac{21.7922}{e^2} \right)^4 - 2 \left( -\frac{21.7922}{e^2} \right)^9 \right)}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{21.7922}{e^2} \right)^3 \right)} \right) =$$

$$\frac{3}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left( -1 + \frac{4.43366 \times 10^{12} - 451057.e^{10} + 43.5843.e^{16} - e^{18}}{e^6 (21.7922 + e^2)(474.898 + e^4)(10349.1 + e^6)} \right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

[Open code](#)

Continued fraction:

Linear form

$$\bullet \quad \begin{aligned} 12 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{23 + \cfrac{1}{23 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{7 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{12 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}$$

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$$\ln((-(-66.0051133800458917738884796 * (((1 - 2((-e^-2 * (-21.79216))^3)) + 2((-e^-2 * (-21.79216))^12)))^2 / (((((1 - ((-e^-2 * (-21.79216))))(((1 - ((-e^-2 * (-21.79216))^2))))((((1 + ((-e^-2 * (-21.79216))^3))))$$

Input interpretation:

$$\log \left( -66.0051133800458917738884796 \times \frac{\left( \left( 1 - 2 \left( -\frac{-21.79216}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.79216}{e^2} \right)^{12} \right)^2}{\left( 1 - \frac{-21.79216}{e^2} \right) \left( 1 - \left( -\frac{-21.79216}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.79216}{e^2} \right)^3 \right)} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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[Result:](#)

- Fewer digits
- More digits

25.54181188272365799971502287103541800516547999711591434900...

This result 25,5418 is very near to the value of black hole entropy (see Tables)

[Alternative representations:](#)

[More](#)

$$\begin{aligned} & \log \left( -\frac{-66.00511338004589177388847960000 \left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - \left( -\frac{-21.7922}{e^2} \right) \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) \\ & = \log_e \left( \frac{66.00511338004589177388847960000 \left( 1 - 2 \left( \frac{21.7922}{e^2} \right)^3 + 2 \left( \frac{21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 - \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right) \end{aligned}$$

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$$\begin{aligned} & \log \left( -\frac{-66.00511338004589177388847960000 \left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - \left( -\frac{-21.7922}{e^2} \right) \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) \\ & = \log(a) \log_a \left( \frac{66.00511338004589177388847960000 \left( 1 - 2 \left( \frac{21.7922}{e^2} \right)^3 + 2 \left( \frac{21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 - \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right) \end{aligned}$$

[Open code](#)

$$\begin{aligned}
& \log \left( -\frac{-66.00511338004589177388847960000 \left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) \\
& = -\text{Li}_1 \left( 1 - \frac{66.00511338004589177388847960000 \left( 1 - 2 \left( \frac{21.7922}{e^2} \right)^3 + 2 \left( \frac{21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - \frac{21.7922}{e^2} \right) \left( 1 - \left( \frac{21.7922}{e^2} \right)^2 \right) \left( 1 + \left( \frac{21.7922}{e^2} \right)^3 \right)} \right)
\end{aligned}$$

Series representations:

More

$$\begin{aligned}
& \log \left( -\frac{-66.00511338004589177388847960000 \left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) \\
& = \log \left( -1 + \frac{66.00511338004589177388847960000 \left( 1 + \frac{2.29421 \times 10^{16}}{e^{24}} - \frac{20698.1}{e^6} \right)^2}{\left( 1 + \frac{10349.1}{e^6} \right) \left( 1 - \frac{474.898}{e^4} \right) \left( 1 - \frac{21.7922}{e^2} \right)} \right) - \\
& \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{66.0051 (2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6)} \right)^{-k}}{k}
\end{aligned}$$

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$$\begin{aligned}
& \log \left( -\frac{-66.00511338004589177388847960000 \left( \left( 1 - 2 \left( -\frac{-21.7922}{e^2} \right)^3 \right) + 2 \left( -\frac{-21.7922}{e^2} \right)^{12} \right)^2}{\left( 1 - -\frac{-21.7922}{e^2} \right) \left( 1 - \left( -\frac{-21.7922}{e^2} \right)^2 \right) \left( 1 + \left( -\frac{-21.7922}{e^2} \right)^3 \right)} \right) \\
& = 2i\pi \left[ -\frac{-\pi + \arg \left( \frac{66.0051 (2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6) z_0} \right) + \arg(z_0)}{2\pi} \right] + \\
& \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{66.0051 (2.29421 \times 10^{16} - 20698.1 e^{18} + e^{24})^2}{e^{36} (-21.7922 + e^2) (-474.898 + e^4) (10349.1 + e^6)} - z_0 \right)^k z_0^{-k}}{k}
\end{aligned}$$

$$\log\left(-\left(-66.00511338004589177388847960000 \right.\right.$$

$$\left.\left. \left(\left(1-2\left(-\frac{-21.7922}{e^2}\right)^3\right)+2\left(-\frac{-21.7922}{e^2}\right)^{12}\right)^2\right)/\right.$$

$$\left.\left.\left(\left(1-\frac{-21.7922}{e^2}\right)\left(1-\left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1+\left(-\frac{-21.7922}{e^2}\right)^3\right)\right)\right)\right)=$$

$$2i\pi \left[ \arg \left( \frac{\frac{66.00511338004589177388847960000 \left(1+\frac{2.29421\times 10^{16}}{e^{24}}-\frac{20.698.1}{e^6}\right)^2}{\left(1+\frac{10.349.1}{e^6}\right)\left(1-\frac{474.898}{e^4}\right)\left(1-\frac{21.7922}{e^2}\right)} - x \right) \right] +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{66.0051(2.29421\times 10^{16}-20.698.1 e^{18}+e^{24})^2}{e^{36} (-21.7922+e^2)(-474.898+e^4)(10.349.1+e^6)} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0$$

Integral representations:

$$\log\left( \frac{-66.00511338004589177388847960000 \left( \left(1-2\left(-\frac{-21.7922}{e^2}\right)^3\right)+2\left(-\frac{-21.7922}{e^2}\right)^{12}\right)^2}{\left(1-\frac{-21.7922}{e^2}\right)\left(1-\left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1+\left(-\frac{-21.7922}{e^2}\right)^3\right)} \right)$$

$$= \int_1^{\frac{66.0051(2.29421\times 10^{16}-20.698.1 e^{18}+e^{24})^2}{e^{36} (-21.7922+e^2)(-474.898+e^4)(10.349.1+e^6)}} \frac{1}{t} dt$$

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$$\log\left(-\left(-66.00511338004589177388847960000 \right.\right.$$

$$\left.\left. \left(\left(1-2\left(-\frac{-21.7922}{e^2}\right)^3\right)+2\left(-\frac{-21.7922}{e^2}\right)^{12}\right)^2\right)/\right.$$

$$\left.\left.\left(\left(1-\frac{-21.7922}{e^2}\right)\left(1-\left(-\frac{-21.7922}{e^2}\right)^2\right)\left(1+\left(-\frac{-21.7922}{e^2}\right)^3\right)\right)\right)\right)=$$

$$\frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{66.0051(2.29421\times 10^{16}-20.698.1 e^{18}+e^{24})^2}{e^{36} (-21.7922+e^2)(-474.898+e^4)(10.349.1+e^6)}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

[Open code](#)

Continued fraction:

• Linear form

$$25 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{2 + \cfrac{1}{83 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{71 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

[Open code](#)

$$\text{For } \phi(q) \quad q = -e^{-t}, t = 0.5 \quad q^n = -21.79216 * e^{-0.5}$$

from the expression on pag. 93, we obtain:

$$\begin{aligned} \phi(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \dots \\ \psi(q) &= \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \dots \\ \chi(q) &= 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \dots \end{aligned}$$

$$\phi(q) = 1.075226 + 0.00572374 = 1.08094974$$

$$\psi(q) = -1.08185 + 1.08232 - 1.08232 = -1.08185$$

$$\chi(q) = 1.081345 + 0.00618954 = 1.08753454$$

The sum of  $\phi(q) + \psi(q) + \chi(q) = 1.08663428$  very near to the value 1.08643 already calculated from Ramanujan

$$((((1 - 2((-e^{-0.5} * (-21.79216))^1) + 2((-e^{-0.5} * (-21.79216))^4) - 2((-e^{-0.5} * (-21.79216))^9))) / (((1 + ((-e^{-0.5} * (-21.79216))^1)) * ((1 + ((-e^{-0.5} * (-21.79216))^2))) * (((1 + ((-e^{-0.5} * (-21.79216))^3))))$$

Input interpretation:

$$1 - 2\left(-\frac{-21.79216}{e^{0.5}}\right)^1 + 2\left(-\frac{-21.79216}{e^{0.5}}\right)^4 - 2\left(-\frac{-21.79216}{e^{0.5}}\right)^9$$

Result:

- More digits

$$-2.46267 \dots \times 10^{10}$$

Open code

$$-\frac{2.46267 \times 10^{10}}{\left(1 - \frac{-21.79216}{e^{0.5}}\right)\left(1 + \left(\frac{-21.79216}{e^{0.5}}\right)^2\right)\left(1 + \left(\frac{-21.79216}{e^{0.5}}\right)^3\right)}$$

Result:

- More digits

$$-4267.24 \dots$$

$$3/2 \ln(-4267.244455443856010826564662484282717644843292832231)$$

Input interpretation:

$$\frac{3}{2} \log(-4267.244455443856010826564662484282717644843292832231)$$

Open code

- $\log(x)$  is the natural logarithm

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Result:

- More digits

$$12.538085356973841583836496939105618470070512336019490\dots$$

This result 12,538 is very near to the value of black hole entropy (see Tables)

Series representations:

- More

$$\frac{1}{2} \log(-4267.2444554438560108265646624842827176448432928322310000)$$

$$3 = \frac{3 \log(4266.2444554438560108265646624842827176448432928322310000)}{2}$$

$$\frac{3}{2} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.3584892005878088134998295670694835221325364493099970758 k}}{k}$$

Open code

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$$\frac{1}{2} \log(-(-4267.2444554438560108265646624842827176448432928322310000))$$

$$3 = 3 i \pi \left| \frac{1}{2 \pi} \arg(4267.2444554438560108265646624842827176448432928322310000 - x) \right| + \frac{3 \log(x)}{2} - \frac{3}{2} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k$$

$$(4267.2444554438560108265646624842827176448432928322310000 - x)^k x^{-k} \text{ for } x < 0$$

[Open code](#)

$$\frac{1}{2} \log(-(-4267.2444554438560108265646624842827176448432928322310000))$$

$$\frac{3}{2} \left| \frac{1}{2 \pi} \arg(4267.2444554438560108265646624842827176448432928322310000 - z_0) \right| \log\left(\frac{1}{z_0}\right) + \frac{3 \log(z_0)}{2} + \frac{3}{2} \left| \frac{1}{2 \pi} \arg(4267.2444554438560108265646624842827176448432928322310000 - z_0) \right| \log(z_0) - \frac{3}{2} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k$$

$$(4267.2444554438560108265646624842827176448432928322310000 - z_0)^k z_0^{-k}$$

Integral representations:

$$\frac{1}{2} \log(-(-4267.2444554438560108265646624842827176448432928322310000))$$

$$3 = \frac{3}{2} \int_1^{4267.2444554438560108265646624842827176448432928322310000} \frac{1}{t} dt$$

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$$\frac{1}{2} \log(-(-4267.2444554438560108265646624842827176448432928322310000))$$

$$3 = \frac{3}{4 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-8.3584892005878088134998295670694835221325364493099970758 s}}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) ds \text{ for } -1 < \gamma < 0$$

$$(72*2-6) * 3/2 \ln(-4267.244455443856010826564662484282717644843292832231)$$

Input interpretation:

$$\frac{3}{2} \log(-(-4267.244455443856010826564662484282717644843292832231))$$

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- $\log(x)$  is the natural logarithm

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Result:

[More digits](#)

- 1730.2557792623901385694365775965753488697307023706896...

This result 1730,2557 is very near to the mass of  $f_0(1710)$  candidate glueball

[Alternative representations:](#)

[More](#)

- $$\begin{aligned} & \frac{1}{2} ((72 \times 2 - 6) \log( \\ & \quad -(-4267.2444554438560108265646624842827176448432928322310000) \\ & \quad )) 3 = 207 \\ & \log_e(4267.2444554438560108265646624842827176448432928322310000) \end{aligned}$$

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$$\begin{aligned} & \frac{1}{2} ((72 \times 2 - 6) \log( \\ & \quad -(-4267.2444554438560108265646624842827176448432928322310000) \\ & \quad )) 3 = 207 \log(a) \\ & \log_a(4267.2444554438560108265646624842827176448432928322310000) \end{aligned}$$

[Open code](#)

$$\begin{aligned} & \frac{1}{2} ((72 \times 2 - 6) \log( \\ & \quad -(-4267.2444554438560108265646624842827176448432928322310000) \\ & \quad )) 3 = -207 \\ & \text{Li}_1(-4266.2444554438560108265646624842827176448432928322310000) \end{aligned}$$

[Open code](#)

[Series representations:](#)

[More](#)

- $$\begin{aligned} & \frac{1}{2} ((72 \times 2 - 6) \log( \\ & \quad -(-4267.2444554438560108265646624842827176448432928322310000) \\ & \quad )) 3 = \\ & 207 \log(4266.2444554438560108265646624842827176448432928322310000) - \\ & 207 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.3584892005878088134998295670694835221325364493099970758 k}}{k} \end{aligned}$$

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$$\begin{aligned} \frac{1}{2} ((72 \times 2 - 6) \log( & -(-4267.244455443856010826564662484282717644843292832231000 \cdot \\ & 00))) 3 = 414 i \pi \left| \frac{1}{2 \pi} \arg( \right. \\ & 4267.2444554438560108265646624842827176448432928322310000 - \\ & x) \left. \right| + 207 \log(x) - 207 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \\ & (4267.2444554438560108265646624842827176448432928322310000 \\ & - x)^k x^{-k} \text{ for } x < 0 \end{aligned}$$

[Open code](#)

$$\begin{aligned} \frac{1}{2} ((72 \times 2 - 6) \log( & -(-4267.2444554438560108265646624842827176448432928322310000) \\ & )) 3 = 207 \left| \frac{1}{2 \pi} \right. \\ & \arg(4267.2444554438560108265646624842827176448432928322310000 - \\ & z_0) \left. \right| \log\left(\frac{1}{z_0}\right) + 207 \log(z_0) + 207 \left| \frac{1}{2 \pi} \right. \\ & \arg(4267.2444554438560108265646624842827176448432928322310000 - \\ & z_0) \left| \log(z_0) - 207 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \right. \\ & (4267.2444554438560108265646624842827176448432928322310000 - \\ & z_0)^k z_0^{-k} \end{aligned}$$

Integral representations:

$$\begin{aligned} \frac{1}{2} ((72 \times 2 - 6) \log( & -(-4267.2444554438560108265646624842827176448432928322310000) \\ & )) 3 = \\ & 207 \int_1^{4267.2444554438560108265646624842827176448432928322310000} \frac{1}{t} dt \end{aligned}$$

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$$\begin{aligned} \frac{1}{2} ((72 \times 2 - 6) \log( & -(-4267.244455443856010826564662484282717644843292832231000 \cdot \\ & 00))) 3 = \frac{207}{2 i \pi} \\ & \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-8.3584892005878088134998295670694835221325364493099970758 s}}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) \\ & ds \text{ for } -1 < \gamma < 0 \end{aligned}$$

[Open code](#)

Continued fraction:

Linear form

$$1730 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{15 + \cfrac{1}{2 + \cfrac{1}{81 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{104 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{25 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}$$

[Open code](#)

$$(89+3) \ln(-4267.244455443856010826564662484282717644843292832231)$$

Alternative representations:

More

$$(89 + 3) \log(-(-1))$$

$$4267.2444554438560108265646624842827176448432928322310000 =$$

$$92 \log_e(4267.2444554438560108265646624842827176448432928322310000)$$

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$$(89 + 3) \log(-(-1))$$

$$4267.2444554438560108265646624842827176448432928322310000 =$$

$$92 \log(a) \log_a($$

$$4267.2444554438560108265646624842827176448432928322310000)$$

[Open code](#)

$$(89 + 3) \log(-(-1))$$

$$4267.2444554438560108265646624842827176448432928322310000 =$$

$$-92 \text{Li}_1(-4266.2444554438560108265646624842827176448432928322310000)$$

[Open code](#)

Series representations:

More

$$(89 + 3) \log(-(-1)) = 4267.2444554438560108265646624842827176448432928322310000 - 92 \log(4266.2444554438560108265646624842827176448432928322310000) - 92 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-8.3584892005878088134998295670694835221325364493099970758 k}}{k}$$

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$$(89 + 3) \log(-(-1)) = 4267.2444554438560108265646624842827176448432928322310000 - 184 i \pi \left\lfloor \frac{1}{2 \pi} \arg(4267.2444554438560108265646624842827176448432928322310000 - x) \right\rfloor + 92 \log(x) - 92 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (4267.2444554438560108265646624842827176448432928322310000 - x)^k x^{-k} \quad \text{for } x < 0$$

[Open code](#)

$$(89 + 3) \log(-(-1)) = 4267.2444554438560108265646624842827176448432928322310000 - 92 \left\lfloor \frac{1}{2 \pi} \arg(4267.2444554438560108265646624842827176448432928322310000 - z_0) \right\rfloor \log\left(\frac{1}{z_0}\right) + 92 \log(z_0) + 92 \left\lfloor \frac{1}{2 \pi} \arg(4267.2444554438560108265646624842827176448432928322310000 - z_0) \right\rfloor \log(z_0) - 92 \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (4267.2444554438560108265646624842827176448432928322310000 - z_0)^k z_0^{-k}$$

[Open code](#)

Integral representations:

$$(89 + 3) \log(-(-1)) = 4267.2444554438560108265646624842827176448432928322310000 - 92 \int_1^{4267.2444554438560108265646624842827176448432928322310000} \frac{1}{t} dt$$

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$$(89 + 3) \log(-(-1)) = 4267.2444554438560108265646624842827176448432928322310000 =$$

$$\frac{46}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-8.3584892005878088134998295670694835221325364493099970758s}}{\Gamma(1-s)} \frac{\Gamma(-s)^2}{\Gamma(1+s)} ds \quad \text{for } -1 < \gamma < 0$$

[Open code](#)

$$((((1 - 2((-e^{-0.5} * (-21.79216))^3)) + 2((-e^{-0.5} * (-21.79216))^12)))^2 / (((((1 - ((-e^{-0.5} * (-21.79216))))((1 - ((-e^{-0.5} * (-21.79216))^2))))(((1 + ((-e^{-0.5} * (-21.79216))^3))))$$

Input interpretation:

$$\frac{\left(1 - 2\left(-\frac{-21.79216}{e^{0.5}}\right)^3\right) + 2\left(-\frac{-21.79216}{e^{0.5}}\right)^{12})^2}{\left(1 - -\frac{-21.79216}{e^{0.5}}\right)\left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^2\right)\left(1 + \left(-\frac{-21.79216}{e^{0.5}}\right)^3\right)}$$

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Result:

- Fewer digits
- More digits

$$6.5960861587739343651331914888154239698536251608680987... \times 10^{20}$$

Alternative representation:

$$\frac{\left(1 - 2\left(-\frac{-21.7922}{e^{0.5}}\right)^3\right) + 2\left(-\frac{-21.7922}{e^{0.5}}\right)^{12})^2}{\left(1 - -\frac{-21.7922}{e^{0.5}}\right)\left(1 - \left(-\frac{-21.7922}{e^{0.5}}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{e^{0.5}}\right)^3\right)} =$$

$$\frac{\left(1 - 2\left(-\frac{-21.7922}{\exp^{0.5}(z)}\right)^3\right) + 2\left(-\frac{-21.7922}{\exp^{0.5}(z)}\right)^{12})^2}{\left(1 - -\frac{-21.7922}{\exp^{0.5}(z)}\right)\left(1 - \left(-\frac{-21.7922}{\exp^{0.5}(z)}\right)^2\right)\left(1 + \left(-\frac{-21.7922}{\exp^{0.5}(z)}\right)^3\right)} \quad \text{for } z = 1$$

[Open code](#)

Series representations:

- More

$$\begin{aligned}
& \frac{\left(1 - 2 \left(-\frac{-21.7922}{e^{0.5}}\right)^3 + 2 \left(-\frac{-21.7922}{e^{0.5}}\right)^{12}\right)^2}{\left(1 - -\frac{-21.7922}{e^{0.5}}\right) \left(1 - \left(-\frac{-21.7922}{e^{0.5}}\right)^2\right) \left(1 + \left(-\frac{-21.7922}{e^{0.5}}\right)^3\right)} = \\
& \left(5.2634 \times 10^{32} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4.5} - 9.49717 \times 10^{20} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^9 + 4.58842 \times 10^{16} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10.5} + \right. \\
& \quad \left. 4.28412 \times 10^8 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{13.5} - 41396.2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{15} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16.5}\right) / \\
& \left(\left(-21.7922 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.5}\right) \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{13.5} \left(-474.898 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^1\right) \left(10349.1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{1.5}\right)\right)
\end{aligned}$$

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$$\begin{aligned}
& \frac{\left(1 - 2 \left(-\frac{-21.7922}{e^{0.5}}\right)^3 + 2 \left(-\frac{-21.7922}{e^{0.5}}\right)^{12}\right)^2}{\left(1 - -\frac{-21.7922}{e^{0.5}}\right) \left(1 - \left(-\frac{-21.7922}{e^{0.5}}\right)^2\right) \left(1 + \left(-\frac{-21.7922}{e^{0.5}}\right)^3\right)} = \\
& \left(2.15589 \times 10^{36} \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{4.5} - 1.71917 \times 10^{23} \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^9 + \right. \\
& \quad \left. 2.93659 \times 10^{18} \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{10.5} + 3.4273 \times 10^9 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{13.5} - \right. \\
& \quad \left. 117086. \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{15} + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{16.5}\right) / \left(\left(-30.8188 + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.5}\right) \right. \\
& \quad \left. \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{13.5} \left(-949.796 + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^1\right) \left(29271.6 + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{1.5}\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(1 - 2 \left(-\frac{-21.7922}{e^{0.5}}\right)^3 + 2 \left(-\frac{-21.7922}{e^{0.5}}\right)^{12}\right)^2}{\left(1 - -\frac{-21.7922}{e^{0.5}}\right) \left(1 - \left(-\frac{-21.7922}{e^{0.5}}\right)^2\right) \left(1 + \left(-\frac{-21.7922}{e^{0.5}}\right)^3\right)} = \\
& \left(5.2634 \times 10^{32} \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{4.5} - 9.49717 \times 10^{20} \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^9 + \right. \\
& \quad \left. 4.58842 \times 10^{16} \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{10.5} + 4.28412 \times 10^8 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{13.5} - \right. \\
& \quad \left. 41396.2 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{15} + \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{16.5}\right) / \\
& \left(\left(-21.7922 + \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.5}\right) \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{13.5} \right. \\
& \quad \left. \left(-474.898 + \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^1\right) \left(10349.1 + \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{1.5}\right)\right)
\end{aligned}$$

$$(-4267.244455443856010826564662484282717644843292832231) * (((1-2((-e^-0.5 * (-21.79216))^3))+2((-e^-0.5 * (-21.79216))^12)))^2 / (4.90283*10^6)$$

[Input interpretation:](#)

$$\frac{\left(1 - 2\left(\frac{-21.79216}{e^{0.5}}\right)^3\right) + 2\left(\frac{-21.79216}{e^{0.5}}\right)^{12}}{4.90283 \times 10^6}$$

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[Result:](#)

- More digits  
 $-2.81471... \times 10^{24}$

$$\frac{1}{(2.61803398)} \ln \left( -(-2.8147095673908046130256778529964805112284843919 \times 10^{24}) \right)$$

[Input interpretation:](#)

$$\frac{1}{2.61803398} \log \left( -(-2.8147095673908046130256778529964805112284843919 \times 10^{24}) \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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[Result:](#)

- More digits  
 $21.5035029...$

This result 21,503 is very near to the value of black hole entropy (see Tables)

$$\frac{1}{2} \ln \left( 6.59608615877393436513319148 \times 10^{20} \right)$$

[Input interpretation:](#)

$$\frac{1}{2} \log \left( 6.59608615877393436513319148 \times 10^{20} \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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[Result:](#)

- More digits  
 $23.969089163384586714340873596...$

This result 23,969 is very near to the value of black hole entropy (see Tables), and equal to 24, are the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

[Alternative representations:](#)

[More](#)

- $$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) =$$
  

$$\frac{\log_e(6.596086158773934365133191480000 \times 10^{20})}{2}$$

[Open code](#)

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$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) =$$

$$\frac{1}{2} \log(a) \log_a(6.596086158773934365133191480000 \times 10^{20})$$

[Open code](#)

$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) =$$

$$-\frac{1}{2} \text{Li}_1(1 - 6.596086158773934365133191480000 \times 10^{20})$$

[Open code](#)

[Series representations:](#)

[More](#)

- $$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) =$$
  

$$\frac{\log(6.596086158773934365123191480000 \times 10^{20})}{2} -$$
  

$$\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-47.938178326769173428680231141385 k}}{k}$$

[Open code](#)

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$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) =$$

$$\frac{i \pi \left[ \arg(6.596086158773934365133191480000 \times 10^{20} - x) \right]}{2 \pi} + \frac{\log(x)}{2} -$$

$$\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (6.596086158773934365133191480000 \times 10^{20} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\begin{aligned} \frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) = \\ \frac{1}{2} \left[ \frac{\arg(6.596086158773934365133191480000 \times 10^{20} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \frac{\log(z_0)}{2} + \\ \frac{1}{2} \left[ \frac{\arg(6.596086158773934365133191480000 \times 10^{20} - z_0)}{2\pi} \right] \log(z_0) - \\ \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (6.596086158773934365133191480000 \times 10^{20} - z_0)^k z_0^{-k}}{k} \end{aligned}$$

[Open code](#)

- $\arg(z)$  is the complex argument

- Integral representations:

$$\begin{aligned} \frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) = \\ \frac{1}{2} \int_1^{6.596086158773934365133191480000 \times 10^{20}} \frac{1}{t} dt \end{aligned}$$

- [Open code](#)
- 

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$$\begin{aligned} \frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) = \\ \frac{1}{4i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-47.938178326769173428680231141385s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0 \end{aligned}$$

- [Open code](#)

((sqrt(5)-1)/2))) ln (6.59608615877393436513319148 \* 10^20)

Input interpretation:

$$\left( \frac{1}{2} (\sqrt{5} - 1) \right) \log(6.59608615877393436513319148 \times 10^{20})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

29.627423564696912369148797588...

