The generation of mass and its directivity

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Abstract:

The mass of matter includes the mass of elementary particles and the interaction mass between the particles. So far there has been a great deal of research on the origin of the mass of the elementary particles, but there are few studies on the origin of the interaction mass. In this paper, we show that the electric field of a uniformly accelerating point charge in vacuum is curved toward the opposite direction of acceleration. When a positive charge system is accelerated, the interaction forces between its charges cannot cancel each other out; therefore the charge system will produce a self-action force opposite to the acceleration, thus generating interaction mass. The interaction mass is not constant, but varies with the direction of the acceleration. Finally, we conclude that the mass of object is related to its self-action and has directivity.

Keywords: Origin of mass; Disturbance electric field; Interaction mass; Self-action; Directivity; Inertial force

1. Introduction

Mass is the intrinsic property of matter and also the measure of the magnitude of the inertia of object. The origin of mass (inertia) is a matter of concern to modern physicists, on which is still no definitive conclusion. The current research is mainly aimed at the origin of the mass of elementary particles. However, the mass of object includes not only the mass of the elementary particles, but also the interaction mass between the particles. This paper will take charges as an example to illustrate how the interaction mass is generated and its directivity.

2. The electric field of uniformly accelerating point charge

The universe is made up of matter and space, which interact and influence each other. The space is the foundation upon which the matter exists, and the matter can not exist without the space. All the properties of the matter must depend on the space for showing up, and therefore it can be said that the properties of the matter are determined by the property of the space in a certain sense. In vacuum, the electric field of a static point charge is sphere-symmetric because of the isotropy and homogeneity of the space, and its electric field lines radiate in straight line. According to Einstein's general relativity, gravitational field is essentially curved space-time; therefore the electric field of a static point charge in the gravitational field should also bend with the curved space-time, and its electric field lines should also become curves. From the equivalence principle, a uniform gravitational field is equivalent to a uniformly accelerated reference frame, and therefore in the uniformly accelerated reference frame, the electric field of a static point charge should also be curved. Now we will first study the electric field of a uniformly accelerating point charge in the vacuum.

Suppose there is a point charge q in the inertial reference frame S in the vacuum, and the charge q has always been stationary at the origin O. At the time $t_0 = 0$, the charge q starts to accelerate at constant acceleration $\boldsymbol{a}$ along the straight line in the positive direction of the y-axis. At time t, the velocity of the charge q is $\boldsymbol{v} = \boldsymbol{a}t$. To make it simple, we suppose that $v << c$ (c is the speed of light). Now let's look at the electric field of the charge q at the time t.

As shown in figure 1, the charge q starts to accelerate at the origin O at the time t_0, and then reaches the point P at the time t. In the meantime, because the charge q is accelerated, the electric field around it is disturbed, and this disturbance is propagated outward at the speed c. At the time t, the front of the disturbance arrives at the sphere centered at the origin O with the radius $r_0 = ct$. According to the theory of relativity that the speed of light is the maximum speed, no change of any information can reach beyond the sphere at this moment; therefore the electric field outside the sphere is still the electrostatic field of the original stationary charge q at the point O, and its electric field lines are along the straight lines radiating from the point O, but the electric field inside the sphere is the disturbance electric field produced by the accelerated charge q during this period. In the situation where the velocity of the charge q is far less than the speed of light, the distribution of the disturbance electric field around the charge q can be regarded as approximate invariant, as if the disturbance electric field accelerates with the charge q. In fact, as time goes on, the disturbed electric field propagates continuously from near to far, while the charge q continues to produce new disturbance electric field.

Because the charge q has been accelerating, the electric field lines inside the sphere are not straight lines but curves; therefore at the time t, the electric field lines inside the sphere should be the curves from the point P. According to Gauss law, the total number of the electric field lines on both sides of the sphere should be equal, and the electric field lines should also be continuous through the sphere; therefore when the entire electric field is described in terms of field lines, the electric field lines on the two sides of the sphere should be connected one to one (see figure 1).

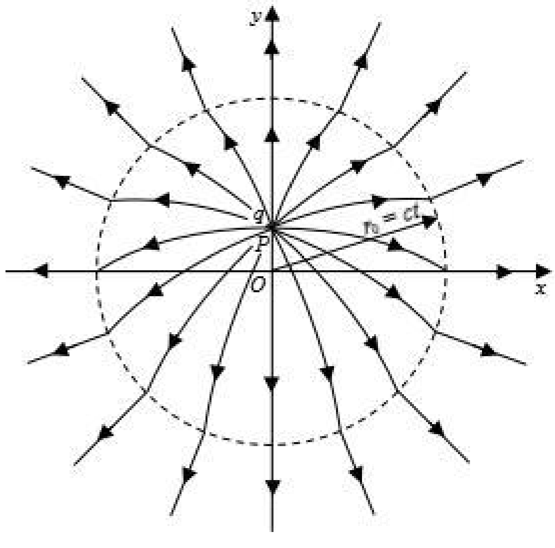

Figure 1. The electric field of the accelerating charge q at time t. A positive point charge q had always been stationary at the origin O, and started to be uniformly accelerated along the positive direction of the y-axis at time t_0, while the disturbance of the electric field that was caused by the accelerated charge q was propagated around at the speed of light c. At time t, the charge q arrives at the point P, while the front of the disturbance reaches the sphere with radius r_0 centered at the origin O. The electric field inside the sphere is the disturbance electric field produced by the accelerated charge q, and that outside the sphere is still the electrostatic field of the original charge q at the point O.

