

ORBITAL ANGULAR MOMENTUM OF ALL ORBITALS

FLUYIEP EXPLANATION

BOHR ATOMIC MODEL WITH VICTORIA THEORY

Javier Silvestre

ABSTRACT

Bohr atomic model is developed with electronic extremes of Victoria Theory. Toyi mechanism is focused on explaining orbital angular and spin movement of electronic extremes. This article summarizes first part of Toyi Mechanism and then proceeds with individualized explanation of angular momentum for all orbitals (named as Fluyiep Explanation). Finally, paper concludes with fundamental relations between angular momentum of all orbitals.

KEYWORDS

Bohr atom, orbital angular momentum, Toyi mechanism.

INTRODUCTION

Atomic model with at least two electronic extremes represents development of Bohr model. First part consists of first nine articles that are dedicated to this new model and are divided into four sections:

- a) Radial distribution with two electronic extremes [1]
- b) From one dimension (radial distribution) to three dimensions [2]
- c) Geometry and probability [3,5]
- d) Geometric and probabilistic coupling according to n [6,9]

Excited electron is treated in second part [10,20]. Introduction to behavior of probability coefficients is carried out in the third part [21,24].

This article develops movements of electronic extremes, global orbital angular momentum of electron resulting from these movements and number of electronic extremes as function of orbital quantum number. For this, article continues Toyi mechanism [25] that proposes:

a) C_{POTI} sweeps are performed for each division of the birth wavelength until division sweep is completed. All this is done with steps towards other lobe when division is infinite and C_{POTI} is limit.

b) There is one of two electronic extremes that remains on axis. This electronic extreme lacks spin charge. The stay in this axis with $c_i = 0$ and consequent zero probability is governed by P86 - Principle of movement equality outside axis by radial unit.

c) All spin charge belongs to electronic extreme that performs C_{POTI} sweep. This first approximation already allows that $g_s = 2$ in spin magnetic moment of electron.

d) This mechanism allows equation $r_B = r_A - \lambda/d$ [1] to be fulfilled each time C_{POTI} sweep is initiated (indicated as t_0). That is, each time division change is made.

Movements of electronic extremes have been seen (especially in [2] and [25]). Important note prior to movements description:

- Geometry and movement of electronic extremes are introduced in [2], but no electronic extreme is in axis because probability concept had not yet been introduced and, consequently, C_{POTI} had not been either. Therefore, electronic extremes only perform division sweeps.

- Bohr atomic model with Equation Victoria and fulfilling of Toyi mechanism is the one exposed in [25] and this mechanism is maintained for Fluyiep Explanation in this article.

Considering previously exposed, first summary for electronic extremes movements (movement due to C_{POTI} sweep is ignored in this first scheme of movements) is:

EE1 (Axis)

1) Orbital angular movement with value of $\hbar/2$ and associated $q/2$ [2]

EE2 (C_{POTI} Sweep)

1) Orbital angular movement with value of $\hbar/2$ and associated $q/2$ [2]

2) Spin movement to perform C_{POTI} sweep with value of $\hbar / 2$ and associated q [25]. Spin movement represented in [2] corresponds to mechanism inverse to Toyi mechanism.

Introduction to orbital angular momentum [2] allows to explain value of $L=\hbar$ in lobe direction in p orbital. This fact is interesting since angular momentum of p_x or p_y orbitals is \hbar in specific directions: p_x and p_y orbitals have angular momentum value of \hbar in direction of X and Y axis respectively and another angular momentum value of $\pm \hbar$ indistinctly in Z axis. However, several unknowns are not closed in [2]:

- * If $L=\hbar$ due to two $\hbar/2$ contributions of orbital angular movement of each of electronic extremes, what movements generate \hbar value on Z axis?
- * How can zero value on Z axis of p_z orbital be explained? If contribution of $\hbar/2$ of one electronic extreme in Z axis is canceled with the other electronic extreme, how can electron still have an orbital angular momentum of $2^{1/2} \hbar$
- * How to explain higher angular moments in higher l orbitals?

Fluyiep explanation is made to solve all these questions.

FLUYIEP EXPLANATION FOR ORBITAL ANGULAR MOMENTUM

(Explanation and application to all existing orbitals)

Fluyiep explanation is focused on study of orbital angular momentum of Bohr atomic model developed with electronic extremes of Victoria Theory [1,9] and is continuation of Toyi Mechanism [25] that delves into movements of the electronic extremes. Steps are following:

- * Enunciated of P87 and P88 that synthesize orbital angular movements of the two electronic extremes.
- * Nomenclature of orbital angular movements that allows a simpler representation in the different orbitals.
- * Enunciated of P89 and P90 (Groups of orbitals: all orbitals are located in 4 groups)
- * Summary table of all orbitals (p, d, f and g) that already allows a first approximation of the analogous behavior of orbitals of each Group and how all orbitals can be described with Explanation Fluyiep.
- * Introduction to P91 (extended at the end of the article) which is started with simple rules of the orbitals and the 4 groups of orbitals.
- * Detailed explanation of all orbitals.
- * P91 continuation.

P87 EE2: Orbital angular movement by C_{POTI}

EE2 performs one additional movement when continuously changes its C_{POTI} . This change of C_{POTI} , already represented in Figure 3 of [25], is performed simultaneously with the simple orbital angular movement [2] and its spin movement that allows it to change its C_{POTI} . Consequently, EE2 adds a second movement of orbital type:

- 1) Orbital angular movement with value of $\hbar/2$ and associated $q/2$ [2]
- 2) Spin movement to perform C_{POTI} sweep with value of $\hbar / 2$ and associated q [25]

3) Orbital angular movement by C_{POTI} sweep with value of $\hbar/2$ and associated $q/2$

P88 EE1: Orbital angular movement of equilibrium

EE of axis (EE1) performs a second intrinsic movement with value of $q/2$ (summing full charge of intrinsic motion and equalizing the situation with spin movement of EE2) that considers the orbital angular movement by C_{POTI} of EE2 to provide vector of global C_{POTI} orbital angular movement that, together with the orbital angular movement due to rotation of electronic extremes, can fulfill (O.A.):

$$(O.A.)L = \sqrt{l(l+1)} = \sqrt{l^2 + l} = \sqrt{L_Z^2 + L_{XY}^2}$$

EE1 therefore has two movements:

- 1) Orbital angular movement with value of $\hbar/2$ and associated $q/2$ [2]. This orbital angular movement of EE1, being static at $c_i = 0$, is comparable to an intrinsic motion [25].
- 2) Orbital angular movement of equilibrium by C_{POTI} with value of $\hbar/2$ and $q/2$ associated. This second completes charge of static orbital angular movement.

OAM nomenclature (Orbital Angular Momentum)

OAM is divided into two types of movement:

1) OAMc (circular OAM): Includes two movements:

- * Movement 1) of EE2: vector of OAM is linear to axis where lobe is located (introduced in [2]) and due to circular orbit rotation. Value of $\hbar/2$. Represented as C2.
- * Movement 1) of EE1. It really is a turn on itself because EE1 is on axis. Value of $\hbar/2$. Represented as C1.

2) OAMp (OAM poti): OAM due to C_{POTI} sweeping movement (movement 3) of EE2) and including equilibrium OAM of EE1 (movement 2) of EE1). Its value is equally of $\hbar/2$ for each electronic extreme. Both represented as P.

P89 Impossibility of free rotation in XYZ planes of angular momentum vectors

No vector OAMc (C1 or C2) or OAMp (P) can have a free movement in any of 3 planes (XY YZ or XZ) because there are lobes located in them.

P90 Groups of orbitals

Orbitals are subdivided into 4 groups (Table 1) whose angular movements are achieved following same model.

Table 1 – Groups of orbitals						
Group	l	L _Z	Orbital	No.	Group characteristic	
I	p	±1	p _x p _y	2	L _Z maximum. Lobes on X or Y axis	
II	d	±2	d _{xy} d _{x²-y²}	2	L _Z maximum. Lobes in XY plane	
	f	±3	f _{y(3x²-y²)} f _{x(x²-3y²)}	2		
	g	±4		2		
III	p	0	p _z	1	L _Z = 0. 2 lobes and rings	
	d	0	d _{z²}	1		
	f	0	f _{z³}	1		
	g	0	g _{z⁴}	1		
IV	A	d	±1	d _{xy} d _{yz}	2	Intermediate L _Z
		f	±2	f _{z(x²-y²)} f _{xyz}	2	
		g	±3		2	
	B	f	±1	f _{yz²} f _{xz²}	2	
		g	±2		2	
		C	g	±1		

Different models of angular momentum for 4 groups that allows to fulfill L and L_Z can already be intuited from general scheme of orbital angular momentum where all the orbitals are included (Table 2). Table 2 is explained and put into practice with the different orbitals.

TABLE 2 – ORBITAL ANGULAR MOMENTUM: TOTAL, ON Z AXIS AND XY PLANE – VECTORS SUMMARY

Group	O	Lz	$L = \sqrt{L_z^2 + L_{XY}^2}$	VU	Lz1	C2	P	VLz1	$L_{XYZ} = \sqrt{L_{ZZ}^2 + L_{XY}^2}$	C	P	VLXYZ	VA= VU-V Lz1-V LXYZ
					Lz1 Analysis				Lxyz Analysis				
I) Lz max Lobe on X or Y axis	p	1	$L = \sqrt{1^2 + 1^2}$	2	1	0→0	2→1	1	$L_{XYZ} = \sqrt{0^2 + 1^2}$	2→1	0→0	1	2-1-1 = 0
II) Lz max. Lobes in XY plane	d	2	$L = \sqrt{2^2 + (2^{1/2})^2}$	4	2	0→0	4→2	2	$L_{XYZ} = \sqrt{0^2 + (2^{1/2})^2}$	4→2 ^{1/2}	0→0	2	4-2-2 = 0
	f	3	$L = \sqrt{3^2 + (3^{1/2})^2}$	6	3	0→0	6→3	3	$L_{XYZ} = \sqrt{0^2 + (3^{1/2})^2}$	4→3 ^{1/2}	0→0	2	6-3-2 = 1
	g	4	$L = \sqrt{4^2 + (4^{1/2})^2}$	8	4	0→0	8→4	4	$L_{XYZ} = \sqrt{0^2 + (4^{1/2})^2}$	4→4 ^{1/2}	0→0	2	8-4-2 = 2
III) Lz = 0 Two lobes and rings	p	0	$L = \sqrt{0^2 + (2^{1/2})^2}$	2	1/2	1→1/2	0→0	1/2	$L_{XYZ} = \sqrt{(0-1/2)^2 + (2^{1/2})^2} = 3/2$	1→1/2	2→1	3/2	2-1/2-3/2 = 0
	d	0	$L = \sqrt{0^2 + (6^{1/2})^2}$	4	1/2	1→1/2	0→0	1/2	$L_{XYZ} = \sqrt{(0-1/2)^2 + (6^{1/2})^2} = 5/2$	1→1/2	4→2	5/2	4-1/2-5/2 = 1
	f	0	$L = \sqrt{0^2 + (12^{1/2})^2}$	6	1/2	1→1/2	0→0	1/2	$L_{XYZ} = \sqrt{(0-1/2)^2 + (12^{1/2})^2} = 7/2$	1→1/2	6→3	7/2	6-1/2-7/2 = 2
	g	0	$L = \sqrt{0^2 + (20^{1/2})^2}$	8	1/2	1→1/2	0→0	1/2	$L_{XYZ} = \sqrt{(0-1/2)^2 + (20^{1/2})^2} = 9/2$	1→1/2	8→4	9/2	8-1/2-9/2 = 3
IV) Intermediate Lz	A	d	$L = \sqrt{1^2 + (5^{1/2})^2}$	4	$\sqrt{2}/2$	2 → $\sqrt{2}/2$	0→0	1	$L_{XYZ} = \sqrt{\left(1 - \frac{\sqrt{2}}{2}\right)^2 + (5^{1/2})^2} \approx 3\frac{\sqrt{2}}{2}$	6 → $3\sqrt{2}/2$		3	4-1-3 = 0
		f	$L = \sqrt{2^2 + (8^{1/2})^2}$	8	$2\sqrt{2}/2$	4 → $\sqrt{2}$	0→0	2	$L_{XYZ} = \sqrt{\left(2 - 2\frac{\sqrt{2}}{2}\right)^2 + (8^{1/2})^2} \approx 4\frac{\sqrt{2}}{2}$	8 → $4\sqrt{2}/2$		4	8-2-4 = 2
		g	$L = \sqrt{3^2 + (11^{1/2})^2}$	12	$3\sqrt{2}/2$	6 → $3\sqrt{2}/2$	0→0	3	$L_{XYZ} = \sqrt{\left(3 - 3\frac{\sqrt{2}}{2}\right)^2 + (11^{1/2})^2} \approx 5\frac{\sqrt{2}}{2}$	10 → $5\sqrt{2}/2$		5	12-3-5 = 4
	B	f	$L = \sqrt{1^2 + (11^{1/2})^2}$	6	$\sqrt{3}/2$	2 → $\sqrt{3}/2$	0→0	1	$L_{XYZ} = \sqrt{\left(1 - \frac{\sqrt{3}}{2}\right)^2 + \left(11^{1/2} + \frac{1}{2}\right)^2} \approx \left(4 + \frac{1}{2}\right)\frac{\sqrt{3}}{2}$	9 → $(9/2)\sqrt{3}/2$		5	$6 - 1 - \frac{1}{2} - \frac{9}{2} = 0$
		g	$L = \sqrt{2^2 + (16^{1/2})^2}$	12	$\sqrt{2}$	4 → $\sqrt{2}$	0→0	2	$L_{XYZ} = \sqrt{(2 - \sqrt{2})^2 + \left(16^{1/2} + 2\frac{\sqrt{2}}{4}\right)^2} \approx 7\frac{\sqrt{2}}{2}$	14 → $7\sqrt{2}/2$		8	12-2-1-7 = 2
		C	g	$L = \sqrt{1^2 + (19^{1/2})^2}$	8	1,307	4 → 1,307	0→0	2	$L_{XYZ} = \sqrt{(1 - 1,307)^2 + (19^{1/2})^2} \approx 6\frac{\sqrt{2}}{2}$	12 → $6\sqrt{2}/2$		6

P91 1, 2 and 3

Orbitals occupy their lobes based on the numbers 1, 2 and 3:

* Angles are the fundamental ones:

$$\cos 90 = \cos \frac{\pi}{2} = 0$$

$$\cos 45 = \cos \frac{\pi}{2^2} = \frac{\sqrt{2}}{2}$$

$$\cos 30 = \cos \frac{\pi}{3 * 2} = \frac{\sqrt{3}}{2}$$

$$\cos 0 = \cos \frac{0\pi}{2} = 1$$

* Number of orbitals in each group and total number of orbitals is reached with the numbers 2 and 3:

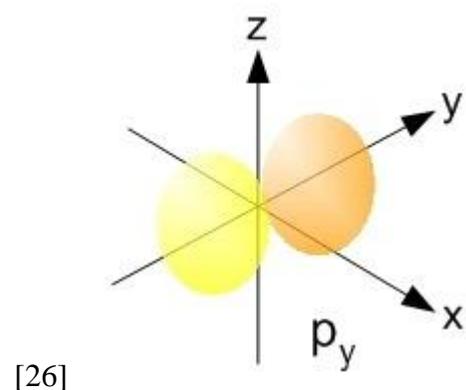
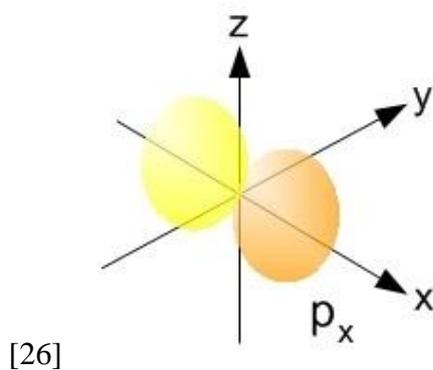
$$\text{Number of orbitals I + III + II + IV} = 2 + 4 + 6 + 12 = 2 + 2^2 + 2*3 + 2^2*3 = 24 = 2^3 * 3$$

$$\text{Number of orbitals I + III + II + IV} = \text{I} + (\text{I} + \text{I}) + (\text{I} + \text{III}) + (\text{I} + \text{III} + \text{II})$$

* In turn, these groups of orbitals are unified by following higher rules (at the end of the article and related to relations between the Z axis and the XY plane) based on the numbers 1, 2 and 3.

1) GROUP I L_Z maximum. Lobes on X or Y axis

1.1) Orbital p_x y p_y $L_z = \pm 1$



$L_z=0$ corresponds to the p_z orbital since if it had $L_z=1$ or $L_z=-1$ then fails P89 (see point 3)). p_x and p_z have $L_z=\pm 1$ indistinctly since vectorially both orbitals can be ± 1 . p_x or p_y orbital locate their two OAMc

vectors linked in direction of orbital (X or Y respectively) and two OAMp vectors on Z axis to +1 or -1. Different columns of Table 2 are now developed with this first group.