This result 29,627 is very near to the value of black hole entropy (see Tables)

Alternative representations:

More

$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) =$$

$$\frac{1}{2} \log_e(6.596086158773934365133191480000 \times 10^{20}) (-1 + \sqrt{5})$$

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$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) =$$

$$\frac{1}{2} \log(a) \log_a(6.596086158773934365133191480000 \times 10^{20}) (-1 + \sqrt{5})$$

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$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) =$$

$$-\frac{1}{2} \text{Li}_1\left(1 - 6.596086158773934365133191480000 \times 10^{20}\right) (-1 + \sqrt{5})$$

[Open code](#)

Series representations:

More

$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) =$$

$$-\frac{\log(6.596086158773934365133191480000 \times 10^{20})}{2} +$$

$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}$$

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$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) =$$

$$\frac{1}{2} \left[ \log(6.596086158773934365123191480000 \times 10^{20}) - \right.$$

$$\left. \sum_{k=1}^{\infty} \frac{(-1)^k e^{-47.938178326769173428680231141385 k}}{k} \right] \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)$$

[Open code](#)

$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) =$$

$$\frac{1}{2} \left[ \log(6.596086158773934365123191480000 \times 10^{20}) - \right.$$

$$\left. \sum_{k=1}^{\infty} \frac{(-1)^k e^{-47.938178326769173428680231141385 k}}{k} \right] \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

[Open code](#)

Integral representations:

$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) = \\ \frac{1}{2} (-1 + \sqrt{5}) \int_1^{6.596086158773934365133191480000 \times 10^{20}} \frac{1}{t} dt$$

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$$\frac{1}{2} \log(6.596086158773934365133191480000 \times 10^{20}) (\sqrt{5} - 1) = \frac{-1 + \sqrt{5}}{4 i \pi} \\ \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-47.938178326769173428680231141385 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

[Open code](#)

Now the point here is that the entropy of a black hole is also known from semi-classical arguments (i.e. neglecting the quantisation of gravity) by a formula known as the Bekenstein-Hawking formula, which in this case becomes  $S = 4\pi\sqrt{k}$ . So for  $k = 1$  we expect the result  $4\pi = 12.566$ , which compares with Witten's 12.190, and for  $k = 4$  we expect  $8\pi = 25.133$  which compares with Witten's 25.118. The comparison for the first 4 values of  $k$  is,

<b>k</b>	<b>Bekenstein-Hawking</b>	<b>Witten</b>	<b>Difference (%)</b>
1	12.566	12.190	3.0%
2	17.772	17.576	1.1%
3	21.766	21.676	0.4%
4	25.133	25.118	0.06%

For  $k = 15$  for  $S = 4\pi\sqrt{k} = 48.66934411168\dots$

$\ln(6.59608615877393436513319148 * 10^{20})$

Input interpretation:

$$\log(6.59608615877393436513319148 \times 10^{20})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Result:  
More digits

$$47.938178326769173428681747192\dots$$

[Alternative representations:](#)

More

$$\log(6.596086158773934365133191480000 \times 10^{20}) = \\ \log_e(6.596086158773934365133191480000 \times 10^{20})$$

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$$\log(6.596086158773934365133191480000 \times 10^{20}) = \\ \log(a) \log_a(6.596086158773934365133191480000 \times 10^{20})$$

[Open code](#)

$$\log(6.596086158773934365133191480000 \times 10^{20}) = \\ -\text{Li}_1(1 - 6.596086158773934365133191480000 \times 10^{20})$$

[Series representations:](#)

More

$$\log(6.596086158773934365133191480000 \times 10^{20}) = \\ \log(6.596086158773934365123191480000 \times 10^{20}) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k e^{-47.938178326769173428680231141385 k}}{k}$$

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$$\log(6.596086158773934365133191480000 \times 10^{20}) = \\ \left[ \frac{\arg(6.596086158773934365133191480000 \times 10^{20} - x)}{2\pi} \right] + \log(x) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k (6.596086158773934365133191480000 \times 10^{20} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\log(6.596086158773934365133191480000 \times 10^{20}) = \\ \left[ \frac{\arg(6.596086158773934365133191480000 \times 10^{20} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \\ \left[ \frac{\arg(6.596086158773934365133191480000 \times 10^{20} - z_0)}{2\pi} \right] \log(z_0) - \\ \sum_{k=1}^{\infty} \frac{(-1)^k (6.596086158773934365133191480000 \times 10^{20} - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\log(6.596086158773934365133191480000 \times 10^{20}) = \int_1^{6.596086158773934365133191480000 \times 10^{20}} \frac{1}{t} dt$$

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$$\log(6.596086158773934365133191480000 \times 10^{20}) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-47.938178326769173428680231141385s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

Now:

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \dots$$
$$\phi(-q) + \chi(q) = 2F(q).$$

$$f(-q) + 2F(q^2) - 2 = \phi(-q^2) + \psi(-q)$$

$$= 2\phi(-q^2) - f(q) = \frac{1 - 2q + 2q^4 - 2q^9 + \dots}{(1-q)(1-q^4)(1-q^6)(1-q^9)\dots}$$
$$\psi(q) - F(q^2) + 1 = q \frac{1 + q^2 + q^6 + q^{12} + \dots}{(1-q^8)(1-q^{12})(1-q^{28})\dots}$$

$$(((1 - (2(-e^{-0.5} * (-21.79216))) + ((2((-e^{-0.5} * (-21.79216))^4)) - ((2((-e^{-0.5} * (-21.79216))^9))))$$

Input interpretation:

$$\left(1 - 2\left(-\frac{-21.79216}{e^{0.5}}\right)^1\right) + \left(2\left(-\frac{-21.79216}{e^{0.5}}\right)^4 - 2\left(-\frac{-21.79216}{e^{0.5}}\right)^9\right)$$

[Open code](#)

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Result:

• Fewer digits  
More digits

$$-2.462670232548926677016218944792315099779228851126416\dots \times 10^{10}$$

$$(1 - ((-e^{-0.5} * (-21.79216)^1))) * (1 - ((-e^{-0.5} * (-21.79216))^4))) * (1 - (-e^{-0.5} * (-21.79216)^6))) * (1 - (-e^{-0.5} * (-21.79216)^9)))$$

Input interpretation:

$$\left(1 - \frac{(-21.79216)^1}{e^{0.5}}\right) \left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^4\right) \left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^6\right) \left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^9\right)$$

Open code

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Result:

- Fewer digits
- More digits

$$2.4483853984140801815532157792827833324243862392592875... \times 10^{22}$$

$$(-2.4626702325489266770162189447923150997792288511 * 10^{10}) / (((((((1-((-e^{0.5} * (-21.79216)^1))) * (1-((-e^{0.5} * (-21.79216))^4))) * (1-((-e^{0.5} * (-21.79216))^6))) * (1-((-e^{0.5} * (-21.79216))^9))))))$$

Input interpretation:

$$\frac{-2.4626702325489266770162189447923150997792288511 \times 10^{10}}{\left(\left(1 - \frac{(-21.79216)^1}{e^{0.5}}\right) \left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^4\right) \left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^6\right) \left(1 - \left(-\frac{-21.79216}{e^{0.5}}\right)^9\right)} \\$$

Open code

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Result:

- Fewer digits
- More digits

$$-1.005834389530381683148320482183214342598410387750360... \times 10^{-12}$$

Thence:

$$\text{Pi} + \ln(-1.005834389530381683148320482183214342598410387 \times 10^{-12})$$

Input interpretation:

$$\pi + \log(-(-1.005834389530381683148320482183214342598410387 \times 10^{-12}))$$

Open code

- $\log(x)$  is the natural logarithm

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Result:

- More digits

$$-24.483611026946235966067286739667167615827087684...$$

This result -24,483 is very near to the value of black hole entropy (see Tables), with minus sign

$$(64+6) * ((\text{Pi} + \ln(-1.005834389530381683148320482183214342598410387 \times 10^{-12})))$$

Input interpretation:

$$(64 + 6) (\pi + \log(-(-1.005834389530381683148320482183214342598410387 \times 10^{-12})))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-1713.8527718862365176247100717767017331078961379\dots$$

This result 1713,852 is very near to the mass of  $f_0(1710)$  candidate glueball

$$32 * ((\text{Pi} + \ln(-1.005834389530381683148320482183214342598410387 \times 10^{-12})))$$

Input interpretation:

$$32 (\pi + \log(-(-1.005834389530381683148320482183214342598410387 \times 10^{-12})))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-783.47555286227955091415317566934936370646680588\dots$$

This value is practically equal to the rest mass of Omega meson with minus sign

$$1/6.626 \ln(-1.005834389530381683148320482183214342598410387 \times 10^{-12}))$$

Input interpretation:

$$\frac{1}{6.626} \log(-(-1.005834389530381683148320482183214342598410387 \times 10^{-12}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Result:

More digits

$$-4.16921\dots$$

This result -4,16921 is in the range of the mass of hypothetical dark matter particles with minus sign

Now:

$$-\frac{-21.79216 \left( \left( 1 + \left( -\frac{-21.79216}{e^{0.5}} \right)^2 \right) + \left( -\frac{-21.79216}{e^{0.5}} \right)^6 \right) + \left( -\frac{-21.79216}{e^{0.5}} \right)^{12}}{e^{0.5}}$$

$$3.7582838213624412933987326486202281989113112090736723... \times 10^{14}$$

$$\left( 1 - \left( -\frac{-21.79216}{e^{0.5}} \right)^8 \right) \left( 1 - \left( -\frac{-21.79216}{e^{0.5}} \right)^{12} \right) \left( 1 - \left( -\frac{-21.79216}{e^{0.5}} \right)^{28} \right)$$

- Fewer digits
- More digits

$$-6.536508342258849896114505619942924476936123331637070... \times 10^{53}$$

$$((-e^{-0.5} * (-21.79216)) (((1 + ((-e^{-0.5} * (-21.79216))^2)) + ((-e^{-0.5} * (-21.79216))^6)) + ((-e^{-0.5} * (-21.79216))^12))) / (-6.53651 * 10^{53})$$

Input interpretation:

$$-\frac{-21.79216 \left( \left( 1 + \left( -\frac{-21.79216}{e^{0.5}} \right)^2 \right) + \left( -\frac{-21.79216}{e^{0.5}} \right)^6 \right) + \left( -\frac{-21.79216}{e^{0.5}} \right)^{12}}{e^{0.5}}$$

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Result:

More digits

$$-5.74968... \times 10^{-40}$$

Result:  $-5,74968 * 10^{-40}$

Or:

$$(3.758283821362441293398732 * 10^{14}) / (-6.5365083422588498961145 * 10^{53})$$

Input interpretation:

$$-\frac{3.758283821362441293398732 \times 10^{14}}{6.5365083422588498961145 \times 10^{53}}$$

[Open code](#)

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Result:

More digits

$$-5.749681059939831189605844527957593202706898451414516... \times 10^{-40}$$

$$(\ln(-5.749681059939831189605844527957593202706898451414516 \times 10^{-40})$$

Input interpretation:

$$\log(-(-5.749681059939831189605844527957593202706898451414516 \times 10^{-40}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-90.3542593343275167200143902227806058724882730681235663\dots$$

Continued fraction:

Linear form

$$\begin{aligned} -90 + & \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-26 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-5 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}} \end{aligned}$$

$$\text{For } k = 52 \text{ for } S = 4\pi\sqrt{k} = 90,617387193111821540$$

Or:

$$1/(\Pi^*(\sqrt{5}+1)/2)) (\ln(-5.749681059939831189605844527 \times 10^{-40})$$

Input interpretation:

$$\frac{1}{\pi \left( \frac{1}{2} (\sqrt{5} + 1) \right)} \log(-(-5.749681059939831189605844527 \times 10^{-40}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-17.7750617137228334461446315225\dots$$

This result -17,775 is very near to the value of black hole entropy (see Tables) with minus sign

$$1.674 \times 5 \ln(-5.74968 \times 10^{-40})$$

$$1.674 \times 5 \log\left(-\left(-\frac{5.74968}{10^{40}}\right)\right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

- Result:  
More digits  
**-756.265...**

This value -756,265 is very near to the rest mass of Charged rho meson with minus sign

- Alternative representations:  
More

$$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = 8.37 \log_e\left(\frac{5.74968}{10^{40}}\right)$$

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$$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = 8.37 \log(a) \log_a\left(\frac{5.74968}{10^{40}}\right)$$

[Open code](#)

$$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = \left(-8.37 \text{Li}_1\left(1 - \frac{5.74968}{10^{40}}\right)\right) = -8.37 \text{Li}_1(1)$$

[Open code](#)

- Series representations:

$$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = 16.74 i \pi \left\lfloor \frac{\arg(5.74968 \times 10^{-40} - x)}{2\pi} \right\rfloor + \\ 8.37 \log(x) - 8.37 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

- [Open code](#)
- 

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$$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = 8.37 \left\lfloor \frac{\arg(5.74968 \times 10^{-40} - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 8.37 \log(z_0) + \\ 8.37 \left\lfloor \frac{\arg(5.74968 \times 10^{-40} - z_0)}{2\pi} \right\rfloor \log(z_0) - 8.37 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - z_0)^k z_0^{-k}}{k}$$

- [Open code](#)
-

- $$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = 16.74 i \pi \left[ -\frac{-\pi + \arg\left(\frac{5.74968 \times 10^{-40}}{z_0}\right) + \arg(z_0)}{2\pi} \right] +$$

$$8.37 \log(z_0) - 8.37 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$1.674 \times 5 \log\left(-\frac{-5.74968}{10^{40}}\right) = 8.37 \int_1^{5.74968 \times 10^{-40}} \frac{1}{t} dt$$

(27-8)  $\ln(-5.74968 * 10^{-40})$

Input interpretation:

$$(27 - 8) \log\left(-\left(-\frac{5.74968}{10^{40}}\right)\right)$$

[Open code](#)

•  $\log(x)$  is the natural logarithm

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Result:

• Fewer digits  
More digits

-1716.73093085482729937796207726050240794490191524210475227...

This result -1716,73 is very near to the mass of  $f_0(1710)$  candidate glueball, with minus sign

Alternative representations:

More

- $$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = 19 \log_e\left(\frac{5.74968}{10^{40}}\right)$$

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$$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = 19 \log(a) \log_a\left(\frac{5.74968}{10^{40}}\right)$$

[Open code](#)

$$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = \left(-19 \operatorname{Li}_1\left(1 - \frac{5.74968}{10^{40}}\right)\right) = -19 \operatorname{Li}_1(1)$$

[Open code](#)

Series representations:

$$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = 38 i \pi \left\lfloor \frac{\arg(5.74968 \times 10^{-40} - x)}{2 \pi} \right\rfloor +$$
$$19 \log(x) - 19 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

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$$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = 19 \left\lfloor \frac{\arg(5.74968 \times 10^{-40} - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 19 \log(z_0) +$$
$$19 \left\lfloor \frac{\arg(5.74968 \times 10^{-40} - z_0)}{2 \pi} \right\rfloor \log(z_0) - 19 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

$$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = 38 i \pi \left\lfloor -\frac{-\pi + \arg\left(\frac{5.74968 \times 10^{-40}}{z_0}\right) + \arg(z_0)}{2 \pi} \right\rfloor +$$
$$19 \log(z_0) - 19 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

Integral representation:

$$(27 - 8) \log\left(-\frac{-5.74968}{10^{40}}\right) = 19 \int_1^{5.74968 \times 10^{-40}} \frac{1}{t} dt$$

[Open code](#)

Continued fraction:

Linear form

$$\bullet$$
$$-1716 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-8 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-42 + \cfrac{1}{-1 + \cfrac{1}{...}}}}}}}}}}}}}}$$

[Open code](#)

$$1/21.676 \ln(-5.74968 * 10^{-40})$$

Input interpretation:

$$\frac{1}{21.676} \log\left(-\left(-\frac{5.74968}{10^{40}}\right)\right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

-4.16840...

This result -4,16840 is in the range of the mass of hypothetical dark matter particles with minus sign

Alternative representations:

More

$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = \frac{\log_e\left(\frac{5.74968}{10^{40}}\right)}{21.676}$$

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$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = \frac{\log(a) \log_a\left(\frac{5.74968}{10^{40}}\right)}{21.676}$$

[Open code](#)

$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = \left( -\frac{\text{Li}_1\left(1 - \frac{5.74968}{10^{40}}\right)}{21.676} = -0.046134 \text{Li}_1(1) \right)$$

[Open code](#)

Series representations:

$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = 0.0922679 i \pi \left[ \frac{\arg(5.74968 \times 10^{-40} - x)}{2 \pi} \right] + 0.046134 \log(x) - 0.046134 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

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$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = 0.046134 \left[ \frac{\arg(5.74968 \times 10^{-40} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$0.046134 \log(z_0) + 0.046134 \left[ \frac{\arg(5.74968 \times 10^{-40} - z_0)}{2\pi} \right] \log(z_0) -$$

$$0.046134 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = 0.0922679 i\pi \left[ -\frac{-\pi + \arg\left(\frac{5.74968 \times 10^{-40}}{z_0}\right) + \arg(z_0)}{2\pi} \right] +$$

$$0.046134 \log(z_0) - 0.046134 \sum_{k=1}^{\infty} \frac{(-1)^k (5.74968 \times 10^{-40} - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

Integral representation:

$$\frac{\log\left(-\frac{-5.74968}{10^{40}}\right)}{21.676} = 0.046134 \int_1^{5.74968 \times 10^{-40}} \frac{1}{t} dt$$

[Open code](#)

Now, from the two results, we have:

$$(-5.749681059939831189605844527 * 10^{-40}) * 1 / (-1.005834389530381683148320482183214342598410387 * 10^{-12})$$

Input interpretation:

$$-\frac{5.749681059939831189605844527}{10^{40}} \left( -\frac{1}{1.005834389530381683148320482183214342598410387 \times 10^{-12}} \right)$$

[Open code](#)

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Result:

More digits

$$5.7163297654043468851807601732326407912784224112699857... \times 10^{-28}$$

[Open code](#)

$$1/0.347835 * (-5.749681059939831189605844527 * 10^{-40}) * 1/(-1.005834389530381683148320482183214342598410387 \times 10^{-12})$$

Input interpretation:

$$\frac{1}{0.347835} \left( -\frac{5.749681059939831189605844527}{10^{40}} \right) \\ \left( -\frac{1}{1.005834389530381683148320482183214342598410387 \times 10^{-12}} \right)$$

[Open code](#)

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Result:

More digits

$$1.6434026953596811376603160042067764288465572502105842... \times 10^{-27}$$

[Open code](#)

Value very near to the mass of Proton

Where  $0.347835\dots$  is a value of special continued fraction  $1/\sqrt{2} - 1/\sqrt{3} + 1/\sqrt{5} - 1/\sqrt{7} + 1/\sqrt{11} - 1/\sqrt{13} + 1/\sqrt{17} - \dots$

$$1/(\sqrt{27}) \ln(1.6434026953596811376603160042067764288465572502105842 \times 10^{-27})$$

Input interpretation:

$$\frac{1}{\sqrt{27}} \log(1.6434026953596811376603160042067764288465572502105842 \times 10^{-27})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-11.86897988879412041807006094440142026101885469876679117\dots$$

This result  $-11.868$  is very near to the value of black hole entropy (see Tables) with minus sign

Continued fraction:

Linear form

$$-11 + \cfrac{1}{-1 + \cfrac{1}{-6 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{-2 + \cfrac{1}{\dots}}}}}}}}}}}}}}$$

[Open code](#)

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$$(8+21)*5/(sqrt(27)) \ln$$

$$(1.6434026953596811376603160042067764288465572502105842 \times 10^{-27})$$

Input interpretation:

$$(8 + 21) \times \frac{5}{\sqrt{27}}$$

$$\log(1.6434026953596811376603160042067764288465572502105842 \times 10^{-27})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-1721.002083875147460620158836938205937847733931321184720\dots$$

This result -1721,002 is very near to the mass of  $f_0(1710)$  candidate glueball, with minus sign

Continued fraction:

Linear form

$$-1721 + \cfrac{1}{-479 + \cfrac{1}{-1 + \cfrac{1}{-7 + \cfrac{1}{-78 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{-14 + \cfrac{1}{-1 + \cfrac{1}{-8 + \cfrac{1}{-3 + \cfrac{1}{-1 + \cfrac{1}{-1 + \frac{1}{\dots}}}}}}}}}}}}}$$

[Open code](#)

$$4\pi * \ln(1.6434026953596811376603160042067764288465572502105842 \times 10^{-27})$$

Input interpretation:

$$4\pi \log(1.6434026953596811376603160042067764288465572502105842 \times 10^{-27})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-775.0061343505257587831732946365737661640479828966709754\dots$$

This value -775,006 is practically equal to the rest mass of Charged rho meson with minus sign

Continued fraction:

Linear form

$$-775 + \cfrac{1}{-163 + \cfrac{1}{-60 + \cfrac{1}{-1 + \cfrac{1}{-4 + \cfrac{1}{-2 + \cfrac{1}{-24 + \cfrac{1}{-1 + \cfrac{1}{-8 + \cfrac{1}{-16 + \cfrac{1}{-1 + \cfrac{1}{-11 + \cfrac{1}{-1 + \cfrac{1}{-2 + \cfrac{1}{\dots}}}}}}}}}}}}$$

Now, we have:

Mock  $\vartheta$ -functions (of 7th order)

$$\begin{aligned}
 \text{(i)} \quad & 1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\
 \text{(ii)} \quad & \frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)} + \dots \\
 \text{(iii)} \quad & \frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)} + \dots
 \end{aligned}$$

From the (i), we have:

$$0.9239078 + 0.000433255 + (- \\ 1.8754140254243246404383299476354805043847163776 \times 10^{-7})$$

Input interpretation:

$$0.9239078 + 0.000433255 - \\ 1.8754140254243246404383299476354805043847163776 \times 10^{-7}$$

[Open code](#)

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Result:

$$0.92434086745859745756753595616700523645194956152836224$$

[Open code](#)

The result is

$$0.92434086745859745756753595616700523645194956152836224$$

Repeating decimal:

$$0.924340867458597457567535956167005236451949561528362240$$

Continued fraction:

Linear form

$$\begin{array}{c}
 & & 1 \\
 & & \hline
 1 + & \frac{1}{12 + } & \frac{1}{1} \\
 & 4 + & \frac{1}{1 + } \frac{1}{1} \\
 & & 1 + \frac{1}{1 + } \frac{1}{1} \\
 & & 8 + \frac{1}{7 + } \frac{1}{1 + } \frac{1}{1} \\
 & & 49 + \frac{1}{5 + } \frac{1}{1 + } \frac{1}{3 + } \frac{1}{2 + } \frac{1}{5 + } \frac{1}{1 + } \frac{1}{3 + } \dots
 \end{array}$$

$2\pi * ((\exp(0.92434086745859745756753595616700523645194956152836224)))$

Input interpretation:

$2\pi \exp(0.92434086745859745756753595616700523645194956152836224)$

[Open code](#)

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Result:

More digits

- 15.834924845326022882101195810364752953481611459791164...

This result 15,8349 is very near to the value of black hole entropy (see Tables)

[Continued fraction:](#)

[Linear form](#)

$$15 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{17 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{15 + \cfrac{1}{138 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{61 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{26 + \cfrac{1}{1 + \cfrac{1}{...}}}}}}}}}}}}}}}}$$

$2\pi * (27*4) ((\exp(0.92434086745859745756753595616700523645194956152836224)))$

[Input interpretation:](#)

$2\pi (27 \times 4) \exp(0.92434086745859745756753595616700523645194956152836224)$

[Open code](#)

[Enlarge Data](#) [Customize A](#) Plaintext [Interactive](#)

[Result:](#)

[More digits](#)

1710.1718832952104712669291475193933189760140376574457...

This result 1710,1718 is very near to the mass of  $f_0(1710)$  candidate glueball

$16\pi + 288 * ((\exp(0.92434086745859745756753595616700523645194956152836224)))$

[Input interpretation:](#)

$16\pi + 288 \exp(0.92434086745859745756753595616700523645194956152836224)$

[Open code](#)

[Enlarge Data](#) [Customize A](#) Plaintext [Interactive](#)

[Result:](#)

[More digits](#)

776.08497249265987625649272094034356243641594928890348...

