The electron as a harmonic electromagnetic oscillator

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Abstract: This paper complements previous papers and the book which would have been published by IOP and WSP if it were not for the casual comments of a critic, who opined our oscillator model is just "casually connecting disparate formulas." This paper explains all the nuances and logical steps in the model in very much detail and we, therefore, hope we succeeded in making the case. Comments, remarks and questions are, obviously, welcome. We suggest such comments, remarks and questions are published as comments on this paper on the viXra.org site so as to ensure some objectivity.

Keywords: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations, realist interpretation of quantum mechanics.

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Introduction

In previous papers[1], we boldly equated the c^2 and $a^2 \cdot \omega^2$ factors in the E = m·c^2 and E = m·$a^2 \cdot \omega^2$ equations to get the Compton scattering radius ($a = \lambda_C/2\pi$)of an electron. We felt entitled to equate these two energy formulas because the Planck-Einstein relation (E = ħ·ω) then allows us to substitute ω for E/ħ, and we get a wonderfully elegant derivation of the Compton radius[2]:

$$\mathrm{E} = \mathrm{m}a^2\omega^2 = \mathrm{m}a^2\frac{\mathrm{E}^2}{\hbar^2} \Leftrightarrow \hbar^2 = \mathrm{m}a^2\mathrm{E} = \mathrm{m}a^2\mathrm{m}c^2 = \mathrm{m}^2a^2c^2$$

$$\Leftrightarrow a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

While our critics seem to feel that we are just “casually connecting disparate formulas” here[3], the only formula that needs to be ‘connected’ or explained here is the E = m·$a^2 \cdot \omega^2$ formula. In our *Zitterbewegung* (or realist, we should say) interpretation of quantum electrodynamics, this formula represents the oscillator model of the electron. Indeed, the energy in an oscillation – think of an electric circuit, or a mass on a spring – is proportional to the square of (i) the amplitude of the oscillation (which we’ll write as a) and (ii) the frequency of the oscillation. So we will have some proportionality coefficient k and we can write the energy as:

$$\mathrm{E} = \mathrm{k}a^2\omega^2$$

For example, if we think of a mass on an ideal spring (no friction), then one can show that the proportionality will be equal to m/2 here, so the formula becomes:

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[1] See http://vixra.org/author/jean_louis_van_belle. For those who don’t have time to read, I have uploaded two YouTube videos that explain the basic approach and, more in particular, the idea of a two-dimensional oscillation (https://www.youtube.com/watch?v=tsliySnOviQ and https://www.youtube.com/watch?v=gD79nBZwxjI).

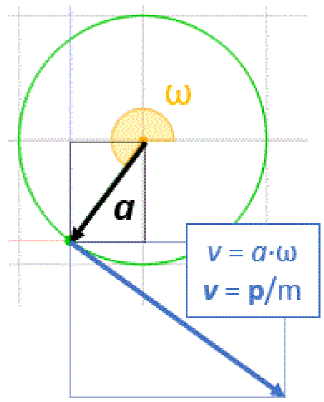
[2] In the literature, one will usually find references to the Compton *wavelength*. References to the *reduced* Compton wavelength (which we effectively interpret as a *radius*) are not so common. The *Zitterbewegung* interpretation of the nature of an electron – of which our model is a variant – is fully consistent with this concept. In our *zbw* interpretation, we add the notion of an elementary cycle. In other words, we assume that one oscillation – one fundamental *cycle* – packs an amount of physical action that is equal to Planck’s quantum of action: $S = h$. This explains why we can substitute ω for E/ħ in the oscillator equations.