Vector units of \hbar (VU)

p orbitals have 2 electronic extremes and have 4 vectors of $\hbar/2$:

* 2 OAMc (C1 and C2)

* 2 OAMp (2 P)

4 vectors of $\hbar/2$ are two vector units of \hbar (2 VU). 1 VU = unit value of $\hbar = 2 * \hbar/2 = \hbar$

L_{Z1}

L_{Z1} is Z component created directly by C or P vectors. For p_x and p_y orbitals, L_{Z1} is made by the two aligned P vectors and, since each has a value of $\hbar/2$, global value is \hbar (2→1 in column P of the L_{Z1} analysis)

C2 is on X (p_x) or Y (p_y) axis and therefore does not participate in the Z axis (0 → 0 in column C2 of L_{Z1} analysis).

L_{XYZ}

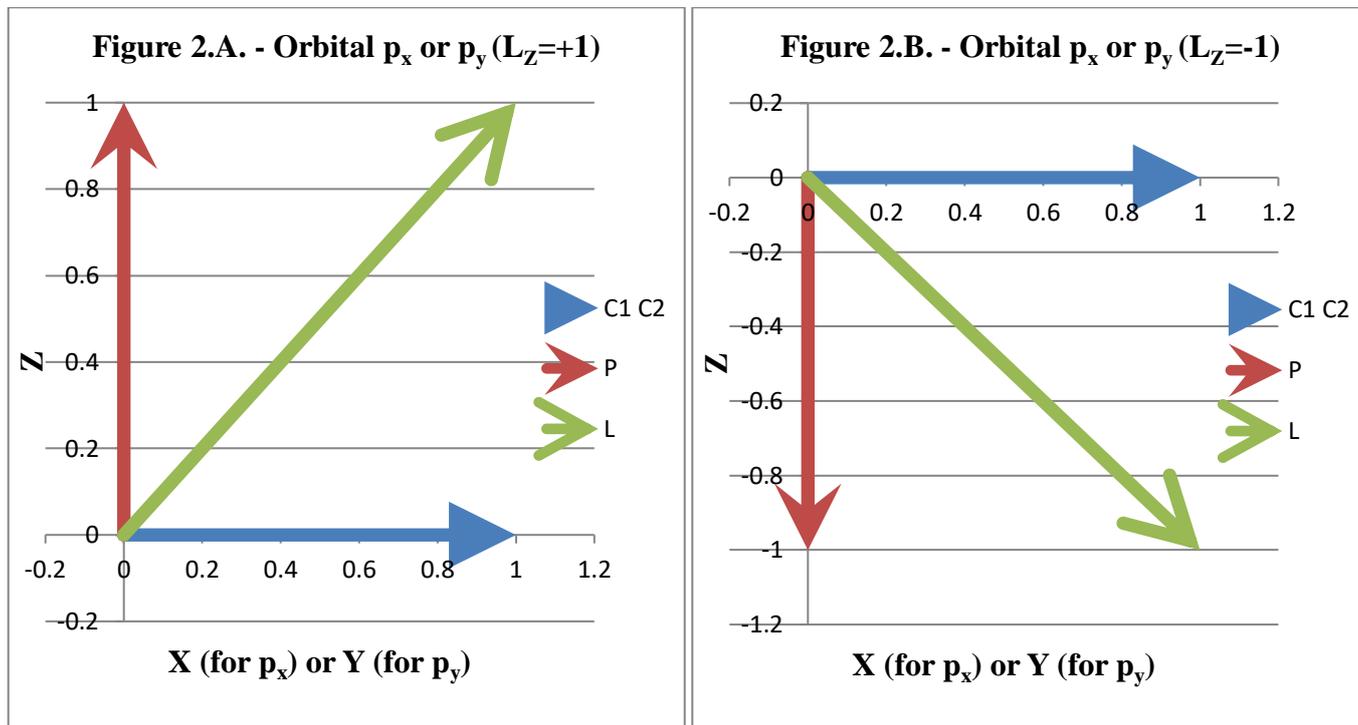
L_{XYZ} is the complementary component to L_{Z1} that allows reaching L_Z and L. L_{XYZ} is subdivided into two orthonormal components (XY and Z (Z is null for Groups I and II)). The two P vectors have been used to provide L_Z and therefore can no longer be used in L_{XYZ} . L_{XYZ} is reached with C2 that is already on X (p_x) or Y (p_y) axis and C1 that is aligned with C2. The two C vectors of $\hbar/2$ create one unit vector \hbar (2→1 in column C of L_{XYZ} analysis) and this unit vector \hbar is registered in column V L_{XYZ}

$$V_A = V_U - V_{L_{Z1}} - V_{L_{XYZ}}$$

V_A is voided vectors, that is, vectors that are voided between them. First of the group never cancels vectors. The next ones follow an ascending order of voided vectors as can be checked in Table 2. Therefore, there is no voided vector because p_x and p_y is the first (and only) type of orbitals of the first group:

$$2 - 1 \text{ (used on Z axis)} - 1 \text{ (used on X or Y axis)} = 0$$

p_x or p_y orbital with $L_z=1$ is represented in **Figure 2.A.** where the two vectors of C can be equally drawn in negative component X or Y. Likewise, vector location for $L_z=-1$ is represented (**Figure 2.B.**)



P92 is previously enunciated to the orbitals with $l > 1$

P92 Number of pairs of electronic extremes per orbital (lobe shape)

Number of pairs of electronic extremes per orbital is equal to number of pairs of lobes separated by one node. In case of p orbital, there are two lobes separated by one node and, according to P92, each p orbital has one unique pair of electronic extremes. Considering that:

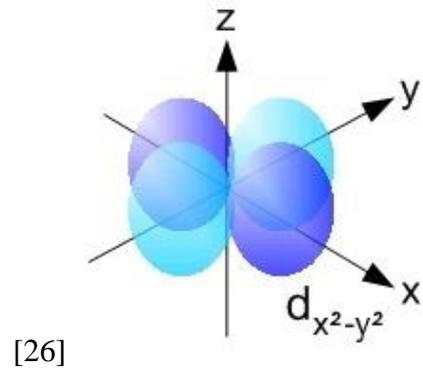
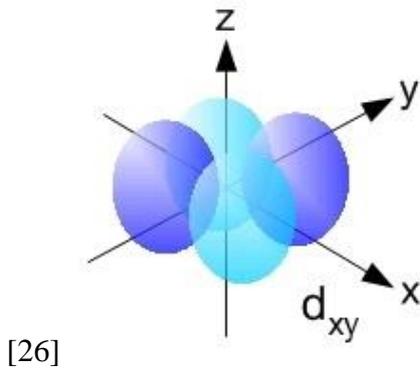
- * Two electronic extremes are in two adjacent lobes
- * Each electronic extreme has one OAMc and one OAMp (both are vectors with value $\hbar/2$) with a global vector value of \hbar given by two units of $\hbar/2$.

Number of vector units \hbar is equal to number of lobes that orbital has (0.B.)

$$(0.B.) \frac{2 \text{ electronic extremes}}{2 \text{ lobes}} \frac{\hbar}{\text{electronic extreme}} = \frac{\hbar}{\text{lobe}}$$

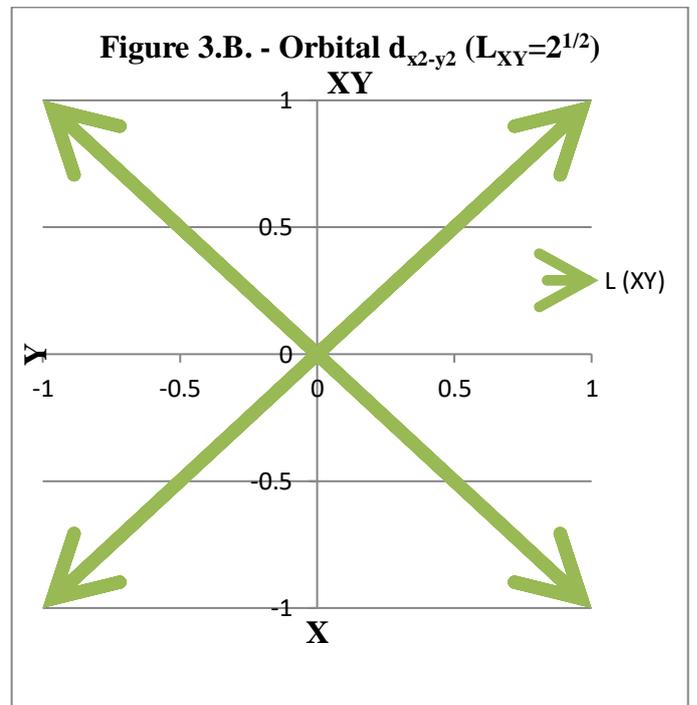
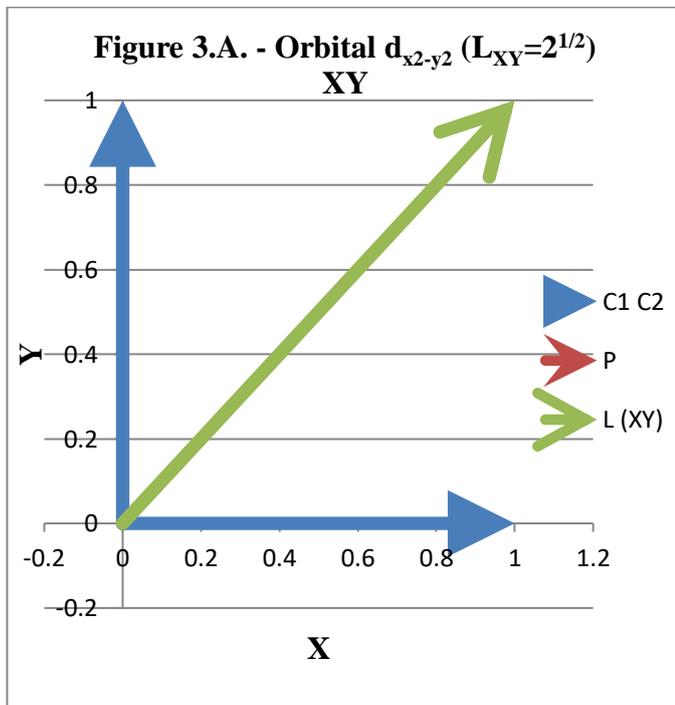
2) GRUPO II L_z máximo. Lóbulos en plano XY

2.1) d_{xy} $d_{x^2-y^2}$ orbitals $L_z = \pm 2$

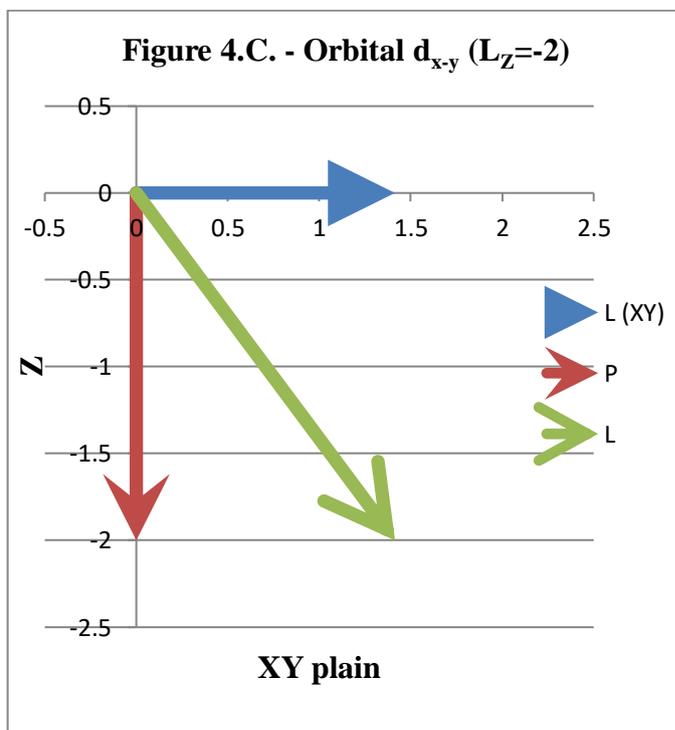
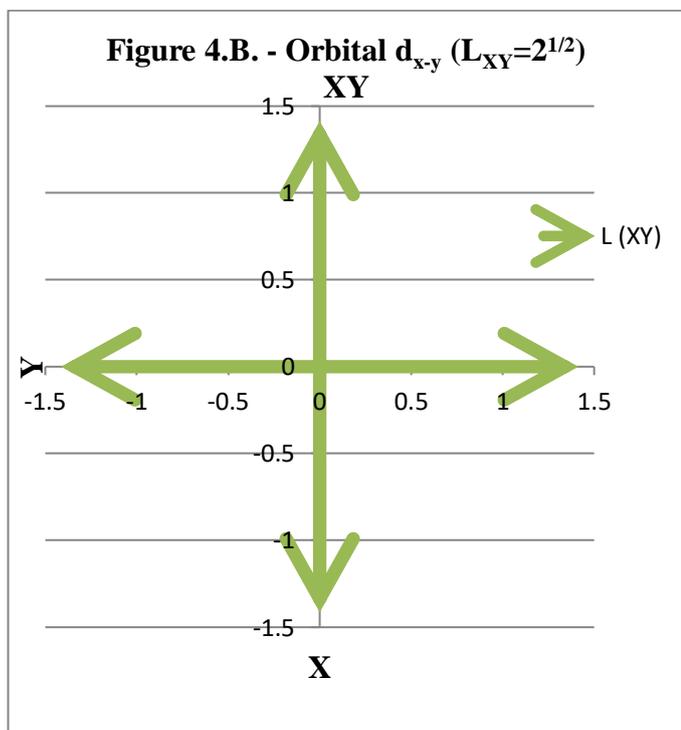
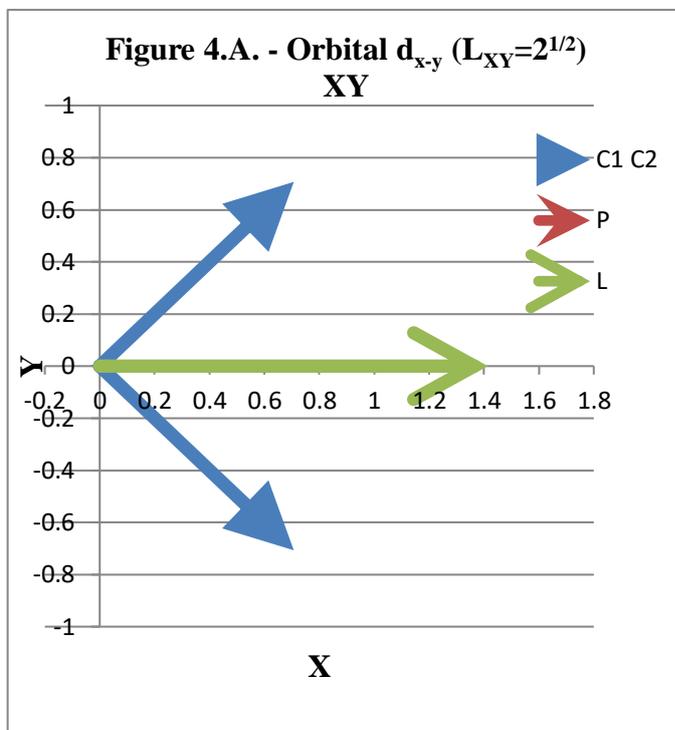
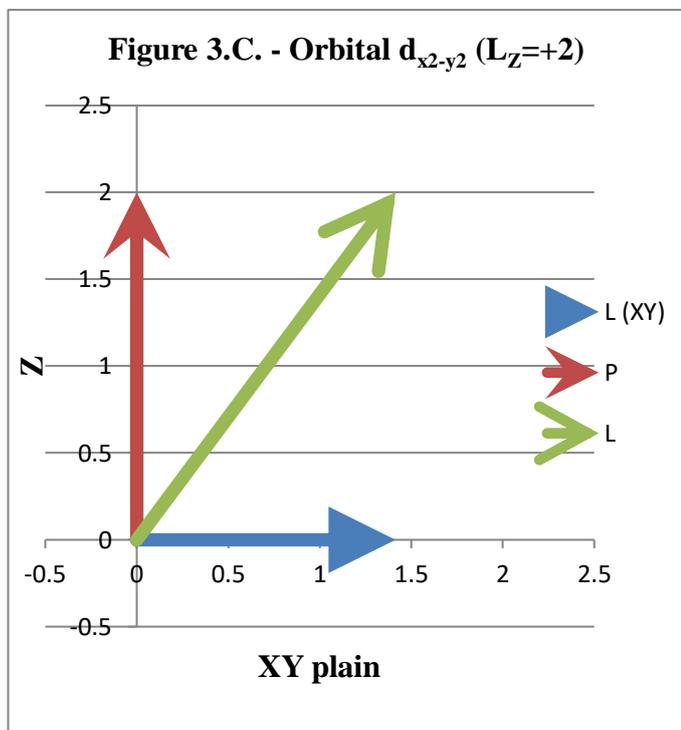


$d_{x^2-y^2}$ orbital is treated before d_{x-y} although, as in all orbital pairs, vector treatment is analogous to other member (in this case other member is d_{x-y})

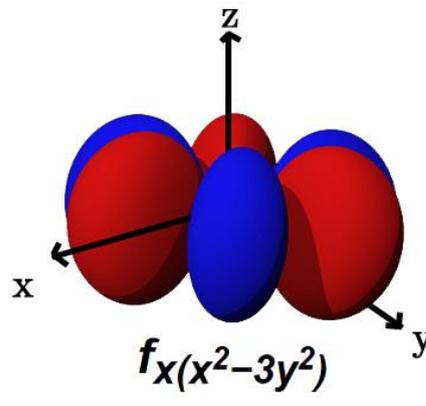
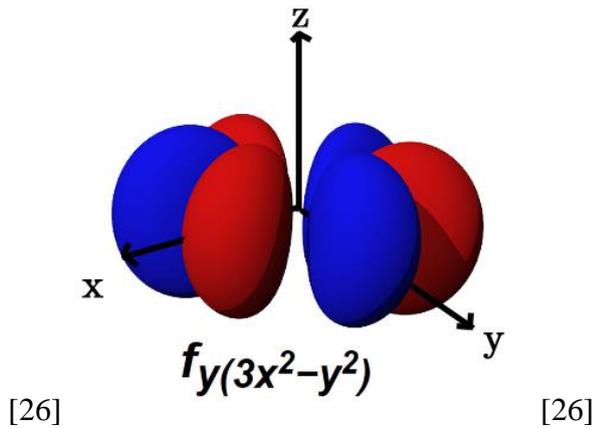
$d_{x^2-y^2}$ orbital has 4 lobes \rightarrow 4 VU of \hbar formed by 4 C and 4 P. Z component is ± 2 and therefore XY plane must be equal to $2^{1/2}$. $L_{XY} = 2^{1/2}\hbar$ is obtained with the 4 C vectors (C2 vectors in lobes direction with corresponding C1 vectors aligned: $4 \rightarrow 2^{1/2}$ in column C of L_{XYZ} and therefore using two vector units $V_{XYZ} = 2$) (Figure 3.A.). Vector result for L_{XY} represented in Figure 3.A. corresponds to first quadrant, but can also be done for remaining 3 quadrants (Figure 3.B.)