This value 776,084 is practically equal to the rest mass of Charged rho meson

$1.6449 ((\exp(0.92434086745859745756753595616700523645194956152836224)))$

[Input interpretation:](#)

1.6449 exp(0.92434086745859745756753595616700523645194956152836224)  
[Open code](#)

[Enlarge](#) [Data](#) [Customize](#) A [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

4.14549...

This result 4,14549 is in the range of the mass of hypothetical dark matter particles

[Continued fraction:](#)

Linear form

$$4 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{13 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{19 + \cfrac{1}{1 + \cfrac{1}{100 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}$$

[Open code](#)

$(8\pi) * (((0.92434086745859745756753595616700523645194956152836224)))$

[Input interpretation:](#)

$(8\pi) \times 0.92434086745859745756753595616700523645194956152836224$

[Open code](#)

[Enlarge](#) [Data](#) [Customize](#) A [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

23.231219828965972384323403382955795246074975012553105...

This result 23,2312 is very near to the value of black hole entropy (see Tables)

$(576\pi) * (((0.92434086745859745756753595616700523645194956152836224)))$

Input interpretation:

$$(576 \pi) \times 0.92434086745859745756753595616700523645194956152836224$$

[Open code](#)

[Enlarge](#) [Data](#) [Customize](#) A [Plaintext](#) [Interactive](#)

Result:

More digits

$$1672.6478276855500116712850435728172577173982009038236\dots$$

This result 1672,6478 is practically equal to the value of the rest mass of Omega baryon  $1672.45 \pm 0.29$

From the (ii), we have:

$$-1.081849047367565973116419938674252971482398018961922 +$$

$$0.0761251367814440464022202749466671971676215118725857$$

$$-0.000433255719961759072744149660169833646052283127278$$

Input interpretation:

$$-1.081849047367565973116419938674252971482398018961922 +$$

$$0.0761251367814440464022202749466671971676215118725857 -$$

$$0.000433255719961759072744149660169833646052283127278$$

[Open code](#)

Result:

$$-1.0061571663060836857869438133877556079608287902166143$$

The result is -1.0061571663...

$$-1.0061571663060836857869438133877556079608287902166143$$

[Open code](#)

Continued fraction:

Linear form

$$\begin{aligned}
 & -1 + \frac{1}{-162 + \frac{1}{-2 + \frac{1}{-2 + \frac{1}{-2 + \frac{1}{-1 + \frac{1}{-4 + \frac{1}{-1 + \frac{1}{-30 + \frac{1}{-1 + \frac{1}{-2 + \frac{1}{-3 + \frac{1}{-1 + \frac{1}{-5 + \frac{1}{-2 + \frac{1}{\dots}}}}}}}}}}}}}}}
 \end{aligned}$$

$4\pi * 10^4 \ln(-1.00615716630608368578694381338775560796082879021661439)$

Input interpretation:

$$4\pi \times 10^4$$

$$\log(-(-1.00615716630608368578694381338775560796082879021661439))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Result:

More digits

$$771.3600706731711996344212483263107355399393482535669\dots$$

This value 771,36 is very near to the rest mass of Charged rho meson

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Series representations:

More

$$4\pi 10^4 \log(-(-1))$$

$$1.006157166306083685786943813387755607960828790216614390000 =$$

$$-40000$$

$$\sum_{k=1}^{\infty} \frac{(-0.006157166306083685786943813387755607960828790216614390000)^k}{k}$$

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$$\begin{aligned}
& 4\pi 10^4 \log(-(-1)) \\
& = 1.006157166306083685786943813387755607960828790216614390000 \\
& = 80\,000 i \pi^2 \left| \frac{1}{2\pi} \arg( \right. \\
& \quad \left. 1.0061571663060836857869438133877556079608287902166143900 \cdot \right. \\
& \quad \left. 00 - x) \right| + 40\,000 \pi \log(x) - 40\,000 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \\
& = (1.0061571663060836857869438133877556079608287902166143900 \cdot \\
& \quad \left. 00 - x)^k x^{-k} \text{ for } x < 0 \right)
\end{aligned}$$

[Open code](#)

$$\begin{aligned}
& 4\pi 10^4 \log(-(-1)) \\
& = 1.006157166306083685786943813387755607960828790216614390000 = \\
& = 80\,000 i \pi^2 \left| -\frac{1}{2\pi} (-\pi + \arg( \right. \\
& \quad \left. 1.0061571663060836857869438133877556079608287902166143 \cdot \right. \\
& \quad \left. 90000/z_0 + \arg(z_0))) \right| + \\
& = 40\,000 \pi \log(z_0) - 40\,000 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \\
& = (1.006157166306083685786943813387755607960828790216614390000 \\
& \quad \left. - z_0)^k z_0^{-k} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& 4\pi 10^4 \log(-(-1)) \\
& = 1.006157166306083685786943813387755607960828790216614390000 = \\
& = 40\,000 \pi \int_1^{1.006157166306083685786943813387755607960828790216614390000} \frac{1}{t} dt
\end{aligned}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

$$\begin{aligned}
& 4\pi 10^4 \log(-(-1)) \\
& = \frac{1.006157166306083685786943813387755607960828790216614390000}{20\,000} \\
& = \frac{i}{\Gamma(1-s)} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{5.0901386225223366536389003362428466688609619239262995361s}}{\Gamma(s)} \frac{\Gamma(-s)^2}{\Gamma(1+s)} s ds \text{ for } -1 < \gamma < 0
\end{aligned}$$

$$9\text{Pi} * 10^4 \ln(-(-1.00615716630608368578694381338775560796082879021661439))$$

Input interpretation:

$$9\pi \times 10^4$$

$$\log(-(-1.00615716630608368578694381338775560796082879021661439))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

1735.560159014635199177447808734199154964863533570525...

This result 1735.56 is very near to the mass of  $f_0(1710)$  candidate glueball

Series representations:

More

$$9\pi 10^4 \log(-1)$$

$$1.006157166306083685786943813387755607960828790216614390000) =$$

-90000

$$\sum_{k=1}^{\infty} \frac{(-0.006157166306083685786943813387755607960828790216614390000)^k}{k}$$

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$$9\pi 10^4 \log(-1)$$

$$1.006157166306083685786943813387755607960828790216614390000)$$

$$= 180000 i \pi^2 \left[ \frac{1}{2\pi} \arg($$

$$1.00615716630608368578694381338775560796082879021661439000)$$

$$00 - x) \Big] + 90000 \pi \log(x) - 90000 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k$$

$$(1.00615716630608368578694381338775560796082879021661439000)$$

$$00 - x)^k x^{-k} \text{ for } x < 0$$

[Open code](#)

$$9\pi 10^4 \log(-(-1))$$

$$1.006157166306083685786943813387755607960828790216614390000) =$$

$$180000 i \pi^2 \left[ -\frac{1}{2\pi} (-\pi + \arg($$

$$1.006157166306083685786943813387755607960828790216614390000)$$

$$90000/z_0) + \arg(z_0)) \Big] +$$

$$90000 \pi \log(z_0) - 90000 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k$$

$$(1.006157166306083685786943813387755607960828790216614390000)$$

$$-z_0)^k z_0^{-k}$$

Integral representations:

$$9\pi 10^4 \log(-(-1) \\ 1.006157166306083685786943813387755607960828790216614390000) = \\ 90000\pi \int_1^{1.006157166306083685786943813387755607960828790216614390000} \frac{1}{t} dt$$

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$$9\pi 10^4 \log(-(-1) \\ 1.006157166306083685786943813387755607960828790216614390000) \\ = \frac{45000}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{5.0901386225223366536389003362428466688609619239262995361s}}{\Gamma(1-s)} \Gamma(-s)^2 ds \\ \text{for } -1 < \gamma < 0$$

$1/144 * 9\text{Pi} * 10^4 \ln -(-$

$1.00615716630608368578694381338775560796082879021661439)$

Input interpretation:

$$\frac{1}{144} \times 9(\pi \times 10^4) \\ \log(-(-1.00615716630608368578694381338775560796082879021661439))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

- More digits

$12.05250110426829999428783200509860524281155231646198\dots$

This result 12,0525 is very near to the value of black hole entropy (see Tables

Series representations:

More

- $\frac{1}{144} (9 \log(-(-1.006157166306083685786943813387755607960828790216614390000)) \pi 10^4 = -625 \pi$

$$\sum_{k=1}^{\infty} \frac{(-0.006157166306083685786943813387755607960828790216614390000)^k}{k}$$

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$$\frac{1}{144} (9 \log(-(-1.0061571663060836857869438133877556079608287902166143900))) \pi 10^4 = 1250 i \pi^2 \left[ \frac{1}{2\pi} \arg(1.0061571663060836857869438133877556079608287902166143900) - 00 - x \right] + 625 \pi \log(x) - 625 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (1.0061571663060836857869438133877556079608287902166143900) - 00 - x)^k x^{-k} \text{ for } x < 0$$

[Open code](#)

$$\frac{1}{144} (9 \log(-(-1.0061571663060836857869438133877556079608287902166143900))) \pi 10^4 = 1250 i \pi^2 \left[ -\frac{1}{2\pi} (-\pi + \arg(1.0061571663060836857869438133877556079608287902166143900) - 90000/z_0 + \arg(z_0)) \right] + 625 \pi \log(z_0) - 625 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (1.006157166306083685786943813387755607960828790216614390000 - z_0)^k z_0^{-k}$$

Integral representations:

$$\frac{1}{144} (9 \log(-(-1.0061571663060836857869438133877556079608287902166143900))) \pi 10^4 = 625 \pi \int_1^{1.006157166306083685786943813387755607960828790216614390000} \frac{1}{t} dt$$

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$$\frac{1}{144} (9 \log(-(-1.0061571663060836857869438133877556079608287902166143900))) \pi 10^4 = \frac{625}{2i} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{5.0901386225223366536389003362428466688609619239262995361s}}{\Gamma(1-s)} \frac{\Gamma(-s)^2}{\Gamma(1+s)} ds \text{ for } -1 < \gamma < 0$$

$$\frac{1}{72} * 9\text{Pi} * 10^4 \ln -(-1.00615716630608368578694381338775560796082879021661439)$$

Input interpretation:

$$\frac{1}{72} \times 9 (\pi \times 10^4)$$

$$\log(-(-1.00615716630608368578694381338775560796082879021661439))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$24.10500220853659998857566401019721048562310463292396\dots$$

This result 24,105 is very near to the value of black hole entropy (see Tables) and equal to 24, the physical degrees of freedom of the bosonic string, that are the 24 transverse coordinates.

Series representations:

More

$$\begin{aligned} & \frac{1}{72} (9 \log(-(-1.006157166306083685786943813387755607960828790216614390000))) \pi 10^4 = -1250 \pi \\ & \sum_{k=1}^{\infty} \frac{(-0.006157166306083685786943813387755607960828790216614390000)^k}{k} \end{aligned}$$

[Open code](#)

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$$\begin{aligned} & \frac{1}{72} (9 \log(-(-1.006157166306083685786943813387755607960828790216614390000))) \pi 10^4 = 2500 i \pi^2 \left[ \frac{1}{2\pi} \arg(1.006157166306083685786943813387755607960828790216614390000) - x \right] + 1250 \pi \log(x) - 1250 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (1.00615716630608368578694381338775560796082879021661439000000 - x^k) x^{-k} \quad \text{for } x < 0 \end{aligned}$$

[Open code](#)

$$\begin{aligned}
& \frac{1}{72} (9 \log(-(-1.006157166306083685786943813387755607960828790216614390 \cdot 10^4)) = \\
& 2500 i \pi^2 \left[ -\frac{1}{2\pi} (-\pi + \arg(1.00615716630608368578694381338775560796082879021661439 \cdot 10^4) \right. \\
& \left. + 90000/z_0 + \arg(z_0)) \right] + \\
& 1250 \pi \log(z_0) - 1250 \pi \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \\
& (1.006157166306083685786943813387755607960828790216614390000 \\
& - z_0)^k z_0^{-k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{1}{72} (9 \log(-(-1.006157166306083685786943813387755607960828790216614390 \cdot 10^4)) = \\
& 1250 \pi \int_1^{1.006157166306083685786943813387755607960828790216614390000} \frac{1}{t} dt
\end{aligned}$$

[Open code](#)

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$$\begin{aligned}
& \frac{1}{72} (9 \log(-(-1.00615716630608368578694381338775560796082879021661439 \cdot 10^4)) = \\
& 0000))) \pi 10^4 = \frac{625}{i} \\
& \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{5.0901386225223366536389003362428466688609619239262995361 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \\
& ds \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

$$\begin{aligned}
& 1/(142*((sqrt(5)+1)/2))^2)) * (8\pi) * 10^4 \ln(- \\
& 1.00615716630608368578694381338775560796082879021661439)
\end{aligned}$$

Input interpretation:

$$\frac{1}{142 \left(\frac{1}{2} (\sqrt{5} + 1)\right)^2} (8 \pi) \times 10^4 \log(-(-1.00615716630608368578694381338775560796082879021661439))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

4.149765203276484724071530319835938309107282758224328...

This result 4,14976 is in the range of the mass of hypothetical dark matter particles

Series representations:

More

$$\frac{1}{142 \left(\frac{1}{2} (\sqrt{5} + 1)\right)^2} (10^4 \log(-(-1.006157166306083685786943813387755607960828790216614390 \cdot 000))) 8 \pi = -\frac{160000 \pi \sum_{k=1}^{\infty} \frac{(-0.006157166306083685786943813387755607960828790216614390000)^k}{k}}{71 \left(1 + \sqrt{4 \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}\right)^2}$$

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$$\frac{1}{142 \left(\frac{1}{2} (\sqrt{5} + 1)\right)^2} (10^4 \log(-(-1.006157166306083685786943813387755607960828790216614390 \cdot 000))) 8 \pi = -\frac{160000 \pi \sum_{k=1}^{\infty} \frac{(-0.006157166306083685786943813387755607960828790216614390000)^k}{k}}{71 \left(1 + \sqrt{4 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)^2}$$

[Open code](#)

$$\frac{1}{142 \left(\frac{1}{2} (\sqrt{5} + 1)\right)^2} (10^4 \log(-(-1.006157166306083685786943813387755607960828790216614390 \cdot 000))) 8 \pi = (160000 \pi \log(1.0061571663060836857869438133877556079608287902166143900 \cdot 00)) / \left(71 \left(1 + \exp\left(i \pi \left[\frac{\arg(5-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^2\right)$$

for ( $x \in \mathbb{R}$  and  $x < 0$ )

Integral representations:

$$\frac{1}{142 \left(\frac{1}{2} (\sqrt{5} + 1)\right)^2} (10^4 \log(-(-1.006157166306083685786943813387755607960828790216614390 \cdot 000))) 8 \pi = \frac{160\,000 \pi}{71 (1 + \sqrt{5})^2}$$

$$\int_1^{1.006157166306083685786943813387755607960828790216614390000} \frac{1}{t} dt$$

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$$\frac{1}{142 \left(\frac{1}{2} (\sqrt{5} + 1)\right)^2} (10^4 \log(-(-1.00615716630608368578694381338775560796082879021661439 \cdot 000))) 8 \pi = \frac{80\,000}{71 i (1 + \sqrt{5})^2}$$

$$\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{5.0901386225223366536389003362428466688609619239262995361 s} \Gamma(-s)^2 \Gamma(1 + s)}{\Gamma(1 - s)} ds \quad \text{for } -1 < \gamma < 0$$

From the (iii), we have:

-0.081849047367565973116419938674252971482398018961922

0.0004357345630640457140757853070834281049705616972466

-1.8762261787851325482986508127679968797519452065 × 10^-7

-0.081849047367565973116419938674252971482398018961922 +

0.0004357345630640457140757853070834281049705616972466 -

1.8762261787851325482986508127679968797519452065 × 10^-7

-0.08141350042711980591559898323225082017711543245919605

The result is:

-0.08141350042711980591559898323225082017711543245919605

Continued fraction:  
Linear form

$$\begin{array}{r}
 & & 1 \\
 & & \hline
 -12 + & & 1 \\
 & -3 + & & 1 \\
 & & -1 + & & 1 \\
 & & & -1 + & & 1 \\
 & & & & -6 + & & 1 \\
 & & & & & -1 + & & 1 \\
 & & & & & & -7 + & & 1 \\
 & & & & & & & -3 + & & 1 \\
 & & & & & & & & -1 + & & 1 \\
 & & & & & & & & & -2 + & & 1 \\
 & & & & & & & & & & -1 + & & 1 \\
 & & & & & & & & & & & -21 + & & 1 \\
 & & & & & & & & & & & & -1 + & & 1 \\
 & & & & & & & & & & & & & -1 + & & 1 \\
 & & & & & & & & & & & & & & ... \\
 \end{array}$$

$$5 \ln(-0.081849047367565973116419938674252971482398018961922+ \\
 0.0004357345630640457140757853070834281049705616972466 - \\
 1.8762261787851325482986508127679968797519452065 \times 10^{-7})$$

Input interpretation:

$$5 \log(-(-0.081849047367565973116419938674252971482398018961922 + \\
 0.0004357345630640457140757853070834281049705616972466 - \\
 1.8762261787851325482986508127679968797519452065 \times 10^{-7}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$-12.541070834063482807190784297649523845515152785009\dots$$

This result -12,541 is very near to the value of black hole entropy (see Tables) with minus sign

$$137 * 5 \ln(-0.081849047367565973116419938674252971482398018961922+ \\
 0.0004357345630640457140757853070834281049705616972466 - \\
 1.8762261787851325482986508127679968797519452065 \times 10^{-7})$$

Input interpretation:

$$137 \times 5 \log(-(-0.081849047367565973116419938674252971482398018961922 + \\
 0.0004357345630640457140757853070834281049705616972466 - \\
 1.8762261787851325482986508127679968797519452065 \times 10^{-7}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

-1718.1267042666971445851374487779847668355759315462...

This result -1718,126 is very near to the mass of  $f_0(1710)$  candidate glueball with minus sign

$$(54+8) * 5 \ln(-0.081849047367565973116419938674252971482398018961922 + 0.0004357345630640457140757853070834281049705616972466 - 1.8762261787851325482986508127679968797519452065 \times 10^{-7})$$

Input interpretation:

$$(54 + 8) \times 5 \\ \log(-(-0.081849047367565973116419938674252971482398018961922 + 0.0004357345630640457140757853070834281049705616972466 - 1.8762261787851325482986508127679968797519452065 \times 10^{-7}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

-777.54639171193593404582862645427047842193947267053...

This value -777,546 is very near to the rest mass of Charged rho meson with minus sign

$$((\sqrt{5})+1)/2 - \ln(-0.081849047367565973116419938674252971482398018961922 + 0.0004357345630640457140757853070834281049705616972466 - 1.8762261787851325482986508127679968797519452065 \times 10^{-7})$$

Input interpretation:

$$\frac{1}{2} (\sqrt{5} + 1) - \log(-(-0.081849047367565973116419938674252971482398018961922 + 0.0004357345630640457140757853070834281049705616972466 - 1.8762261787851325482986508127679968797519452065 \times 10^{-7}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

- Result:  
More digits  
4.12624815556259140964274369389554288682333973680747...

This result 4,12624 is in the range of the mass of hypothetical dark matter particles

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$$\begin{aligned} & 1/(-0.081849047367565973116419938674252971482398018961922+ \\ & 0.0004357345630640457140757853070834281049705616972466 - \\ & 1.8762261787851325482986508127679968797519452065 \times 10^{-7}) \end{aligned}$$

Input interpretation:  
 $1/(-0.081849047367565973116419938674252971482398018961922 +$   
 $0.0004357345630640457140757853070834281049705616972466 -$   
 $1.8762261787851325482986508127679968797519452065 \times 10^{-7})$

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- Result:  
More digits  
-12.2829751178084476360250362583088142452273794012236292807...

[Open code](#)

This result -12,2829 is very near to the value of black hole entropy (see Tables) with minus sign

$$((-1/(-0.081849047367565973116419938674252971482398018961922+ \\ 0.0004357345630640457140757853070834281049705616972466 - \\ 1.8762261787851325482986508127679968797519452065 \times 10^{-7}))^{1/5}$$

Input interpretation:  
 $(-1/(-0.081849047367565973116419938674252971482398018961922 +$   
 $0.0004357345630640457140757853070834281049705616972466 -$   
 $1.8762261787851325482986508127679968797519452065 \times$   
 $10^{-7}))^{1/5}$

[Open code](#)

- Result:  
More digits  
1.65143207109614550635944059863711361652099172351972...

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We take all the three results (i), (ii) and (iii) and multiply:

Input interpretation:

$$0.92434086745859745756753595616700523645194956152836224 \times \\ (-1.0061571663060836857869438133877556079608287902166143) \times \\ (-0.08141350042711980591559898323225082017711543245919605)$$

[Open code](#)

Result:

More digits

$$0.075717175927080104333280313003357627560491481124352895669\dots$$

[Open code](#)

Repeating decimal:

More digits

$$0.075717175927080104333280313003357627560491481124352895669\dots$$

Now:

Input interpretation:

$$1.772453 / (0.92434086745859745756753595616700523645194956152836224 \times \\ (-1.0061571663060836857869438133877556079608287902166143) \times \\ (-0.08141350042711980591559898323225082017711543245919605))$$

[Open code](#)

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Result:

More digits

$$23.40886302609822439151210039876532117647838546703822548593\dots$$

[Open code](#)

This result 23,4088 is very near to the value of black hole entropy (see Tables)

Input interpretation:

$$-(0.945308 / (0.92434086745859745756753595616700523645194956152836224 \times \\ (-1.0061571663060836857869438133877556079608287902166143) \times \\ (-0.08141350042711980591559898323225082017711543245919605)))$$

[Open code](#)

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Result:

More digits

$$-12.4847234253742470480692693141967929365091371162309922224\dots$$

[Open code](#)

This result -12,4847 is very near to the value of black hole entropy (see Tables) with minus sign

Input interpretation:

$$2.363271 / (0.92434086745859745756753595616700523645194956152836224 \times (-1.0061571663060836857869438133877556079608287902166143) \times (-0.08141350042711980591559898323225082017711543245919605))$$

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Result:

More digits

$$31.21182177047863997293761415478465118232034953878771069381\dots$$

[Open code](#)

This result 31,2118 is very near to the value of black hole entropy (see Tables)

Input interpretation:

$$1.329340 / (0.92434086745859745756753595616700523645194956152836224 \times (-1.0061571663060836857869438133877556079608287902166143) \times (-0.08141350042711980591559898323225082017711543245919605))$$

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Result:

More digits

$$17.55665057133442388182518551639715809262066578733122664887\dots$$

[Open code](#)

This result 17,5566 is very near to the value of black hole entropy (see Tables)

From the sum of all results that we have obtained, we have:

$$\text{Pi}/6 \ln (( -4267.24 + (6.5960861587 * 10^{20}) - (1.0058343895 * 10^{-12}) - (5.74968 * 10^{-40}) - 0.163229799274606034135006840453001191685994661147448110))$$

Input interpretation:

$$\frac{\pi}{6} \log(-4267.24 + 6.5960861587 \times 10^{20} - 1.0058343895 \times 10^{-12} - 5.74968 \times 10^{-40} - 0.163229799274606034135006840453001191685994661147448110)$$

[Open code](#)

•  $\log(x)$  is the natural logarithm

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Result:

More digits

$$25.10037147630\dots$$

This result 25,10 is very near to the value of black hole entropy (see Tables)

Multiplying all results obtained, we have:

$$1/((5+\sqrt{5})/2))) \ln -(( -4267.24*(6.5960861587*10^{20})*(1.0058343895*10^{-12})*(5.74968*10^{-40})*0.163229799274606034135006840453001191685994661147448110))$$

Input interpretation:

$$\frac{1}{\frac{1}{2} (5 + \sqrt{5})} \log(-(-4267.24 \times 6.5960861587 \times 10^{20} \times 1.0058343895 \times 10^{-12} \times 5.74968 \times 10^{-40} \times 0.163229799274606034135006840453001191685994661147448110))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

-17.549630...

This result -17,5496 is very near to the value of black hole entropy (see Tables) with minus sign

We have (pag.4 paper):

Mock  $\vartheta$ -functions (of 5th order)

$$\begin{aligned} f(q) &= 1 + \frac{q^2}{1+q} + \frac{q^6}{(1+q)(1+q^2)} + \frac{q^{12}}{(1+q)(1+q^2)(1+q^3)} + \dots \\ \phi(q) &= q + q^4(1+q) + q^9(1+q)(1+q^3) + \dots \\ \psi(q) &= 1 + q(1+q) + q^3(1+q)(1+q^2) + q^6(1+q)(1+q^2)(1+q^3) + \dots \end{aligned}$$

$$f(q) = (13.2879 + 2134.55 - (4.93121 \times 10^6))$$

Input interpretation:

$$13.2879 + 2134.55 - 4.93121 \times 10^6$$

[Open code](#)

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Result:

$-4.9290621621 \times 10^6$

The result is  $-4.9290621621 \times 10^6$

Input interpretation:

$$\log(-(13.2879 + 2134.55 - 4.93121 \times 10^6))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Result:  
Fewer digits  
More digits

15.41065929711133801879098175296564648543508664754251474353...

