The number $v=\phi-\sqrt{\phi}$

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Abstract. This note presents some formulas related with the number:
$v=\phi-\sqrt{\phi}$, $\phi=\left(1+\sqrt{5}\right)/2$.

Resumen. Esta nota muestra algunas fórmulas relacionadas con el número:
$v=\phi-\sqrt{\phi}$, $\phi=\left(1+\sqrt{5}\right)/2$.

1. Introducción

El número $v=\phi-\sqrt{\phi}$, donde $\phi=\left(1+\sqrt{5}\right)/2$ es una raíz de la ecuación:

$$x^4-2x^3-2x^2-2x+1=0 \tag{1}$$

En esta nota recordamos algunas fórmulas relacionas con $v=\phi-\sqrt{\phi}$.

2. Fórmulas

Entry 1.

$$\frac{1}{1-2x-2x^2-2x^3+x^4}=\sum_{n=0}^{\infty} c_n\, x^n \tag{2}$$

$$c_n=2c_{n-1}+2c_{n-2}+2c_{n-3}-c_{n-4} \quad , c_0=1, c_1=2, c_2=6, c_3=18 \tag{3}$$

$$c_n=\{1,2,6,18,51,148,428,1236,3573,10326,29842,\ldots\} \tag{4}$$

$$\lim_{n\to\infty}\frac{c_n}{c_{n+1}}=v=\phi-\sqrt{\phi} \tag{5}$$

Entry 2.

$$x^4-2x^3-2x^2-2x+1= \\ =\left(x-\phi+\sqrt{\phi}\right)\left(x-\phi-\sqrt{\phi}\right)\left(x+\frac{1}{\phi}+i\sqrt{\frac{1}{\phi}}\right)\left(x+\frac{1}{\phi}-i\sqrt{\frac{1}{\phi}}\right) \tag{6}$$

Entry 3.

$$v=\phi-\sqrt{\phi}=\frac{1}{\phi+\sqrt{\phi}}=\sqrt{\frac{\sqrt{\phi}-1}{\sqrt{\phi}+1}} \tag{7}$$

Entry 4.

$$\pi = 8\tan^{-1} v + 4\tan^{-1} v^2 \tag{8}$$

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\left(2v^{2n+1} + v^{4n+2}\right) \tag{9}$$

Entry 5.

$$\pi = 8\sum_{n=0}^{\infty}\sum_{k=0}^{n} \binom{2k}{k}\binom{2n}{2n-2k}\frac{v^{4n-2k+1}}{2k+1} \tag{10}$$

$$\pi = 8\sum_{n=0}^{\infty} v^{2n+1}\sum_{k=0}^{[n/2]} \binom{2n-4k}{n-2k}\binom{2n-2k}{2k}\frac{1}{2n-4k+1} \tag{11}$$

$$\pi = 4\sum_{n=0}^{\infty} v^{2n+1}\sum_{k=0}^{n} \binom{2k}{k}\binom{n+k}{n-k}\frac{1+(-1)^{n+k}}{2k+1} \tag{12}$$

$$\begin{aligned}\pi = 8\sum_{n=0}^{\infty} v^{4n+1}\sum_{k=0}^{n} \binom{4k}{2k}\binom{2n+2k}{2n-2k}\frac{1}{4k+1} + \\ + 8\sum_{n=0}^{\infty} v^{4n+3}\sum_{k=0}^{n} \binom{4k+2}{2k+1}\binom{2n+2k+2}{2n-2k}\frac{1}{4k+3}\end{aligned} \tag{13}$$

$$\pi = 8v + 4\sum_{n=0}^{\infty} (-1)^n v^{2n+3} a_n \tag{14}$$

$$a_n = \sum_{k=0}^{[n/3]} \frac{2^{6k+3}}{2k+1}\binom{n+3k+2}{n-3k} - \sum_{k=0}^{n+1} \frac{2^{2k+1}}{2k+1}\binom{n+1+k}{n+1-k} \quad , n = 0,1,2,3,\ldots \tag{15}$$

$$\pi = 4\sum_{n=0}^{\infty} (-1)^n v^{2n+1}\sum_{k=0}^{n} \frac{(-1)^k 2^{2k+1}}{2k+1}\binom{n+k}{n-k} b_k \tag{16}$$

$$b_k = \left(\frac{1+i\sqrt{3}}{2}\right)\left(\frac{1-i\sqrt{3}}{2}\right)^k + \left(\frac{1-i\sqrt{3}}{2}\right)\left(\frac{1+i\sqrt{3}}{2}\right)^k \quad , k = 0,1,2,3,\ldots \tag{17}$$

Referencias

1. Ramanujan, S.: Notebooks (2 volumes), Tata Institute of Fundamental Research, Bombay , 1957.