Some Problems About The Gravitational Wave

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Abstract - We think about the contribution of the dipole moment whether it is useful or not. In this article, we will see that the contribution of the dipole moment nonzero. In addition, we also discuss the possibility of gravitational waves escaping the black hole.

Keywords: gravitational wave, binary system, black hole

I. Introduction

Recently, gravitational waves have been detected [1-5] that the long-term predictions of gravitational waves have been proven. In general relativity, gravitational waves are the ripples of spacetime. The gravity described by Einstein's general relativity is a phenomenon induced by the curvature of spacetime [6-11]. In principle, mass and energy can cause this curvature [6-11]. As the massive body moves in space, the nearby spacetime curvature changes. The change in curvature induced by such movement produces a wave propagating at the speed of light. This phenomenon is that gravitational waves propagate outward from the position of the massive body. The amplitude of the gravitational wave will decrease as the propagation distance increases, and the frequency of the gravitational wave will perform red-shift or the Doppler’s effect due to the expansion of the universe. Gravitational waves even focus at a strong gravitational field and also exhibit diffraction behavior.

Because the interaction between gravitational waves and matter is very weak, it can truly transmit valuable information from the far source to the Earth. The study of the gravitational wave collects the data from gravitational-wave sources, such as the binary systems of white dwarfs, neutron stars, and black holes. In 1974, the Hulse–Taylor binary pulsar was discovered [6]. When the binary star system revolves around each other, they gradually lose energy and approach each other because they continuously emit gravitational waves. This phenomenon provides the first indirect evidence for the existence of the gravitational waves. In the later, scientists used the gravitational-wave detectors to observe the gravitational wave, such as the LIGO's laser interference instrument at the gravitational wave observatory [6,7,10].

It is traditionally believed that gravitational waves are generated from asymmetric movements which causes the quadrupole moments changing with time [6-11]. The general statement is that the gravitational waves can be generated as long as the shape of a system changes during motion. The two celestial bodies moving around each other can also produce gravitational waves as mentioned previously. The higher the asymmetry of a system is or the higher the speed of motion is, the stronger the gravitational waves it emits. However, we have to think about the contribution of the dipole moment whether it is useful or not. In this article, we will see that the contribution of the dipole moment nonzero. In addition, we also discuss the possibility of the gravitational waves escaping the black hole.

II. The Changes of The Spacetime Of The Black Hole

Considering the first example an object of mass Δm is very close to the black hole, and the surrounding space and time is denoted as Spacetime 1, as shown in Figure 1(a). When this object is inhaled by the black hole and disappears outside the event horizon, the spacetime changes to Spacetime 2, and it is redistributed with the center of the black hole as the gravitational center, instead of staying at the Spacetime 1 before the object enters the black hole, as shown in Figure 1 (b). When more objects are close to the black hole and are attracted by its gravity, the space and time will be redistributed with the center of the black hole as the gravitational center again [1,2]. Then it is a new spacetime, Spacetime 3, as shown in Figure 1(c). Whenever objects enter the black hole, the mass of the black hole changes, and the spacetime outside the horizon is also changed with the center of the black hole as the gravitational center at the same time. If no gravitational waves escape from the inside of a black hole, how does the black hole change the surrounding spacetime? This makes us feel unsatisfied that gravitational waves cannot leave the black hole to the space outside its event horizon. Without gravitational waves escaping the black hole, how does the outside spacetime reconstruct? Therefore, the gravitational waves have to be able to spread out from the inside of a black hole to the outer space, as shown in Figure 1(d). The black hole reconstructing the spacetime in the external space is not instantaneous, and it has to take a certain time to reach the observation point. The time for affecting the observation point is determined by the speed of the gravitational wave. That is, the change of the spacetime depends on the speed of the gravitational wave. Hence, by the gravitational waves, the spacetime away from the black hole can be influenced [6].