This result 15,41 is very near to the value of black hole entropy (see Tables)

$$\varphi(q) = (4.33962 \times 10^5) + (4.04437 \times 10^{14})$$

Input interpretation:

$$4.33962 \times 10^5 + 4.04437 \times 10^{14}$$

[Open code](#)

Result:  
**404437000433962**

Scientific notation:  
 $4.04437000433962 \times 10^{14}$

The result is:  $4.04437 \times 10^{14}$

[Open code](#)

Input:

$$\frac{1}{\sqrt{8}} \log(404437000433962)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Exact result:

$$\frac{\log(404437000433962)}{2\sqrt{2}}$$

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Decimal approximation:

More digits

11.89124400599553129874872846872635917675038539612929535882...

[Open code](#)

This result 11,8912 is very near to the value of black hole entropy (see Tables)

Property:

$$\frac{\log(404437000433962)}{2\sqrt{2}}$$
 is a transcendental number

Continued fraction:

Linear form

$$11 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{58 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{6 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{22 + \cfrac{1}{44 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

Series representations:

More

$$\frac{\log(404437000433962)}{\sqrt{8}} = \frac{\log(404437000433961)}{2\sqrt{2}} - \frac{\sum_{k=1}^{\infty} \left(-\frac{1}{404437000433961}\right)^k}{2\sqrt{2}}$$

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$$\frac{\log(404437000433962)}{\sqrt{8}} = \frac{i\pi \left\lfloor \frac{\arg(404437000433962-x)}{2\pi} \right\rfloor}{\sqrt{2}} + \frac{\log(x)}{2\sqrt{2}} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (404437000433962-x)^k x^{-k}}{k}}{2\sqrt{2}} \quad \text{for } x < 0$$

[Open code](#)

$$\frac{\log(404437000433962)}{\sqrt{8}} = \frac{1}{2\sqrt{2}} \\ \left( \log(z_0) + \left\lfloor \frac{\arg(404437000433962-z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (404437000433962-z_0)^k z_0^{-k}}{k} \right)$$

Integral representations:

$$\frac{\log(404437000433962)}{\sqrt{8}} = \frac{1}{2\sqrt{2}} \int_1^{404437000433962} \frac{1}{t} dt$$

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$$\frac{\log(404437000433962)}{\sqrt{8}} = -\frac{i}{4\sqrt{2}\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{404437000433961^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

$$144 * 1/(\text{sqrt8}) \ln(404437000433962)$$

Input:

$$144 \times \frac{1}{\sqrt{8}} \log(404437000433962)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Exact result:

$$36\sqrt{2} \log(404437000433962)$$

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Decimal approximation:

More digits

$$1712.339136863356507019816899496595721452055497042618531671\dots$$

This result 1712,339 is very near to the mass of  $f_0(1710)$  candidate glueball

Property:

$$36\sqrt{2} \log(404437000433962) \text{ is a transcendental number}$$

[Open code](#)

Alternate forms:

$$36\sqrt{2} (\log(2) + \log(139) + \log(809) + \log(9883) + \log(181957))$$

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$$36\sqrt{2} \log(2) + 36\sqrt{2} \log(139) +$$

$$36\sqrt{2} \log(809) + 36\sqrt{2} \log(9883) + 36\sqrt{2} \log(181957)$$

Continued fraction:

Linear form

$$1712 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{18 + \cfrac{1}{2 + \cfrac{1}{11 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}$$

Series representations:

More

$$\begin{aligned} \frac{144 \log(404437000433962)}{\sqrt{8}} = \\ 36\sqrt{2} \log(404437000433961) - 36\sqrt{2} \sum_{k=1}^{\infty} \frac{(-\frac{1}{404437000433961})^k}{k} \end{aligned}$$

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$$\begin{aligned} \frac{144 \log(404437000433962)}{\sqrt{8}} = 72i\sqrt{2}\pi \left\lfloor \frac{\arg(404437000433962-x)}{2\pi} \right\rfloor + \\ 36\sqrt{2} \log(x) - 36\sqrt{2} \sum_{k=1}^{\infty} \frac{(-1)^k (404437000433962-x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned}$$

[Open code](#)

$$\begin{aligned} \frac{144 \log(404437000433962)}{\sqrt{8}} = \\ 36\sqrt{2} \left( \log(z_0) + \left\lfloor \frac{\arg(404437000433962-z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \\ \left. \sum_{k=1}^{\infty} \frac{(-1)^k (404437000433962-z_0)^k z_0^{-k}}{k} \right) \end{aligned}$$

Integral representations:

$$\frac{144 \log(404437000433962)}{\sqrt{8}} = 36\sqrt{2} \int_1^{404437000433962} \frac{1}{t} dt$$

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$$\frac{144 \log(404437000433962)}{\sqrt{8}} = -\frac{18 i \sqrt{2}}{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{404437000433961^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$(64+2)/(\sqrt{8}) \ln(404437000433962)$$

Input:

$$\frac{64+2}{\sqrt{8}} \log(404437000433962)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Exact result:

$$\frac{33 \log(404437000433962)}{\sqrt{2}}$$

Decimal approximation:

More digits

$$784.8221043957050657174160789359397056655254361445334936826\dots$$

[Open code](#)

This value 784,82 is very near to the rest mass of Omega meson

Property:

$$\frac{33 \log(404437000433962)}{\sqrt{2}} \text{ is a transcendental number}$$

Continued fraction:

Linear form

$$784 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

Series representations:

More

$$\begin{aligned} \frac{\log(404437000433962)(64+2)}{\sqrt{8}} &= \\ \frac{33 \log(404437000433961)}{\sqrt{2}} - \frac{33 \sum_{k=1}^{\infty} \frac{(-1)^k (404437000433961)^k}{k}}{\sqrt{2}} \end{aligned}$$

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$$\begin{aligned} \frac{\log(404437000433962)(64+2)}{\sqrt{8}} &= 33 i \sqrt{2} \pi \left[ \frac{\arg(404437000433962 - x)}{2\pi} \right] + \\ \frac{33 \log(x)}{\sqrt{2}} - \frac{33 \sum_{k=1}^{\infty} \frac{(-1)^k (404437000433962 - x)^k x^{-k}}{k}}{\sqrt{2}} &\quad \text{for } x < 0 \end{aligned}$$

[Open code](#)

$$\begin{aligned} \frac{\log(404437000433962)(64+2)}{\sqrt{8}} &= \frac{1}{\sqrt{2}} \\ 33 \left( \log(z_0) + \left[ \frac{\arg(404437000433962 - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (404437000433962 - z_0)^k z_0^{-k}}{k} \right) \end{aligned}$$

Integral representations:

$$\frac{\log(404437000433962)(64+2)}{\sqrt{8}} = \frac{33}{\sqrt{2}} \int_1^{404437000433962} \frac{1}{t} dt$$

[Open code](#)

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$$\frac{\log(404437000433962)(64+2)}{-\frac{33i}{2\sqrt{2}\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{404437000433961^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \text{ for } -1 < \gamma < 0$$

And

$$\begin{aligned}\psi(q) &= (188.923) + (5.76860 * 10^6) + (3.07735 * 10^{13}) = \\ &= 3.0773505768788923 \times 10^{13}\end{aligned}$$

Input interpretation:

$$\frac{1}{\sqrt{2\pi}} \log(3.0773505768788923 \times 10^{13})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

• 12.3902197815835696...

This result 12,39 is very near to the value of black hole entropy (see Tables)

Input interpretation:

$$\frac{138}{\sqrt{2\pi}} \log(3.0773505768788923 \times 10^{13})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

• 1709.85032985853260...

This result 1709,85 is very near to the mass of  $f_0(1710)$  candidate glueball

Continued fraction:

Linear form

$$1709 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{7 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{56 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}$$

Input interpretation:

$$\frac{55 + 8}{\sqrt{2 \pi}} \log(3.0773505768788923 \times 10^{13})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

780.583846239764883...

This value 780,583 is very near to the rest mass of Omega meson

We have also (pag. 5 paper):

$$\begin{aligned}\chi(q) &= \frac{1}{1-q} + \frac{q}{(1-q^2)(1-q^3)} + \frac{q^2}{(1-q^3)(1-q^4)(1-q^5)} \\ &\quad + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)(1-q^7)} + \dots \\ F(q) &= \frac{1}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^{12}}{(1-q)(1-q^3)(1-q^5)} + \dots\end{aligned}$$

Thence:  $\chi(q) =$

$$(-0.0818490) + (0.0000329662) + (-6.14714 \cdot 10^{-12}) + (4.99008 \cdot 10^{-22})$$

The result is: -0.081816033806147139999500992;

$$F(q) = (-0.0818490) + (1.08232) + (-2499.28) = -2498.279529$$

The result is: -2498.279529.

The sum of all five results, is:

$$(-4.9290621621 * 10^6) + (4.04437 * 10^{14}) + (3.0773505768788923 * 10^{13}) + (-0.081816033806147139999500992) + (-2498.279529)$$

Input interpretation:

$$-4.9290621621 \times 10^6 + 4.04437 \times 10^{14} + 3.0773505768788923 \times 10^{13} - 0.081816033806147139999500992 - 2498.279529$$

[Open code](#)

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Result:

$$4.35210500837228399554966193852860000499008 \times 10^{14}$$

[Open code](#)

Input interpretation:

$$\frac{1}{2} \log(4.35210500837228399554966193852860000499008 \times 10^{14})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

16.853425469970600484924629536865716030796991...

This result 16,853 is very near to the value of black hole entropy (see Tables)

Input interpretation:

$$\frac{64 + 38}{2} \log(4.35210500837228399554966193852860000499008 \times 10^{14})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

1719.0493979370012494623122127603030351412931...

This result 1719,049 is very near to the mass of  $f_0(1710)$  candidate glueball

Continued fraction:

Linear form

$$1719 + \cfrac{1}{20 + \cfrac{1}{4 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{38 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{75 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{}}}}}}}}}}}}}}}}}}}$$

[Input interpretation:](#)

$$\frac{54 - 8}{2} \log(4.35210500837228399554966193852860000499008 \times 10^{14})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$775.25757161864762230653295869582293741666159\dots$$

This value 775,257 is practically equal to the rest mass of Charged rho meson

The product of the five results is:

$$(-4.9290621621 * 10^6) * (4.04437 * 10^{14}) * (3.0773505768788923 * 10^{13}) * (-0.081816033806147139999500992) * (-2498.279529)$$

[Input interpretation:](#)

$$(-4.9290621621 \times 10^6)(4.04437 \times 10^{14})(3.0773505768788923 \times 10^{13}) \times (-0.081816033806147139999500992) \times (-2498.279529)$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$-1.253925117438473458100264852971283358888143723063549\dots \times 10^{37}$$

$$(55+8)/\pi * \ln(-(-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))$$

Input interpretation:  
 $\frac{55 + 8}{\pi}$   
 $\log(-(-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))$   
[Open code](#)

- $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:  
 More digits

• 1713.01056657493400634514241940683193778085705254390955...

This result 1713.01 is very near to the mass of  $f_0(1710)$  candidate glueball

$$(-(-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))^{1/12}$$

Input interpretation:  
 $\sqrt[12]{-(-1.253925117438473458100264852971283358888143723063549 \times 10^{37})}$   
[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:  
 More digits

• 1234.5896547875960518590887724000691554296376974340245...

This value 1234,589 is practically equal to the rest mass of Delta baryon  $1232 \pm 2$

$$(-(-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))^{1/34}$$

Input interpretation:  
 $\sqrt[34]{-(-1.253925117438473458100264852971283358888143723063549 \times 10^{37})}$   
[Open code](#)

Result:  
 More digits

• 12.3346160489457212096954783004550310113394241087746250...

This result 12,33461 is very near to the value of black hole entropy (see Tables)

$$\frac{1}{(\sqrt{27})} (1.08643 \times 10^{11}) / ((-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))$$

Input interpretation:

$$\frac{1}{\sqrt{27}} \left( -\frac{1.08643 \times 10^{11}}{1.253925117438473458100264852971283358888143723063549 \times 10^{37}} \right)$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$-1.66743... \times 10^{-27}$$

Result very near to the mass of proton

$$1/(\text{sqrt}(29)) (1.08643*10^{19}) / ((-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))$$

[Input interpretation:](#)

$$\frac{1}{\sqrt{29}} \left( -\frac{1.08643 \times 10^{19}}{1.253925117438473458100264852971283358888143723063549 \times 10^{37}} \right)$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$-1.60891... \times 10^{-19}$$

Result very near to the electric charge of electron

$$1/(\text{sqrt}(48)) \ln ((((-0.72999480077443047538362776991420567540346048553554829328)/(-1.253925117438473458100264852971283358888143723063549 \times 10^{37})))$$

[Input interpretation:](#)

$$\frac{1}{\sqrt{48}} \log \left( \frac{-0.72999480077443047538362776991420567540346048553554829328}{-1.253925117438473458100264852971283358888143723063549 \times 10^{37}} \right)$$

[Open code](#)

•  $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$-12.3750187723532620489764739439417118591696884684175630...$$

This result -12,35750 is very near to the value of black hole entropy (see Tables) with minus sign

$$139/(\text{sqrt}(48)) \ln ((((-0.72999480077443047538362776991420567540346048553554829328)/(-1.253925117438473458100264852971283358888143723063549 \times 10^{37})))$$

[Input interpretation:](#)

$$\frac{139}{\sqrt{48}} \log\left(\frac{-0.72999480077443047538362776991420567540346048553554829328}{-1.253925117438473458100264852971283358888143723063549 \times 10^{37}}\right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

-1720.12760935710342480772987820789794842458669711004126...

This result -1720.1276 is very near to the mass of  $f_0(1710)$  candidate glueball with minus sign

$$8 - \frac{139}{\sqrt{48}} \ln ((((-0.72999480077443047538362776991420567540346048553554829328)/(-1.253925117438473458100264852971283358888143723063549 \times 10^{37})))$$

Input interpretation:

$$8 - \frac{139}{\sqrt{48}} \log\left(\frac{-0.72999480077443047538362776991420567540346048553554829328}{-1.253925117438473458100264852971283358888143723063549 \times 10^{37}}\right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

1728.12760935710342480772987820789794842458669711004126...

This result 1728.1276 is very near to the mass of  $f_0(1710)$  candidate glueball

$$\sqrt{37} * (((((169/(\sqrt{48})) \ln ((((-0.72999480077443047538362776991420567540346048553554829328)/(-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))))))^16$$

Input interpretation:

$$\sqrt{37} \left( \frac{169}{\sqrt{48}} \log\left(\frac{-0.72999480077443047538362776991420567540346048553554829328}{-1.253925117438473458100264852971283358888143723063549 \times 10^{37}}\right) \right)^{16}$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$8.1474555250402646646723462934432481544841699733603052... \times 10^{53}$$

Comparisons:

$\approx$  the size of the Monster group ( $\approx 8.1 \times 10^{53}$ )

$$\begin{aligned} & \text{sqrt}(37)/196884 * (((((169/(sqrt(48)) \ln ((((-} \\ & 0.72999480077443047538362776991420567540346048553554829328)/(-} \\ & 1.253925117438473458100264852971283358888143723063549 \times 10^{37}))))))^16 \end{aligned}$$

Input interpretation:

$$\frac{\sqrt{37}}{196884} \left( \frac{169}{\sqrt{48}} \log \left( \frac{-0.72999480077443047538362776991420567540346048553554829328}{-1.253925117438473458100264852971283358888143723063549 \times 10^{37}} \right) \right)^{16}$$

Open code

- $\log(x)$  is the natural logarithm

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Result:

More digits

$$4.1382009330571629307980060814709413433718179097134887... \times 10^{48}$$

This result 4,1382 is a multiple of the mass of hypothetical dark matter particles

$$\begin{aligned} & 2/\text{gamma } (7/2) * \ln ((((-} \\ & 0.72999480077443047538362776991420567540346048553554829328)/(-} \\ & 1.253925117438473458100264852971283358888143723063549 \times 10^{37})))))^{28} \end{aligned}$$

Input interpretation:

$$\frac{2}{\Gamma\left(\frac{7}{2}\right)} \log^{28} \left( \frac{-0.72999480077443047538362776991420567540346048553554829328}{-1.253925117438473458100264852971283358888143723063549 \times 10^{37}} \right)$$

Open code

- $\Gamma(x)$  is the gamma function
- $\log(x)$  is the natural logarithm

Result:

More digits

$$8.093108635057017220776239185289761354907255581285566... \times 10^{53}$$

Comparisons:

≈ the size of the Monster group ( $\approx 8.1 \times 10^{53}$ )

In conclusion, we have also that:

$$\frac{1}{34} \zeta(2) \gamma(2) * \ln ((((( -0.72999480077443047538362776991420567540346048553554829328) * (-1.253925117438473458100264852971283358888143723063549 \times 10^{37}) )))))$$

Input interpretation:

$$\frac{1}{34} \zeta(2) \Gamma(2) \log(-0.72999480077443047538362776991420567540346048553554829328 \\ (-1.253925117438473458100264852971283358888143723063549 \times 10^{37}))$$

[Open code](#)

- $\zeta(s)$  is the Riemann zeta function
- $\Gamma(x)$  is the gamma function
- $\log(x)$  is the natural logarithm

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Result:

More digits

4.11752199736943834292796697826889090908182207984549507...

This result 4,11752 is in the range of the mass of hypothetical dark matter particles

Continued fraction:

Linear form

$$4 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{27 + \cfrac{1}{7 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{19 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{}}}}}}}}}}}}}}}}}}}}}$$

$$((2 \zeta(2) \gamma(2))/(1.6220877440446798)^8 (((-0.72999480077443)) * ((-1.2539251174384734581 \times 10^{37})))$$

Input interpretation:

$$\frac{2 \zeta(2) \Gamma(2)}{1.6220877440446798^8} (-0.72999480077443 (-1.2539251174384734581 \times 10^{37}))$$

[Open code](#)

- $\zeta(s)$  is the Riemann zeta function
- $\Gamma(x)$  is the gamma function

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Result:

More digits

- $6.2831165538163\dots \times 10^{35}$

This result is practically a multiple of a length of a circle

$$((2 \zeta(2) \gamma(2))/(1.611349)^8 (((-0.72999480077443)) * ((-1.2539251174384734581 \times 10^{37})))$$

Input interpretation:

$$\frac{2 \zeta(2) \Gamma(2)}{1.611349^8} (-0.72999480077443 (-1.2539251174384734581 \times 10^{37}))$$

[Open code](#)

- $\zeta(s)$  is the Riemann zeta function
- $\Gamma(x)$  is the gamma function

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Result:

More digits

- $6.62602\dots \times 10^{35}$

This result can be considered a multiple of a Planck's constant

Now, we have:

## Mock $\vartheta$ -functions (of 5th order)

$$\begin{aligned}
 f(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1+q^2)(1+q^3)} + \dots \\
 \phi(q) &= 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \dots \\
 \psi(q) &= q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^2)(1+q^3) + \dots \\
 \chi(q) &= 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\
 &= 1 + \left\{ \frac{q}{1-q} + \frac{q^3}{(1-q^2)(1-q^3)} + \frac{q^5}{(1-q^3)(1-q^4)(1-q^5)} + \dots \right\}
 \end{aligned}$$

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \dots$$

$$\begin{aligned}
 \chi(q) &= 1 + (-1.08185) + (0.00575937) + (-1.41949 \times 10^{-8}) = -0.0760906441949 \\
 &= 1 + (-0.0760922) + (2.47992 \times 10^{-6}) + (-3.51705 \times 10^{-14}) = \\
 &= 0.9239102799199648295
 \end{aligned}$$

$$F(q) = 1 + (-14.2995) + (33034.4) = 33021.1005$$

Note that:

$$3\pi \sqrt{33021.1005}$$

Input interpretation:  
 $3\pi\sqrt{33021.1005}$   
[Open code](#)

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Result:

More digits

1712.64322...

This result 1712.64322 is very near to the mass of  $f_0(1710)$  candidate glueball

Series representations:  
[More](#)

$$3\pi\sqrt{33021.1} = 3\pi\sqrt{33020.1} \sum_{k=0}^{\infty} e^{-10.4049k} \binom{\frac{1}{2}}{k}$$

[Open code](#)

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$$3\pi\sqrt{33021.1} = 3\pi\sqrt{33020.1} \sum_{k=0}^{\infty} \frac{(-0.0000302846)^k \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

$$3\pi\sqrt{33021.1} = \frac{3\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} e^{-10.4049s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}$$

Continued fraction:

Linear form

$$\bullet \quad 1712 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{13 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\ddots}}}}}}}}}}$$

$$f(q) = (1.075226 + 12.2180 - 2135.48) = -2122.186775$$

$$\varphi(q) = (188.923) + (1.00250 \times 10^9) + (1.63161 \times 10^{20}) =$$

$$= 1.63161000001002500188923 \times 10^{20}$$

$$\psi(q) = (32844.3) + (1.33208 \times 10^{10}) + (9.39267 \times 10^{17}) =$$

$$= 9.392670133208328443 \times 10^{17}$$

The sum of the three functions is:

$$(-2122.186775) + (9.392670133208328443 \times 10^{17}) + (1.63161000001002500188923 \times 10^{20}) = 1.64100267014323330911036225 \times 10^{20}$$

The sum of the six functions is:

$$(-0.0760906441949 + 0.9239102799199648295 + 33021.1005) + (-2122.186775) + (9.392670133208328443 \times 10^{17}) + (1.63161000001002500188923 \times 10^{20})$$

Input interpretation:

$$(-0.0760906441949 + 0.9239102799199648295 + 33021.1005) - 2122.186775 + 9.392670133208328443 \times 10^{17} + 1.63161000001002500188923 \times 10^{20}$$

[Open code](#)

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Result:

$$1.641002670143233639329845446357250648295 \times 10^{20}$$

$$1.641002670143233639329845446357250648295 \times 10^{20}$$

We note that this value can be considered a multiple near to the  $\zeta(2)$

This is the total sum, also of the precedent results:

$$(1.641002670143233639329845446357250648295 \times 10^{20}) + (-4.9290621621 * 10^6) + (4.04437 * 10^{14}) + (3.0773505768788923 * 10^{13}) + (-0.081816033806147139999500992) + (-2498.279529)$$

Input interpretation:

$$1.641002670143233639329845446357250648295 \times 10^{20} - 4.9290621621 \times 10^6 + 4.04437 \times 10^{14} + 3.0773505768788923 \times 10^{13} - 0.081816033806147139999500992 - 2498.279529$$

[Open code](#)

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Result:

$$1.64100702224824201161384099601918917689500499008 \times 10^{20}$$

[Open code](#)

Total of Mock 9-functions (of 5th order) =

$$= 1.64100702224824201161384099601918917689500499008 \times 10^{20}$$

another value that can be considered a multiple near to the  $\zeta(2)$

Note that from:

$$(((1.64100702224824201161384099601918917689500499008 \times 10^{20}))^{10/\Gamma(1.63975513^2 \times 1/5)})$$

Input interpretation:

$$(1.64100702224824201161384099601918917689500499008 \times 10^{20})^{10/\Gamma(1.63975513^2 \times 1/5)}$$

[Open code](#)

- $\Gamma(x)$  is the gamma function

[Enlarge](#) [Data](#) [Customize](#) A [Plaintext](#) [Interactive](#)

Result:

More digits

$$2.60000\dots \times 10^{122}$$

We obtain a result practically equal to the entropy of cosmic event horizon  $S_{\text{CEH}} = 2.6 \times 10^{122} \text{ k}$  (from: **Dark Energy and the Entropy of the Observable Universe**  
Charles H. Lineweaver and Chas A. Egan)

From the inverse of the result obtained on pag.126, we obtain the following expression:

$$-[(1/(-5.74968 * 10^{-40})) \\ (1.64100702224824201161384099601918917689500499008 \times \\ 10^{20})]^*1/(1.624*15)^4$$

Input interpretation:

$$-\left( -\frac{1}{\frac{5.74968}{10^{-40}}} \times 1.64100702224824201161384099601918917689500499008 \times 10^{20} \right) \times \\ \frac{1}{(1.624 \times 15)^4}$$

[Open code](#)

Result:

More digits

•  $8.1050860323194458929813551374360349927299793897165808... \times 10^{53}$

[Open code](#)

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Comparisons:

$\approx$  the size of the Monster group ( $\approx 8.1 \times 10^{53}$ )

Total of Mock 9-functions (of 7th order)

-0.08141350042711980591559898323225082017711543245919605

-1.0061571663060836857869438133877556079608287902166143

0.92434086745859745756753595616700523645194956152836224

-0.08141350042711980591559898323225082017711543245919605 -

1.0061571663060836857869438133877556079608287902166143 -

0.92434086745859745756753595616700523645194956152836224

Result:

-2.01191153419180094927007875278701166458989378420417259

[Open code](#)

$\ln(-2.01191153419180094927007875278701166458989378420417259 * 1.64100702224 \\ 824201161384099601918917689500499008 \times 10^{20})$

47.246097233410359177767214822126853043140038647709

Continued fraction:  
Linear form

$$\bullet \quad 47 + \cfrac{1}{4 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{2 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{10 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{8 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

More

$$\frac{2621132213\pi}{174290157} \approx 47.2460972334103591710232$$

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$$\frac{94930 + 3\sqrt{1284494290}}{4285} \approx 47.246097233410359190352$$

$$\sqrt{\frac{1}{11}(3917 + 4381e + 2127\pi + 2952\log(2))} \approx 47.2460972334103591749299$$

From:

### Three-Dimensional Gravity Reconsidered

Edward Witten

School of Natural Sciences, Institute for Advanced Study  
Princeton, New Jersey 08540

<https://arxiv.org/abs/0706.3359v1>

The classical BTZ black hole is characterized by its mass  $M$  and angular momentum  $J$ . In terms of the Virasoro generators,

$$\begin{aligned} M &= \frac{1}{\ell}(L_0 + \bar{L}_0) \\ J &= (L_0 - \bar{L}_0), \end{aligned} \tag{3.2}$$

so  $L_0 = (\ell M + J)/2$ ,  $\bar{L}_0 = (\ell M - J)/2$ . The classical BTZ black hole obeys  $M\ell \geq |J|$ , or  $L_0, \bar{L}_0 \geq 0$ . The BTZ black hole is usually studied in the absence of the gravitational Chern-Simons coupling, that is for  $k_L = k_R = k$ . Its entropy is  $S = \pi(\ell/2G)^{1/2} (\sqrt{M\ell - J} + \sqrt{M\ell + J})$ . (This entropy was first expressed in terms of two-dimensional conformal field theory in [13].) With  $\ell/G = 16k$  as in (2.19), this is equivalent to  $S = 4\pi\sqrt{k} (\sqrt{L_0} + \sqrt{\bar{L}_0})$ . For the holomorphic sector, the entropy is therefore

$$S_L = 4\pi\sqrt{k_L L_0}, \tag{3.3}$$

and similarly for the antiholomorphic sector.