[3] While the Institute of Physics (IOP) and World Science Publishing (WSP) were initially interested in publishing my book on the *Zitterbewegung* interpretation of quantum mechanics (http://vixra.org/abs/1901.0105), an academic reviewer effectively claimed that I am just "casually connecting disparate formulas to try to build up credibility." IOP and WSP then decided to *not* publish the book. David Hestenes himself, who revived the *zbw* interpretation of quantum mechanics in the 1980s and 1990s, had similar negative comments in his exchanges with me. In fact, I don’t exclude he was the reviewer. The comments are, obviously, everything but objective or scientific. They illustrate the rather sorry state of academic physics: references and bibliographies are far more important than original content. While David Hestenes would not want to be associated with our writings, we continue to refer to our realist interpretation of quantum mechanics as a *Zitterbewegung* interpretation. However, we’d rather refer to Dirac’s original writings on it now, if only because the models of Hestenes and others are couched in complicated math. To be precise, Hestenes invented a new geometric calculus, which I think over-complicates the matter at hand.

$$E = \frac{1}{2}ma^2\omega^2$$

It is easy to see that one can get the E = m·a^2·ω^2 by combining *two* oscillators in a 90-degree angle. This is visualized below: think of the green dot in Figure 1 as the mass on the springs (plural!): it will now go round and round with some *constant* tangential velocity $v = a \cdot \omega$, and we can add the potential and kinetic energy in both oscillators to get the E = m·a^2·ω^2 = m·v^2 equation.

Figure 1: E = m$a^2\omega^2$ = mv^2



The only question we need to answer now is why we would equate *v* to *c*, especially because our oscillator model seems to be non-relativistic. Before we address these questions – in a way that will, hopefully, satisfy our critics – we would like to quickly review the key results of our model. These results – which include a geometric explanation of the spin and orbital angular momentum of an electron – are, effectively, at least as significant as our geometric explanation of the Compton scattering radius. After this short review, we will get into the nitty-gritty of the model, which – as mentioned – will, hopefully, demonstrate convincingly that we are *not* just "casually connecting disparate formulas".

Key results of the oscillator model

The oscillator model for our electron shares some similarities with the idea of a photon. Indeed, we get the k = m equation quite naturally when thinking of photons. The *de Broglie* equation for a photon gives us the following formula for the wavelength:

$$h = \mathrm{p} \cdot \lambda = \frac{E}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{E}$$

We can then write:

$$E = \mathrm{k}a^2\omega^2 = \mathrm{k}\lambda^2\frac{E^2}{h^2} = \mathrm{k}\frac{h^2c^2}{E^2}\frac{E^2}{h^2} = \mathrm{k}c^2 = \mathrm{m}c^2 \Leftrightarrow \mathrm{k} = \mathrm{m} \text{ and } E = \mathrm{m}c^2$$

The logic behind our oscillator model for the electron is basically the same. We assume that one oscillation – one fundamental *cycle* – packs an amount of physical action that is equal to Planck's quantum of action. We write: $S = h$. Physical action is the product of force, time and distance. The distance here is the distance along the *zbw* circumference, which is equal to $\lambda_C = 2\pi \cdot a = h/\mathrm{m}c$. The cycle time is T = 1/f = h/E. Hence, we can calculate the force from the $S = \mathrm{F} \cdot \lambda_C \cdot \mathrm{T}$ formula:

$$\mathrm{F} = \frac{S}{\lambda_C \cdot \mathrm{T}} = \frac{h}{\frac{h}{\mathrm{m}c} \cdot \frac{h}{\mathrm{E}}} = \frac{\mathrm{m}^2 \cdot c^3}{h}$$

The energy is the force times the distance over the loop, so it's equal to:

$$\mathrm{E} = \mathrm{F} \cdot \lambda_C = \frac{\mathrm{m}^2 \cdot c^3}{h} \cdot \frac{h}{\mathrm{m}c} = \mathrm{m} \cdot c^2$$

The mass m is the rest mass of our electron—it is, obviously, *not* the mass of the pointlike charge. Our pointlike charge has zero (rest) mass: it's just an electric charge with no other attributes. The *Zitterbewegung* model is Wheeler's theory of 'mass without mass': the mass of the electron is the equivalent mass of the energy in the oscillation of the pointlike charge.

