A simple representation for Pi

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Abstract. We recall a simple representation for Pi.

The number Pi is defined by: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.1415926535...$, in this note we recall a simple formula for Pi.

Formula

Entry 1.

$$\begin{aligned}\pi = 3+\sin 3+\sin(3+\sin 3)+\sin\left(3+\sin 3+\sin(3+\sin 3)\right)+ \\ +\sin\left(3+\sin 3+\sin(3+\sin 3)+\sin\left(3+\sin 3+\sin(3+\sin 3)\right)\right)+...\end{aligned} \tag{1}$$

Explanation

Entry 2. Iteration:

$$x_{n+1} = x_n + \sin x_n \quad , x_0 = 3 \Rightarrow x_n \to \pi \tag{2}$$

Entry 3. Convergence: if $I = [3, 3.2], F(x) = x + \sin x$, then

$$F'(x) = 1+\cos x \tag{3}$$

$$\pi \in I \tag{4}$$

$$F(I) \subset I \tag{5}$$

$$|F'(x)| < 1 \ \ \forall x \in I \tag{6}$$

$$\lambda = \max_I |F'(x)| = 1+\cos 3 < 1 \tag{7}$$

$$|x_n - \pi| \le \frac{\lambda^n}{1-\lambda}|x_0 - x_1| = (1+\cos 3)^n(-\tan 3) \quad , n = 0,1,2,3,... \tag{8}$$

Entry 4. If $a_0 = 3, x_n = a_0 + a_1 + a_2 + \ldots + a_n$, then

$$a_{n+1} = \sin\left(\sum_{k=0}^{n} a_k\right) \quad , n = 0,1,2,3,\ldots \tag{9}$$

Other Formulas

Entry 5.

$$\begin{aligned}\frac{\pi}{2} = \frac{3}{2} + \cos\left(\frac{3}{2}\right) + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right)\right) + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right) + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right)\right)\right) + \\ + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right) + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right)\right) + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right) + \cos\left(\frac{3}{2} + \cos\left(\frac{3}{2}\right)\right)\right)\right) + \ldots\end{aligned} \tag{10}$$

$$\frac{\pi}{4} = \frac{3}{4} \cdot \cot\left(\frac{3}{4}\right) \cdot \cot\left(\frac{3}{4} \cdot \cot\left(\frac{3}{4}\right)\right) \cdot \cot\left(\frac{3}{4} \cdot \cot\left(\frac{3}{4}\right) \cdot \cot\left(\frac{3}{4} \cdot \cot\left(\frac{3}{4}\right)\right)\right) \cdot \ldots \tag{11}$$

$$\begin{aligned}\frac{\pi}{4} = 1 \cdot \cot(1) \cdot \cot\left(1 \cdot \cot(1)\right) \cdot \cot\left(1 \cdot \cot(1) \cdot \cot\left(1 \cdot \cot(1)\right)\right) \cdot \\ \cdot \cot\left(1 \cdot \cot(1) \cdot \cot\left(1 \cdot \cot(1)\right) \cdot \cot\left(1 \cdot \cot(1) \cdot \cot\left(1 \cdot \cot(1)\right)\right)\right) \cdot \ldots\end{aligned} \tag{12}$$

References

1. Frank W.J. Olver , Daniel W. Lozier , Ronald F. Boisvert , and Charles W. Clark : NIST Handbook of Mathematical Functions. Cambridge University Press, 2010.
2. Borwein, J.M. and Bailey , D.H. : Mathematics by Experiment: Plausible reasoning in the 21-st century, 1st edition. A.K, Peters, 2003.