There is no classical BTZ black hole with  $L_0 < 0$ , and the entropy of such a black hole is zero if  $L_0 = 0$ . We will take this as a suggestion that quantum states corresponding to black holes exist only if  $L_0 > 0$ , that is  $L_0 \geq 1$ . This means that the exact partition function  $Z(q)$  should differ from the function  $Z_0(q)$  in (3.1) by terms of order  $q$ :

$$Z(q) = q^{-k} \prod_{n=2}^{\infty} \frac{1}{1 - q^n} + \mathcal{O}(q). \tag{3.4}$$

Admittedly, we are here trying to squeeze more information from the classical result than is justified. But it turns out that a modular-invariant partition function of this form exists and is unique. This result (which is due to Höhn [31]) follows from the fact that the moduli space  $\mathcal{M}_1$  of Riemann surfaces of genus 1 is itself a Riemann surface of genus 0, in fact parametrized by the  $j$ -function. If  $E_4$  and  $E_6$  are the usual Eisenstein series of weights 4 and 6, then  $j = 1728E_4^3/(E_4^3 - E_6^2)$ . Its expansion in powers of  $q$  is

$$j(q) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \dots. \tag{3.5}$$

Actually, it is more convenient to use the function

$$J(q) = j(q) - 744 = q^{-1} + 196884q + 21493760q^2 + \dots, \tag{3.6}$$

which likewise parametrizes the moduli space.

In our interpretation, the 196883 primaries are operators that (when combined with suitable anti-holomorphic factors) create black holes. It is illuminating to compare the number 196883 to the Bekenstein-Hawking formula. An exact quantum degeneracy of 196883 corresponds to an entropy of  $\ln 196883 \simeq 12.19$ . By contrast, the Bekenstein-Hawking entropy at  $k = 1$  and  $L_0 = 1$  is  $4\pi \simeq 12.57$ . We should not expect perfect agreement, because the Bekenstein-Hawking formula is derived in a semiclassical approximation which is valid for large  $k$ .

Agreement improves rapidly if one increases  $k$ . For example, at  $k = 4$ , and again taking  $L_0 = 1$ , the exact quantum degeneracy of primary states is 81026609426, according to eqn. (3.8). (Two of the states at this level are descendants.) This corresponds to an entropy  $\ln 81026609426 \simeq 25.12$ , compared to the Bekenstein-Hawking entropy  $8\pi \simeq 25.13$ . Shortly we will compute the entropy in the large  $k$  limit.

Our interpretation is that a primary state  $|\Lambda\rangle$  represents a black hole, while a descendant  $\prod_{n=1}^{\infty} L_n^{s_n} |\Lambda\rangle$  describes a black hole embellished by boundary excitations. These boundary excitations are the closest we can come in  $2+1$  dimensions to the gravitational waves that a black hole can interact with in a larger number of dimensions. If this interpretation is correct, then for an exact count of black hole states, we should count primaries only. However, as the above examples illustrate, in practice this issue has only a very slight effect on the black hole degeneracies. The boundary excitations contribute to the entropy an amount that is independent of  $k$  and thus negligible in the regime where we compare to the Bekenstein-Hawking entropy, and also, numerically, negligible even for small  $k$ . The separation between the black hole and the boundary excitations is best-motivated for large  $k$ .

Finally, for  $k^* = 10$ , we find

$$\begin{aligned} F_{10} &= K^{10} - 2760K^8 - 20479K^7 + 2554141K^6 + 33421429K^5 - 793639831K^4 \\ &\quad - 14145708496K^3 + 30831695165K^2 + 1166011724825K + 2482063616019 \\ &= q^{-5} + q^{-7/2} + q^{-3} + q^{-5/2} + q^{-2} + 2q^{-3/2} + 3q^{-1} + 3q^{-1/2} + 3 \\ &\quad + 389274233q^{1/2} + 842231630010q + 341925580784341q^{3/2} + \dots \\ H_{10} &= -261 + 1652836102144q + 112692628289650688q^2 + 630520566901614002176q^3 + \dots \end{aligned}$$

From  $H_{10}$  we take the sum of coefficients of  $q$  and calculate the  $\ln$ :

$$\ln ((1652836102144 + 112692628289650688 + 630520566901614002176))$$

Input:

$\log(1652836102144 + 112692628289650688 + 630520566901614002176)$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Exact result:

$$\log(630\,633\,261\,182\,739\,755\,008)$$

Decimal approximation:

More digits

$$47.89325616490734310752554062193900949670489094140506994433\dots$$

[Open code](#)

Property:

$\log(630\,633\,261\,182\,739\,755\,008)$  is a transcendental number

[Open code](#)

Alternate forms:

$$12 \log(2) + \log(153\,963\,198\,530\,942\,323)$$

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$$12 \log(2) + \log(938\,117) + \log(164\,119\,399\,319)$$

[Open code](#)

Continued fraction:

Linear form

$$47 + \cfrac{1}{1 + \cfrac{1}{8 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{13 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{14 + \cfrac{1}{1 + \cfrac{1}{11 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{32 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{}}}}}}}}}}}}}}}}}}}}}$$

...

Alternative representations:

More

$$\log(1\,652\,836\,102\,144 + 112\,692\,628\,289\,650\,688 + 630\,520\,566\,901\,614\,002\,176) =$$

$$\log_e(630\,633\,261\,182\,739\,755\,008)$$

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$\log(1652836102144 + 112692628289650688 + 630520566901614002176) =$   
 $\log(a) \log_a(630633261182739755008)$

[Open code](#)

$\log(1652836102144 + 112692628289650688 + 630520566901614002176) =$   
 $-Li_1(-630633261182739755007)$

[Open code](#)

- Integral representations:

$\log(1652836102144 + 112692628289650688 + 630520566901614002176) =$   
 $\int_1^{630633261182739755008} \frac{1}{t} dt$

- [Open code](#)
- 

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$\log(1652836102144 + 112692628289650688 + 630520566901614002176) =$   
 $-\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{630633261182739755007^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$

- [Open code](#)

The result is 47.893256 another value of the black hole entropy, very near to the 47.2460972334... previously calculated

From the sum of Mock 9-function (of 3th and 5th order), we calculate the ln and obtain:

$\ln(((6.596086158 * 10^{20}) + (-4267.24) + (1.64100702224824201161384099601918917689500499008 \times 10^{20})))$

Input interpretation:

$\log(6.596086158 \times 10^{20} - 4267.24 + 1.64100702224824201161384099601918917689500499008 \times 10^{20})$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

Fewer digits  
More digits

48.16034937217718692950483134865563144497601328513350730897...

value very near to the precedent values of black hole entropy

Continued fraction:

- Linear form

$$48 + \cfrac{1}{6 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{1 + \cfrac{1}{16 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{...}}}}}}}}}}}}}}}}$$

Note that:

$$36 * (((\ln (((6.596086158 * 10^{20}) + (-4267.24) + (1.64100702224824201161384099601918917689500499008 * 10^{20}))))))$$

Input interpretation:

$$36 \log(6.596086158 \times 10^{20} - 4267.24 + 1.64100702224824201161384099601918917689500499008 \times 10^{20})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

1733.77257740...

This result 1733.7725 is very near to the mass of  $f_0(1710)$  candidate glueball

$$64 * (((\ln (((6.596086158 * 10^{20}) + (-4267.24) + (1.64100702224824201161384099601918917689500499008 * 10^{20}))))))$$

Input interpretation:

$$64 \log(6.596086158 \times 10^{20} - 4267.24 + 1.64100702224824201161384099601918917689500499008 \times 10^{20})$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

3082.26235982...

This value 3082.26 is a good approximation of the rest mass of J/Psi meson

$3096.916 \pm 0.011$

From the  $S = 4\pi \sqrt{k} = 4\pi \sqrt{14} = 47,0190534$  that is black hole entropy for  $k = 14$ .

As a last and fundamental deepening, let's multiply and add algebraically various solutions of the 21 Mock theta functions. We'll have:

$$[(1.63161 \times 10^{20})(9.39267 \times 10^{17})(6.5960861587 \times 10^{20})(4.04437000433962 \times 10^{14}) \\ (3.0773505768788923 \times 10^{13})(0.923910279 + 0.924340867458)] / (0.081816 + 0.07609 + 0.0814135 + 1.006157 + 1.08185 + 1.08753 + 1.0809 + 4.85773)$$

Input interpretation:

$$(1.63161 \times 10^{20} \times 9.39267 \times 10^{17} \times 6.5960861587 \times 10^{20} \times 4.04437000433962 \times 10^{14} \times \\ 3.0773505768788923 \times 10^{13} (0.923910279 + 0.924340867458)) / \\ (0.081816 + 0.07609 + 0.0814135 + 1.006157 + \\ 1.08185 + 1.08753 + 1.0809 + 4.85773)$$

[Open code](#)

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Result:

More digits

2.4860339674506047571235568994210729337306033506999466...  $\times 10^{86}$

[Open code](#)

Value very near to the entropy of Relic Gravitons  $\approx 2.3 \times 10^{86}$

$$[(1.63161 \times 10^{20})(9.392 \times 10^{17})(6.59608 \times 10^{20})(4.04437 \times 10^{14})(3.07735 \times 10^{13}) \\ (4.929 \times 10^6)(33021.10)(2498.27)(2122.18)] / (0.081816 + 0.07609 + 0.0814135 + 1.00615 \\ 7 + 1.08185 + 1.08753 + 1.0809 + 4.85773)^2$$

Input interpretation:

$$1.63161 \times 10^{20} \times 9.392 \times 10^{17} \times 6.59608 \times 10^{20} \times \\ 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times 4.929 \times 10^6 \times 33021.10 \times \\ 2498.27 \times 2122.18 / (0.081816 + 0.07609 + 0.0814135 + 1.006157 + \\ 1.08185 + 1.08753 + 1.0809 + 4.85773)^2$$

[Open code](#)

[Enlarge](#) [Data](#) [Customize](#) A [Plaintext](#) [Interactive](#)

Result:

More digits

1.2408321207041088963380961774320869895869655907042159...  $\times 10^{103}$

[Open code](#)

This important and beautifully result is very near to the entropy of SMBHs  
(supermassive Black Hole)  $\approx 1.2 \times 10^{103}$

$$((2.4860339674506047571235568994210729337306033506999466 \times 10^{86}) * (\sqrt{2\pi}))$$

Input interpretation:

$$2.4860339674506047571235568994210729337306033506999466 \times 10^{86} \sqrt{2\pi}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

$$6.2315630345047702650888005180122216175976846206065610... \times 10^{86}$$

This result is very near to the entropy of Dark Matter  $\approx 6 \times 10^{86}$

$$[(1.63161 \times 10^{20})(9.392 \times 10^{17})(6.59608 \times 10^{20})(4.04437 \times 10^{14})(3.07735 \times 10^{13})(4.929 \times 10^6)(33021.10)(2498.27)(2122.18)(4267.24)](1.00583 \times 10^{-12})/(1.006157 + 0.9239 + 0.07609 + 0.08181)$$

Input interpretation:

$$\frac{(1.63161 \times 10^{20} \times 9.392 \times 10^{17} \times 6.59608 \times 10^{20} \times 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times 4.929 \times 10^6 \times 33021.10 \times 2498.27 \times 2122.18 \times 4267.24) \times 1.00583 \times 10^{-12}}{1.006157 + 0.9239 + 0.07609 + 0.08181}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

$$2.2315682741425171676039766571641207790444544820858809... \times 10^{96}$$

This value is very near to the entropy of stellar BHs (2.5-15 solar mass)  $\approx 2.2 \times 10^{96}$

$$[(1.63161 \times 10^{20})(9.392 \times 10^{17})(6.59608 \times 10^{20})(4.04437 \times 10^{14})(3.07735 \times 10^{13})(4.929 \times 10^6)(33021.10)(2498.27)(2122.18)(4267.24)(1/(1.0058343895 \times 10^{-12})) * (1.08663428)^{48}]$$

Input interpretation:

$$1.63161 \times 10^{20} \times 9.392 \times 10^{17} \times 6.59608 \times 10^{20} \times 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times 4.929 \times 10^6 \times 33021.10 \times 2498.27 \times 2122.18 \times 4267.24 \left( \frac{1}{1.0058343895 \times 10^{-12}} \times 1.08663428^{48} \right)$$

[Open code](#)

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Result:

More digits

$$2.4847570983272032366407660380481283715700342763729692... \times 10^{122}$$

This result can be considered a good approximation to the value of entropy of Cosmic Event Horizon  $2.6 \pm 0.3 * 10^{122}$

$$[(1.63161*10^{20})(9.392*10^{17})(6.59608*10^{20})(4.04437*10^{14})(3.07735*10^{13})(1.08663428)^{34}]$$

[Input interpretation:](#)

$$1.63161 \times 10^{20} \times 9.392 \times 10^{17} \times 6.59608 \times 10^{20} \times \\ 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times 1.08663428^{34}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$2.1209175404816740728327077813952760972851937504697911... \times 10^{88}$$

[Open code](#)

This result is very near to the entropy of Photons  $2.03 \pm 0.15 * 10^{88}$

$$[(1.63161*10^{20})(9.392*10^{17})(6.59608*10^{20})(4.04437*10^{14})(3.07735*10^{13})(1.08663428)^{33}]$$

[Input interpretation:](#)

$$1.63161 \times 10^{20} \times 9.392 \times 10^{17} \times 6.59608 \times 10^{20} \times \\ 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times 1.08663428^{33}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

$$1.9518227793086686652593987568616702367287674289732431... \times 10^{88}$$

[Open code](#)

This result is very near to the entropy of Relic Neutrinos  $1.93 \pm 0.15 * 10^{88}$

$\text{sqrt}((\ln$

$$((1.63161*10^{20})(9.392*10^{17})(6.59608*10^{20})(4.04437*10^{14})(3.07735*10^{13})(4.929*10^6)(33021.10)(2498.27)(2122.18)(4267.24))))$$

[Input interpretation:](#)

$$\sqrt{\log(1.63161 \times 10^{20} \times 9.392 \times 10^{17} \times 6.59608 \times 10^{20} \times 4.04437 \times 10^{14} \times \\ 3.07735 \times 10^{13} \times 4.929 \times 10^6 \times 33021.10 \times 2498.27 \times 2122.18 \times 4267.24)}$$

[Open code](#)

•  $\log(x)$  is the natural logarithm

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[Result:](#)

More digits

$$15.81810...$$

This result 15.81810 is very near to the value of black hole entropy (see Tables)

Continued fraction:

Linear form

$$15 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{99 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{23 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{11 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

More

$$\frac{1}{53} (120 e^\pi + 83 \pi - 1648 \log(\pi) + 106 \log(2 \pi) - 402 \tan^{-1}(\pi)) \approx$$

$$15.818099369021165226496866$$

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$$\text{root of } 51 x^4 - 559 x^3 - 4139 x^2 + 3800 x - 4939 \text{ near } x = 15.8181 \approx$$

$$15.818099369021165234347$$

$$\sec\left(\csc\left(\frac{1452871841}{482220326}\right)\right) \approx 15.8180993690211652244686$$

From the simple multiplication of the previously terms, we obtain:

$$((1.63161 \times 10^{20})(9.39267 \times 10^{17})(6.596086 \times 10^{20})(4.04437 \times 10^{14})(3.07735 \times 10^{13})(4.929062 \times 10^6)(33021.10)(2498.27)(2122.18)(4267.24)))$$

Input interpretation:

$$1.63161 \times 10^{20} \times 9.39267 \times 10^{17} \times 6.596086 \times 10^{20} \times 4.04437 \times 10^{14} \times \\ 3.07735 \times 10^{13} \times 4.929062 \times 10^6 \times 33021.10 \times 2498.27 \times 2122.18 \times 4267.24$$

Open code

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Result:

$$4.6328 \times 10^{108}$$

(2Pi) \*

$$(((1.63161 \times 10^{20})(9.39267 \times 10^{17})(6.596086 \times 10^{20})(4.04437 \times 10^{14})(3.07735 \times 10^{13})(4.929062 \times 10^6)(33021.10)(2498.27)(2122.18)(4267.24))) \times 10^{13}$$

Input interpretation:

$$(2\pi) \left( 1.63161 \times 10^{20} \times 9.39267 \times 10^{17} \times 6.596086 \times 10^{20} \times 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times 4.929062 \times 10^6 \times 33021.10 \times 2498.27 \times 2122.18 \times 4267.24 \right) \times 10^{13}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

$$2.91088\dots \times 10^{122}$$

the maximum possible value of Cosmic Event Horizon

Multiplying all 21 values obtained, we have the following final expressions:

$$((-1.0058343895 \times 10^{-12}) * (-5.74968 \times 10^{-40}) * (-1.08663428) * (-0.081816) * (-0.07609064) * (0.92391) * (-0.0814135) * (-1.00615716) * (0.9243408))$$

Input interpretation:

$$-1.0058343895 \times 10^{-12} (-5.74968 \times 10^{-40}) \times (-1.08663428) \times (-0.081816) \times (-0.07609064) \times 0.92391 \times (-0.0814135) \times (-1.00615716) \times 0.9243408$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

$$-2.736825291832791665509199539149494356652196487772300\dots \times 10^{-55}$$

[Open code](#)

$$((( -2.73682529183279166550919 \times 10^{-55}) * (-4.92906 \times 10^6) * (4.04437 \times 10^{14}) * (3.07735 \times 10^{13}) * (-2498.279) * (33021.10) * (-2122.186) * (1.63161 \times 10^{20}) * (9.39267 \times 10^{17}) * (-4267.24) * (6.596086 \times 10^{20}))$$

Input interpretation:

$$-2.73682529183279166550919 \times 10^{-55} (-4.92906 \times 10^6) \times 4.04437 \times 10^{14} \times 3.07735 \times 10^{13} \times (-2498.279) \times 33021.10 \times (-2122.186) \times 1.63161 \times 10^{20} \times 9.39267 \times 10^{17} \times (-4267.24) \times 6.596086 \times 10^{20}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-1.267925315513541562416051980712673446144837786881388 \dots \times 10^{54}$

[Open code](#)

The final result is  $-1.26792531 * 10^{54}$

$1/8 (\ln(-1.267925315513541562416051980712673446144837786881388 \times 10^{54}))$

Input interpretation:

$$\frac{1}{8} \log(-(-1.267925315513541562416051980712673446144837786881388 \times 10^{54}))$$

[Open code](#)

- $\log(x)$  is the natural logarithm

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

- More digits

$15.5721221220651988860234713021056265411100478947193221\dots$

This result 15.5721 is very near to the value of black hole entropy (see Tables)

Continued fraction:

- Linear form

$$15 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{29 + \cfrac{1}{5 + \cfrac{1}{45 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{22 + \cfrac{1}{30 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}$$

Note that  $(1.2678106049501570296300918814275121667096448751540617 \times 10^{54}) / (1/0.6449)$  where  $0.6449$  is  $1 - \zeta(2) = 1 - 1.6449$ , give us:

Input interpretation:

$$\frac{1.2678106049501570296300918814275121667096448751540617 \times 10^{54}}{0.6449}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

- $8.1761105913235626840844625433260259631104997998685439\dots \times 10^{53}$

$\approx$  the size of the Monster group ( $\approx 8.1 \times 10^{53}$ )

$$((-1.267925315513541562416051980712673446144837786881388 \times 10^{54}))^2$$

[Input interpretation:](#)

$$(-1.267925315513541562416051980712673446144837786881388 \times 10^{54})^2$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

$$1.607634605720113919847654673731700619660517613224198 \times 10^{108}$$

$$(1.607634605720113919847654673731700619660517613224198 \times 10^{108}) / (33021.1 \\ 0 * 1.08663428^{16})$$

[Input interpretation:](#)

$$\frac{1.607634605720113919847654673731700619660517613224198 \times 10^{108}}{33021.10 \times 1.08663428^{16}}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

- $1.2884316496159661343073291050577187770453894858832274\dots \times 10^{103}$

[Open code](#)

$$(1.607634605720113919847654673731700619660517613224198 \times 10^{108}) / (6.59608 \\ * 10^{20} * 1.086634^{28})$$

[Input interpretation:](#)

$$\frac{1.607634605720113919847654673731700619660517613224198 \times 10^{108}}{6.59608 \times 10^{20} \times 1.086634^{28}}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result:](#)

More digits

- $2.3799581536691101825973019439047070402315311531908581\dots \times 10^{86}$

[Open code](#)

$$(1.607634605720113919847654673731700619660517613224198 \times 10^{108}) / (6.59608 \times 10^{20} \times 1.086634^{28}) * 2.61803398$$

Input interpretation:

$$\frac{1.607634605720113919847654673731700619660517613224198 \times 10^{108}}{6.59608 \times 10^{20} \times 1.086634^{28}} \times$$

2.61803398

[Open code](#)

[Enlarge Data](#) [Customize A](#) Plaintext [Interactive](#)

Result:

More digits

$$6.2308113172837921344037411454625769132713756264822520\ldots \times 10^{86}$$

$$(((1.607634605720113919847654673731700619660517613224198 \times 10^{108}) * (4.04437 \times 10^{14})) * 1 / (2.61803398 - 0.081816))$$

Input interpretation:

$$\frac{(1.607634605720113919847654673731700619660517613224198 \times 10^{108}) \times}{4.04437 \times 10^{14} \times \frac{1}{2.61803398 - 0.081816}}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) Plaintext [Interactive](#)

Result:

More digits

$$2.5636081841578369119574884225054969585604813113877339\ldots \times 10^{122}$$

[Open code](#)

From the previously analyzed

Mock  $\vartheta$ -functions (of 5th order)

$$\begin{aligned} f(q) &= 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1+q^2)(1+q^3)} + \dots \\ \phi(q) &= 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \dots \\ \psi(q) &= q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^2)(1+q^3) + \dots \\ \chi(q) &= 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\ &= 1 + \left\{ \frac{q}{1-q} + \frac{q^3}{(1-q^2)(1-q^3)} + \frac{q^5}{(1-q^3)(1-q^4)(1-q^5)} + \dots \right\} \end{aligned}$$

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \dots$$

we have obtained that

$$\begin{aligned}\psi(q) &= (32844.3) + (1.33208 \times 10^{10}) + (9.39267 \times 10^{17}) = \\ &= 9.392670133208328443 \times 10^{17}\end{aligned}$$

From:

## Magnetic Monopoles and Dark Matter

**V. V. Burdyuzha**

*Astrospace Center, Lebedev Physical Institute, Russian Academy of Sciences,*

*Profssoyuznaya ul. 84/32, Moscow, 117997 Russia*

*e-mail: burdyuzh@asc.rssi.ru*

Received October 8, 2017; in final form, May 29, 2018

Dirac's theory does not predict the magnetic monopole mass, but it is often assumed that the monopole mass can be

$$\begin{aligned}m_g &= (g/e)^2 m_e \\ &= 4692.25 m_e \approx 2.56 m_p \approx 2.4 \text{ GeV}/c^2.\end{aligned}\tag{6}$$

from our electric world. The minimum mass of the magnetic charge in Schwinger's symmetric world must probably be

$$\begin{aligned}m_g &= (g/e)^2 m_e \\ &= 18769 m_e \approx 10.24 m_p \approx 9.6 \text{ GeV}/c^2.\end{aligned}\tag{10}$$

Here, as in Dirac's case, the classical magnetic monopole radius is equal to the classical electron radius. Before the annihilation of Schwinger magnetic charges with a minimum mass of  $9.6 \text{ GeV}/c^2$ , they could also form an atomic system—monopolium ( $g^+g^-$ ). In this

theory [62]. As has been mentioned, the relation between the masses  $m_g$  and  $m_e$  in the case where the classical radii  $r_g$  and  $r_e$  are equal is  $m_g = m_e(g^2/e^2)$  and then  $m_g = 2.4 \text{ GeV}/c^2$ . However, nature could choose a different definition of the masses. As was shown by Caruso [61], the relation between the charges in the Born–Infeld electromagnetic theory is different,  $m_g = m_e(g^2/e^2)^{3/4}$ , and then  $m_g = 0.29 \text{ GeV}/c^2$ . This point

In this work we have three fundamental values:

$$m_g = 2.4 \text{ GeV}/c^2 ; m_g = 9.6 \text{ GeV}/c^2 ; m_g = 0.29 \text{ GeV}/c^2 .$$

We observe that utilizing the following 5th order Ramanujan's Mock Theta function  $\psi(q) = 9.392670133208328443 \times 10^{17}$ , we have obtained some new interesting mathematical connections.