Let us summarize the key results we get out of these model—in case you would also think that we are just "casually connecting disparate formulas". We can see that the magnitude of the force (about 0.034 N) is rather enormous in light of the sub-atomic scale[4]:

$$\mathrm{F} = \frac{\mathrm{E}}{\lambda_C} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\ \mathrm{N}$$

The associated current is equally humongous. It is a household-level current (about 2 ampere) but, again, this is huge at the sub-atomic scale:

$$\mathrm{I} = \mathrm{q_e} f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

However, these results are consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. As the oscillator model implies the effective mass of the electron will be spread over the circular disk, we should use the 1/2 form factor for the moment of inertia (*I*). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We have also augmented the Bohr-Rutherford model to calculate all of the values that relate to the electron (atomic) *orbitals*. Table 1 summarizes these results.

[4] A force of 0.033743 N is equivalent to a force that gives a mass of about 34 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second.

Table 1: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	Orbital electron (Bohr orbitals)
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I \cdot \omega = \frac{1}{2} \cdot \mathrm{m}a^2 \cdot \omega = \frac{\mathrm{m}}{2} \cdot \frac{\hbar^2}{\mathrm{m}^2 c^2} \cdot \frac{\mathrm{E}}{\hbar} = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu = \mathrm{I} \cdot \pi r_\mathrm{C}{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi r_n{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}} n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

Last but not least, we argue[5] that the anomalous magnetic moment of an electron might be explained by a very classical coupling between the two moments because of the Larmor precession of the electron in the Penning trap.

While these results should impress, our critics seem to want to know whether or not the $\mathrm{E} = \mathrm{m}\cdot a^2\cdot\omega^2$ makes sense in a relativistically correct analysis. Before we answer that question, we will first reflect some more about the *nature* of the force that keeps our pointlike charge in orbit.

We will show it is just the electromagnetic force: the same force that causes persistent or perpetual currents in superconducting materials. The only difference is that the scale ensure we do *not* need the superconducting material to actually hold the electron(s): it is a persistent current in free space.

The electromagnetic *Zitterbewegung* force

We think of the green dot in Figure 1 as a pointlike charge with *zero* rest mass. As such, it is its circular or oscillatory motion – its *Zitterbewegung*, in other words[6] - that gives the electron its mass. Hence, we

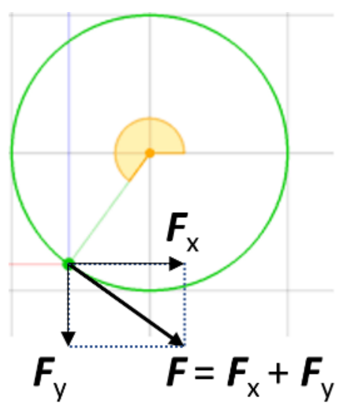
[5] Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018, http://vixra.org/pdf/1812.0233v3.pdf.
[6] *Zitter* is German for shaking or trembling. It refers to a presumed local oscillatory motion which Erwin Schrödinger stumbled upon when he was exploring solutions to Dirac's wave equation for free electrons. Schrödinger shared the 1933 Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, Theory of Electrons and Positrons, Nobel Lecture, December 12, 1933) Dirac refers to the phenomenon of

have a hybrid description of a charged particle here: it combines the idea of a pointlike charge and perpetual motion, and the $c^2 = a^2 \cdot \omega^2$ hypothesis defines both the frequency as well as the amplitude of what we will refer to as *the rest energy oscillation*. It is that what gives mass to our electron: its *rest* mass is nothing but the equivalent mass of the energy in the oscillation. As such, the only way we can interpret it, is as the velocity of the pointlike charge in its *Zitterbewegung*.