Another example is to consider black holes in motion. We know that many black holes are located in the centers of the galaxies. According to the theory of the universal expansion, the galaxies are leaving away from each other, and the black holes in the galaxies are also moving in space. The moving black hole also causes the changes in spacetime as shown in Figure 1(e). If there is no gravitational wave spreading from the inside of a black hole, how does this moving black hole reconstruct new spacetime in space?

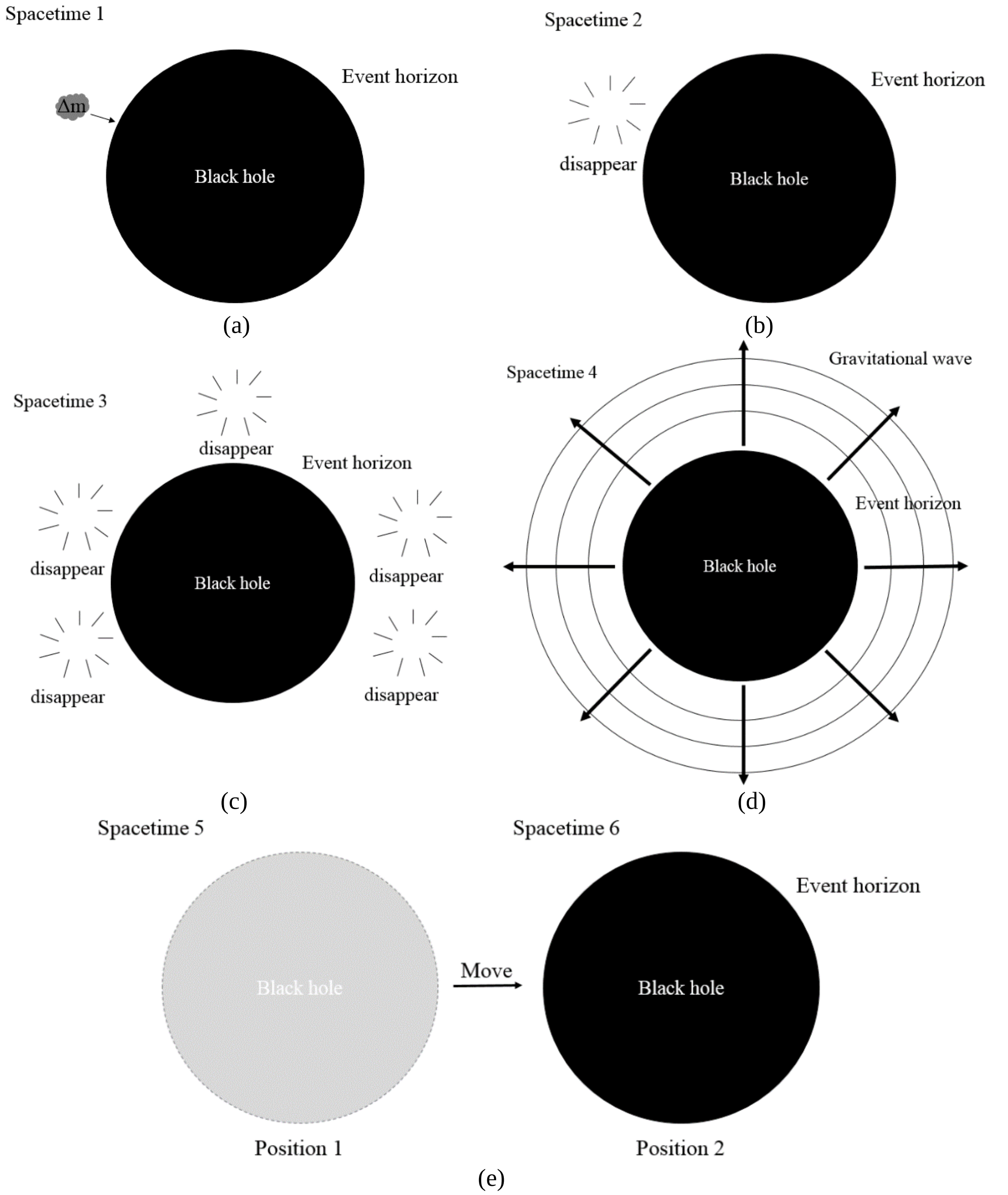

Figure 1. (a) Spacetime before something enters into the black hole. (b) Spacetime after something enters into the black hole without gravitational wave escaping from it. (c) Spacetime after a lot of things enter into the black hole without the gravitational wave escaping it. (d) The spacetime 4 after something enters into the black hole with the gravitational wave escaping from it. (e) The spacetime 5 before the movement of the black hole and the spacetime 6 after the movement.