The 4 P vectors of $\hbar/2$ are aligned on Z axis to give two units of \hbar on Z axis ($4 \rightarrow 2$ on column P of L_{Z1} and using two unit vectors of \hbar). Final result is fulfillment of L and L_Z (**Figure 3.C.**) Situation is analogous to d_{x-y} orbital since d_{x-y} also has its 4 lobes in XY plane although with 45-degree offset from $d_{x^2-y^2}$ orbital. Therefore, **Figures 4.A. 4.B and 4.C.** (where are represented for $L_Z = -2$) are equivalent to Figures 3.



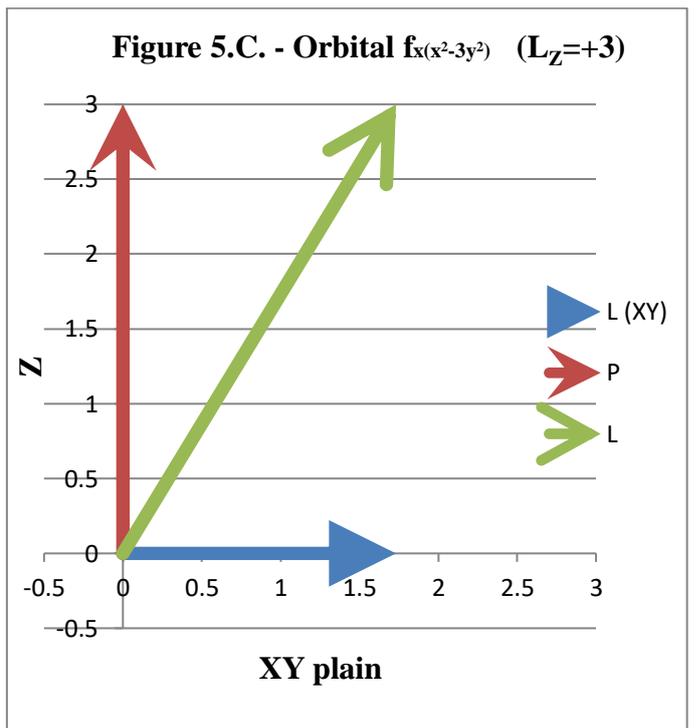
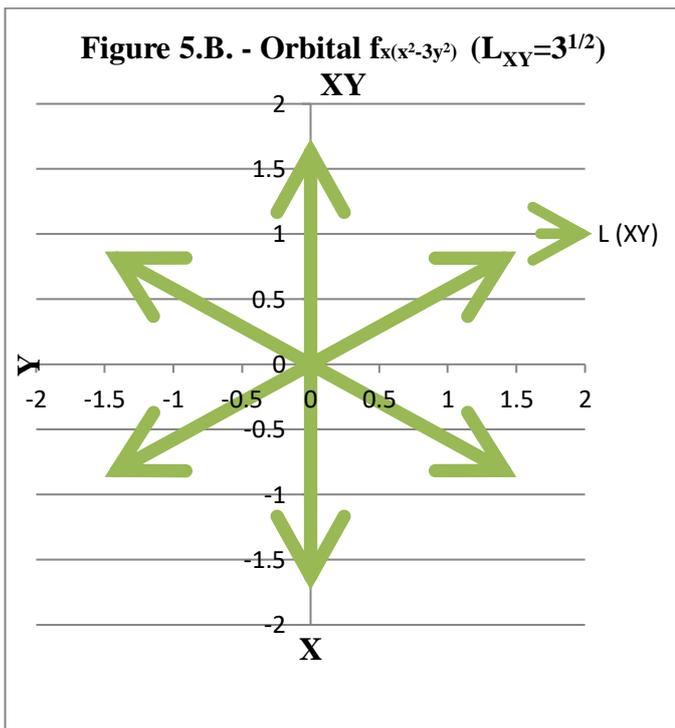
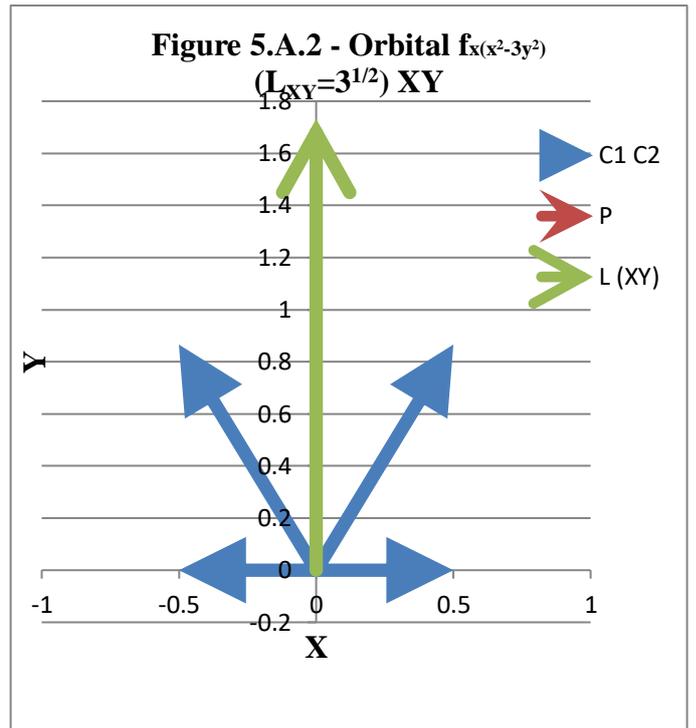
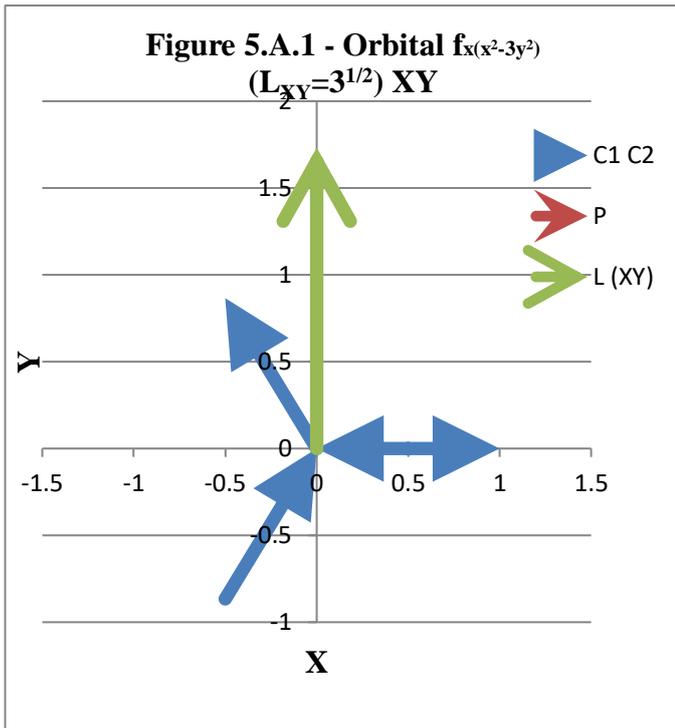
2.2) $f_{y(3x^2-y^2)}$ $f_{x(x^2-3y^2)}$ Orbitals $L_Z = \pm 3$



Vector treatment is equivalent for all pairs of orbitals as indicated in 2.1). In this case, study is initiated with $f_{y(3x^2-y^2)}$. $f_{y(3x^2-y^2)}$ orbitals has 6 lobes \rightarrow 6 VU of \hbar formed by 6 C and 6 P. Z component is ± 3 and therefore XY plane must be equal to $3^{1/2}$. $L_{XY} = 3^{1/2}$ is obtained by 2 pairs of C1 and C2 aligned (4 C vectors) (**Figure 5.A.**). The 6 lobes are located in XY plane and separated by 60 degrees ($360^\circ / 6 \text{ lobes} = 60^\circ$) and resultant, located between them, is at 30° : $2 * \cos 30^\circ = 2 * 3^{1/2} / 2 = 3^{1/2}$. The other 2 C vectors of the remaining lobe are mutually annulled. Orbital cancellations follow an ascending order when increasing orbital quantum number l (0, 1 and 2 for d, f and g respectively in this group II - see Table 2) as previously indicated.

Figure 5.A.1 considers that each pair of electronic extremes of one electron is located in two adjacent lobes (P92) and **Figure 5.A.2** considers the vectors sum. In following cases, vectorial sum representation is the one used preferably (Figure 5.A.2), although lobes contribution located in lower quadrants should also be considered. Vectors sum in XY plane (L_{XY} represented $L(XY)$ in the figures) can have several orientations depending on selected lobes (**Figure 5.B.**)

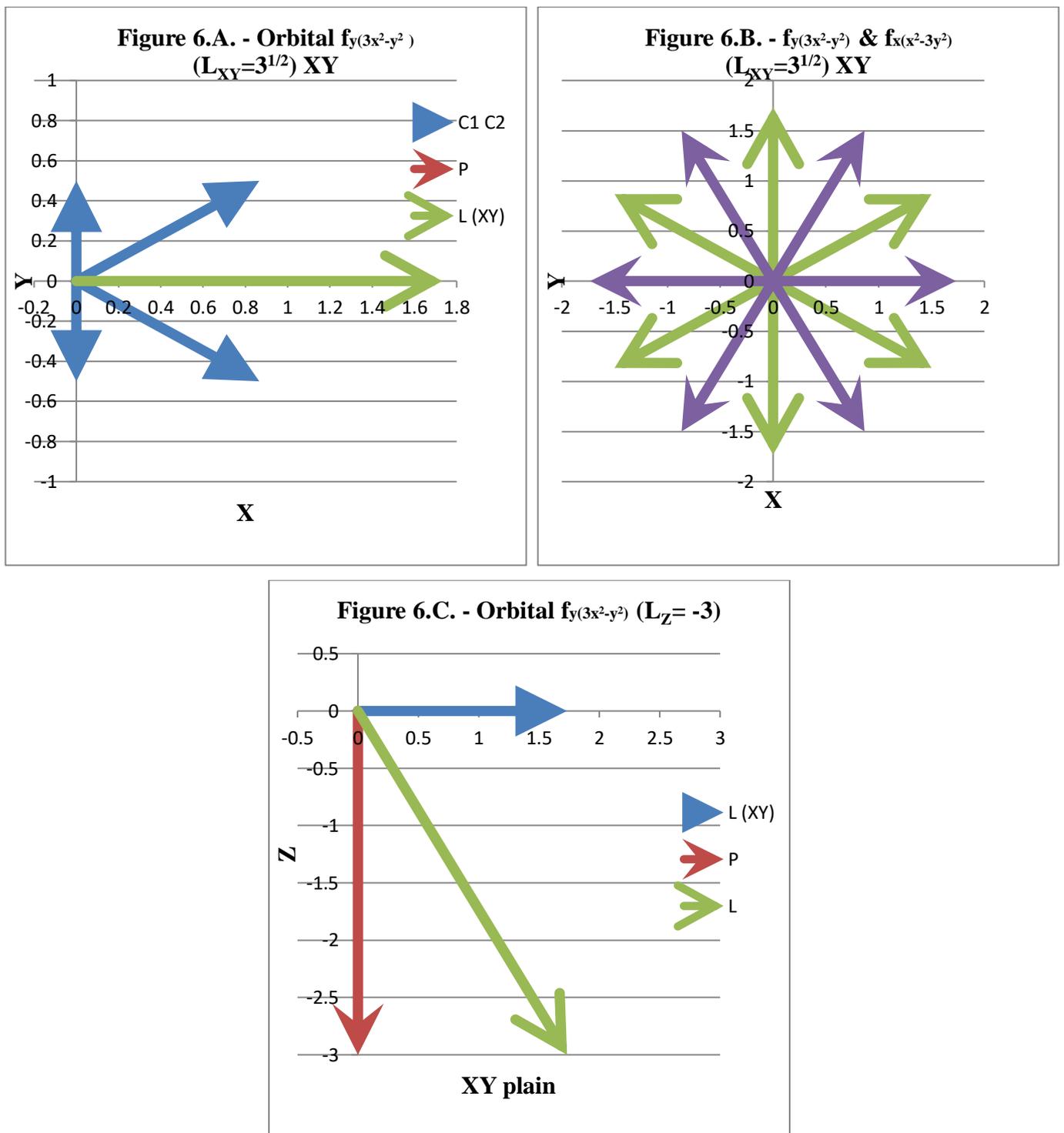
As in case 1.1) and 2.1), the vectors P of aligned form are those that cause L_Z . In this case, the 6 P vectors of $\hbar/2$ are aligned on the Z axis to give three units of \hbar on the Z axis ($6 \rightarrow 3$ on column P of L_{Z1} and using three unit vectors of \hbar). Final result is fulfillment of L and L_Z (**Figure 5.C.**). The 6 orientations of L_{XY} vector are jointly represented as $L(XY)$ on the axis indicated as XY Plane.



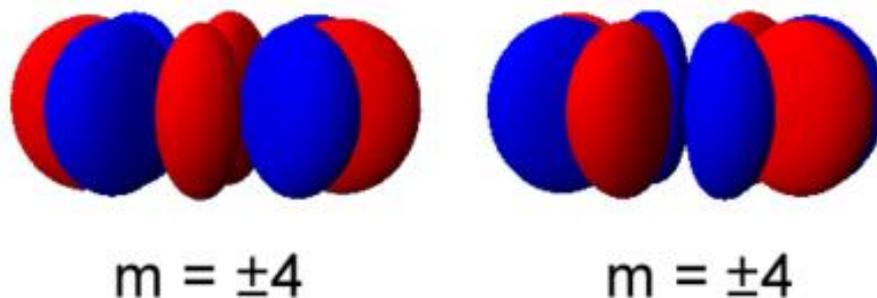
Difference between total unit vectors (6) and unit vectors used in the XY plane (2) and the Z axis (3) is equal to $VA = 1$ (voided vectors). Cancellation of this vector allows L_{XY} value required ($3^{1/2}$), continuation of Group II model for angular movements that allows L and L_Z to be fulfilled and start of progressive increase of voided vectors as orbital quantum number increases in same group. Number of vectors annulled globally (considering all Groups) also has relation with numbers 2 and 3 as seen at end of this article.