The geometry of the model (Figure 1) gives us two formulas: $v = a \cdot \omega$ and $\boldsymbol{v} = \mathbf{p}/\mathrm{m}$. These formulas are relativistically correct. Our oscillator model only needs to explain why we can equate v to c. It also needs to explain the nature of the force that keeps the pointlike charge in its orbit.[7] Figure 2 shows the circular motion must be driven by a tangential force **F** whose $\mathbf{F_x}$ and $\mathbf{F_y}$ components depend on the position of the pointlike charge (the green dot in Figure 2). That (x, y) position is given by the implicit $x^2 + y^2 = a^2$ equation, which describes a circle with radius a. The geometry of the situation allows us to describe the components of **F** as the following functions of the *magnitude* (F) and the x and y coordinates[8]:

- $\mathrm{F_x} = \mathrm{F} \cdot y = \mathrm{F} \cdot \sin(\theta) = \mathrm{F} \cdot \cos(\theta - \pi/2) = \mathrm{F} \cdot \cos(\omega t - \pi/2)$
- $\mathrm{F_y} = \mathrm{F} \cdot x = \mathrm{F} \cdot \cos(\theta) = -\mathrm{F} \cdot \sin(\theta - \pi/2) = -\mathrm{F} \cdot \sin(\omega t - \pi/2)$

Figure 2: The *Zitterbewegung* model of an electron



We thus get the following formula for the force:

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y} = \mathrm{F} \cdot \cos(\theta - \pi/2) - i \cdot \mathrm{F} \cdot \sin(\theta - \pi/2) = \mathrm{F} \cdot e^{-i(\theta - \pi/2)}$$

This formula uses the geometry of complex numbers and the formula for the complex conjugate: $e^{-i\theta} = \cos(\theta) - i \cdot \sin(\theta) = \cos(-\theta) + i \cdot \sin(-\theta)$. Of course, we can – and should – relate this force formula to the

Compton scattering of light by an electron here and it is, therefore, highly significant that we get the formula for the Compton radius of an electron out of our model.

[7] This echoes one of Dr. Burinskii's very first communications to me. Dr. Alexander Burinskii is an eminent quantum physicist who has worked almost all of his life on various electron models. He wrote the following to me when I first contacted him on the viability on my model: "I know many people who considered the electron as a toroidal photon and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about Zitterbewegung [because of ideological reasons], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?"

[8] The $\sin(\theta) = \cos(\theta - \pi/2)$ identity is easy to remember when one draws the graphs. In contrast, the $\cos(\theta) = -\sin(\theta - \pi/2)$ requires some more manipulation: $\cos(\theta) = \sin(\pi/2 - \theta) = -\sin(\theta - \pi/2)$. We also use the $\cos(\theta) = \cos(-\theta)$ and $\sin(\theta) = -\sin(-\theta)$ formulas. We can, therefore, write: $\cos(\theta - \pi/2) = \cos(\pi/2 - \theta) = \sin(\theta)$.

elementary wavefunction, which describes the physical position (i.e. the x- and y-coordinates) of our pointlike charge (the green dot in Figure 1 and Figure 2) and whose motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\theta) + i \cdot a \cdot \sin(\theta) = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

However, this does not answer the more fundamental question: what is the *nature* of the force? The answer looks remarkably easy. Because the force has only a pointlike charge to grab onto, it must be electromagnetic. It must be the Lorentz force $\mathbf{F} = q_e \cdot \mathbf{E} + q_e \cdot \boldsymbol{v} \times \mathbf{B}$. Of course, this is interesting because, using complex-number geometry once again, we may write the magnetic field vector as $\mathbf{B} = -i \cdot \mathbf{E}/c$ and, hence, the $\mathbf{F} = q_e \cdot \mathbf{E} + q_e \cdot \boldsymbol{v} \times \mathbf{B}$ becomes:

$$\mathbf{F} = q_e \cdot \mathbf{E} + q_e \cdot \boldsymbol{v} \times \mathbf{B} = q_e \cdot \mathbf{E} - i \cdot q_e \cdot (\boldsymbol{v}/c) \times \mathbf{E} = \mathbf{F_x} + \mathbf{F_y} = \mathrm{F} \cdot e^{-i(\theta - \pi/2)}$$

