# $E=mc^2$ Dimensional Descriptor Usage Error

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**Abstract –** $E=mc^2$ is Einstein's mass-energy equation. It is an algebraic equation that uses letters to represent single or composite numbers. Each number type on the right side of the equal sign, mass, distance and a unit time duration are defined as System International basic units. As described, they correctly represent a dimension type. Over a centuries ago, the unit time duration dimension, the second, was improperly used to represent the time duration for the longest wavelength an electromagnetic wave can have and it would have a frequency of "one". As a result, all current physical law equations that contain a value for electromagnetic wave frequency or the speed of light are using a non-linear scale.

## Introduction

The length of the time duration that establishes the base frequency (**f**) used for various cyclic phenomenon, such as sound waves, electromagnetic (EM) waves and tidal waves had been determined centuries ago and currently are never questioned whether they had been properly determined. The hertz (Hz) is the derived unit of frequency in the International System of Units (SI) and is defined as one cycle per second. How frequency is actually defined for various cyclic phenomena is not defined by SI. Some frequencies are relatively easy to establish and reasonably easy to correct.

Mathematically, frequency is defined as $\mathbf{f} = 1/T$, where T is the longest duration of time to encompass the longest wave of a cyclic process. For EM waves, T is the SI second. EM frequency used in conjunction with an EM wavelength, provides a value for the speed of light (**c**)**.** Many physical law equations (PLE) include **f** and **c**. The velocity of light is presented by the formula $\mathbf{c} = \mathbf{f}\,\boldsymbol{\lambda}$, where $\boldsymbol{\lambda}$ is the wavelength. An improperly defined time duration presents a problem for physical law equations that include **f** or **c**.

The method for determining the base frequency of a cyclic phenomenon is established by observing the longest wave duration and using that time duration as the starting frequency with a value of 1. For a wave phenomena produced by nature, it is necessary to constantly measure the waves to determine the longest period. The accuracy of the longest wave determination is dependent upon the detection and time measurement instruments that were available at the time a cyclic frequency is established.

Several centuries ago, the second was the shortest time period where instruments were available to provide a reasonably accurate measurement for that small of time segment. The original duration of the second was based upon the segment of $1/86{,}400^{th}$ rotation of the Earth. The precision of the duration of the second was improved in 1950 by ephemeris time. The current System International second is based upon the duration of the ephemeris second even though it is described as being the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

## Longest Electromagnetic Wave

How did the early researchers of the EM phenomenon determine that the duration of the second encompassed the longest wavelength that EM waves could have? They did not. The earliest researchers could not have created or identified the longest possible EM wave nor can we do that today. Selecting an improper time duration for the longest EM wave has consequences because that will result in a nonlinear frequency scale if there are EM waves longer than will fit in a 1 second time duration.

Magnetotelluric transmitters are producing frequencies below 1 Hz and the frequencies are being identified by using a decimal notation, 0.100, 0.0100, 0.00100 Hz etc. This means we have a non-linear EM frequency scale. It appears that all contemporary PLE that contain **f** or **c** as a value are based upon the assumption that the EM frequency scale was linear.

It is not known why the process that determined the longest time period of a propagating EM wave was not examined by other individuals many decades earlier. Over the years, one might think that those individuals that developed a PLE that uses EM frequency, directly or indirectly, as one of the parameters would have examined the validity of that and all the other dimensional descriptors used.

There is no compelling reason to change the current frequency scale for general public use but a nonlinear scale effects the validity of every PLE that uses **f** or **c**. Arbitrarily increasing the length of the time duration to establish a longer EM wavelength than we can currently detect or generate does not provide a real solution, that just masks the non-linear frequency scale for some future generation to solve.

## Geometric Wavelength and Frequency

What we have been taught how wavelength and frequency can be expressed mathematically is not complete. The relationship between EM wavelength and frequency can be represented by Euclidean geometry. A pair of linked right triangles can be used to represent wavelength and frequency, and the angle, representing a time duration, is common to both triangles. The tangent of the angle represents the time duration and at 45° the duration had a value of 1. A paper on the paired triangle concept was published in the Aug-Sept 2011 issue of IEEE Potentials titled "A Methodology to Define Physical Constants Using Mathematical Constants."[1] *(Methodology*) A change in the angle, represents a change in the basic time duration, and produces a direct translation for wavelength and frequency for different time durations. The *Methodology* identified that two mathematical constants can define a physical constant, the velocity of EM waves. At 45°, the value of **c** has a numeric value of $2\pi\sqrt{2}$, which would be mathematically friendly in a PLE. The concepts presented in the *Methodology* may help to find a solution for the improperly defined base frequency for EM waves. The mathematics of the *Methodology* suggests that the frequency at 45° represents the middle of the EM spectrum.

It must be noted that the *Methodology* applied a scaling factor of $10^6$ to adjust the frequency at 45° to match the contemporary frequency in SI units; this is noted in the *Methodology* section "Intrinsic units in SI units."

The *Methodology* does not identify the longest EM wavelength, but the mathematics identifies how to get closer to it.

## Conclusion

All equations that contain **c** or **f** should be examined to determine whether they are impacted by the improper determination of the longest EM wave.

It is desirable that a solution to the EM frequency scale problem preserves the relationships produced by hetereodynes, harmonics and sub-harmonics.

## Reference

[1] Makinson, F. H. IEEE, "A methodology to define physical constants using mathematical constants."IEEE Potentials Vol: 30, Issue: 4, July-Aug. 2011 Page(s)**:** 39 - 43 Date: 25 July 2011 ISSN: 0278-6648 INSPEC Accession Number: 12136007 DOI: 10.1109/MPOT.2011.940377