$f_{y(3x^2-y^2)}$ is obtained analogously to $f_{x(x^2-3y^2)}$ orbital:

- * **Figure 6.A.** is like Figure 5.A.2.
- * **Figure 6.B.** includes different L_{XY} orientations of $f_{y(3x^2-y^2)}$ that are added to the views in Figure 5.B.
- * **Figure 6.C.** is like Figure 5.C. but for $L_Z = -3$



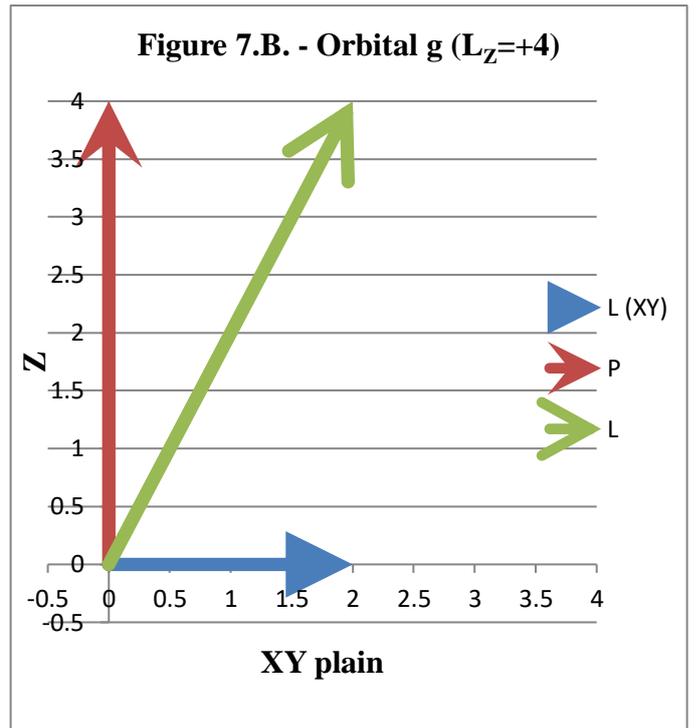
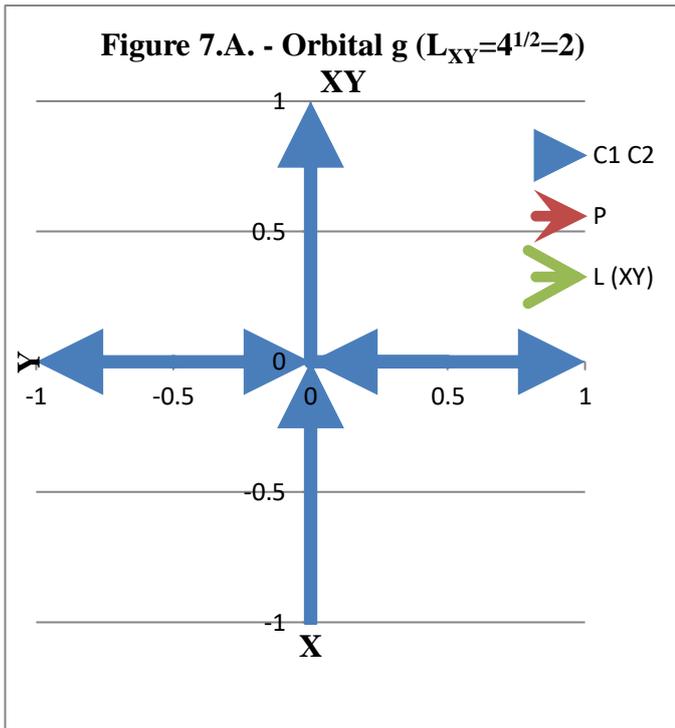
2.3) g Orbitals $L_Z = \pm 4$



[26]

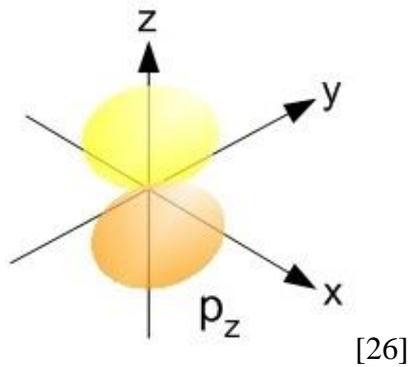
g orbital with $L_Z = \pm 4$ follows the model of the angular movements that allows to fulfill L and L_Z for group II (group with maximum L_Z and that has its lobes in XY plane). Follow-up of Group II model is as follows:

- * There are 8 lobes (lobes number progressively increases 4, 6 and 8) → 8 VU of \hbar (unit vectors). Therefore, there are 8 C vectors and 8 P vectors each with a value of $\hbar/2$
- * 8 lobes imply 4 electronic extremes located according to P92, that is, each pair of electronic extremes is located in 2 adjacent lobes separated by one node.
- * Canceled vectors (VA) increases to 2: d (0), f (1) and g (2). Both voided vectors are located in the XY plane and therefore cancellation is performed with C vectors located in the direction of the orbital axis.
- * The other 4 non-canceled C vectors are aligned to give $L_{XY} = 4 \frac{1}{2} \hbar = 2\hbar$ which is required value in XY plane. (**Figure 7.A.**). Possible orientations in XY plane are 8.
- * The 8 P vectors are aligned on Z axis to add $L_Z = 8 \frac{1}{2} \hbar = 4\hbar$ and thus can be fulfilled that $L = (4^2 + (4^2)^{1/2})^{1/2} = 20^{1/2}$ (**Figure 7.B.**).



3) GROUP III $L_Z = 0$ Two lobes and rings

3.1) p_z Orbital



p_z orbital is initiated with discarded possibilities by non-compliance of P89 or L (point 3.1.1) and correct option (3.1.2) that is maintained in following orbitals with $L_Z = 0$

3.1.1) First option discarded due to incompatibility with P89

p_z orbital has OAM of $\pm \hbar/2$ on Z axis contributed by movement 1) of EE2 (C2). First idea can be to locate the other vector C of $\hbar/2$ (C1) on Z axis and place the two P vectors of $\hbar/2$ together and perpendicularly. Result is $L_Z = \pm \hbar$ or 0 (depending on where these two C vectors are placed on Z axis). L

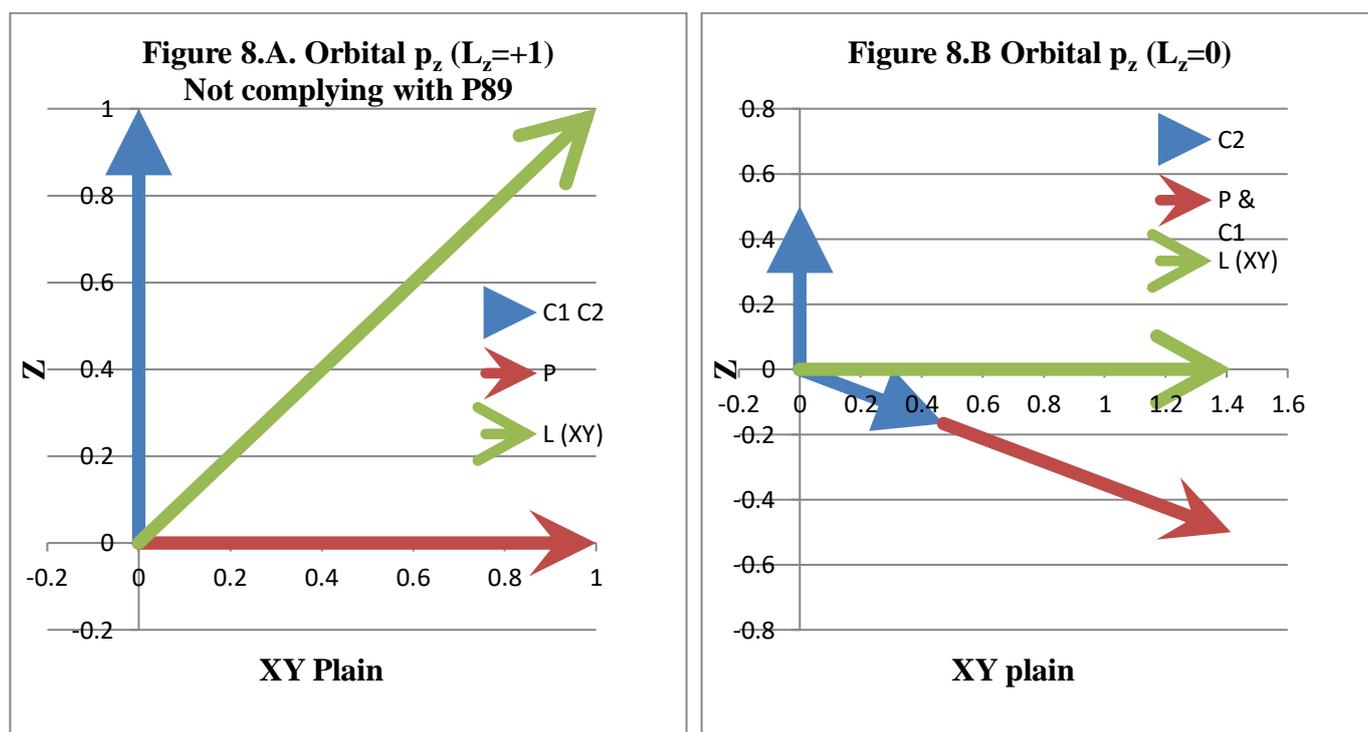
consequent is: $L = (1^2+1^2)^{1/2} \hbar = 2^{1/2} \hbar$ with $L_z = \pm \hbar$ (**Figure 1**) or $L = (0^2+1^2)^{1/2} \hbar = \hbar$ with $L_z = 0$. L is not correct when $L_z = 0$. L is correct for orbitals p with $L_z = \pm 1$, but resulting vector perpendicular to Z axis can be located in any direction of XY plane providing $L = 2^{1/2} \hbar$ and this fact fails to comply with P89.

3.1.2) p_z Orbital with $L_z = 0$

As commented in 1.1.1), p_z orbital has OAM of $\pm \hbar/2$ on Z axis contributed by movement 1) of EE2 (C2 vector). The 3 remaining vectors of $\hbar/2$ must achieve two objectives: cancel Z component and reach L value. This fact is not trivial and is achieved because 3 values allow the right triangle (1): $\hbar/2$ is value to be canceled on Z axis, $L = 2^{1/2} \hbar$ must be obtained in XY plane and for this remaining 3 contributions of $\hbar/2$ ($3/2$) are used.

$$(1) \left(\frac{1}{2}\right)^2 + (2^{1/2})^2 = \left(\frac{3}{2}\right)^2$$

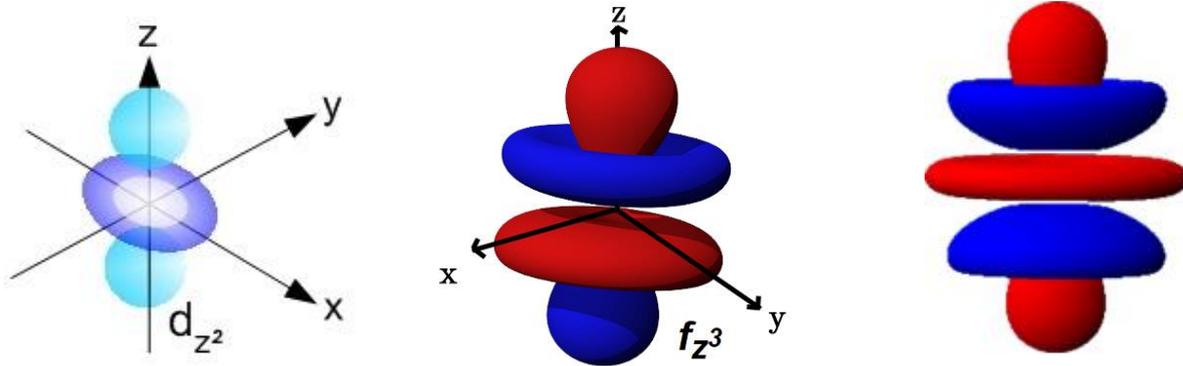
Following orbital quantum numbers exhibit identical behavior (for example with d_z^2) and compliance with (1) for which they require a specific orbital type with a ring shape.



P93 Number of electronic extremes pairs per orbital (ring shape ($L_z=0$))

Orbital region that is ring-shaped (orbitals d_z^2 f_z^3 and g_z^4) presents one pair of electronic extremes for each ring. These electronic extremes cancel each other's C2 of MOAc because they circulate with the same sense in half-turn. Orbital ring should be expanded in attached article.

3.2) z^2 , f_z^3 , y , g_z^4 Orbitals (Orbitals with ring and $L_z=0$) – Concepts for all orbital with main lobe in Z axis.



[26]

Considering p_z (point 3.1), P89, P92 and P93 can be demonstrated that orbitals with one single pair of lobes separated by a node in Z axis require $(l-1)$ rings (2) to meet their corresponding equation of right triangle (1). The orbital p has 0 rings because $l = 1$.

$$(2) \text{ Number of rings in orbital } z = l - 1$$

Analysis of requirement indicated in (2):

- Z component that must be canceled = $1/2 \hbar$ (coming from C2 of EE2 on Z axis).
- Value in XY plane that must be achieved = total angular momentum = $L = (l(l+1))^{1/2} \hbar$
- OAM vectors (each with value of $\hbar/2$) usable for hypotenuse vector (**Figure 9** for d_z^2 and Figure 8.B for p_z orbital):
 - OAMc of EE1 of lobe located on Z axis (C1): 1
 - OAMp of EE1 and EE2 of lobe located on Z axis (P): 2
 - OAMp of EE1 and EE2 of each ring in function of l (2) and as there are two electronic extremes in each ring (P93): $(l-1)*2$

For to be correct and right triangle to be fulfilled, sum of two sides $a)^2$ y $b)^2$ must equal the hypotenuse $c)^2$ (3):

$$(3) \left(\frac{1}{2}\right)^2 + \sqrt{l(l+1)}^2 = \left(\frac{1}{2}(1+2+2(l-1))\right)^2$$

(3) development leads to both sides being equal (4):

$$(4) \frac{1}{4} + l^2 + l = \frac{1}{4} + l^2 + l \quad (4.B.) \left(l + \frac{1}{2}\right)^2 = \left(l + \frac{1}{2}\right)^2$$

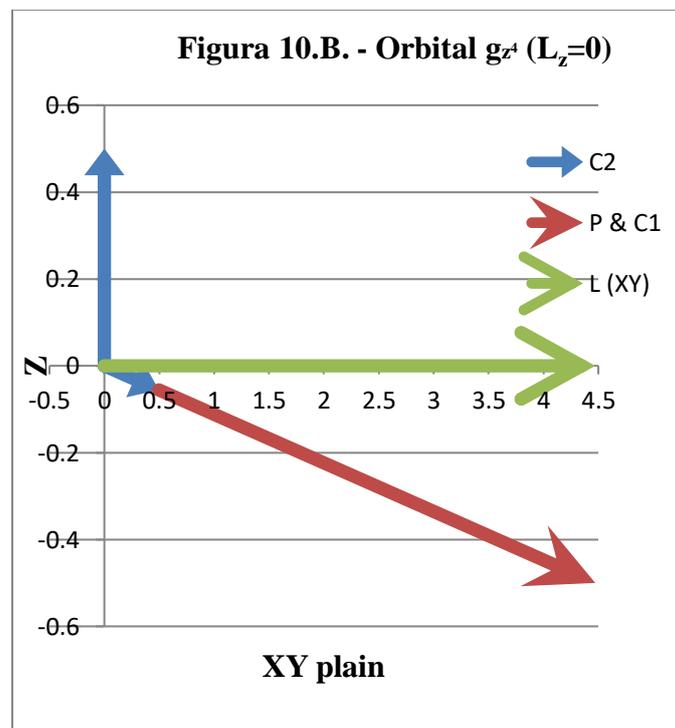
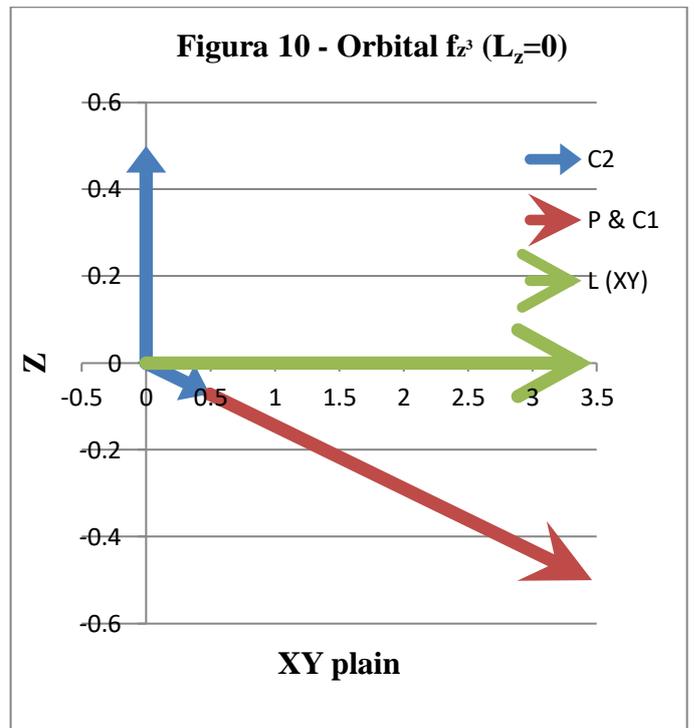
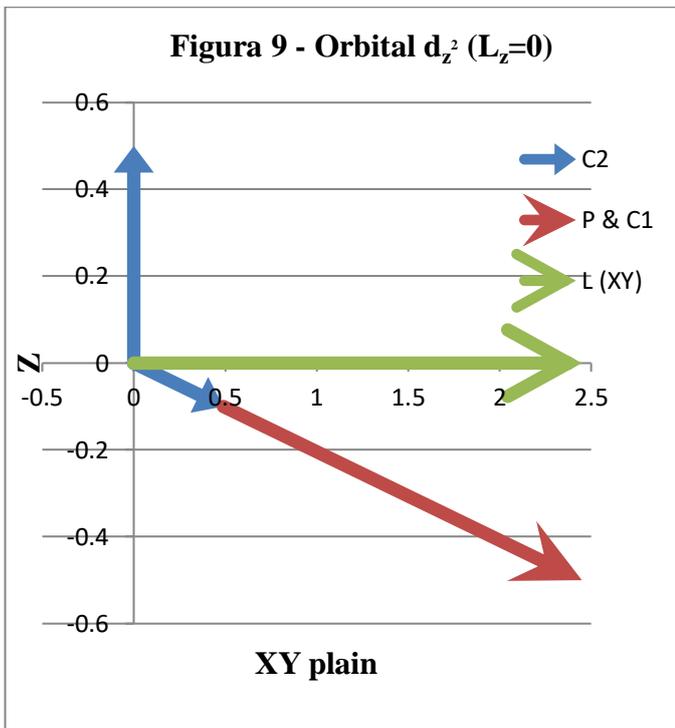
Therefore it is fulfilled:

* Experimental angular momentums. Total angular momentum $L = (l(l+1))^{1/2} \hbar$ and $L_z = 0$ for p_z , d_z^2 , f_z^3 and g_z^4

* Experimental number of rings. Rings number is consistent with experimental: 0 rings for p_z , 1 ring for d_z^2 , 2 rings for f_z^3 and 3 rings for g_z^4

Vectors number (with $\hbar/2$ value) joined in d_z^2 orbital are therefore 5 for allow it to be fulfilled:

$$(1/2)^2 + (6^{1/2}/2)^2 = (5/2)^2$$



Likewise, f_{z^3} orbital can be represented (**Figure 10**) where there are 7 vectors of $\hbar/2$ (2 additional with respect to d_{z^2} from OAMp of EE1 and EE2 of second ring) to compensate $1/2 \hbar$ on the Z axis (from OAMc of EE2) and reach $L=12^{1/2}\hbar$ in XY plane. Increase in orbital quantum number causes angle between OAMc and OAMp vectors to decrease, but without reaching XY plane and consequently P89 is always fulfilled independently of orbital quantum number. Orbital g with $L_z = 0$ is represented in **Figure 10.B**.

4) GROUP IV Orbitals with L_z intermediate

Group IV has similarities with angular movements model that allows L and L_z to be met for group III, as can be foreseen with Table 2. Some coincidence points of both groups:

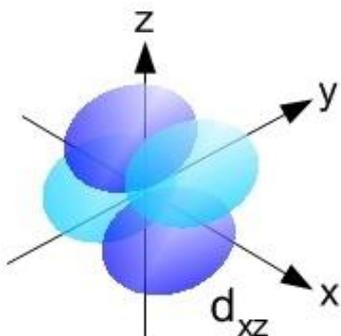
- * L_{z1} is obtained with vectors C2, that is, with the OAMc of EE2 which is vector with lobe direction.
- * L_{xy} component and L_z component completed are provided by vectors C1 and P.

Possibly, the most outstanding differential element between both groups III and IV is how L_{xyz} is achieved:

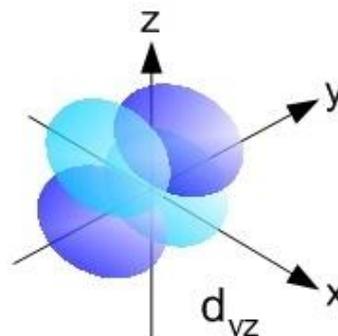
- * Group III: vectors C1 and P are all aligned together since they strictly comply with necessary L_{xyz} value (3) and (4).
- * Group IV: Vectors C1 and P are superior to those necessary if they are located linearly. Vectors C1 and P are placed at fundamental angles (45 or 30 degrees fulfilling P91 1 2 and 3) with respect to necessary L_{xyz} vector. This value of 45 and 30 degrees is the same as angle formed by C2 and Z axis in each orbital. In this way, all orbitals of this group IV except orbital f with $L_z=\pm 1$ have angles of 45 degrees between C2 and Z and between L_{xyz} and C1 and P. These angles for orbital f with $L_z=\pm 1$ are 30 degrees.

4.1) Group IV: Subgroup A

4.1.1) Orbital d_{xy} d_{yz} $L_Z = \pm 1$ (first orbital of Group IV, subgroup A)

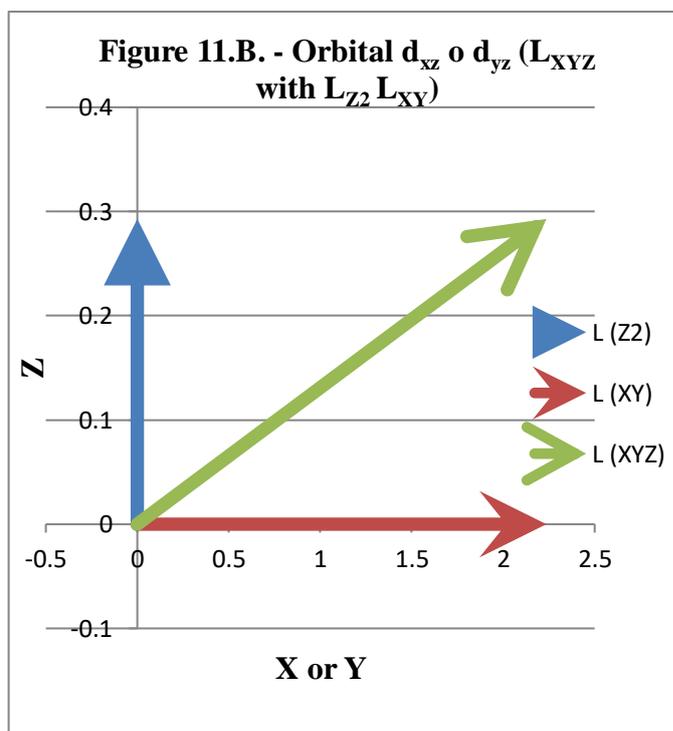
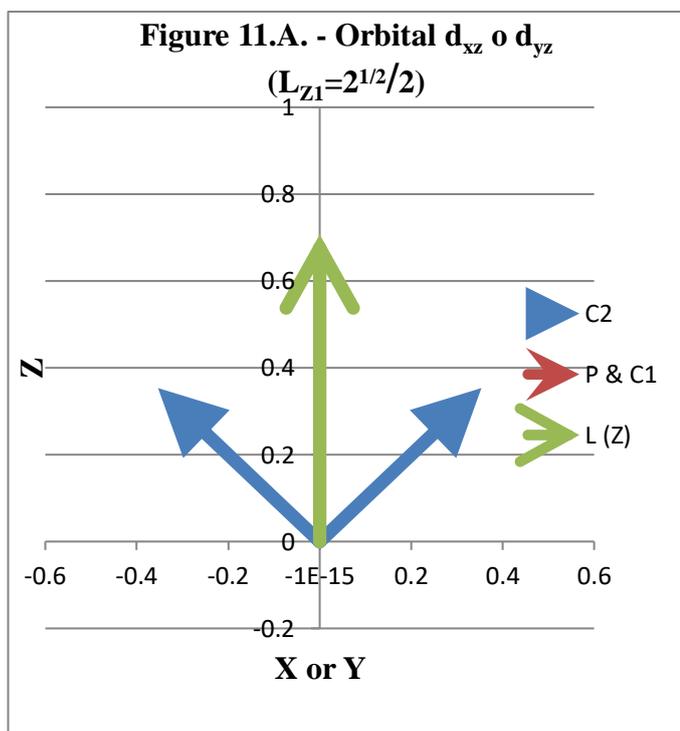


[26]

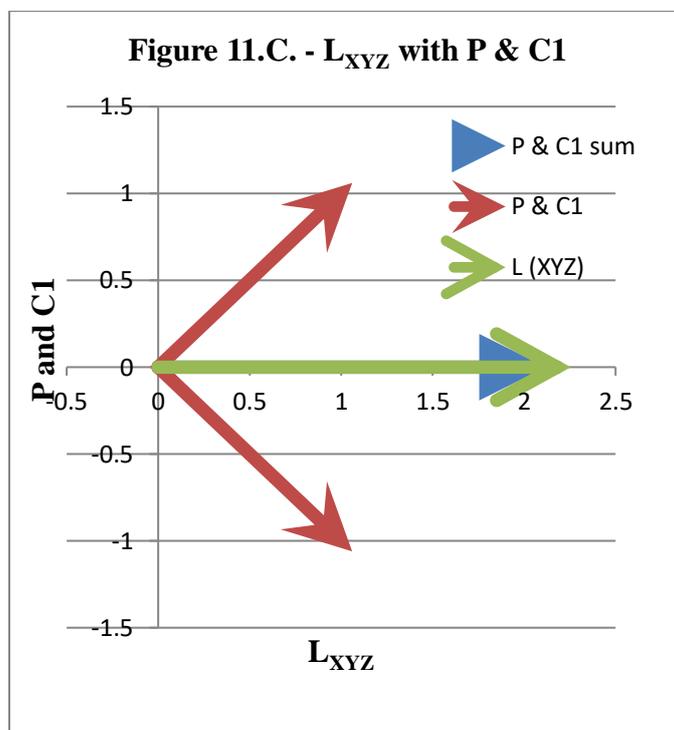


[26]

Orbitals d_{xy} d_{yz} have 4 lobes and therefore 4 unit vectors or vector units \hbar since there are as many VU as lobes the orbital has (P92). As indicated in the introduction of Group IV, the 2 C2 vectors of $\hbar/2$ are those that give the vector L_{Z1} (one unit vector is used in $V L_{Z1}$ since two C2 vectors of $\hbar/2$ have been required: $V L_{Z1}=1$). Each of the vectors C2 provides $(2^{1/2}\hbar)/4$ because they are at an angle of 45° with respect to Z axis. Consequently, L_{Z1} result on Z axis is $(2^{1/2}\hbar)/2$ ($2 \rightarrow 2^{1/2}/2$) (**Figure 11.A.**). L_{XYZ} has to fulfill two objectives: complete L_{Z1} up to L_Z value (in this case, ± 1) through L_{Z2} vector and provide L_{XY} as indicated in L_{XYZ} column of Table 2 (**Figure 11.B.**).



The necessary value of L_{XYZ} (5) can be approximated by $3\frac{\sqrt{2}}{2}$ with 3 vector units of \hbar ($V L_{XYZ} = 3$) located at an angle of 45 degrees with respect to necessary L_{XYZ} vector (**Figure 11.C.**) This fact can be done since there are 3 unit vectors (there are 4 in total and only one has been used in L_{Z1}): 6 vectors of $\hbar/2$ corresponding to 2 C1 and 4 P.

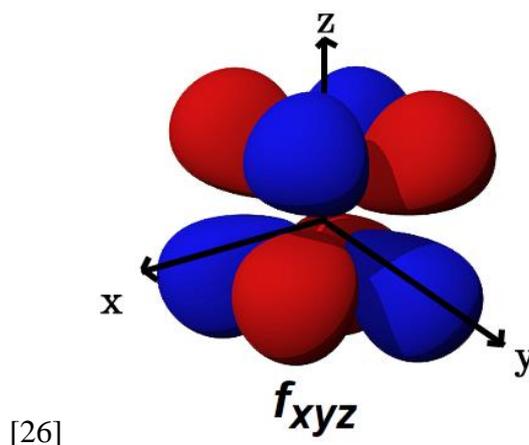
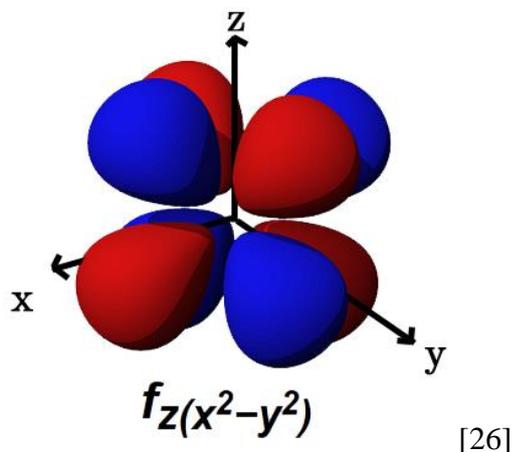


$$(5)L_{XYZ} = \sqrt{\left(1 - \frac{\sqrt{2}}{2}\right)^2 + (5^{1/2})^2} \approx 3\frac{\sqrt{2}}{2}$$

4.1.2) Orbital $f_{z(x^2-y^2)}$ y f_{xyz} $L_Z = \pm 2$ and type g with $L_Z = \pm 3$ (second and third orbital of Group IV, subgroup A)

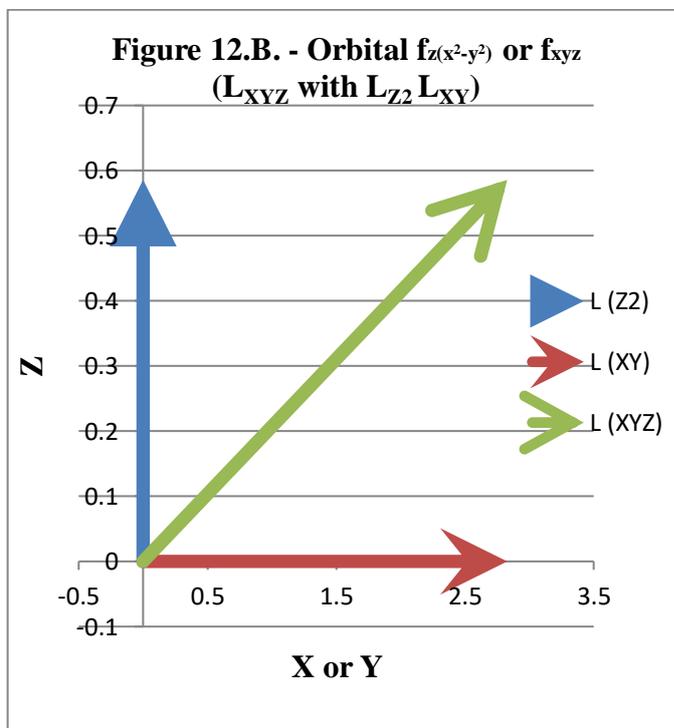
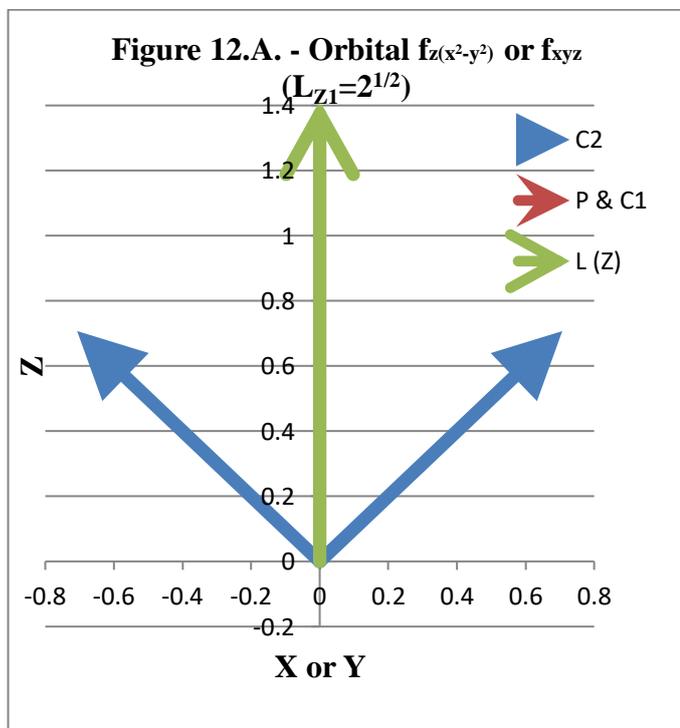
The other orbitals of Group IV, subgroup A, type f with $L_Z = \pm 2$ and type g with $L_Z = \pm 3$, are the same as seen in 4.1.1) with the only exception of the increase in lobes and consequently unit vectors of \hbar which serve to achieve L and L_Z . Therefore, these orbitals are explained in a more schematic way.