This is interesting ! The picture that is emerging here makes us think of (i) the horizontal component of the electromagnetic force on the *zbw* charge as an electric force ($\mathbf{F_x} = q_e \cdot \mathbf{E}$) and (ii) the vertical component as the magnetic force ($\mathbf{F_y} = q_e \cdot \boldsymbol{v} \times \mathbf{B} = -i \cdot q_e \cdot (\boldsymbol{v}/c) \times \mathbf{E}$). Needless to say, the velocity vector here is *not* the tangential velocity but the velocity of the *zbw* charge along the $\boldsymbol{x}$-direction. Let us briefly examine the dynamics by looking at what happens at the usual special angles θ = 0, π/2, π and 3π/4:

- The horizontal velocity $\boldsymbol{v}$ is zero when the position of our charge is equal to $(x, y) = (\pm a, 0)$. Hence, θ = 0 or π, at which point $\mathrm{F} \cdot \cos(0 - \pi/2) = \mathrm{F} \cdot \cos(-\pi/2) = \mathrm{F} \cdot \cos(\pi/2) = \mathrm{F} \cdot \cos(\pi - \pi/2) = 0$. Hence, $q_e \cdot \mathbf{E} = \mathbf{F_x} = 0$ and all of **F** is magnetic – and magnetic only – at these two points. In other words, the direction of the force is all vertical and all magnetic, and the electric force is zero: $\mathbf{F} = \mathbf{F_y} = -\boldsymbol{i} \cdot \mathrm{F} \cdot \sin(\theta - \pi/2)$ [9]. To be specific, we get the following:

  (1) At θ = 0, we get $\mathbf{F} = \mathbf{F_y} = -\boldsymbol{i} \cdot \mathrm{F} \cdot \sin(-\pi/2) = \boldsymbol{i} \cdot \mathrm{F}$.

  (3) At θ = π, we get $\mathbf{F} = \mathbf{F_y} = -\boldsymbol{i} \cdot \mathrm{F} \cdot \sin(+\pi/2) = -\boldsymbol{i} \cdot \mathrm{F}$.

- In contrast, we assume $\boldsymbol{v}$ equals the speed of light – the highest possible speed ($\boldsymbol{v} = \boldsymbol{c}$) – when the pointlike charge passes the $x = 0$ point along the $\boldsymbol{x}$-axis. The position of our charge is then equal to $(x, y) = (0, \pm a)$ and, hence, the argument of the wavefunction is equal to θ = ±π/2 here. The magnetic force is equal to $-i \cdot \mathrm{F} \cdot \sin(\pm\pi/2 - \pi/2) = -i \cdot \mathrm{F} \cdot \sin(0) = -i \cdot \mathrm{F} \cdot \sin(-\pi) = 0$. Hence, all of **F** is electric – and electric only – at these two points. In other words, the direction of the force is all horizontal and all electric, and the magnetic force is zero. To be specific, we get the following:

  (2) At θ = +π/2, we get $\mathbf{F} = \mathbf{F_x} = \mathrm{F} \cdot \mathbf{cos}(+\pi/2 - \pi/2) = \mathrm{F} \cdot \mathbf{cos}(0) = +\mathbf{1} \cdot \mathrm{F}$.

  (4) At θ = –π/2, we get $\mathbf{F} = \mathbf{F_x} = \mathrm{F} \cdot \mathbf{cos}(-\pi/2 - \pi/2) = \mathrm{F} \cdot \mathbf{cos}(-\pi) = \mathbf{-1} \cdot \mathrm{F}$.