## Conclusion

We observe how the results of the proposed equations (based on Ramanujan's mathematics) are either transcendental numbers or numbers that can be represented as infinite continued fractions. Both sets are an uncountable infinity. This, physically, could mean that the supersymmetric vacuum and the Hilbert space contained in it are also uncountable infinities. These numbers, therefore, could be identified in the infinite information, of which only a part (the one re-elaborated at each cycle of the universe, given the increasingly evident cyclical nature of the multiverse) passes into the Hilbert fractal space, coming to constitute the infinite frequencies that collapse in well-defined physical properties of strings / particles and physical-mathematical constants.

## Appendix A

From:

### **Monstrous Moonshine and the Entropy of the Smallest Black Hole**

Last Update: 14th September 2008

<b>k</b>	<b>Bekenstein-Hawking</b>	<b>Witten</b>	<b>Difference (%)</b>
1	12.566	12.190	3.0%
2	17.772	17.576	1.1%
3	21.766	21.676	0.4%
4	25.133	25.118	0.06%

TABLE 1

From:

Published by Institute of Physics Publishing for SISSA

Received: September 7, 2007 -Accepted: October 28, 2007 - Published: November 9, 2007 “Three-dimensional AdS gravity and extremal CFTs at  $c = 8m$ ”

Spyros D. Avramis, Alex Kehagias and Constantina Mattheopoulou

$m$	$L_0$	$d$	$S$	$S_{BH}$
3	1	196883	12.1904	12.5664
	2	21296876	16.8741	17.7715
	3	842609326	20.5520	21.7656
4	2/3	139503	11.8458	11.8477
	5/3	69193488	18.0524	18.7328
	8/3	6928824200	22.6589	23.6954
5	1/3	20619	9.9340	9.3664
	4/3	86645620	18.2773	18.7328
	7/3	24157197490	23.9078	24.7812

$m$	$L_0$	$d$	$S$	$S_{BH}$
6	1	42987519	17.5764	17.7715
	2	40448921875	24.4233	25.1327
	3	8463511703277	29.7668	30.7812
7	2/3	7402775	15.8174	15.6730
	5/3	33934039437	24.2477	24.7812
	8/3	16953652012291	30.4615	31.3460
8	1/3	278511	12.5372	11.8477
	4/3	13996384631	23.3621	23.6954
	7/3	19400406113385	30.5963	31.3460

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of  $m$  and  $L_0$ .

TABLE 2

From:

**Dark Energy and the Entropy of the Observable Universe**

Charles H. Lineweaver and Chas A. Egan

## A LARGER ESTIMATE OF THE ENTROPY OF THE UNIVERSE

CHAS A. EGAN

Research School of Astronomy and Astrophysics, Australian National University,  
Canberra, Australia 1

CHARLES H. LINEWEAVER

Planetary Science Institute, Research School of Astronomy and Astrophysics and  
Research School of Earth Sciences, Australian National University,  
Canberra, Australia

*Received 2009, September 22; accepted 2010, January 11.*

TABLE 1. The entropy of the universe including the Gibbons-Hawking entropy of the cosmic event horizon as well as the entropy of the dominant components contained within the cosmic event horizon. See Egan & Lineweaver (2009) for details.

Component	Entropy $S [k]$
Cosmic Event Horizon	$2.6 \pm 0.3 \times 10^{122}$
SMBHs	$1.2_{-0.7}^{+1.1} \times 10^{103}$
*Stellar BHs ( $42 - 140 M_\odot$ )	$1.2 \times 10^{98_{-1.6}^{+0.8}}$
Stellar BHs ( $2.5 - 15 M_\odot$ )	$2.2 \times 10^{96_{-1.2}^{+0.6}}$
Photons	$2.03 \pm 0.15 \times 10^{88}$
Relic Neutrinos	$1.93 \pm 0.15 \times 10^{88}$
Dark Matter	$6 \times 10^{86 \pm 1}$
Relic Gravitons	$2.3 \times 10^{86_{-3.1}^{+0.2}}$
ISM & IGM	$2.7 \pm 2.1 \times 10^{80}$
Stars	$3.5 \pm 1.7 \times 10^{78}$
<b>Total</b>	<b><math>2.6 \pm 0.3 \times 10^{122}</math></b>

TABLE 3

TABLE 2  
 ENTROPY OF THE EVENT HORIZON AND THE  
 MATTER WITHIN IT (SCHEME 2 ENTROPY  
 BUDGET)

Component	Entropy $S [k]$
Cosmic Event Horizon	$2.6 \pm 0.3 \times 10^{122}$
SMBHs	$1.2_{-0.7}^{+1.1} \times 10^{103}$
Stellar BHs ( $2.5 - 15 M_\odot$ )	$2.2 \times 10^{96_{-1.2}^{+0.6}}$
Photons	$2.03 \pm 0.15 \times 10^{88}$
Relic Neutrinos	$1.93 \pm 0.15 \times 10^{88}$
WIMP Dark Matter	$6 \times 10^{86 \pm 1}$
Relic Gravitons	$2.3 \times 10^{86_{-3.1}^{+0.2}}$
ISM and IGM	$2.7 \pm 2.1 \times 10^{80}$
Stars	$3.5 \pm 1.7 \times 10^{78}$
<b>Total</b>	<b><math>2.6 \pm 0.3 \times 10^{122}</math></b>
Tentative Components:	
Massive Halo BHs ( $10^5 M_\odot$ )	$10^{104}$
Stellar BHs ( $42 - 140 M_\odot$ )	$1.2 \times 10^{98_{-1.6}^{+0.8}}$

TABLE 4

From: “SQUARE SERIES GENERATING FUNCTION TRANSFORMATIONS”  
 MAXIE D. SCHMIDT - <https://arxiv.org/abs/1609.02803v2>

**Corollary 4.7** (Special Values of Ramanujan's  $\varphi$ -Function). *For any  $k \in \mathbb{R}^+$ , the variant of the Ramanujan  $\varphi$ -function,  $\varphi(e^{-k\pi}) \equiv \vartheta_3(e^{-k\pi})$ , has the integral representation*

$$\varphi(e^{-k\pi}) = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{k\pi} (e^{2\pi} - \cos(\sqrt{2\pi}kt))}{e^{4k\pi} - 2e^{2k\pi} \cos(\sqrt{2\pi}kt) + 1} \right] dt. \quad (33)$$

Moreover, the special values of this function corresponding to the particular cases of  $k \in \{1, 2, 3, 5\}$  in (33) have the respective integral representations

$$\begin{aligned} \frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} &= 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^\pi (e^{2\pi} - \cos(\sqrt{2\pi}t))}{e^{4\pi} - 2e^{2\pi} \cos(\sqrt{2\pi}t) + 1} \right] dt \\ \frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} \cdot \frac{\sqrt{\sqrt{2}+2}}{2} &= 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{2\pi} (e^{4\pi} - \cos(2\sqrt{\pi}t))}{e^{8\pi} - 2e^{4\pi} \cos(2\sqrt{\pi}t) + 1} \right] dt \\ \frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} \cdot \frac{\sqrt{\sqrt{3}+1}}{2^{1/4}3^{3/8}} &= 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{3\pi} (e^{6\pi} - \cos(\sqrt{6\pi}t))}{e^{12\pi} - 2e^{6\pi} \cos(\sqrt{6\pi}t) + 1} \right] dt \\ \frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} \cdot \frac{\sqrt{5+2\sqrt{5}}}{5^{3/4}} &= 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^{5\pi} (e^{10\pi} - \cos(\sqrt{10\pi}t))}{e^{20\pi} - 2e^{10\pi} \cos(\sqrt{10\pi}t) + 1} \right] dt. \end{aligned} \quad (34)$$

From the first of (34):

$$\frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[ \frac{4e^\pi (e^{2\pi} - \cos(\sqrt{2\pi}t))}{e^{4\pi} - 2e^{2\pi} \cos(\sqrt{2\pi}t) + 1} \right] dt$$

we have:

$$\Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)} = \frac{4,44288293815}{3,625609908} = 1,2254167025$$

$$\frac{\pi^{1/4}}{\Gamma(\frac{3}{4})} = \frac{1,3313353638}{1,2254167025} = 1,08643481 \dots$$

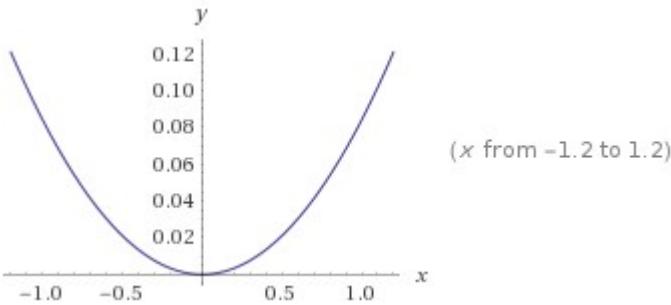
For the integral, we have calculate as follows:

integrate [(2.71828^0.89)/(sqrt6.283185307)][4e^3.14159265 \* (e^6.283185307 - cos(sqrt6.283185307)1.33416)]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)1.33416))+1]x

Indefinite integral:

$$\int \frac{2.71828^{0.89} \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos(\sqrt{6.283185307} \cdot 1.33416) \right) \right) x}{\sqrt{6.283185307} \left( e^{12.56637} - (2 e^{6.283185307}) \left( \cos(\sqrt{6.283185307}) \cdot 1.33416 \right) + 1 \right)} dx = 0.0837798 x^2 + \text{constant}$$

Plot of the integral:



Alternate form assuming x is real:

$$0.0837798 x^2 + 0 + \text{constant}$$

$$\text{Thence: } 1 + 0.0837798 = 1.0837798$$

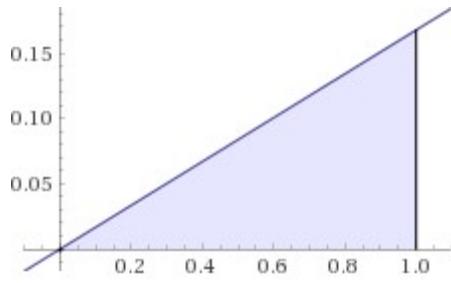
and:

$$\int [ (2.71828^{0.89}) / (\sqrt{6.283185307}) ] [ 4e^{3.14159265} * (e^{6.283185307} - \cos((\sqrt{6.283185307})1.33416)) ] / [ e^{12.56637} - 2e^{6.283185307} (\cos(\sqrt{6.283185307})1.33416) + 1 ] \, dx, [0, 1]$$

Definite integral:

$$\int_0^1 \frac{2.71828^{0.89} \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos(\sqrt{6.283185307} 1.33416) \right) \right) x}{\sqrt{6.283185307} \left( e^{12.56637} - (2 e^{6.283185307}) \left( \cos(\sqrt{6.283185307}) 1.33416 \right) + 1 \right)} \, dx = 0.0837798$$

Visual representation of the integral:



[Open code](#)

Riemann sums:

left sum	$0.0837798 - \frac{0.0837798}{n} = 0.0837798 - \frac{0.0837798}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$
----------	--

(assuming subintervals of equal length)

Indefinite integral:

$$\int \frac{2.71828^{0.89} \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos\left(\sqrt{6.283185307} 1.33416\right) \right) x \right)}{\sqrt{6.283185307} \left( e^{12.56637} - (2 e^{6.283185307}) \left( \cos\left(\sqrt{6.283185307}\right) 1.33416 \right) + 1 \right)} dx = 0.0837798 x^2 + \text{constant}$$

Thence:  $1 + 0.0837798 = 1.0837798$

With regard the integral, from 0 to 0.58438 for t = 2, where  $(2.71828^2)/(\sqrt{6.283185307}) = 2.94780$  for t=2, we have:

integrate  $(2.94780)[4e^{3.14159265} * (e^{6.283185307} - \cos((\sqrt{6.283185307})2))]/[e^{12.56637} - 2e^{6.283185307} (\cos(\sqrt{6.283185307})2)+1] x, [0,0.58438]$

$$\int_0^{0.58438} \frac{2.94780 \left( 4 e^{3.14159265} \left( e^{6.283185307} - \cos\left(\sqrt{6.283185307} 2\right) \right) x \right)}{e^{12.56637} - (2 e^{6.283185307}) \left( \cos\left(\sqrt{6.283185307}\right) 2 \right) + 1} dx = 0.0864364$$

Thence,  $1 + 0.0864364 = 1.0864364$ ;  $1,08643481 \cong 1,0864364$ .

In conclusion, the value of this, defined by us, "New Ramanujan's Constant" is 1.08643.

In this and others our papers, we have used 1.08643 as a new "Ramanujan's constant" and we can see as this constant is fundamental for some results that we have obtained in various equations analyzed and developed.

Remember that,  $1729 = 12^3 + 1 = 1728 + 1$ , is also the fundamental number that is in the range of the mass of the candidate "glueball"  $f_0(1710)$ :

### $f_0(1710)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
<b><math>1723^{+6}_{-5}</math> OUR AVERAGE</b>				Error includes scale factor of 1.6. See the ideogram below.
$1720 \pm 10$	$\pm 10$	<sup>9</sup> BALTRUSAIT..87	MRK3	$J/\psi \rightarrow \gamma K^+ K^-$
$1742 \pm 15$		<sup>8</sup> WILLIAMS 84	MPSF	$200 \pi^- N \rightarrow 2K_S^0 X$

$1726 \pm 7$	74	$^{13}$ CHEKANOV	04	ZEUS	$e p \rightarrow K_S^0 K_S^0 X$
$1732+15$		$^{14}$ ANISOVICH	03	RVUE	
$1726 \perp 7$	74	$^{13}$ CHEKANOV	04	ZEUS	$e p \rightarrow K_S^0 K_S^0 X$
$1732+15$		$^{14}$ ANISOVICH	03	RVUE	
$1744 \pm 15$		$^{22}$ ALDE	92D	GAM2	$38 \pi^- p \rightarrow \eta \eta n$
$1730 \begin{matrix} +2 \\ -10 \end{matrix}$	27	LONGACRE	86	RVUE	$22 \pi^- p \rightarrow n2K_S^0$

Very important is the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$

$$\left( \sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}} \right)^3 = 1164,269601267364$$

For the value 1181,81 we have that:

$$\sqrt[14]{\left( \sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}} \right)^3} = 1,65578 \dots$$

## List of Baryons

Various result are very near or also equal to the rest masses of following baryons:

$J^P = \frac{1}{2}^+$  baryons

Particle name	Symbol	Quark content	Rest mass (MeV/c <sup>2</sup> )	$J$	$J^P$	$Q(\Theta)$	$S$	$C$	$B'$	Mean lifetime (s)	Commonly decays to
proton <sup>[9]</sup>	$\underline{p}/\underline{p}^+/\underline{N}^+$	$\underline{u}\underline{d}$	938.272 046(21) <sup>[a]</sup>	$\frac{1}{2}$	$\frac{1}{2}^+$	+1	0	0	0	Stable <sup>[a]</sup>	Unobserved
neutron <sup>[10]</sup>	$\underline{n}/\underline{n}^0/\underline{N}^0$	$\underline{u}\underline{d}\underline{d}$	939.565 379(21) <sup>[a]</sup>	$\frac{1}{2}$	$\frac{1}{2}^+$	0	0	0	0	$(8.800 \pm 0.009) \times 10^{-2}$ <sup>[a]</sup>	$\underline{p}^+ + \underline{e}^- + \bar{\nu}_e$
Lambda <sup>[10]</sup>	$\underline{\Lambda}^0$	$\underline{u}\underline{d}\underline{s}$	1 115.683 $\pm 0.006$	0	$\frac{1}{2}^+$	0	-1	0	0	$(2.632 \pm 0.020) \times 10^{-10}$	$\underline{p}^+ + \underline{\pi}^-$ or $\underline{n}^0 + \underline{\Lambda}^0$
charmed Lambda <sup>[11]</sup>	$\underline{\Lambda}_c^+$	$\underline{u}\underline{d}\underline{c}$	2 286.46 $\pm 0.14$	0	$\frac{1}{2}^+$	+1	0	+1	0	$(2.00 \pm 0.06) \times 10^{-13}$	See $\underline{\Lambda}_c^+$ decay modes ( <a href="http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-lambdac-plus.pdf">http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-lambdac-plus.pdf</a> )
bottom Lambda <sup>[12]</sup>	$\underline{\Lambda}_b^0$	$\underline{u}\underline{d}\underline{b}$	5 619.4 $\pm 0.6$	0	$\frac{1}{2}^+$	0	0	0	-1	$(1.429 \pm 0.024) \times 10^{-12}$	See $\underline{\Lambda}_b^0$ decay modes ( <a href="http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-lambdab-zero.pdf">http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-lambdab-zero.pdf</a> )
Sigma <sup>[13]</sup>	$\underline{\Sigma}^+$	$\underline{u}\underline{u}\underline{s}$	1 189.37 $\pm 0.07$	1	$\frac{1}{2}^+$	+1	-1	0	0	$(8.018 \pm 0.026) \times 10^{-11}$	$\underline{p}^+ + \underline{\pi}^0$ or $\underline{n}^0 + \underline{\pi}^+$
Sigma <sup>[14]</sup>	$\underline{\Sigma}^0$	$\underline{u}\underline{d}\underline{s}$	1 192.642 $\pm 0.024$	1	$\frac{1}{2}^+$	0	-1	0	0	$(7.4 \pm 0.7) \times 10^{-20}$	$\underline{\Lambda}^0 + \underline{y}$
Sigma <sup>[15]</sup>	$\underline{\Sigma}^-$	$\underline{d}\underline{d}\underline{s}$	1 197.449 $\pm 0.030$	1	$\frac{1}{2}^+$	-1	-1	0	0	$(1.479 \pm 0.011) \times 10^{-10}$	$\underline{n}^0 + \underline{\pi}^-$
charmed Sigma <sup>[16]</sup>	$\underline{\Sigma}_{cc}^{++}$	$\underline{u}\underline{u}\underline{c}\underline{c}$	2 453.98 $\pm 0.16$	1	$\frac{1}{2}^+$	+2	0	+1	0	$(2.91 \pm 0.32) \times 10^{-22}$ <sup>[b]</sup>	$\underline{\Lambda}_c^+ + \underline{\pi}^+$
charmed Sigma <sup>[16]</sup>	$\underline{\Sigma}_c^+$	$\underline{u}\underline{d}\underline{c}$	2 452.9 $\pm 0.4$	1	$\frac{1}{2}^+$	+1	0	+1	0	$>1.43 \times 10^{-22}$ <sup>[b]</sup>	$\underline{\Lambda}_c^+ + \underline{\pi}^0$
charmed Sigma <sup>[16]</sup>	$\underline{\Sigma}_c^0$	$\underline{d}\underline{d}\underline{c}$	2 453.74 $\pm 0.16$	1	$\frac{1}{2}^+$	0	0	+1	0	$(3.05 \pm 0.37) \times 10^{-22}$ <sup>[b]</sup>	$\underline{\Lambda}_c^+ + \underline{\pi}^-$
bottom Sigma <sup>[17]</sup>	$\underline{\Xi}_b^+$	$\underline{u}\underline{u}\underline{b}$	5 811.3 $^{+0.9}_{-0.8} \pm 1.7$	1	$\frac{1}{2}^+$	+1	0	0	-1	$6.8^{+2.7}_{-3.5} \times 10^{-23}$ <sup>[b]</sup>	$\underline{\Lambda}_b^0 + \underline{\pi}^+$
bottom Sigma <sup>[17]</sup>	$\underline{\Xi}_b^0$	$\underline{u}\underline{d}\underline{b}$	Unknown	1	$\frac{1}{2}^+$	0	0	0	-1	Unknown	Unknown
bottom Sigma <sup>[17]</sup>	$\underline{\Xi}_b^-$	$\underline{d}\underline{d}\underline{b}$	5 815.5 $^{+0.6}_{-0.5} \pm 1.7$	1	$\frac{1}{2}^+$	-1	0	0	-1	$1.34^{+0.87}_{-1.15} \times 10^{-23}$ <sup>[b]</sup>	$\underline{\Lambda}_b^0 + \underline{\pi}^-$
Xi <sup>[18]</sup>	$\underline{\Xi}^0$	$\underline{u}\underline{s}\underline{s}$	1 314.86 $\pm 0.20$	$\frac{1}{2}$	$\frac{1}{2}^+$	0	-2	0	0	$(2.00 \pm 0.09) \times 10^{-10}$	$\underline{\Lambda}^0 + \underline{\pi}^0$
Xi <sup>[19]</sup>	$\underline{\Xi}^-$	$\underline{d}\underline{s}\underline{s}$	1 321.71 $\pm 0.07$	$\frac{1}{2}$	$\frac{1}{2}^+$	-1	-2	0	0	$(1.639 \pm 0.015) \times 10^{-10}$	$\underline{\Lambda}^0 + \underline{\pi}^-$
charmed Xi <sup>[20]</sup>	$\underline{\Xi}_c^+$	$\underline{u}\underline{s}\underline{c}$	2 467.8 $^{+0.4}_{-0.6}$	$\frac{1}{2}$	$\frac{1}{2}^+$	+1	-1	+1	0	$(4.42 \pm 0.26) \times 10^{-13}$	See $\underline{\Xi}_c^+$ decay modes ( <a href="http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xic-plus.pdf">http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xic-plus.pdf</a> )
charmed Xi <sup>[21]</sup>	$\underline{\Xi}_c^0$	$\underline{d}\underline{s}\underline{c}$	2 470.88 $^{+0.34}_{-0.80}$	$\frac{1}{2}$	$\frac{1}{2}^+$	0	-1	+1	0	$1.12^{+0.15}_{-0.10} \times 10^{-13}$	See $\underline{\Xi}_c^0$ decay modes ( <a href="http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xic-zero.pdf">http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xic-zero.pdf</a> )
charmed Xi prime <sup>[22]</sup>	$\underline{\Xi}_c^{*+}$	$\underline{u}\underline{s}\underline{c}$	2 575.6 $\pm 3.1$	$\frac{1}{2}$	$\frac{1}{2}^+$	+1	-1	+1	0	Unknown	$\underline{\Xi}_c^{*+} + \underline{y}$ (seen)
charmed Xi prime <sup>[23]</sup>	$\underline{\Xi}_c^0$	$\underline{d}\underline{s}\underline{c}$	2 577.9 $\pm 2.9$	$\frac{1}{2}$	$\frac{1}{2}^+$	0	-1	+1	0	Unknown	$\underline{\Xi}_c^0 + \underline{y}$ (seen)
double charmed Xi <sup>[24]</sup>	$\underline{\Xi}_{cc}^{++}$	$\underline{u}\underline{c}\underline{c}$	3 621.40 $\pm 0.78$	$\frac{1}{2}$	$\frac{1}{2}^+$	+2	0	+2	0	Unknown	$\underline{\Lambda}_c^+ + \underline{K}^- + \underline{\pi}^+ + \underline{\pi}^+$ (seen)
double charmed Xi <sup>[25]</sup>	$\underline{\Xi}_{cc}^+$	$\underline{d}\underline{c}\underline{c}$	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	+1	0	+2	0	Unknown	Unknown
bottom Xi <sup>[25]</sup> (or Cascade B)	$\underline{\Xi}_b^0$	$\underline{u}\underline{s}\underline{b}$	5 787.8 $\pm 5.0 \pm 1.3$	$\frac{1}{2}$	$\frac{1}{2}^+$	0	-1	0	-1	Unknown	See $\underline{\Xi}_b^0$ decay modes ( <a href="http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xib-zero-xi-b-minus.pdf">http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xib-zero-xi-b-minus.pdf</a> )
bottom Xi <sup>[25]</sup> (or Cascade B)	$\underline{\Xi}_b^-$	$\underline{d}\underline{s}\underline{b}$	5 791.1 $\pm 2.2$	$\frac{1}{2}$	$\frac{1}{2}^+$	-1	-1	0	-1	$(1.56^{+0.27}_{-0.25} \pm 0.02) \times 10^{-12}$	See $\underline{\Xi}_b^-$ decay modes ( <a href="http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xib-zero-xi-b-minus.pdf">http://pdg.lbl.gov/2013/listing_gs/rpp2013-list-xib-zero-xi-b-minus.pdf</a> )
bottom Xi prime <sup>[26]</sup>	$\underline{\Xi}_b^0$	$\underline{u}\underline{s}\underline{b}$	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	0	-1	0	-1	Unknown	Unknown
bottom Xi prime <sup>[26]</sup>	$\underline{\Xi}_b^-$	$\underline{d}\underline{s}\underline{b}$	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	-1	-1	0	-1	Unknown	Unknown
doubtful	$\underline{\Xi}_{bb}^0$	$\underline{u}\underline{b}\underline{b}$	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	0	0	0	-2	Unknown	Unknown

bottom Xi <sup>†</sup>											
	<u>b</u> <sub>bb</sub>	<u>db</u> <sub>bb</sub>	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	-1	0	0	-2	Unknown	Unknown
charmed bottom Xi <sup>†</sup>	<u>c</u> <sub>cb</sub>	<u>uc</u> <sub>b</sub>	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	+1	0	+1	-1	Unknown	Unknown
charmed bottom Xi <sup>†</sup>	<u>c</u> <sub>cb</sub>	<u>dc</u> <sub>b</sub>	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	0	0	+1	-1	Unknown	Unknown
charmed bottom Xi prime <sup>†</sup>	<u>c</u> <sub>ca</sub>	<u>uc</u> <sub>b</sub>	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	+1	0	+1	-1	Unknown	Unknown
charmed bottom Xi prime <sup>†</sup>	<u>c</u> <sub>ca</sub>	<u>dc</u> <sub>b</sub>	Unknown	$\frac{1}{2}$	$\frac{1}{2}^+$	0	0	+1	-1	Unknown	Unknown
charmed Omega <sup>271</sup>	<u>cc</u>	<u>cc</u>	2.695 2 + 1.7	0	$\frac{1}{2}^+$	0	-2	+1	0	$(6.9 \pm 1.2) \times 10^{-14}$	See $\Omega^0$ decay modes ( <a href="http://pdg.lbl.gov/2013/list-onegac-zero.pdf">http://pdg.lbl.gov/2013/list-onegac-zero.pdf</a> )
bottom Omega <sup>271</sup>	<u>c</u> <sub>b</sub>	<u>sc</u> <sub>b</sub>	6071 + 40	0	$\frac{1}{2}^+$	-1	-2	0	-1	$(1.13^{+0.55}_{-0.42} \pm 0.02) \times 10^{-12}$	$\Omega^- + J/\psi$ (scen)
double charmed Omega <sup>†</sup>	<u>cc</u>	<u>cc</u>	Unknown	0	$\frac{1}{2}^+$	+1	-1	+2	0	Unknown	Unknown
charmed bottom Omega <sup>†</sup>	<u>c</u> <sub>cb</sub>	<u>sc</u> <sub>b</sub>	Unknown	0	$\frac{1}{2}^+$	0	-1	+1	-1	Unknown	Unknown
charmed bottom Omega prime <sup>†</sup>	<u>c</u> <sub>cb</sub>	<u>sc</u> <sub>b</sub>	Unknown	0	$\frac{1}{2}^+$	0	-1	+1	-1	Unknown	Unknown
double bottom Omega <sup>†</sup>	<u>c</u> <sub>ba</sub>	<u>sbb</u>	Unknown	0	$\frac{1}{2}^+$	-1	-1	0	-2	Unknown	Unknown
double charmed bottom Omega <sup>†</sup>	<u>ccb</u>	<u>ccb</u>	Unknown	0	$\frac{1}{2}^+$	+1	0	+2	-1	Unknown	Unknown
charmed double bottom Omega <sup>†</sup>	<u>c</u> <sub>ccb</sub>	<u>cbb</u>	Unknown	0	$\frac{1}{2}^+$	0	0	+1	-2	Unknown	Unknown