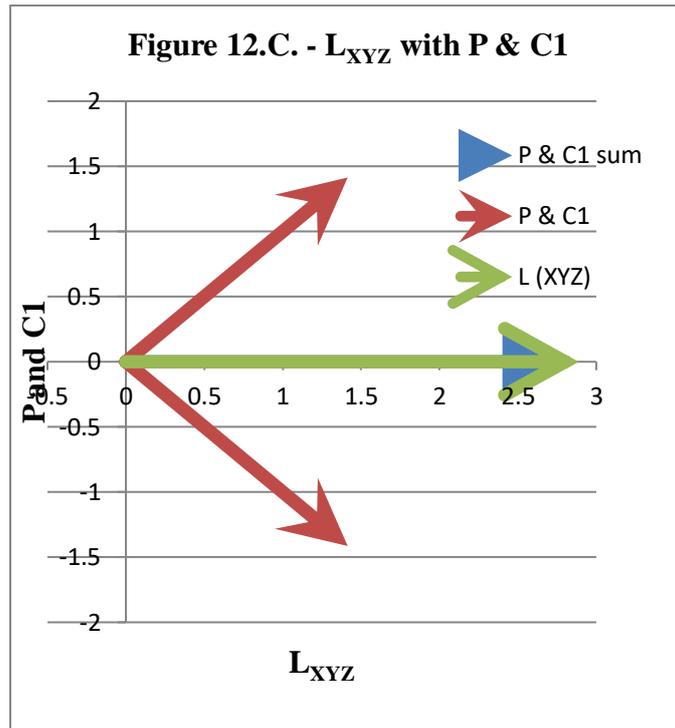
Orbital $f_{z(x^2-y^2)}$ and f_{xyz} $L_Z = \pm 2$



8 lobes \rightarrow 8 unit vectors (VU) \rightarrow all 4 C2 vectors of $\hbar/2$ (2 from above and 2 from below in 45 degrees with respect to the Z axis) $\rightarrow L_{Z1} = 2^{1/2} \hbar$ with consumption of 2 unit vectors (V $L_{Z1} = 2$) (Figure 12.A.) There are 6 VU left.

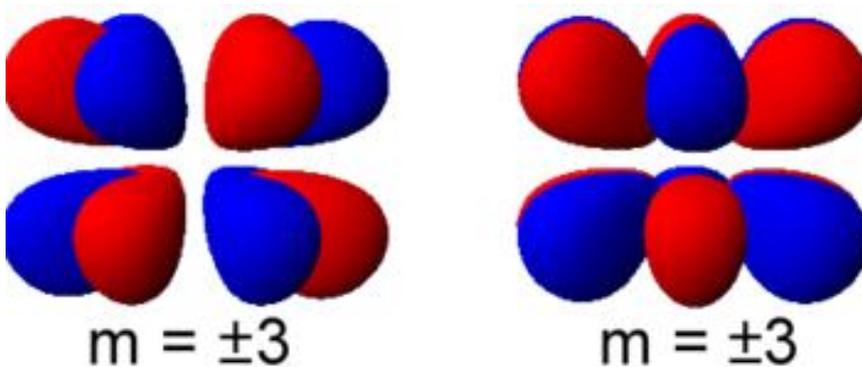
L_{XYZ} must meet L_{XY} and complete L_Z (6) (Figure 12.B.) \rightarrow 8 vectors of $\hbar/2$ located at 45 degrees are used to meet L_{XYZ} (Figure 12.C.) \rightarrow Consumption of 4 unit vectors (V $L_{XYZ} = 4$)





$${}^{(6)}L_{XYZ} = \sqrt{\left(2 - \frac{2\sqrt{2}}{2}\right)^2 + (8^{1/2})^2} \approx 4 \frac{\sqrt{2}}{2}$$

Orbital type g $L_Z = \pm 3$

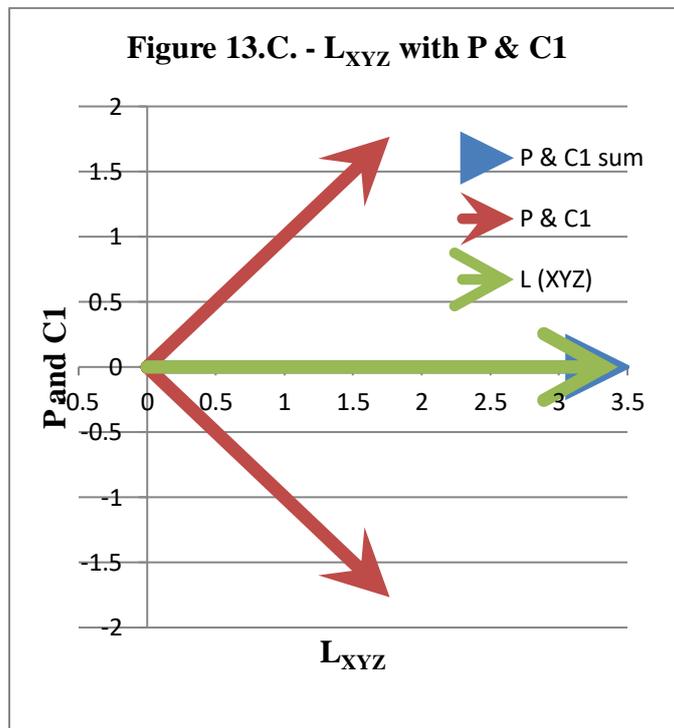
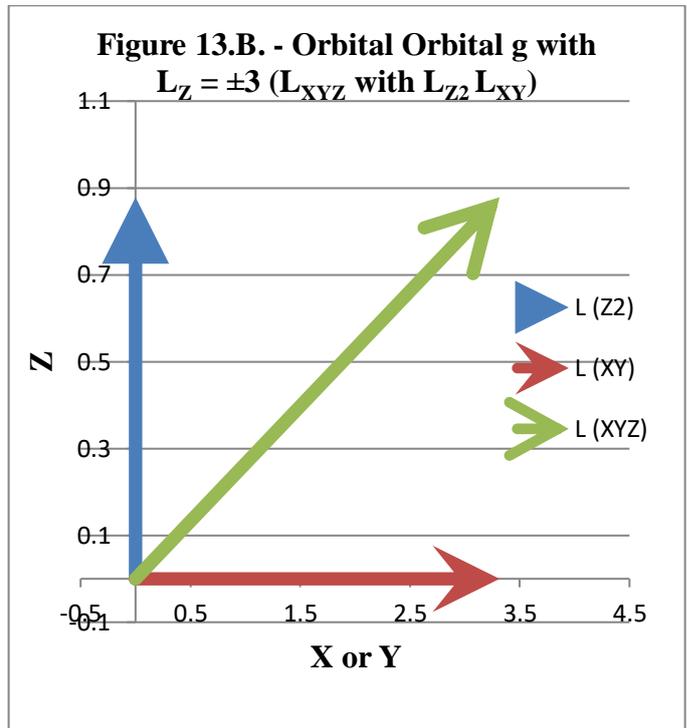
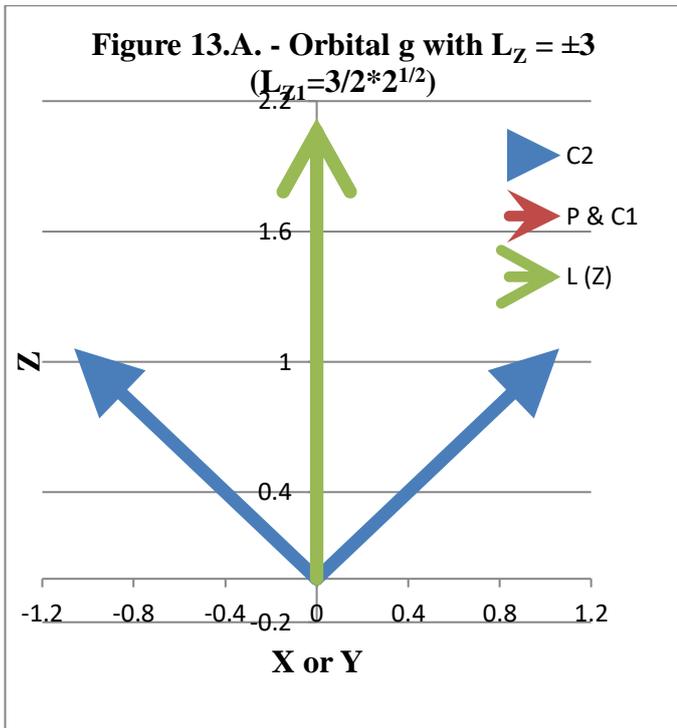


[26]

12 lobes \rightarrow 12 unit vectors (VU) \rightarrow all 6 C2 vectors of $\hbar/2$ (3 from above and 3 from below in 45 degrees with respect to the Z axis) $\rightarrow L_{Z1} = 3/2 * 2^{1/2} \hbar$ with consumption of 3 unit vectors ($V L_{Z1} = 3$) (**Figure 13.A.**) There are 9 VU left.

L_{XYZ} must meet L_{XY} and complete L_Z (7) (**Figure 13.B.**) \rightarrow 10 vectors of $\hbar/2$ located at 45 degrees are used to meet L_{XYZ} (**Figure 13.C.**) \rightarrow Consumption of 5 unit vectors ($V L_{XYZ} = 5$)

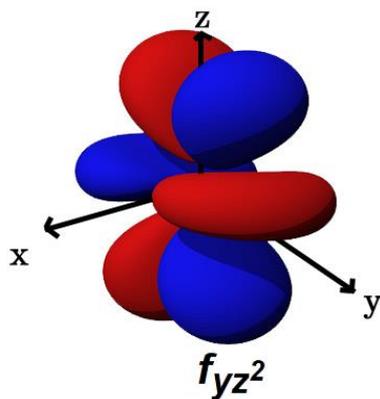
$$(7)L_{XYZ} = \sqrt{\left(3 - 3\sqrt{2}/2\right)^2 + (11^{1/2})^2} \approx 5 \frac{\sqrt{2}}{2}$$



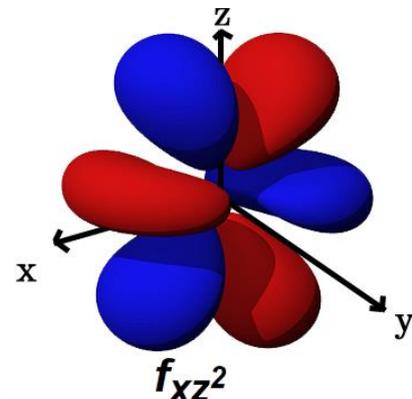
4.2) Group IV: Subgroup B

Orbitals of Group IV, subgroup B, have one unique peculiarity with respect to all other orbitals of Groups I to IV: these orbitals have C2 vectors to give L_{Z1} (as Group III and rest of Group IV), but they also have lobes with their corresponding C2 in the XY plane thus affecting L_{XY} (Group I and II have C2 in XY (giving L_{XY} value), but said groups I and II only have vectors C2 in XY and therefore C2 has no influence on L_Z). That is, orbitals of Group IV, subgroup B, are the only ones whose C2 vectors affect L_{XYZ} and L_{Z1} .

4.2.1) Orbital f_{yz^2} f_{xz^2} $L_Z = \pm 1$ (first orbital of Group IV, subgroup B)



[26]



[26]

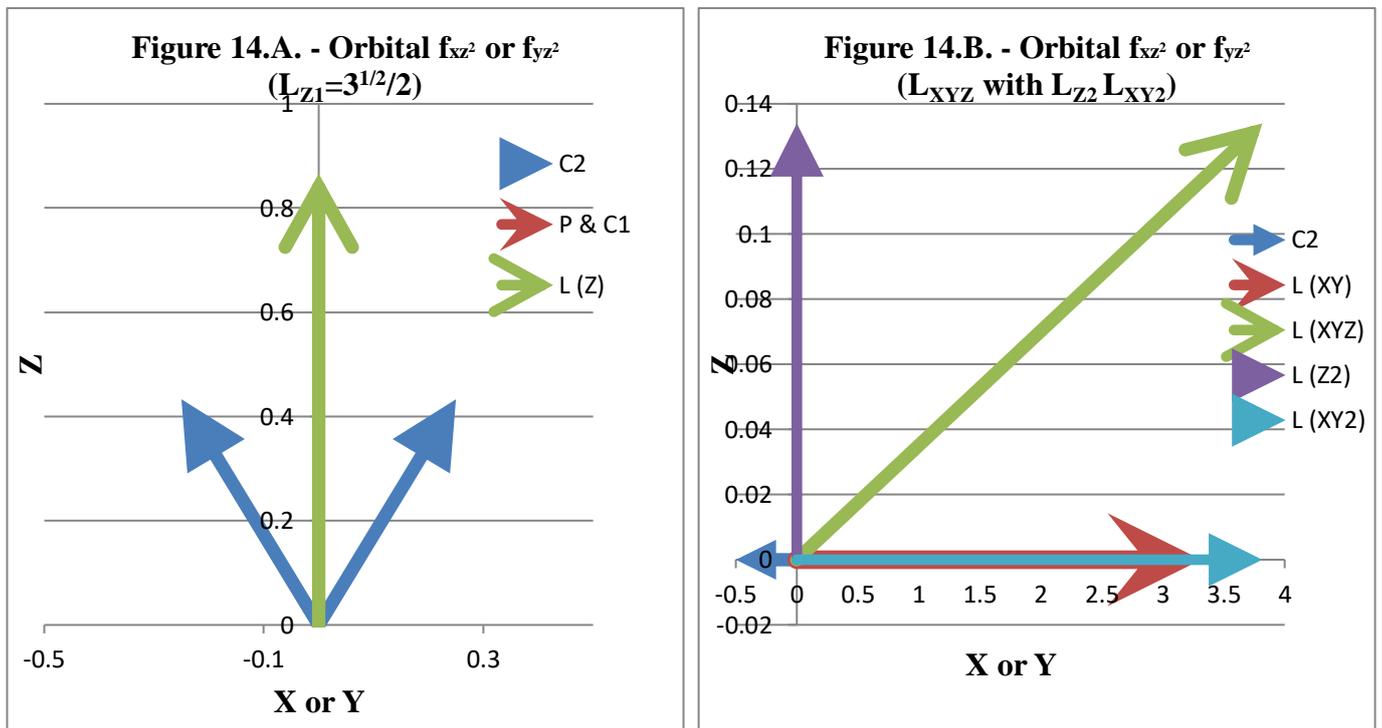
f_{yz^2} f_{xz^2} have lobes on the Y and X axis respectively that affect L_{XY} and the remaining lobes imply L_{Z1} . These two orbitals also have one differential detail: the remaining lobes that imply L_{Z1} have an angle of 30 degrees with respect to Z axis. 30 degrees cosine is $3^{1/2}/2$ and is the only one that works with this value (See Table 2, columns of L_{Z1} and L_{XYZ}). Orbitals of Group IV, subgroup B, type f with $L_Z = \pm 1$ have 6 lobes \rightarrow 6 unit vectors of \hbar (12 of $\hbar / 2$)

L_{Z1} situation: 2 C2 vectors of $\hbar/2$ at an angle of 30 degrees with the Z axis $\rightarrow L_Z = 3^{1/2}/2 \hbar$ and consumption of 1 VU (**Figure 14.A.**)

L_{XYZ} situation: The remaining 9 vectors of $\hbar/2$ (12 in total minus 2 in L_{Z1} and one in plane XY whose behavior with respect to L_{XYZ} must be studied) must provide $L_{XY} = 11^{1/2}$ (considering that situation of the vector $\hbar/2$ of C2 must be studied in the XY plane) and, together with L_{Z1} , reach the value of $L_Z = \pm 1$. L_{XYZ} (8) completes L_Z with $L_{Z2} = 1 - \sqrt{3}/2$ and L_{XY2} has value of $11^{1/2} + 1/2$. 1/2 is vector C2 of the lobe that is on X axis or Y axis (depending on which orbital is selected from the two possible) and that is therefore opposite to sense of vector L_{XY2} . Opposition of vector C2 to L_{XY2} is represented in **Figure 14.B.** This

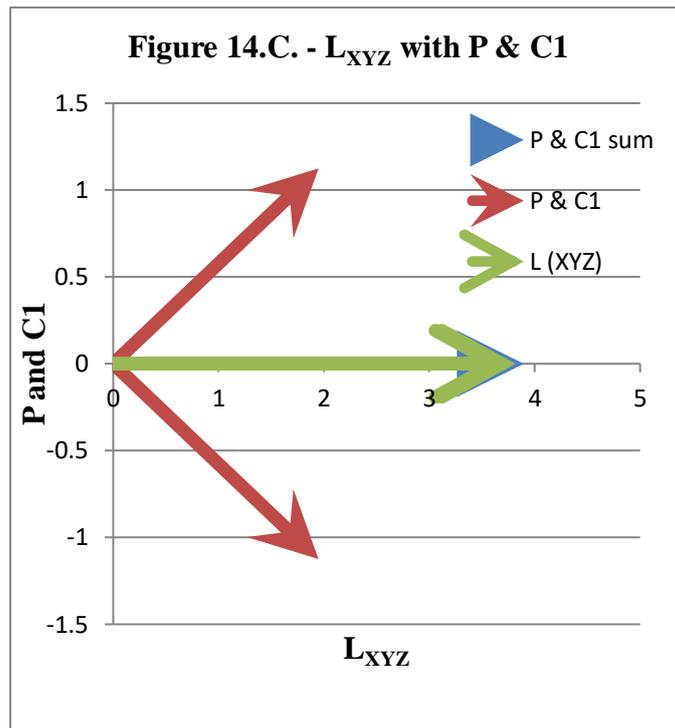
opposition is comparable to orbital g of this subgroup B where there are two vectors C2 opposed to L_{XY2} and also allows proportionality between L_{Z1} and L_{XYZ} (this fact is included together with the rest of the rules of behavior at the end of article). In **Figure 14.B.** There are following vectors:

- * $L_{XY} = 11^{1/2}$ (as $L(XY)$)
- * C2 of $\hbar/2$ opposed to L_{XY2} (as C2)
- * L_{XY2} needed to reach $11^{1/2}$ and counteract C2 (as $L(XY2)$)
- * $L_{Z2} = 1-3^{1/2}/2$

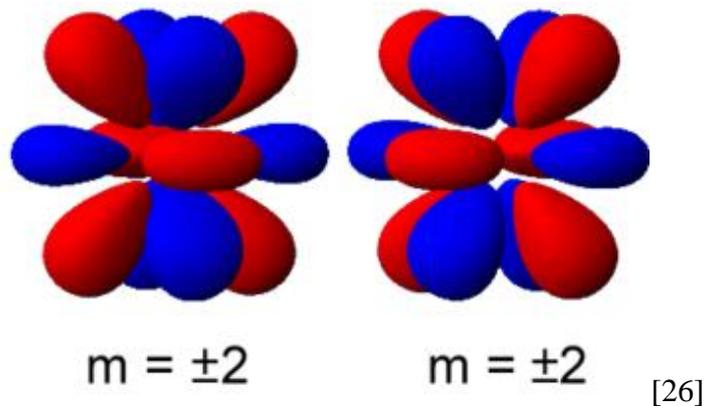


Consequently, L_{XYZ} (8) includes L_{XY2} for subgroup B of Group IV. Necessary L_{XYZ} vector (as $L(XYZ)$) (**Figure 14.C.**) has value very similar to that achieved by the 9 angled vectors that have not yet been used (6 P and 3 C1) located at an angle of 45 degrees respect to L_{XYZ} . Therefore, and as in all other four first orbitals seen, there is no voided vector.

$$(8)L_{XYZ} = \sqrt{\left(1 - \frac{\sqrt{3}}{2}\right)^2 + \left(11^{1/2} + \frac{1}{2}\right)^2} \approx \left(4 + \frac{1}{2}\right) \frac{\sqrt{3}}{2}$$



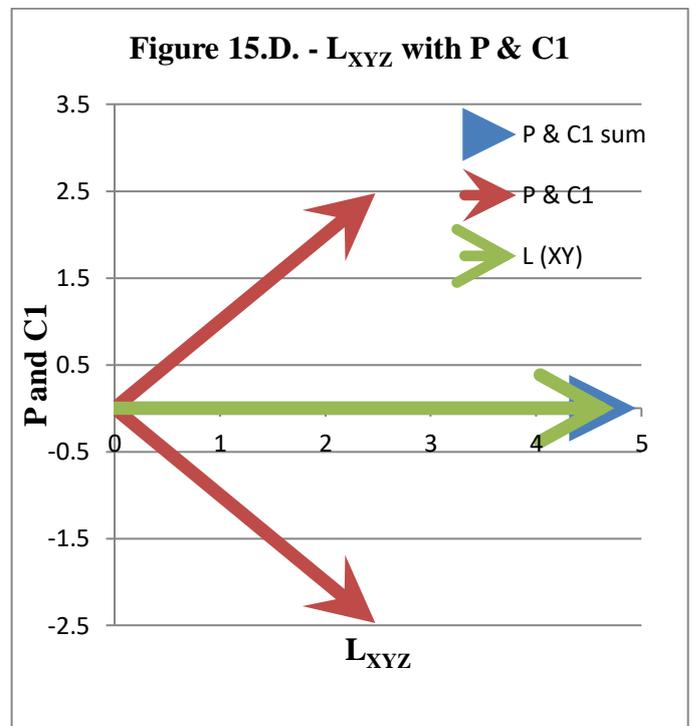
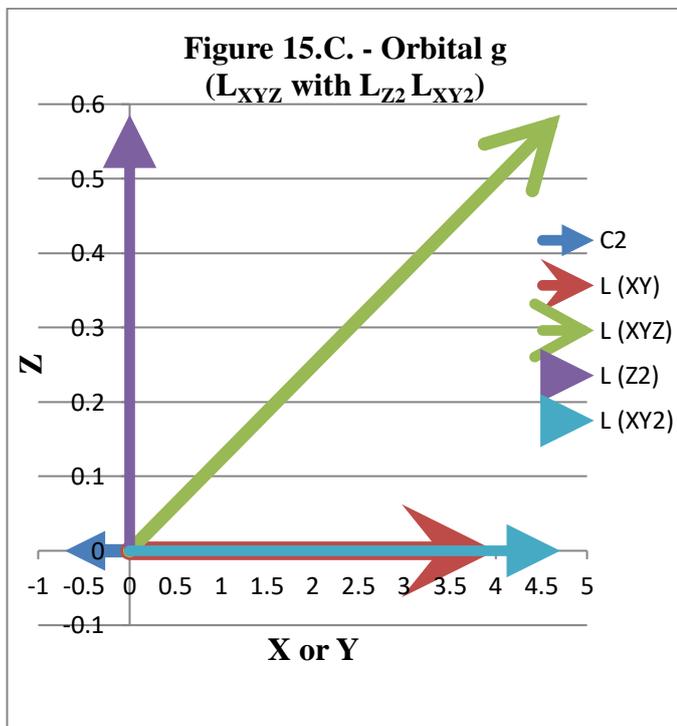
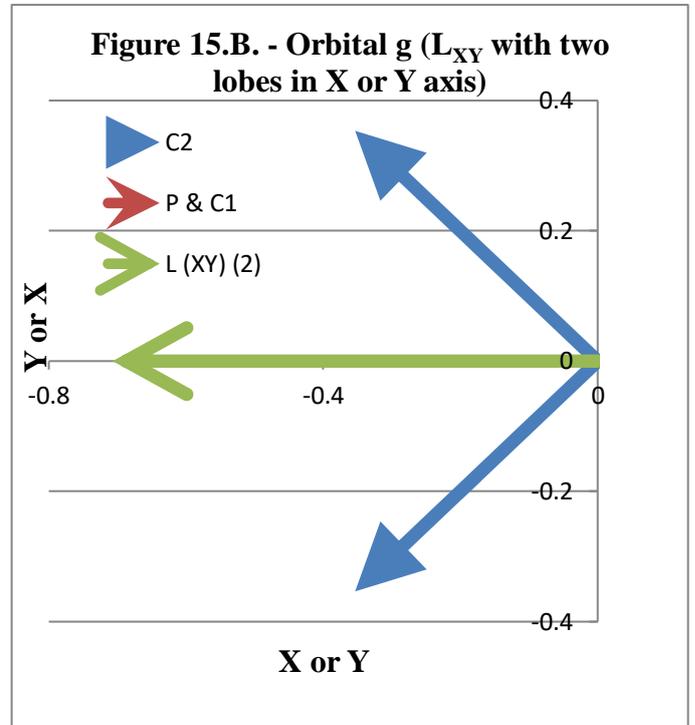
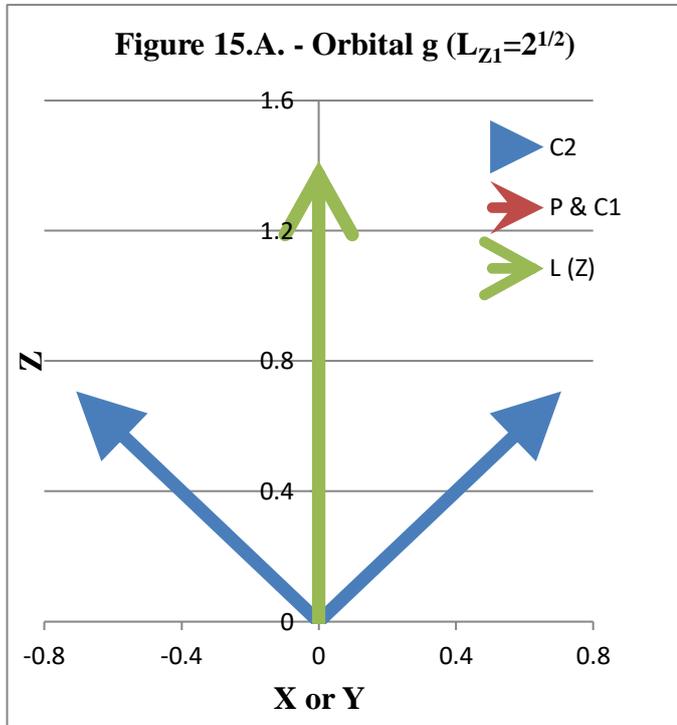
4.2.2) Orbitals g $L_Z = \pm 2$ (second orbital of Group IV, subgroup B)



Lobes increase is remarkable between both orbitals sets of subgroup B: orbitals f have 6 and g have 12 lobes and, consequently, their number of unit vectors \hbar is 12. Scheme is very similar to that seen in 4.2.1.:

12 lobes \rightarrow 12 unit vectors (VU) \rightarrow all 4 C2 vectors of $\hbar/2$ not located in XY plane (2 from above and 2 from below in 45 degrees with respect to Z axis) $\rightarrow L_{Z1} = 2 * 2^{1/2} / 2 \hbar = 2^{1/2} \hbar$ with consumption of 2 unit vectors ($V L_{Z1} = 2$) (**Figure 15.A.**) There remain 10 VU.

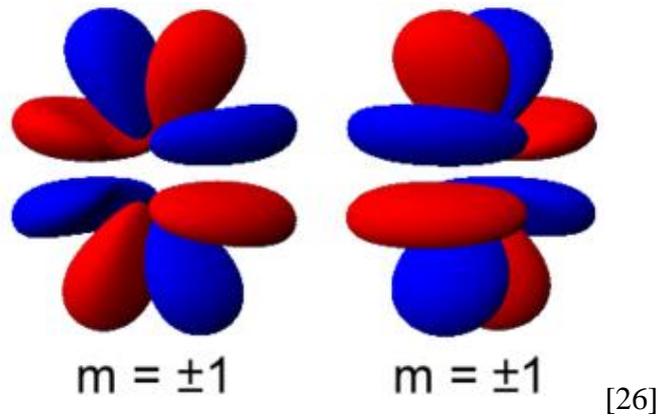
The two vectors C2 of XY plane, orthogonal to each other, are oppositely located with respect to the vector L_{XYZ} as it happens with the only vector C2 of the XY plane seen in 4.2.1) (**Figure 15.B.**). Likewise, L_{XY2} , which is greater than L_{XY} because the two C2 vectors of XY plane must be contrasted, is included in (9) and represented in **Figure 15.C.**



L_{XYZ} must meet L_{XY2} and complete L_Z (9) \rightarrow 14 vectors of $\hbar/2$ located at 45 degrees are used to meet L_{XYZ} ((**Figure 15.D.**) \rightarrow Consumption of 7 unit vectors ($V L_{XYZ} = 7$. In total 8 because the two vectors C2 of $\hbar/2$ placed in the XY plane must be considered. For the same reason $V L_{XYZ} = 5$ in the previous case because vector C2 of $\hbar/2$ placed in the XY plane must be included).

$$(9)L_{XYZ} = \sqrt{(2 - \sqrt{2})^2 + \left(16^{1/2} + 2\sqrt{2}/4\right)^2} \approx 7\frac{\sqrt{2}}{2}$$

4.3.1) Orbital g $L_Z = \pm 1$ (first and only orbital of Group IV, subgroup C)

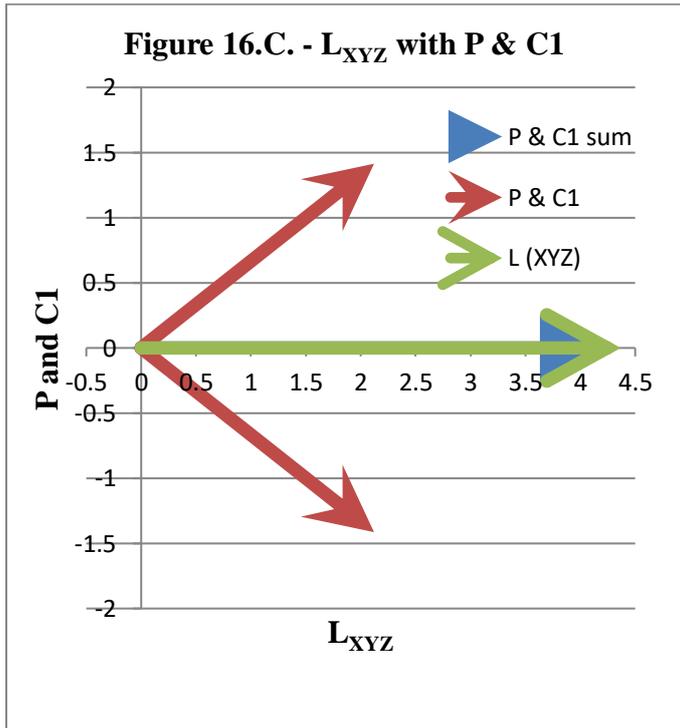
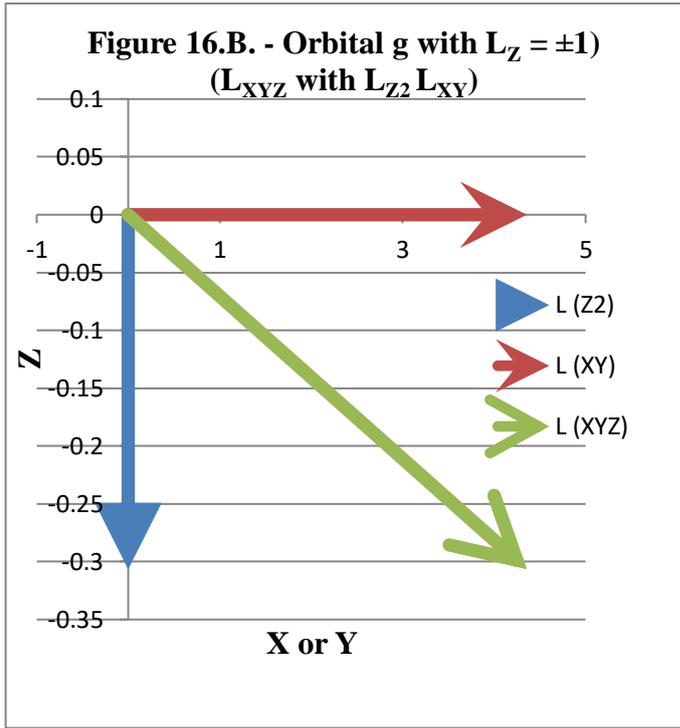
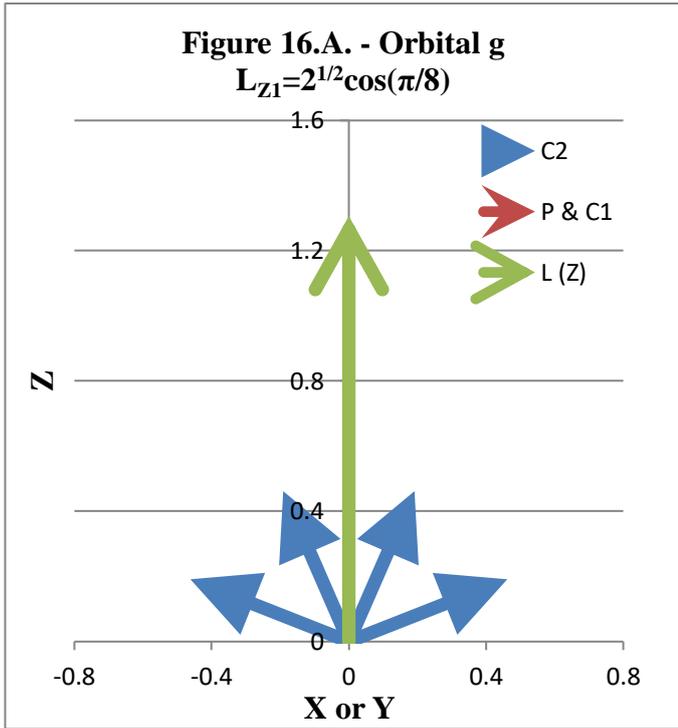


This orbital presents 8 lobes (with its 8 associated unit vectors \hbar) going around Z axis without touching it. Angle between 8 lobes is $360/8$ and therefore with respect to Z axis the angles are 22.5 ($\pi/8$) and $22.5+45$ degrees ($\pi/8 + \pi/4 = 3\pi/8$).