The notation is somewhat unusual here, as we use boldface for the + or – sign, or for **1**. We do write **1** as a vector quantity here. This is a logical consequence of us writing $\mathbf{B} = -i \cdot \mathbf{E}/c$. The minus sign is there because we need to combine several conventions here: there is the classical *physical* right-hand rule for **E** and **B,** but we also need to combine the right-hand rule for the coordinate system with the convention

[9] Notation becomes somewhat special here. Vector quantities are represented in boldface type (e.g. **E** or $\mathbf{F_x}$). Here, we write the imaginary unit *i* as a vector quantity ($\boldsymbol{i}$), which we then multiply by the (constant) *magnitude* (F) of the force (**F**). The force can, effectively, point in one or the other direction, so that explains the ± sign in the $\mathbf{F} = \pm \boldsymbol{i} \cdot \mathrm{F}$ formula. It may take the reader a while to think this through but we took care to ensure consistent notation.

that multiplication with the imaginary unit amounts to a *counter*clockwise rotation by 90 degrees. Hence, the minus sign is necessary for the consistency of the description. It ensures that we can associate the $a·e^{i\theta}$ and $a·e^{-i\theta}$ functions with left- and right-handed polarization respectively.[10]

Now that we have described the nature of the *Zitterbewegung* force – it is nothing but the electromagnetic force on a pointlike charge with zero rest mass – all that is left to do is to analyze how this works in terms of a harmonic oscillation. As speeds go from 0 to *c*, we might need the formulas for a relativistic oscillator. Let's see.

Further analysis: introducing relativity

We know the electric and magnetic field are not to be thought of as independent things: they are apparent manifestations, so to speak, of one and the same electromagnetic field. That's obvious, as Richard Feynman points out, from the $F_{magnetic} = q_e·\boldsymbol{v}\times\mathbf{B}$ force formula. This formula tells us the magnetic force on our charge will be proportional to its velocity, but we should immediately wonder: "What velocity? With respect to which reference frame?" Both ***v*** as well as **B** will depend on our reference frame, which defines what is up or down, front or back, and left and right.

We choose a reference frame when writing $\boldsymbol{r} = a·e^{i\theta} = (x, y)$. The zero point for θ = ω·t and our geometric convention for the imaginary unit (a counterclockwise rotation by 90 degrees) define our three-dimensional space as well as our t = 0 point and the *coordinate time* that is associated with this reference frame.[11] To paraphrase Feynman[12]: "If we had chosen another coordinate system, we would find a different mixture of **E** and **B** fields. Electric and magnetic forces are part of one physical phenomenon – electromagnetic interaction – but the separation of this interaction into electric and magnetic parts depends very much on the reference frame chosen for the description: **E** and **B** appear in different mixtures if we change our frame of reference.

We shouldn't worry too much about this right now. Let us first think about the relativistically correct equation for the motion of our pointlike charge. It is just Newton's Law **F** = d**p**/dt = d(m***v***)/dt. The only difference with the non-relativistic force law is that we are *not* assuming that m is some constant. Instead, we use the $\mathbf{p} = m_v\boldsymbol{v} = \gamma m_0 v$ formula to write the force law like this:

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[10] For a more detailed discussion, see: Jean Louis Van Belle, *Euler's wavefunction: the double life of –1* (http://vixra.org/abs/1810.0339), in which I argue that it is a mistake to *not* use the plus/minus sign of the imaginary unit in the $a·e^{\pm i\theta}$ function to include spin in the mathematical description of an elementary particle. Indeed, most introductory courses in quantum mechanics will show that both $a·e^{-i·\theta} = a·e^{-i·(\omega t - kx)}$ and $a·e^{+i·\theta} = a·e^{+i·(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). If there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations, and the one obvious characteristic that would distinguish the two physical situations is the spin direction. Hence, we do *not* agree with the mainstream view that the choice is a matter of convention. Instead, we dare to suggest that the two mathematical possibilities represent identical particles with opposite spin. It ensures the weird 720-degree symmetries of fermions and, therefore, invalidates the most important objection to a physical interpretation of the wavefunction.