$J^P = \frac{3}{2}^+$  baryons

Particle name	Symbol	Quark content	Rest mass (MeV/c <sup>2</sup> )	$I$	$J^P$	$Q(\text{e})$	$S$	$C$	$B'$	Mean lifetime (s)	Commonly decays to
Delta <sup>[29]</sup>	$\Delta^{++}(1232)$	$uuu$	$1232 \pm 2$	$\frac{3}{2}$	$\frac{3}{2}^+$	+2	0	0	0	$(5.63 \pm 0.14) \times 10^{-24}[\text{h}]$	$\underline{p}^+ + \underline{\pi}^+$
Delta <sup>[29]</sup>	$\Delta^+(1232)$	$uud$	$1232 \pm 2$	$\frac{3}{2}$	$\frac{3}{2}^+$	+1	0	0	0	$(5.63 \pm 0.14) \times 10^{-24}[\text{h}]$	$\underline{\pi}^+ + \underline{n}^0 \text{ or } \underline{\Lambda}^0 + \underline{p}^+$
Delta <sup>[29]</sup>	$\Delta^0(1232)$	$udd$	$1232 \pm 2$	$\frac{3}{2}$	$\frac{3}{2}^+$	-1	0	0	0	$(5.63 \pm 0.14) \times 10^{-24}[\text{h}]$	$\underline{\pi}^- + \underline{n}^0 \text{ or } \underline{\Lambda}^- + \underline{\pi}^+$
Delta <sup>[29]</sup>	$\Delta^-(1232)$	$ddd$	$1232 \pm 2$	$\frac{3}{2}$	$\frac{3}{2}^+$	-1	0	0	0	$(5.63 \pm 0.14) \times 10^{-24}[\text{h}]$	$\underline{\pi}^- + \underline{n}^0$
Sigma <sup>[30]</sup>	$\Sigma^{**}(1380)$	$uus$	$1382.8 \pm 0.4$	1	$\frac{3}{2}^+$	+1	-1	0	0	$(1.039 \pm 0.0041) \times 10^{-23}[\text{h}]$	$\underline{\Lambda}^0 + \underline{\pi}^0 \text{ or } \underline{\Sigma}^+ + \underline{\pi}^0 \text{ or } \underline{\Sigma}^0 + \underline{\pi}^+$
Sigma <sup>[30]</sup>	$\Sigma^{*0}(1385)$	$uds$	$1383.7 \pm 1.0$	1	$\frac{3}{2}^-$	0	-1	0	0	$(1.83 \pm 0.25) \times 10^{-23}[\text{h}]$	$\underline{\Lambda}^0 + \underline{\pi}^0 \text{ or } \underline{\Sigma}^+ + \underline{\pi}^- \text{ or } \underline{\Sigma}^0 + \underline{\pi}^0$
Sigma <sup>[30]</sup>	$\Sigma^{*-}(1385)$	$dds$	$1387.2 \pm 0.5$	1	$\frac{3}{2}^+$	-1	-1	0	0	$(1.671 \pm 0.089) \times 10^{-23}[\text{h}]$	$\underline{\Lambda}^0 + \underline{\pi}^- \text{ or } \underline{\Sigma}^0 + \underline{\pi}^- \text{ or } \underline{\Sigma}^+ + \underline{\pi}^0$
charmed Sigma <sup>[31]</sup>	$\Sigma_c^{**}(2520)$	$uuc$	$2517.9 \pm 0.6$	1	$\frac{3}{2}^+$	+2	0	+1	0	$(4.42 \pm 0.44) \times 10^{-23}[\text{h}]$	$\underline{\Lambda}_c^+ + \underline{\pi}^+$
charmed Sigma <sup>[31]</sup>	$\Sigma_c^{*0}(2520)$	$udc$	$2517.5 \pm 2.3$	1	$\frac{3}{2}^+$	+1	0	+1	0	$> 3.87 \times 10^{-23}[\text{h}]$	$\underline{\Lambda}_c^+ + \underline{\pi}^0$
charmed Sigma <sup>[31]</sup>	$\Sigma_c^{*0}(2520)$	$ddc$	$2518.8 \pm 0.6$	1	$\frac{3}{2}^+$	0	0	+1	0	$(4.54 \pm 0.47) \times 10^{-23}[\text{h}]$	$\underline{\Lambda}_c^+ + \underline{\pi}^-$
bottom Sigma <sup>[32]</sup>	$\Xi_b^{**}$	$uub$	$5832.1 \pm 0.7^{+1.7}_{-1.8}$	1	$\frac{3}{2}^+$	+1	0	0	-1	$(5.7 \pm 1.8) \times 10^{-23}[\text{h}]$	$\underline{\Lambda}_b^0 + \underline{\pi}^+$
bottom Sigma <sup>[32]</sup>	$\Xi_b^{*0}$	$udb$	Unknown	1	$\frac{3}{2}^+$	0	0	0	-1	Unknown	Unknown
bottom Sigma <sup>[32]</sup>	$\Xi_b^{*-}$	$ddb$	$5835.1 \pm 0.6^{+1.7}_{-1.8}$	1	$\frac{3}{2}^+$	-1	0	0	-1	$8.8^{+3.7}_{-3.6} \times 10^{-23}[\text{h}]$	$\underline{\Lambda}_b^0 + \underline{\pi}^-$
$\Xi$ <sup>[33]</sup>	$\Xi^{*0}(1530)$	$uss$	$1531.80 \pm 0.32$	$\frac{1}{2}$	$\frac{3}{2}^+$	0	-2	0	0	$(7.23 \pm 0.40) \times 10^{-23}[\text{h}]$	$\underline{\Xi}^0 + \underline{\pi}^0 \text{ or } \underline{\Xi}^+ + \underline{\pi}^-$
$\Xi$ <sup>[33]</sup>	$\Xi^{*-}(1530)$	$dss$	$1535.0 \pm 0.6$	$\frac{1}{2}$	$\frac{3}{2}^+$	-1	-2	0	0	$6.6^{+1.3}_{-1.1} \times 10^{-23}[\text{h}]$	$\underline{\Xi}^0 + \underline{\pi}^- \text{ or } \underline{\Xi}^- + \underline{\pi}^0$
charmed $\Xi$ <sup>[34]</sup>	$\Xi_c^{**}(2645)$	$usc$	$2645.9^{+0.5}_{-0.6}$	$\frac{1}{2}$	$\frac{3}{2}^+$	+1	-1	+1	0	$> 2.1 \times 10^{-22}[\text{h}]$	$\underline{\Xi}_c^+ + \underline{\pi}^0 \text{ (seen)}$
charmed $\Xi$ <sup>[34]</sup>	$\Xi_c^{*0}(2645)$	$usd$	$2645.0 \pm 0.5$	$\frac{1}{2}$	$\frac{3}{2}^+$	0	-1	+1	0	$> 1.2 \times 10^{-22}[\text{h}]$	$\underline{\Xi}_c^+ + \underline{\pi}^- \text{ (seen)}$
double charmed $\Xi$ <sup>[35]</sup>	$\Xi_{cc}^{**}$	$ucc$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	+2	0	+2	0	Unknown	Unknown
double charmed $\Xi$ <sup>[35]</sup>	$\Xi_{cc}^{*+}$	$dcc$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	+1	0	+2	0	Unknown	Unknown
bottom $\Xi$ <sup>[35]</sup>	$\Xi_b^{*0}$	$ush$	$5945.5 \pm 0.8 \pm 2.2$	$\frac{1}{2}$	$\frac{3}{2}^+$	0	-1	0	-1	$(3.1 \pm 2.5) \times 10^{-22}[\text{h}]$	$\underline{\Xi}_b^0 + \underline{\pi}^+ \text{ (seen)}$
bottom $\Xi$ <sup>[35]</sup>	$\Xi_b^{*-}$	$dsb$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	-1	-1	0	-1	Unknown	Unknown
double bottom $\Xi$ <sup>[35]</sup>	$\Xi_{bb}^{*0}$	$ubb$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	0	0	0	-2	Unknown	Unknown
double bottom $\Xi$ <sup>[35]</sup>	$\Xi_{bb}^{*-}$	$dbb$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	-1	0	0	-2	Unknown	Unknown
charmed bottom $\Xi$ <sup>[35]</sup>	$\Xi_{cc}^{**}$	$uch$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	+1	0	+1	-1	Unknown	Unknown
charmed bottom $\Xi$ <sup>[35]</sup>	$\Xi_{cc}^{*0}$	$dcb$	Unknown	$\frac{1}{2}$	$\frac{3}{2}^+$	0	0	+1	-1	Unknown	Unknown
Omega <sup>[36]</sup>	$\Omega^-$	$sss$	$1672.45 \pm 0.29$	0	$\frac{3}{2}^+$	-1	-3	0	0	$(8.21 \pm 0.11) \times 10^{-11}[\text{h}]$	$\underline{\Lambda}^0 + \underline{K}^- \text{ or } \underline{\Xi}^0 + \underline{\pi}^- \text{ or } \underline{\Xi}^- + \underline{\pi}^0$
charmed Omega <sup>[37]</sup>	$\Omega_c^{*0}(2770)$	$ssc$	$2765.9 \pm 2.0$	0	$\frac{3}{2}^+$	0	-2	+1	0	Unknown	$\underline{\Omega}^0 + \underline{y}$
bottom Omega <sup>+</sup>	$\Omega_b^{*-}$	$ssh$	Unknown	0	$\frac{3}{2}^+$	-1	-2	0	-1	Unknown	Unknown
double charmed Omega <sup>+</sup>	$\Omega_{cc}^{*+}$	$scc$	Unknown	0	$\frac{3}{2}^+$	+1	-1	+2	0	Unknown	Unknown
charmed bottom Omega <sup>+</sup>	$\Omega_{cb}^{*0}$	$scb$	Unknown	0	$\frac{3}{2}^+$	0	-1	+1	-1	Unknown	Unknown

Omega <sup>+</sup>											
double bottom Omega <sup>+</sup>	$\Omega_{bb}^{*-}$	$sbb$	Unknown	0	$\frac{3}{2}^+$	-1	-1	0	-2	Unknown	Unknown
triple charmed Omega <sup>+</sup>	$\Omega_{ccc}^{++}$	$ccc$	Unknown	0	$\frac{3}{2}^+$	+2	0	+3	0	Unknown	Unknown
double charmed bottom Omega <sup>+</sup>	$\Omega_{ccb}^{*+}$	$ccb$	Unknown	0	$\frac{3}{2}^+$	+1	0	+2	-1	Unknown	Unknown
charmed double bottom Omega <sup>+</sup>	$\Omega_{ccb}^{*0}$	$cbb$	Unknown	0	$\frac{3}{2}^+$	0	0	+1	-2	Unknown	Unknown
triple bottom Omega <sup>+</sup>	$\Omega_{bbb}^{*-}$	$bbb$	Unknown	0	$\frac{3}{2}^+$	-1	0	0	-3	Unknown	Unknown

## List of Mesons

Pseudoscalar mesons

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c <sup>2</sup> )	I <sup>G</sup>	J <sup>PC</sup>	S	C	B'	Mean lifetime (s)	Commonly decays to (>5% of decays)
Pion <sup>[23]</sup>	$\pi^+$	$\pi^-$	ud	139.57018 ± 0.00035	1 <sup>-</sup>	0 <sup>-</sup>	0	0	0	(2.6033 ± 0.0005) × 10 <sup>-8</sup>	$\mu^+ + \nu_\mu$
Pion <sup>[24]</sup>	$\pi^0$	Self	$\frac{\bar{u}d - \bar{d}u}{\sqrt{2}}$ [4]	134.9766 ± 0.0006	1 <sup>-</sup>	0 <sup>-+</sup>	0	0	0	(8.4 ± 0.6) × 10 <sup>-17</sup>	$\gamma + \gamma$
Eta meson <sup>[25]</sup>	$\eta$	Self	$\frac{\bar{u}\bar{d} + \bar{s}\bar{d} + \bar{u}\bar{s}}{\sqrt{3}}$ [5]	547.853 ± 0.024	0 <sup>+</sup>	0 <sup>-+</sup>	0	0	0	(5.0 ± 0.3) × 10 <sup>-19</sup> <sup>[6]</sup>	$\pi^0 + \pi^0 + \pi^0$ or $\pi^+ + \pi^0 + \pi^-$
Eta prime meson <sup>[26]</sup>	$\eta'(058)$	Self	$\frac{\bar{u}\bar{d} + \bar{s}\bar{d} + \bar{c}\bar{s}}{\sqrt{3}}$ [6]	957.66 ± 0.24	0 <sup>+</sup>	0 <sup>-+</sup>	0	0	0	(3.2 ± 0.2) × 10 <sup>-21</sup> <sup>[6]</sup>	$\pi^+ + \pi^- + \eta$ or $(\rho^0 + \gamma) / (\pi^+ + \pi^- - \gamma)$ or $\pi^0 + \pi^0 + \eta$
Charmed eta meson <sup>[27]</sup>	$\eta_c(1S)$	Self	$cc$	2980.3 ± 1.2	0 <sup>+</sup>	0 <sup>-+</sup>	0	0	0	(2.5 ± 0.3) × 10 <sup>-23</sup> <sup>[6]</sup>	See $\eta_c$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m17.pdf">http://pdg.lbl.gov/2008/listing/s/m17.pdf</a> )
Bottom eta meson <sup>[28]</sup>	$\eta_b(1S)$	Self	$bb$	9300 ± 40	0 <sup>+</sup>	0 <sup>-+</sup>	0	0	0	Unknown	See $\eta_b$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m17.pdf">http://pdg.lbl.gov/2008/listing/s/m17.pdf</a> )
Kaon <sup>[29]</sup>	$K^+$	$K^-$	$u\bar{s}$	493.677 ± 0.016	1/2	0 <sup>-</sup>	1	0	0	(1.2380 ± 0.0021) × 10 <sup>-8</sup>	$\mu^+ + \nu_\mu$ or $\pi^+ + \pi^0$ or $\pi^0 + \pi^- + \nu_e$ or $\pi^+ + \pi^0$
Kaon <sup>[30]</sup>	$K^0$	$\bar{K}^0$	$d\bar{s}$	497.614 ± 0.024	1/2	0 <sup>-</sup>	1	0	0	[6]	[6]
K Short <sup>[31]</sup>	$K_s^0$	Self	$\frac{\bar{d}\bar{s} - \bar{s}\bar{d}}{\sqrt{2}}$ [7]	497.014 ± 0.024 <sup>[6]</sup>	1/2	0 <sup>-</sup>	1 <sup>0</sup>	0	0	(0.953 ± 0.005) × 10 <sup>-11</sup>	$\pi^+ + \pi^-$ or $\pi^0 + \pi^0$
K Long <sup>[32]</sup>	$K_L^0$	Self	$\frac{\bar{d}\bar{s} + \bar{s}\bar{d}}{\sqrt{2}}$ [8]	497.614 ± 0.024 <sup>[6]</sup>	1/2	0 <sup>-</sup>	1 <sup>0</sup>	0	0	(5.116 ± 0.020) × 10 <sup>-8</sup>	$\pi^2 + \pi^0 + \nu_\alpha$ or $\pi^2 + \mu^2 + \nu_\mu$ or $\pi^0 + \pi^0 + \pi^0$ or $\pi^+ + \pi^0 + \pi^-$
D meson <sup>[33]</sup>	$D^+$	$D^-$	$cd$	1889.62 ± 0.20	1/2	0 <sup>-</sup>	0	+1	0	(1.040 ± 0.007) × 10 <sup>-12</sup>	See $D^+$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m31.pdf">http://pdg.lbl.gov/2008/listing/s/m31.pdf</a> )
D meson <sup>[34]</sup>	$D^0$	$\bar{D}^0$	$cu$	1854.84 ± 0.17	1/2	0 <sup>-</sup>	0	+1	0	(4.101 ± 0.015) × 10 <sup>-13</sup>	See $D^0$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m32.pdf">http://pdg.lbl.gov/2008/listing/s/m32.pdf</a> )
Strange D meson <sup>[35]</sup>	$D_s^+$	$D_s^-$	$\bar{c}\bar{s}$	1958.49 ± 0.34	0	0 <sup>-</sup>	+1	+1	0	(5.00 ± 0.01) × 10 <sup>-13</sup>	See $D_s^+$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m34.pdf">http://pdg.lbl.gov/2008/listing/s/m34.pdf</a> )
B meson <sup>[36]</sup>	$B^+$	$B^-$	$u\bar{b}$	5279.15 ± 0.31	1/2	0 <sup>-</sup>	0	0	+1	(1.638 ± 0.011) × 10 <sup>-12</sup>	See $B^+$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m041.pdf">http://pdg.lbl.gov/2008/listing/s/m041.pdf</a> )
B meson <sup>[37]</sup>	$B^0$	$\bar{B}^0$	$d\bar{b}$	5279.53 ± 33	1/2	0 <sup>-</sup>	0	0	+1	(1.530 ± 0.009) × 10 <sup>-12</sup>	See $B^0$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m042.pdf">http://pdg.lbl.gov/2008/listing/s/m042.pdf</a> )
Strange D meson <sup>[38]</sup>	$B_s^0$	$\bar{B}_s^0$	$s\bar{b}$	5306.3 ± 0.0	0	0 <sup>-</sup>	-1	0	+1	1.470 <sup>+0.026</sup> <sub>-0.027</sub> × 10 <sup>-12</sup>	See $B_s^0$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m080.pdf">http://pdg.lbl.gov/2008/listing/s/m080.pdf</a> )
Charmed B meson <sup>[39]</sup>	$B_c^+$	$B_c^-$	$c\bar{b}$	6276 ± 4	0	0 <sup>-</sup>	0	+1	+1	(4.6 ± 0.7) × 10 <sup>-13</sup>	See $B_c^+$ decay modes ( <a href="http://pdg.lbl.gov/2008/listing/s/m091.pdf">http://pdg.lbl.gov/2008/listing/s/m091.pdf</a> )

Vector mesons												
Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c <sup>2</sup> )	J <sub>G</sub>	J <sub>PC</sub>	S	C	B'	Mean lifetime (s)		Commonly decays to (>5% of decays)
Charged rho meson <sup>[40]</sup>	$\rho^+(770)$	$\rho^-(770)$	$u\bar{d}$	$775.4 \pm 0.4$	1 <sup>+</sup>	1 <sup>-</sup>	0	0	0	$\sim 4.5 \times 10^{-24} \text{ [fs]}$		$\pi^\pm + \pi^0$
Neutral rho meson <sup>[40]</sup>	$\rho^0(770)$	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	$775.49 \pm 0.34$	1 <sup>+</sup>	1 <sup>--</sup>	0	0	0	$\sim 4.5 \times 10^{-24} \text{ [fs]}$		$\pi^+ + \pi^-$
Omega meson <sup>[41]</sup>	$\omega(782)$	Self	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	$782.65 \pm 0.12$	0 <sup>-</sup>	1 <sup>--</sup>	0	0	0	$(7.75 \pm 0.07) \times 10^{-23} \text{ [f]}$		$\pi^0 - \gamma$
Psi meson <sup>[42]</sup>	$\psi(1020)$	Self	$s\bar{s}$	$1019.115 \pm 0.020$	0 <sup>-</sup>	1 <sup>--</sup>	0	0	0	$(1.55 \pm 0.01) \times 10^{-22} \text{ [f]}$		$K^+ + K^-$ or $K_L^0 + K_L^0$ or $(\rho^+ - \rho^-) / (\pi^+ + \pi^0 + \pi^-)$
J/Psi <sup>[43]</sup>	$J/\psi$	Self	$c\bar{c}$	$3096.916 \pm 0.011$	0 <sup>-</sup>	1 <sup>--</sup>	0	0	0	$(7.1 \pm 0.2) \times 10^{-21} \text{ [f]}$		See J/Psi decay modes ( <a href="http://pdg.lbl.gov/2009/listings/m0370.pdf">http://pdg.lbl.gov/2009/listings/m0370.pdf</a> )
Upsilon meson <sup>[44]</sup>	$\Upsilon(1S)$	Self	$b\bar{b}$	$9460.30 \pm 0.26$	0 <sup>-</sup>	1 <sup>--</sup>	0	0	0	$(1.22 \pm 0.03) \times 10^{-20} \text{ [f]}$		See Upsilon decay modes ( <a href="http://pdg.lbl.gov/2009/listings/m049.pdf">http://pdg.lbl.gov/2009/listings/m049.pdf</a> )
Kaon <sup>[45]</sup>	$K^{*+}$	$K^{*-}$	$u\bar{s}$	$991.66 \pm 0.026$	$\frac{1}{2}$	1 <sup>-</sup>	1	0	0	$\sim 7.35 \times 10^{-20} \text{ [f]}$		See K <sup>*</sup> (892) decay modes ( <a href="http://pdg.lbl.gov/2008/listings/m010.pdf">http://pdg.lbl.gov/2008/listings/m010.pdf</a> )
Kaon <sup>[45]</sup>	$K^{*0}$	$\bar{K}^{*0}$	$d\bar{s}$	$899.00 \pm 0.025$	$\frac{1}{2}$	1 <sup>-</sup>	1	0	0	$(7.346 \pm 0.002) \times 10^{-20} \text{ [f]}$		See K <sup>*</sup> (892) decay modes ( <a href="http://pdg.lbl.gov/2008/listings/m010.pdf">http://pdg.lbl.gov/2008/listings/m010.pdf</a> )
D meson <sup>[40]</sup>	$D^{*+}(2010)$	$D^{*-}(2010)$	$c\bar{d}$	$2013.27 \pm 0.17$	$\frac{5}{2}$	1 <sup>-</sup>	0	+1	0	$(6.9 \pm 1.0) \times 10^{-21} \text{ [f]}$		$D^0 + \pi^+$ or $D^+ + \pi^0$
D meson <sup>[47]</sup>	$D^{*0}(2007)$	$\bar{D}^{*0}(2007)$	$c\bar{u}$	$2006.97 \pm 0.16$	$\frac{5}{2}$	1 <sup>-</sup>	0	+1	0	$> 3.1 \times 10^{-22} \text{ [f]}$		$D^0 + \pi^0$ or $D^0 + \gamma$
strange D meson <sup>[48]</sup>	$D_s^{*+}$	$D_s^{*-}$	$c\bar{s}$	$2112.3 \pm 0.5$	0	1 <sup>-</sup>	-1	+1	0	$> 3.4 \times 10^{-22} \text{ [f]}$		$D^{*+} + \gamma$ or $D^{*+} + \pi^0$
B meson <sup>[49]</sup>	$B^{*+}$	$B^{*-}$	$u\bar{b}$	$5325.1 \pm 0.5$	$\frac{5}{2}$	1 <sup>-</sup>	0	0	+1	Unknown		$R^+ + \gamma$
B meson <sup>[49]</sup>	$B^{*0}$	$\bar{B}^{*0}$	$d\bar{b}$	$5325.1 \pm 0.5$	$\frac{5}{2}$	1 <sup>-</sup>	0	0	+1	Unknown		$R^0 + \gamma$
Strange B meson <sup>[50]</sup>	$B_s^{*0}$	$\bar{B}_s^{*0}$	$s\bar{b}$	$5412.8 \pm 1.3$	0	1 <sup>-</sup>	-1	0	+1	Unknown		$H^0 + \gamma$
Charmed B meson <sup>[1]</sup>	$B_c^{*+}$	$B_c^{*-}$	$c\bar{b}$	Unknown	0	1 <sup>-</sup>	0	+1	+1	Unknown		Unknown

Various result are very near or also equal to the rest masses of following mesons:

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c <sup>2</sup> )	J <sub>G</sub>	J <sub>PC</sub>	S	C	B'	Mean lifetime (s)	Commonly decays to (>5% of decays)
Charged rho meson <sup>[40]</sup>	$\rho^+(770)$	$\rho^-(770)$	$u\bar{d}$	$775.4 \pm 0.4$	1 <sup>+</sup>	1 <sup>-</sup>	0	0	0	$\sim 4.5 \times 10^{-24} \text{ [fs]}$	$\pi^\pm + \pi^0$
Neutral rho meson <sup>[40]</sup>	$\rho^0(770)$	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	$775.49 \pm 0.34$	1 <sup>+</sup>	1 <sup>--</sup>	0	0	0	$\sim 4.5 \times 10^{-24} \text{ [fs]}$	$\pi^+ + \pi^-$
Omega meson <sup>[41]</sup>	$\omega(782)$	Self	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	$782.65 \pm 0.12$	0 <sup>-</sup>	1 <sup>--</sup>	0	0	0	$(7.75 \pm 0.07) \times 10^{-23} \text{ [f]}$	$\pi^0 + \gamma$

From:

### Phenomenological consequences of superuid dark matter with baryon-phonon coupling

Lasha Berezhiani

Max-Planck-Institut fur Physik, Fohringer Ring 6, 80805 Munchen, Germany

Benoit Famaey

Universite de Strasbourg, CNRS UMR 7550, Observatoire astronomique de Strasbourg,  
11 rue de l'Universite, F-67000 Strasbourg, France

Justin Khoury

Center for Particle Cosmology, Department of Physics and Astronomy,  
University of Pennsylvania, Philadelphia PA 19104, USA

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Using (22) this translates to an upper bound on the mass  
of the DM particle:

$$m \lesssim 4.2 \left( \frac{\sigma/m}{\text{cm}^2/\text{g}} \right)^{1/4} \text{ eV}. \quad (24)$$

Smaller and less massive galaxies result in a somewhat  
weaker bound.