8 lobes \rightarrow 8 unit vectors (VU) \rightarrow all 4 C2 vectors of $\hbar/2$ (2 above and 2 below in $\pi/8$ and $3\pi/8$ with respect to the Z axis) $\rightarrow L_{Z1} = 2^{1/2}\cos(\pi/8) \hbar$ with consumption of 2 unit vectors ($V L_{Z1} = 2$) (**Figure 16.A.**) There remain 6 VU.

L_{XYZ} must meet L_{XY} and complete L_Z (10) (**Figure 16.B.**) \rightarrow The remaining 12 vectors of $\hbar/2$ (4 C1 and 8 P) located at 45 degrees are used to meet L_{XYZ} (**Figure 16.C.**) \rightarrow Consumption of 6 unit vectors ($V L_{XYZ} = 6$)

$$(10)L_{XYZ} = \sqrt{\left(1 - 2^{1/2}\cos(\pi/8)\right)^2 + (19^{1/2})^2} = \sqrt{(1 - 1,307)^2 + (19^{1/2})^2} \approx 6\frac{\sqrt{2}}{2}$$



5) P91 1, 2 y 3

Orbitals occupy their lobes based on numbers 1, 2 and 3. First two relationships have been seen in introduction and corroborated with orbitals study which is the expansion of what was initially presented in Table 2.

5.1) Fundamental angles

Angles are the fundamental ones:

$$\cos 90 = \cos \frac{\pi}{2} = 0$$

$$\cos 45 = \cos \frac{\pi}{2^2} = \frac{\sqrt{2}}{2}$$

$$\cos 30 = \cos \frac{\pi}{3 * 2} = \frac{\sqrt{3}}{2}$$

$$\text{sen } 90 = \text{sen } \frac{\pi}{2} = 1$$

5.2) Orbitals number

Number of orbitals in each group and total number of orbitals is reached with the numbers 2 and 3:

$$\text{Number of orbitals I + III + II + IV} = 2 + 4 + 6 + 12 = 2 + 2^2 + 2*3 + 2^2*3 = 24 = 2^3 * 3$$

$$\text{Number of orbitals I + III + II + IV} = \text{I} + (\text{I} + \text{I}) + (\text{I} + \text{III}) + (\text{I} + \text{III} + \text{II})$$

5.3) Number of voided vectors (VA)

Number of voided vectors (VA) is equal to 0 for orbital with the lowest orbital quantum number of each Group and increases in each group as orbital quantum number increases.

* Group I: Group I only has one orbital quantum number (type p) and therefore its VA = 0

* Group II and III: VA increases one unit for each unit of increase of orbital quantum number (VA = 0, 1 and 2)

* Group IV: VA increases two units for each unit of increase of orbital quantum number (VA = 0, 2 and 4)

Sum of VA of each group and sum of VA of the Groups according to relationship between VA and l is expressed as function of 2 and 3:

* Group II and III (**Tabla 3**)

Group	Group II			Group III				Sum
L	D	f	g	p	d	f	g	
VA	0	1	2	0	1	2	3	9 = 3 ²
Sum	3			6 = 3*2				

* Group IV (**Tabla 4**)

Subgroup	A			B		C	Sum
L	d	f	g	f	g	g	
VA	0	2	4	0	2	0	8 = 2 ³
Sum	6 = 3*2			2		0	

From 5.4) to 5.6) Relationship by Groups and Global $\sum \left(\frac{L_{Z1}}{L_{XYZ}} \right)_{orbital\ grupo\ x}$

These points study sum of quotients L_{Z1} / L_{XYZ} of orbitals of the different groups and global relation of all the groups. Relation of said quotient with the numbers 1, 2 and 3 is verified.

5.4) L_{Z1} / L_{XYZ} ratio of Group II

L_{Z1} / L_{XYZ} ratio of Group II can be expressed as function of 2 and 3 (11):

$$(11) Ratio II = \sum \left(\frac{L_{Z1}}{L_{XYZ}} \right)_{orbital\ grupo\ II} = \frac{2}{\sqrt{2}} + \frac{3}{\sqrt{3}} + \frac{3}{\sqrt{4}} = 5,14626 \dots \approx \left(\frac{3^2}{2^2} \right)^2 = 5,0625$$

Approximation is good and differential must be analyzed when the rest of the Groups are studied.

5.5) L_{Z1} / L_{XYZ} ratio of Group IV

L_{Z1} / L_{XYZ} ratio of Group II can also be expressed function of 2 and 3 (12). In its simplest form, L_{XYZ} is considered as approximation indicated in Table 2.

$$\begin{aligned}
 (12) \text{Ratio IV} &= \sum \left(\frac{L_{Z1}}{L_{XYZ}} \right)_{\text{orbital grupo IV}} = \\
 &= \frac{\sqrt{2}/2}{3\sqrt{2}/2} + \frac{2\sqrt{2}/2}{4\sqrt{2}/2} + \frac{3\sqrt{2}/2}{5\sqrt{2}/2} + \frac{\sqrt{3}/2}{9\sqrt{3}/4} + \frac{2\sqrt{2}/2}{14\sqrt{2}/4} + \frac{2^{\frac{1}{2}}\cos(\pi/8)}{6\sqrt{2}/2} = \\
 &= \frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{2}{9} + \frac{2}{7} + \frac{2^{\frac{1}{2}}\cos(\pi/8)}{3\sqrt{2}} = 2,2492 \dots \approx \frac{3^2}{2^2} = 2,25
 \end{aligned}$$

5.5) L_{Z1} / L_{XYZ} ratio of Group I and III

L_{Z1} / L_{XYZ} ratio of Group I is equal to 1 because is the quotient of two vectors equal to 1.

L_{Z1} / L_{XYZ} ratio of Group III is approximate to ratio between 3 and 2 (13) in order to obtain an overall relation of all the orbitals (point 5.6)):

$$\begin{aligned}
 (13) \text{Ratio III} &= \sum \left(\frac{L_{Z1}}{L_{XYZ}} \right)_{\text{orbital grupo III}} = \frac{1/2}{3/2} + \frac{1/2}{5/2} + \frac{1/2}{7/2} + \frac{1/2}{9/2} = \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} = 0,7873 \dots \\
 &\approx \frac{3^2}{2^{7/2}} = 0,7954
 \end{aligned}$$

5.6) Global ratio of L_{Z1} / L_{XYZ} of all Groups

Global Ratio of L_{Z1} / L_{XYZ} of all Groups (14) is ratio of L_{Z1} / L_{XYZ} between groups with orbitals in X and Y (Groups I and II) and those that do not (Groups III and IV). Global ratio of the quotient between L_{Z1} and L_{XYZ} is expressed as an extremely simple relationship between 2 and 3: $2^{3/2}$

$$(14) \text{Global Ratio} \sum \left(\frac{L_{Z1}}{L_{XYZ}} \right)_{\text{orbital grupo I a IV}} = \frac{GI * GII}{GIII * GIV} = \frac{1 * \left(\frac{3^2}{2^2}\right)^2}{\frac{3^2}{2^{7/2}} * \frac{3^2}{2^2}} = 2^{3/2}$$

These relations of quotient L_{Z1} / L_{XYZ} for each orbital and subsequent sum of said quotients (seen from point 5.4) to 5.6)) are complemented in point 5.11). Point 5.11 calculates quotient but between sum all L_{Z1} components and all L_{XYZ} components

From 5.7) to 5.9) Relationship by Groups and Global of the angle between L_{Z1orZ2} and L_{XY} :

These points study Relationship of angle formed by two perpendicular vectors: L_{Z1orZ2} and L_{XY} of the different groups and globally. Angular relation with numbers 1, 2 and 3 is verified. Calculated arctangent considers where lobes are located:

- * Groups I and II (lobes organized on X and Y axes or XY plane): arctangent (L_{XY}/L_{Z1})
- * Groups III and IV (unorganized lobes on X and Y axes or XY plane): arctangent (L_{Z2}/L_{XY})

5.7) Angular relations. Calculation of the angle formed by L_{XY} y L_{Z1orZ2} in the different Groups:

Angular relations of all the Groups are 45 ($\pi/2^2$) or 90 ($\pi/2$) degrees. Angular relations for Groups I, II, III and IV are in (15), (16), (17) and (18) respectively:

$$(15) \text{Angular Relation Group I} = \arctan\left(\frac{L_{XY}}{L_{Z1}}\right) = \arctan\left(\frac{1}{1}\right) = 45^\circ = \frac{\pi}{4} = \frac{\pi}{2^2}$$

$$(16) \text{Angular Relation Group II} = \arctan\left(\frac{L_{XY}}{L_{Z1}}\right) = \arctan\left(\frac{\sqrt{2}}{2}\right) + \arctan\left(\frac{\sqrt{3}}{3}\right) + \arctan\left(\frac{\sqrt{4}}{4}\right) \\ = 91,829 \dots^\circ \approx \frac{\pi}{2}$$

$$(17) \text{Angular Relation Group III} = \arctan\left(\frac{L_{Z2}}{L_{XY}}\right) = \\ \arctan\left(\frac{1/2}{\sqrt{2}}\right) + \arctan\left(\frac{1/2}{\sqrt{6}}\right) + \arctan\left(\frac{1/2}{\sqrt{12}}\right) + \arctan\left(\frac{1/2}{\sqrt{20}}\right) = 45,601 \dots^\circ \approx \frac{\pi}{4} = \frac{\pi}{2^2}$$

$$(18) \text{Angular Relation Group IV} = \arctan\left(\frac{L_{Z2}}{L_{XY}}\right) = \\ \arctan\left(\frac{1 - \sqrt{2}/2}{\sqrt{5}}\right) + \arctan\left(\frac{2 - 2\sqrt{2}/2}{\sqrt{8}}\right) + \arctan\left(\frac{3 - 3\sqrt{2}/2}{\sqrt{11}}\right) + \arctan\left(\frac{1 - \sqrt{3}/2}{\sqrt{11}}\right) \\ + \arctan\left(\frac{2 - 2\sqrt{2}/2}{\sqrt{16}}\right) - \arctan\left(\frac{1 - 2^{1/2}\cos(\pi/8)}{\sqrt{19}}\right) = 48,670 \dots^\circ \approx \frac{\pi}{4} = \frac{\pi}{2^2}$$

5.8) Global angular relation

Global angular relation (GAR) (19) is expressed with the numbers 2 and 3. Global angular relation (19) is sum of all angular relations (AR) seen in previous point considering that, except for Group III that has $L_z=0$, remaining Groups have two possibilities depending on whether L_z is positive or negative.

$$(19)GAR = 2AR(1) + 2AR(2) + AR(3) + 2AR(4) = 2\frac{\pi}{2^2} + 2\frac{\pi}{2} + \frac{\pi}{2^2} + 2\frac{\pi}{2^2} = \frac{3^2}{2^2}\pi$$

Considering GAR, Groups I and II (lobes on X and Y) have twice Angular Ratio (AR) than Groups III and IV (20):

$$(20)2AR(1) + 2AR(2) = 2(AR(3) + 2AR(4))$$

$$\frac{2 + 4}{2^2}\pi = 2\frac{1 + 2}{2^2}$$

$$\frac{2 * 3}{2^2}\pi = 2\frac{3}{2^2}$$

5.9) Relation of differentials between RA and the ideal angle

Relation of differentials between AR and ideal angle suggests that there is constant value of differential for all orbital (21). Demonstration of said relation of differentials between RA and ideal angle (**Table 5**) requires considering total number of orbitals per Group and that Group III orbitals have an identical double differential (L_{XY} with respect to L_{Z1} and L_{Z2}):

Group	AR	Ideal Angle	AR-Ideal	No. Orbitals	(21) $\frac{AR - Ideal}{No. orbitales}$
II	91,82944	90	1,82944	6	0,30491
III	45,60076	45	2*0,60076	4	0,30038
IV	48,66970	45	3,66970	12	0,30581

Relation of differentials is very similar (≈ 0.30) that can be seen in a more similar way if two blocks (22) that have been seen in present article are considered (Group I and II on the one hand and Group III and IV for another. Group I does not participate in this calculation since AR and ideal are equal).

$$(22) \frac{0,30491 * 6}{6} \approx \frac{0,30038 * 4 + 0,30581 * 12}{16}$$

$$(22) 0,30491 \approx 0,30445$$

5.10) Global relation of angular momentum modules

Total angular momentum modulus (TAMM) is result of multiplying number of orbitals of each orbital quantum number by its angular momentum L. Total angular momentum, conceived as summation value of modulus of the vectors, is equal to $3^4=3^{2*2}$ (23)

$$(23) TAMM = 3\sqrt{2} + 5\sqrt{6} + 7\sqrt{12} + 9\sqrt{20} = 80,988 \approx 81 = 3^{2*2} = 3^2^2$$

Average value of L_Z (AVL_Z) is average value of sum of all angular moments on Z axis of existing orbitals (23.B.). Values of L_Z , according to Table 2, multiplied by number of orbitals in each case (1 when $L_Z=0$ and 2 in rest) are included in 23.B. AVL_Z is also described by numbers 2 and 3:

$$(23.B.) AVL_Z = \frac{\sum(L_Z)_{orbital}}{No. orbitals} = \frac{2(1 + 2 + 3 + 4 + 1 + 2 + 3 + 1 + 2 + 1) + 1(0 + 0 + 0 + 0)}{24} = \frac{40}{24}$$

$$= 1 + \frac{2}{3}$$

Average value of L_{XY} (AVL_{XY}) is calculated (23.C.) in the same way:

$$(23.C.) AVL_{XY} = \frac{\sum(L_{XY})_{orbital}}{No. orbitals}$$

$$= \frac{2(1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{8} + \sqrt{11} + \sqrt{11} + \sqrt{16} + \sqrt{19}) + 1(\sqrt{2} + \sqrt{6} + \sqrt{12} + \sqrt{20})}{24}$$

$$= \frac{64,21 \dots}{24} = 2,67 \dots \approx 2 + \frac{2}{3}$$

Therefore, difference between (23.C) and (23.B) is ≈ 1 that considering that there are 24 orbitals, difference is equal to total difference of $24 = 3*2^3$

5.11) Global Relation $\frac{\Sigma(L_{XYZ})_{orbital}}{\Sigma(L_{Z1})_{orbital}}$:

This Global Ratio is quotient between sum of all L_{XYZ} modules and sum of all L_{Z1} modules (24). 5.11) takes values considered as real and not approximate values for Group IV. In fact, and this is remarkable, result is optimal with the real ones (that is, with the part located to left of \approx in equations from (5) to (10) and less correct with the approximation (right part)).

$$\begin{aligned}
 (24) \text{Global Relation } \frac{\Sigma(L_{XYZ})_{orbital}}{\Sigma(L_{Z1})_{orbital}} = & \\
 = \frac{1 + \sqrt{2} + \sqrt{3} + \sqrt{4} + \frac{3}{2} + \frac{5}{2} + \frac{7}{2} + \frac{9}{2} + \sqrt{\left(1 - \frac{\sqrt{2}}{2}\right)^2 + \left(5\frac{1}{2}\right)^2} + \sqrt{\left(2 - \frac{2\sqrt{2}}{2}\right)^2 + \left(8\frac{1}{2}\right)^2}}{1 + 2 + 3 + 4 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + \frac{2\sqrt{2}}{2} + 2\frac{1}{2}\cos(\pi/8)} & \\
 + \frac{\sqrt{\left(3 - \frac{3\sqrt{2}}{2}\right)^2 + (11^{1/2})^2} + \sqrt{\left(1 - \frac{\sqrt{3}}{2}\right)^2 + \left(11\frac{1}{2} + \frac{1}{2}\right)^2} + \sqrt{(2 - \sqrt{2})^2 + \left(16^{1/2} + 2\frac{\sqrt{2}}{4}\right)^2}}{1 + 2 + 3 + 4 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + \frac{2\sqrt{2}}{2} + 2\frac{1}{2}\cos(\pi/8)} & \\
 + \frac{\sqrt{\left(1 - 2^{1/2}\cos(\pi/8)\right)^2 + (19^{1/2})^2}}{1 + 2 + 3 + 4 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + \frac{2\sqrt{2}}{2} + 2\frac{1}{2}\cos(\pi/8)} = 2,000 \approx 2 &
 \end{aligned}$$

Whole equation (24) can be summarized in (25):

$$(25) \sum (L_{XYZ})_{orbital} = 2 \sum (L_{Z1})_{orbital}$$

Relation has high sensitivity to changes. Thus, even keeping all the orbitals, the mere fact of placing vectors C2 on XY plane of orbitals of Group IV, subgroup B, in the same sense (instead of opposite) to L_{XYZ} implies going from 2,000 to 1,879 and therefore, abandon Relations of P91 1, 2 and 3.

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