[11] It may be good to remind oneself of the relation between the coordinate time t and the proper time τ (*tau*). They are related through the Lorentz factor γ, which varies with speed:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}} = \frac{dt}{d\tau}$$

As an exercise, the reader may want to think about the concept of *proper* time here and what it means, *exactly*.

[12] See: Feynman's Lectures, *The relativity of electric and magnetic fields*, Vol. II, Lecture 13, Section 6.

$$\frac{d}{dt}\left(\frac{\mathrm{m}_0\boldsymbol{v}}{\sqrt{1-v^2/c^2}}\right) = \mathbf{F} = \mathrm{q}(\mathbf{E} + \boldsymbol{v} \times \mathbf{B})$$

As mentioned above, we shouldn't worry too much about the details of the **E**, ***v*** and **B** vectors. They depend on the orientation of our reference frame:—on how we choose to define what is up or down, front or back, and left and right, in other words. Because of our particular choice, Needless to say, the velocity vector ***v*** is *not* the tangential velocity but the velocity of the *zbw* charge along the ***x***-axis.

So let us look at the math. We wrote the force as the sum of two *component* forces—one along the ***x***-axis and one along the ***y***-axis:

$$\mathbf{F} = \mathbf{F_x} + \mathbf{F_y} = \mathrm{F}\cdot\cos(\theta-\pi/2) - i\cdot\mathrm{F}\cdot\sin(\theta-\pi/2) = \mathrm{F}\cdot e^{-i(\theta-\pi/2)}$$

These two components *behave* like classical mechanical oscillators—like a mass on a spring: if the magnitude of the oscillation is equal to a, then the motion of the piston (or the mass on a spring) will be described by $x = a\cdot\cos(\omega\cdot t + \Delta)$. The Δ is just the phase factor which defines our $t = 0$ point, and ω is the *natural* or *resonant* (angular) frequency of our oscillator. A classical mechanical oscillator involves a restoring force, which will linearly depend on the position—on the *displacement* from the center or equilibrium position, to be precise. If we think of *two* oscillations – along the ***x***- and ***y***-axis respectively – we write:

$$\mathrm{F_x} = \mathrm{dp_x}/\mathrm{dt} = -\kappa\cdot x$$

$$\mathrm{F_y} = \mathrm{dp_y}/\mathrm{dt} = -\kappa\cdot y$$

Note that we use a Greek *kappa* (κ) to distinguish this constant from the proportionality constant k that we used in the $\mathrm{E} = \mathrm{k}\cdot a^2\cdot\omega^2$ formula in our introduction. Also note that these formulas are relativistically correct: the difference between a relativistically correct and a non-relativistic analysis is in how we'd write the momentum. In a non-relativistic analysis, we'd just use the $\mathrm{p} = \mathrm{m}\cdot v$ formula and, importantly, we would assume m is constant. In contrast, in a relativistically correct analysis, we'd write p as $\mathrm{p} = \mathrm{m}_v v = \gamma \mathrm{m}_0 v$. Hence, m would depend on the velocity through the Lorentz factor γ. However, the restoring force constant would *not* depend on the velocity and, hence, the mass. It is always the same: κ. That's why it's a constant. This constant can be written as:

$$\kappa = \mathrm{m}\cdot\omega^2$$

We get that from the *solution* we find for ω when solving the differential equation. What differential equation? The one we wrote above:

$$\mathrm{F} = \frac{d\mathrm{p}}{d\mathrm{t}} = \frac{d(\mathrm{m}\cdot v)}{d\mathrm{t}} = \mathrm{m}\cdot\frac{dv}{d\mathrm{t}} = \mathrm{m}\cdot a = -\kappa\cdot a$$

But this differential equation models the non-relativistic oscillator only, right? Yes. We treat m as a constant and that's why we can take that factor out of the time derivative: $\mathrm{d}(\mathrm{m}\cdot v)/\mathrm{dt} = \mathrm{m}\cdot\mathrm{d}v/\mathrm{dt} = \mathrm{m}\cdot a$. But… Well… That's OK. We are looking for a problem for which we have a solution here !