The bound (24) on the DM particle mass is the main  
result of this Section. It shows that for values of  $\sigma/m$   
satisfying the merging-cluster bound  $\sim 1 \text{ cm}^2/\text{g}$  [85–88],  
 $m$  must be somewhat below 4 eV. The dependence on the  
cross section is rather weak, however, scaling as the 1/4  
power. It should be mentioned that the upper bound (24)  
would be somewhat tighter had we assumed a  $\rho \propto r^{-2}$   
transition density profile outside the superfluid core, in-  
stead of  $\rho \propto r^{-3}$ .

From J. Polchinski “String Theory Vol II”:

#### *Useful facts for grand unification*

The exceptional group  $E_8$  is connected to the groups appearing in grand  
unification through a series of subgroups. This will play a role in the com-

Table 11.3. Dimensions and Coxeter numbers for simple Lie algebras.

	$SU(n)$	$SO(n), n \geq 4$	$Sp(k)$	$E_6$	$E_7$	$E_8$	$F_4$	$G_2$
$\dim(g)$	$n^2 - 1$	$n(n-1)/2$	$2k^2 + k$	78	133	248	52	14
$h(g)$	$n$	$n-2$	$k+1$	12	18	30	9	4

pactification of the heterotic string, and so we record without derivation the necessary results.

The first subgroup is

$$E_8 \rightarrow SU(3) \times E_6 . \quad (11.4.23)$$

We have not described  $E_6$  explicitly, but the reader can reproduce this and the decomposition (11.4.24) from the known properties of spinor representations, as well as the further decomposition of the  $E_6$  representations in table 11.4 (exercise 11.5). In simple compactifications of the  $E_8 \times E_8$  string, the fermions of the Standard Model can all be thought of as arising from the **248**-dimensional adjoint representation of one of the  $E_8$ s. It is therefore interesting to trace the fate of this representation under the successive symmetry breakings. Under  $E_8 \rightarrow SU(3) \times E_6$ ,

$$\mathbf{248} \rightarrow (\mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{78}) + (\mathbf{3}, \mathbf{27}) + (\bar{\mathbf{3}}, \bar{\mathbf{27}}) . \quad (11.4.24)$$

That is, the adjoint of  $E_8$  contains the adjoints of the subgroups, with half the remaining 162 generators transforming as a triplet of  $SU(3)$  and a complex **27**-dimensional representation of  $E_6$  and half as the conjugate of this. Further subgroups are shown in table 11.4. The first three subgroups correspond to successive breaking of  $E_6$  down to the Standard Model group through smaller grand unified groups; the fourth is an alternate breaking pattern.

It is a familiar fact from grand unification that precisely one  $SU(3) \times SU(2) \times U(1)$  generation of quarks and leptons is contained in the **10** plus **5** of  $SU(5)$ . Tracing back further, we see that a generation fits into the single representation **16** of  $SO(10)$ , together with an additional state **1**<sub>-5</sub>. This extra state is neutral under  $SU(5)$ , and so under  $SU(3) \times SU(2) \times U(1)$ , and can be regarded as a right-handed neutrino. Going back to  $E_6$ , the **27** contains the 15 states of a single generation plus 12 additional states. Relative to  $SU(5)$  unification,  $SO(10)$  and  $E_6$  are more unified in the sense that a generation is contained within a single representation, but less economical in that the representation contains additional unseen states as well. In fact, the latter may not be such a

Table 11.4. Subgroups and representations of grand unified groups.

$E_6 \rightarrow SO(10) \times U(1)$
$\mathbf{78} \rightarrow \mathbf{45}_0 + \mathbf{16}_{-3} + \bar{\mathbf{16}}_3 + \mathbf{1}_0$
$\mathbf{27} \rightarrow \mathbf{1}_4 + \mathbf{10}_{-2} + \mathbf{16}_1$
$SO(10) \rightarrow SU(5) \times U(1)$
$\mathbf{45} \rightarrow \mathbf{24}_0 + \mathbf{10}_4 + \bar{\mathbf{10}}_{-4} + \mathbf{1}_0$
$\mathbf{16} \rightarrow \mathbf{10}_{-1} + \bar{\mathbf{5}}_3 + \mathbf{1}_{-5}$
$\mathbf{10} \rightarrow \mathbf{5}_2 + \bar{\mathbf{5}}_{-2}$
$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$
$\mathbf{10} \rightarrow (\mathbf{3}, \mathbf{2})_1 + (\bar{\mathbf{3}}, \mathbf{1})_{-4} + (\mathbf{1}, \mathbf{1})_6$
$\bar{\mathbf{5}} \rightarrow (\bar{\mathbf{3}}, \mathbf{1})_2 + (\mathbf{1}, \mathbf{2})_{-3}$
$E_6 \rightarrow SU(3) \times SU(3) \times SU(3)$
$\mathbf{78} \rightarrow (\mathbf{8}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{8}) + (\mathbf{3}, \mathbf{3}, \mathbf{3}) + (\bar{\mathbf{3}}, \bar{\mathbf{3}}, \bar{\mathbf{3}})$
$\mathbf{27} \rightarrow (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) + (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})$

difficulty. To see why, consider the decomposition of the **27** of  $E_6$  under  $SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO(10) \subset E_6$ :

$$\begin{aligned} \mathbf{27} \rightarrow & (\mathbf{3}, \mathbf{2})_1 + (\bar{\mathbf{3}}, \mathbf{1})_{-4} + (\mathbf{1}, \mathbf{1})_6 + (\bar{\mathbf{3}}, \mathbf{1})_2 + (\mathbf{1}, \mathbf{2})_{-3} \\ & + [\mathbf{1}_0] \\ & + [(\bar{\mathbf{3}}, \mathbf{1})_2 + (\mathbf{3}, \mathbf{1})_{-2}] + [(\mathbf{1}, \mathbf{2})_{-3} + (\mathbf{1}, \mathbf{2})_3] + [\mathbf{1}_0]. \end{aligned} \quad (11.4.25)$$

The first line lists one generation, the second the extra state appearing in the **16** of  $SO(10)$ , and the third the additional states in the **27** of  $E_6$ . The subset within each pair of square brackets is a real representation of  $SU(3) \times SU(2) \times U(1)$ . The significance of this is that for a real representation  $r$ , the  $CPT$  conjugate also is in the representation  $r$ , and so the combined gauge plus  $SO(2)$  helicity representation for the particles plus their antiparticles is  $(r, +\frac{1}{2}) + (r, -\frac{1}{2})$ . This is the same as for a massive spin- $\frac{1}{2}$  particle in representation  $r$ , so it is consistent with the gauge and spacetime symmetries for these particles to be massive. In the most general invariant action, all particles in [ ] brackets will have large (of order the unification scale) masses. It is notable that for any of the **10** + **5** of  $SU(5)$ , the **16** of  $SO(10)$ , or the **27** of  $E_6$ , the natural  $SU(3) \times SU(2) \times U(1)$  spectrum is precisely a standard generation of quarks and leptons.

In this paper we have considered the number 16, fundamental in string theory (see above reference) and 0,5 i.e. 1/2, also important in Number Theory. Indeed:

We take  $n = 16$  in the already previously analyzed formula, and developing, we obtain:

$$-\left[ \frac{\exp\left(\pi\sqrt{\frac{16}{6} - \frac{1}{144}}\right)}{2\sqrt{16 - \frac{1}{24}}} + \frac{\exp\left(\frac{\pi}{2}\sqrt{\frac{\pi}{6} - \frac{1}{144}}\right)}{\sqrt{16 - \frac{1}{24}}} \right]$$

the value  $-21.79216$  (that is the coefficient).

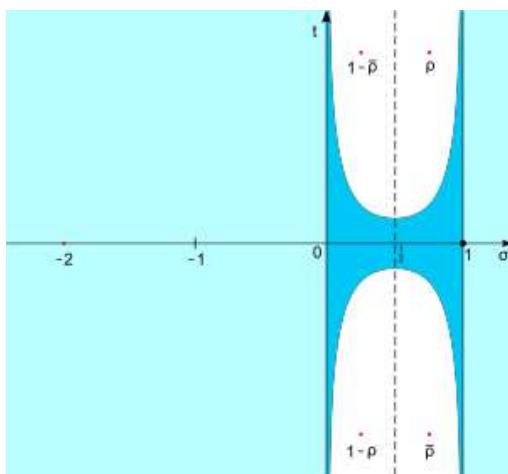
Thence  $q = \text{coefficient} * -e^{-t}$ ; for  $t = 1/2 = 0.5$ ,  $q = (-e^{-0.5}) * -21.79216$  for each  $q$ .

For example:  $q^5 = ((-e^{-0.5}) * -21.79216)^5$  and so on.

With regard  $1/2 = 0.5$  we remember that:

“The Riemann hypothesis, asserts that any non-trivial zero  $s$  has  $\operatorname{Re}(s) = 1/2$ . In the theory of the Riemann zeta function, the set  $\{s \in \mathbb{C} : \operatorname{Re}(s) = 1/2\}$  is called the **critical line**.”

“Apart from the trivial zeros, the Riemann zeta function has no zeros to the right of  $\sigma = 1$  and to the left of  $\sigma = 0$  (neither can the zeros lie too close to those lines). Furthermore, the non-trivial zeros are symmetric about the real axis and the line  $\sigma = 1/2$  and, according to the Riemann hypothesis, they all lie on the line  $\sigma = 1/2$ .”



## Ramanujan original last letter

(Ramanujan's last letter to Hardy, featuring examples of mock modular forms. Credit: Ken Ono)

If we consider a  $\theta$ -function, i.e.  
the transformed form Eulerian, e.g.

$$(A) \quad 1 + \frac{v}{(1-v)^2} + \frac{v^4}{(1-v)^2(1-v^2)^2} + \frac{v^9}{(1-v)^2(1-v^2)^2(1-v^4)^2}$$

$$(B) \quad 1 + \frac{v}{1-v} + \frac{v^4}{(1-v)(1-v^2)} + \frac{v^7}{(1-v)(1-v^2)(1-v^4)}$$

and consider determine the nature of  
the singularities at the points  $v=1$ ,  
 $v^2=1$ ,  $v^3=1$ ,  $v^4=1$ , ... we know how  
beautifully the asymptotic nature  
form of the function can be expressed  
in a very neat and closed form ex-  
ponential form. For instance

when  $v = e^{-t}$  and  $t \rightarrow 0$

$$(A) = \sqrt{\frac{t}{2\pi}} e^{\frac{7t^2}{6} - \frac{5t^4}{24}} + o(1)$$

$$(B) = \frac{e^{\frac{7t^2}{6} - \frac{5t^4}{60}}}{\sqrt{\frac{t}{e-1}}} + o(1)$$

and similar results at other singu-  
larities. It is not necessary that  
there should be only one term left this.  
There may be many terms but the number  
of terms must be finite. Also  $o(1)$   
may turn out to be  $O(1)$ . That is all.  
For instance when  $v \rightarrow 1$  the function

$$\left[ (1-v)(v)(1-v^2) \dots \right]^{1/20}$$

is equivalent to the sum of five  
terms like (\*) together with  $O(1)$  in-  
stead of  $o(1)$ .

If we take a number of functions  
like (A) and (B) it is only in a limited  
number of cases the terms close as  
above; but in the majority of cases they  
never close as above. For instance

when  $v = e^{-t}$  and  $t \rightarrow 0$

$$(C) \quad 1 + \frac{v}{(1-v)^2} + \frac{v^4}{(1-v)^2(1-v^2)^2} + \frac{v^9}{(1-v)^2(1-v^2)^2(1-v^4)^2}$$

$$= \sqrt{\frac{t}{2\sqrt{5}}} e^{\frac{7t^2}{6} + a_1 t + a_2 t^2 + \dots + o(a_n t^n)}$$

where  $a_1 = \frac{1}{8\sqrt{5}}$ , and so on.

The function (2) is a simple function  
belonging in one example of a) enclosed  
form at the singularities.

If the coefft.  $\frac{t_0}{e}$  in the index of  $e$   
happens to be  $\frac{\pi i}{n}$  in the particular  
case. It may be some other transcen-  
dental numbers in other cases.

The coeffts. of  $t_1, t_2, \dots$  happen to  
be  $\frac{1}{n}, \dots$  in this case. In other cases  
they may turn out to be some other  
algebraic numbers.

Now a very interesting question  
arises. Is the converse of the state-  
ments concerning the forms (A)  
and (B) true? That is to say

Suppose there is a function in the  
Eulerian form and suppose that  
all or an infinity of points  $e^{\frac{2\pi i m}{n}}$   
are exponential singularities and  
also suppose that all these points  
to be asymptotic form of the function  
closed as neatly as in the cases of  
(A) and (B). The question is: - is the  
function taken ~~as~~ the sum  
of two functions one of which is  
an ordinary  $\Omega$ -function and the  
other a (transcendental) function which  
is  $O(1)$  at all the points  $e^{\frac{2\pi i m}{n}}$ ?

The answer is it is not necessarily so.  
When it is not so I call the function  
Mock  $\Omega$ -function. I have not proved  
rigorously that it is not necessarily  
so. But I have constructed a number  
of examples in which it is not im-  
conceivable to construct a  $\Omega$  func-  
tion to cut out the singularities

(3)

of the original function. Also I have shown if it is necessarily so that it leads to the following assertion; — viz. it is possible to construct two power series in  $x$  namely  $\sum a_n x^n$  and  $\sum b_n x^n$  both of which have essential singularities on the unit circle, while  $\sum a_n x^n$  exists inside the unit circle and  $\sum b_n x^n$  one pair convergent when  $|x|=1$ , and tend to finite limits at every point  $x = e^{2i\pi k/p}$  and that at the same time the limit of  $\sum a_n x^n$  at the point  $x = e^{2i\pi k/p}$  is equal to the limit of  $\sum b_n x^n$  at the point  $x = e^{-2i\pi k/p}$ .

This assertion seems to be untrue. Any how we shall go to the examples and see how far our assertions are true.

I have proved that if

$$f(z) = 1 + \frac{z}{(1+z)^2} + \frac{z^4}{(1+z)^2(1+z^2)^2} + \dots$$

$$\text{then } f(z) + (1-z)(1-z^3)(1-z^5)\dots (1-2z+2z^4 - 2z^6 + \dots)$$

$$\text{at all the points } z = 0(1)$$

$$\text{at all the points } z = -1, z^3 = -1, z^5 = -1, \dots$$

$$\text{and at the same time}$$

$$f(z) \underset{z=0}{\cancel{\rightarrow}} (1-z)(1-z^3)(1-z^5)\dots (1-2z+2z^4)$$

$$= O(1)$$

$$\text{at all the points } z = -1, z^3 = -1, z^5 = -1, \dots$$

$$\text{Also obviously } f(z) = O(1)$$

$$\text{at all the points } z = 1, z^3 = 1, z^5 = 1, \dots$$

where  $v = -e^{-t}$  and  $t \rightarrow 0$

$$f(v) + \sqrt{\frac{\pi}{2}} e^{\frac{\pi i v^2}{8}} - \frac{v^2}{24} \rightarrow 1.$$

The coefft of  $v^n$  in  $f(v)$  is

$$(-1)^{n-1} \frac{e^{\pi i \frac{v^2}{8} - \frac{n\pi}{4}}}{2\sqrt{n-\frac{1}{24}}} + O\left(\frac{\pi \sqrt{\frac{v^2}{8} - \frac{n\pi}{4}}}{\sqrt{n-\frac{1}{24}}}\right)$$

It is inconceivable that a single  $v$  function could be found to cut out the singularities of  $f(v)$ .

### Mock $\mathcal{D}$ -functions

$$\phi(v) = 1 + \frac{v}{1+v} + \frac{v^4}{(1+v)(1+2v)} + \dots$$

$$\psi(v) = \frac{v}{1-v} + \frac{v^4}{(1-v)(1-v)} + \frac{v^7}{(1-v)(1-v^2)(1-v^3)} + \dots$$

These are

$$\chi(v) = 1 + \frac{v}{1-v+2v^2} + \frac{v^4}{(1-v+2v^2)(1-v^2+2v^3)}$$

These are related to  $f(v)$  as shown below.

$$2\phi(-v) - f(v) = f(v) + \psi(-v)$$

$$= \frac{1-2v+2v^4-2v^7}{(1+v)(1+v^2)(1+v^3)} + \dots$$

There are of the 3rd order

Mock  $\mathcal{D}$ -functions of 5th order

$$f(v) = 1 + \frac{v}{1+v} + \frac{v^4}{(1+v)(1+v^2)} + \frac{v^9}{(1+v)(1+v^2)(1+v^3)} + \dots$$

$$\phi(v) = 1 + v(1+v) + v^4(1+v)(1+v^2) + v^9(1+v)(1+v^2)(1+v^3) + \dots$$

$$\psi(v) = v + v^2(1+v) + v^6(1+v)(1+v^2) + v^{10}(1+v)(1+v^2)(1+v^3) + \dots$$

$$\chi(v) = 1 + \frac{v}{1-v} + \frac{v^4}{(1-v^2)(1-v^4)} + \frac{v^9}{(1-v^4)(1-v^6)(1-v^8)} + \dots$$

$$= 1 + \left\{ \frac{v}{1-v} + \frac{v^4}{(1-v^2)(1-v^4)} + \frac{v^9}{(1-v^4)(1-v^6)(1-v^8)} \right\} + \dots$$

$$F(v) = 1 + \frac{v^2}{1-v} + \frac{v^8}{(1-v)(1-v^2)} + \dots \quad (5)$$

$$\phi(-v) + X(v) = 2 F(v).$$

$$f(v) + 2 F(v^2) - 2 = \phi(-v^2) + \psi(v)$$

$$= 2 \phi(-v) - f(v) = \frac{1 - v + 2v^4 - 2v^8 + \dots}{(1-v)(1-v^2)(1-v^4)(1-v^8)}.$$

$$\psi(v) - F(v) + 1 = v \cdot \frac{1 + v + v^6 + v^{12} + \dots}{(1-v^2)(1-v^4)(1-v^8)}.$$

Muck or fractions (of 7th order)

$$f(v) = 1 + \frac{v^2}{1-v} + \frac{v^6}{(1-v)(1+v)} + \frac{v^{12}}{(1+v)(1+v^2)} + \dots$$

$$\phi(v) = v + v^6(1+v) + v^8(1+v)(1+v^2) + \dots$$

$$\psi(v) = 1 + v(1+v) + v^6(1+v)(1+v^2) + \dots$$

$$X(v) = \frac{1}{1-v} + \frac{v^2}{(1-v^2)(1-v)} + \frac{v^8}{(1-v^8)} + \dots$$

$$F(v) = \frac{1}{1-v} + \frac{v^4}{(1-v)(1-v^2)} + \frac{v^{12}}{(1-v)(1-v^2)(1-v^4)}$$

have polynomials relations (of 7th order)

Muck or fractions (of 7th order)

$$(i) 1 + \frac{v^2}{1-v^2} + \frac{v^4}{(1-v^2)(1-v^4)}$$

$$(ii) \frac{v}{1-v} + \frac{v^4}{(1-v)(1-v^2)} + \frac{v^8}{(1-v^2)(1-v^4)(1-v^8)}$$

$$(iii) \frac{1}{1-v} + \frac{v^2}{(1-v^2)(1-v)} + \frac{v^6}{(1-v^2)(1-v^4)(1-v^8)}$$

These are not related to each other.

Ever yours sincerely  
S. P. —————

## Appendix B

We have also:

From:

Input interpretation:  
$$-1.0058343895 \times 10^{-12} - 5.74968 \times 10^{-40} - 1.08663428 - 0.081816 - 0.07609064 + 0.92391 - 0.0814135 - 1.00615716 + 0.9243408$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

-0.48386078000100583438950000000000000000000574968

$$\frac{1}{e} * [[[(-(0.48386078) * (-4.92906 * 10^6) * (4.04437 * 10^{14}) + (3.07735 * 10^{13}) + (-2498.279) + (33021.10) + (-2122.186) + (1.63161 * 10^{20}) - (9.39267 * 10^{17}) - (-4267.24) - (6.596086 * 10^{20}))]]^5$$

Input interpretation:

$$\frac{1}{e} \left( -(-4.92906 \times 10^6) \times (-0.48386078) - 4.04437 \times 10^{14} + 3.07735 \times 10^{13} - 2498.279 + 33021.10 - 2122.186 + 1.63161 \times 10^{20} - 9.39267 \times 10^{17} - -4267.24 - 6.596086 \times 10^{20} \right)^5$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$-1.11990... \times 10^{103}$

$(16^2 - 12) * \sqrt{-(-1.11990 \times 10^{103})}$

Input interpretation:

$$(16^2 - 12) \sqrt{-(-(1.11990 \times 10^{103}))}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

$8.16544... \times 10^{53}$

$$[[[-(0.48386078)*(-4.92906*10^6)-(4.04437*10^{14})+(3.07735*10^{13})+(-2498.279)+(33021.10)+(-2122.186)+(1.63161*10^{20})-(9.39267*10^{17})-(-4267.24)-(6.596086*10^{20}))]]^4 * 4267.24$$

Input interpretation:

$$\left( -(-4.92906 \times 10^6) \times (-0.48386078) - 4.04437 \times 10^{14} + 3.07735 \times 10^{13} - 2498.279 + 33021.10 - 2122.186 + 1.63161 \times 10^{20} - 9.39267 \times 10^{17} - -4267.24 - 6.596086 \times 10^{20} \right)^4 \times 4267.24$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

- $2.6117140794551119094178581348892784715227931187749319\dots \times 10^{86}$

In conclusion:

$$(0.08)^2 * [[[(-(0.48386078)*(-4.92906*10^6)-(4.04437*10^{14})+(3.07735*10^{13})+(-2498.279)+(33021.10)+(-2122.186)+(1.63161*10^{20})-(9.39267*10^{17})-(-4267.24)-(6.596086*10^{20})))]]^4 * 4267.24)$$

Input interpretation:

$$0.08^2 \left( \left( -(-4.92906 \times 10^6) \times (-0.48386078) - 4.04437 \times 10^{14} + 3.07735 \times 10^{13} - 2498.279 + 33021.10 - 2122.186 + 1.63161 \times 10^{20} - 9.39267 \times 10^{17} - -4267.24 - 6.596086 \times 10^{20} \right)^4 \times 4267.24 \right)$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

Result:

More digits

- $1.6714970108512716220274292063291382217745875960159564\dots \times 10^{84}$

## References

Wikipedia

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<https://www.imsc.res.in/~rao/ramanujan/newnow/lastletter.pdf>