III. The Equation For Producing Gravitational Waves

The gravitational wave in the nearly flat spacetime has been discussed in many textbooks, and it is a mathematical formula to prove the previous statements. The gravitational wave evaluated at the retarded time *t-r* is [6]

$$\phi^{kl}(t,\vec{x}) = -\left[\frac{\kappa}{8\pi r}\frac{\partial^2}{\partial t^2}\int \rho(\vec{x}')\, x'^k x'^l dx'^3\right]_{t-r}, \qquad (1)$$

where k, l=1,2,3. It can be rearranged as

$$\phi^{kl}(t,\vec{x}) = -\frac{\kappa}{8\pi r}\frac{1}{3}\frac{\partial^2}{\partial t^2}\left[Q^{kl} + \delta_k^l \int r'^2 \rho(\vec{x}')dx'^3\right]_{t-r}, \tag{2}$$

and

$$Q^{kl} = \int \left(3x'^k x'^l - r'^2 \delta_k^l\right)\rho(\vec{x}')dx'^3. \tag{3}$$

The gravitational wave has two transverse polarizations, and one expressed in terms of tensor is

$$\epsilon_{\oplus}^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

and the other is

$$\epsilon_{\otimes}^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{5}$$

At the same time, the tensor δ_k^l is

$$\delta_k^l = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

Actually, the inner product of two tensors δ_k^l and $\epsilon_{\oplus}^{kl}$ is nonzero, and that is,

$$\langle \delta_k^l | \epsilon_{\oplus}^{kl} \rangle = \epsilon_{\oplus}^{kl}, \tag{7}$$

so the second term in Eq. (2) clearly tells us that the change of mass can also induce the gravitational wave.

IV. The Cases Of The Symmetrical Change In Mass

The other case can be an explicit evidence for the second term in Eq. (2). It has been mentioned that no gravitational energy is emitted when a supernova produces a completely symmetric stellar collapse [10]. If so, then there exists some contradiction. First, we think about the movement of a single electron with mass of m_e. Can this electron radiate gravitational wave? If it wouldn't be, then the gravitation field established by it will be independent of its movement and position. The gravitational field established by this electron was a very long time ago since it appeared in the universe. It is maybe at ten billion light years away from the Earth. However, as we know, it is not true because the gravitational field depends on the position and movement of the electron, and so does the gravitational wave. Even single electron, it can radiate gravitational wave as long as it moves. Otherwise, its movement cannot affect the surrounding gravitational field because of no gravitational radiation.

For an example of a symmetrically explosive supernova, we consider a point A inside and the other point B outside this star before the supernova explosion. When the supernova goes off, the radius of the star changes from r_o to r_i as shown in Fig. 2. Due to supernova explosion, the mass of the star is changed, and the position of point A becomes outside the star. Obviously, the gravitational fields at the two observation points are significantly different before and after supernova explosion. This change in the gravitational field, and the change in the spacetime structure, comes from the change of the star mass. This change is not instantaneous and must be delivered by gravitational waves to these two observation points. Therefore, a symmetrically explosive supernova also generates gravitational waves in order to make the variation of the surrounding spacetime structure due to the change of mass.