$$x = a\cdot\cos(\omega\cdot t + \Delta)$$

This *solution* for the motion of the pointlike charge is a solution to our differential equation. We do not need to make things complicated by introducing relativity. We just need to equate ω with some value and then we can see what values for m and k make sense by using the formula for the solution for ω, which is:

$$\omega = \sqrt{\frac{\kappa}{\mathrm{m}}}$$

We know that ω is equal to ω = E/ħ. We also know that F_x and F_y will be equal to F = $\mathrm{m}^2 \cdot c^3/\hbar$ if the displacement is equal to a = ħ/m·c. If we don't think about the direction of the force, we can write:

$$\kappa \cdot a = \kappa \cdot \frac{\hbar}{\mathrm{m} \cdot c} = \mathrm{F} = \frac{\mathrm{m}^2 \cdot c^3}{\hbar} \Leftrightarrow \kappa = \frac{\mathrm{m}^2 \cdot c^3}{\hbar} \cdot \frac{\mathrm{m} \cdot c}{\hbar} = \frac{\mathrm{m}^3 \cdot c^4}{\hbar^2}$$

We can now calculate the mass concept we need to use here:

$$\mathrm{m} = \frac{\kappa}{\omega^2} = \frac{\mathrm{m}^3 \cdot c^4}{\hbar^2} \cdot \frac{\hbar^2}{\mathrm{E}^2} = \frac{\mathrm{m}^3 \cdot c^4}{\hbar^2} \cdot \frac{\hbar^2}{\mathrm{m}^2 \cdot c^4} = \mathrm{m}$$

It is all perfectly consistent—except our force formula: F = $\mathrm{m}^2 \cdot c^3/\hbar$. Didn't we say it was equal to F = $\mathrm{m}^2 \cdot c^3/h$? So why do we use the *reduced* Planck constant in the formulas above? The explanation here might seem to be a bit *ad hoc*, but it is a sensible one: we calculated the force along the loop—along the *zbw* circumference λ_C = 2π·a. When we're analyzing the oscillation along the ***x***- or ***y***-axis, then we analyze only one component of the force, and we think of it working along the axis—not along the loop. We also argued that we think of each of the two linear oscillations carrying *half* of the total energy of our *zbw* electron. We write:

$$\mathrm{E}_x = \mathrm{E}_y = \frac{1}{2}\mathrm{m}a^2\omega^2 = \frac{1}{2}\mathrm{m}c^2$$

Also, the force *components* cannot be considered to be constants along the ***x***- or ***y***-axis. The *equivalent* force can, therefore, be calculated using an *effective* distance, which we write as a fraction α of the radius a. This fraction has to be equal to ½ to get the correct value for the *effective* or equivalent force:

$$\mathrm{F} = \frac{energy}{distance} = \frac{\frac{1}{2}\mathrm{m} \cdot c^2}{\frac{1}{2}a} = \mathrm{m} \cdot c^2 \cdot \frac{\mathrm{m} \cdot c}{\hbar} = \frac{\mathrm{m}^2 c^3}{\hbar}$$

Conclusion

We wrote this paper to address criticism that our oscillator model just "casually connects disparate formulas." We took care to explain all the nuances and logical steps in the model. We hope we succeeded in making the case. Comments, remarks and questions are, obviously, welcome. We suggest such comments, remarks and questions are published as comments on this paper on the viXra.org site.

Jean Louis Van Belle, 28 May 2019

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to:

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.
4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.
5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

We would like to mention the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html).

In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

We should, perhaps, also mention the following critical appraisal of the quantum-mechanical framework:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

It is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Finally, I would like to thank Prof. Dr. Alex Burinskii for taking me seriously. He singlehandedly provided the main inspiration for my work.

The illustrations in this paper were made by the author.