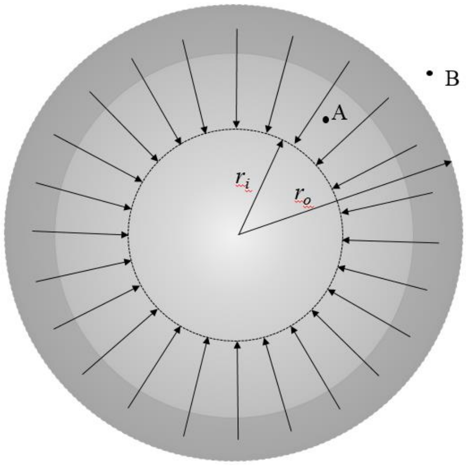

Figure 2. The radius changes during supernova explosion. Two points A and B experiences different gravitational fields respectively before and after supernova explosion.

Another example is to consider the planets in the solar system. When the mass of a certain planet symmetrically increases in a very short time, the radius increases from r_1 to r_2 at the same time as shown in Fig. 3. After that, the earth will feel the gravitational field changes originated from this planet. However, this change is not instantaneous, and it exists a time delay. The change in the gravitational field or the spacetime structure transmitted by gravitational waves is due to the increase in mass of this planet. The delivered time from the planet to the earth is the propagation time of the gravitational waves.

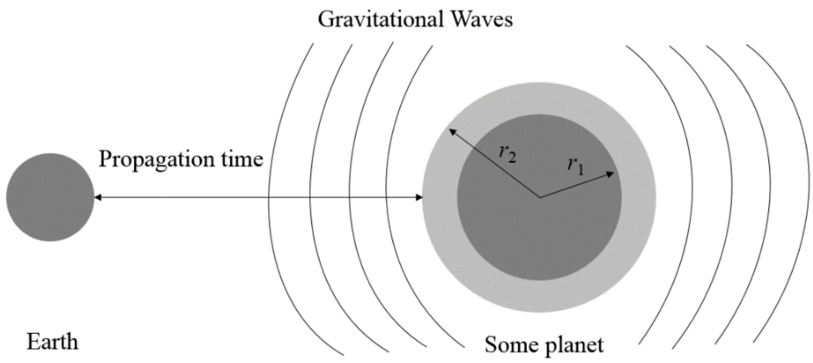

Figure 3. Gravitational wave propagating from a mass-increased planet to the Earth. The delay time exists and such case is an example in Eq. (2) where the second term is useful.

V. Conclusion

About the generation of gravity wave, we first discuss whether the gravitational wave can escape the black hole or not from the viewpoint of the absorption by the black hole. If the gravitational wave cannot escape the black hole, the space-time structure around the black hole does not change although the mass of the black hole changes. Moreover, we know that the most black holes are located in the center of the galaxies, and it is well-known the redshift phenomenon that the galaxies are moving away from the Earth. This also means that the black hole is in moving. If the moving black hole cannot radiate gravitational waves, the space-time structure around it cannot be changed. In addition, from the example of a symmetrical supernova burst, it logically proves that the symmetrical change in mass can produce gravitational waves, which affects the surrounding space-time structure. Another example is the symmetrical increase in mass of a planet in the solar system. It always exists a delay time for the gravitational wave influencing the earth. This delay time is the fact that the change of the space-time structure delivering by the gravitational wave is not instantaneous. Without the information transmitted by the gravitational wave, the earth will not feel the change of the gravity. These are the examples of the generation of gravitational waves based on the Einstein’s general relativity.

We also derive the formula of gravitational waves generating from the weak gravity sources and review the radiation mechanism of gravitational wave. Not only the quadrupole radiation, but the gravitational waves can also be generated as long as the mass changes no matter it is symmetrical or not. Even more, the gravitational waves can be generated by moving massive objects, because the corresponding space-time structure changes depending on the position of the object. For example, the Earth's gravitational field is different for the sun at the near and far points, and the space-time structure is also slightly different at these two points.

Indeed, gravitational waves are generated as long as the position or mass of the object changes. The gravitational waves are not only from the merge of a binary black holes, but also from the motion of a single black hole. The gravitational wave consists in transmitting information about the change of the spacetime structure. It also coincides with the results of our logical deduction, which proves that the first term in the generation of the gravitational wave is not from the quadrupole source.

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