Now we use field line pattern to analyze the disturbance electric field at the sphere. In figure 2, M is any point on the sphere, $\boldsymbol{r}$ is the radius vector from P to M, and the angle between $\boldsymbol{r}$ and the y-axis is φ. The electric field line through M inside the sphere is the curve from P to M, and that outside the sphere is along the straight line through the points O and M. The angle between the radius vector $\boldsymbol{r}_0$ from O to M and the y-axis is θ. The distance from O to P is that $OP = vt/2$. Since $r_0 = ct$, and $v << c$, so that $OP << r_0$.

Let's find out the disturbance electric field $\boldsymbol{E}$ at the point M. $\boldsymbol{E}$ is the disturbance electric field which was produced by the accelerated charge q at the point O and has been propagated to the point M at this time. $\boldsymbol{E}$ is in the tangent line of the curve PM at the point M. $\boldsymbol{E}$ can be divided into two components $\boldsymbol{E}_{r0}$ and $\boldsymbol{E}_\theta$ (see figure 2-a). According to Gauss law that electric flux is only related to the electric field component perpendicular to Gauss surface, therefore the continuous electric field line at the sphere means that the component $\boldsymbol{E}_{r0}$ is still the original electrostatic field at the point M, that is

$$E_{r_0} = \frac{q}{4\pi\varepsilon_0 r_0^2} \tag{1}$$

The component $\boldsymbol{E}_\theta$ is just the transverse electric field produced by the accelerated charge q, that is[1]

$$E_\theta = \frac{qa\sin\theta}{4\pi\varepsilon_0 c^2 r_0} = \frac{v\sin\theta}{c} E_{r_0} \tag{2}$$

Since $v << c$, so that $E_\theta << E_{r0}$, and therefore we can get

$$E = \sqrt{E_{r_0}^2 + E_\theta^2} \approx E_{r_0} = \frac{q}{4\pi\varepsilon_0 r_0^2} \tag{3}$$

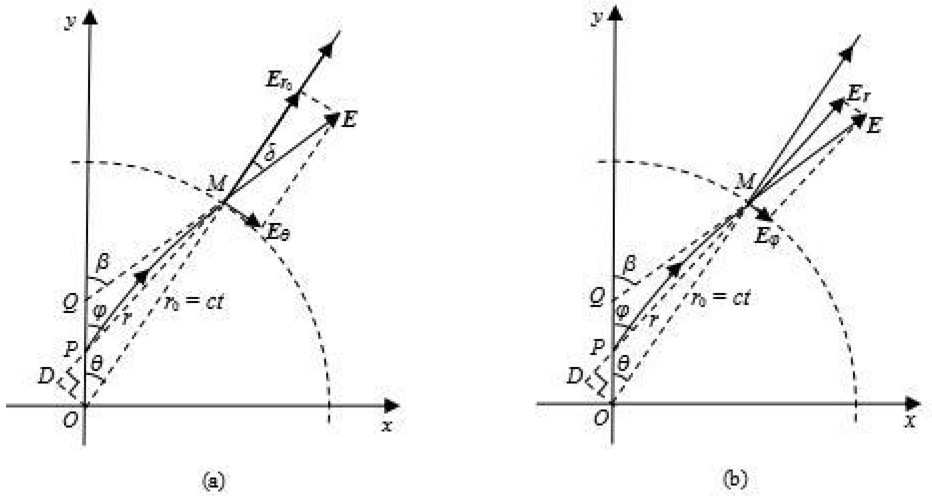

Figure 2. The disturbance electric field $\boldsymbol{E}$ at the point M on the sphere. The electric field line through the point M inside the sphere is the curve PM, and that outside the sphere is along the straight line through the points O and M. The disturbance electric field $\boldsymbol{E}$ at the point M is tangent to the curve PM. (**a)** $\boldsymbol{E}$ is divided into two components $\boldsymbol{E}_{r0}$ and $\boldsymbol{E}_\theta$. (**b**) $\boldsymbol{E}$ is divided into two components $\boldsymbol{E}_r$ and $\boldsymbol{E}_\varphi$.

As shown in figure 2, the straight line *OD* is perpendicular to the straight line *PM* and intersects the extension line of the straight line *PM* at point *D*. The extension line of ***E*** intersects the *y*-axis at point *Q* with an angle β. The angle between ***E*** and $\boldsymbol{E}_{r0}$ is δ. Thus we can get that

$$\sin\angle OMD = \frac{OD}{r_0} = \frac{\frac{vt}{2}\sin\varphi}{ct} = \frac{v\sin\varphi}{2c} \tag{4}$$

$$\cos\angle OMD = \sqrt{1-\sin^2\angle OMD} = \sqrt{1-\left(\frac{v\sin\varphi}{2c}\right)^2} \approx 1 \tag{5}$$

$$\begin{aligned}\sin\delta = \frac{E_\theta}{E} \approx \frac{v\sin\theta}{c} = \frac{v}{c}\sin(\varphi-\angle OMD) \\ \approx \frac{v}{c}\left(\sin\varphi - \frac{v\sin\varphi\cos\varphi}{2c}\right) \approx \frac{v\sin\varphi}{c}\end{aligned} \tag{6}$$

$$\cos\angle\delta = \sqrt{1-\sin^2\angle\delta} = \sqrt{1-\left(\frac{v\sin\varphi}{c}\right)^2} \approx 1 \tag{7}$$

$$\sin\angle QMP = \sin(\delta-\angle OMD) \approx \frac{v\sin\varphi}{c} - \frac{v\sin\varphi}{2c} = \frac{v\sin\varphi}{2c} \tag{8}$$

$$\cos\angle QMP = \sqrt{1-\sin^2\angle QMP} = \sqrt{1-\left(\frac{v\sin\varphi}{2c}\right)^2} \approx 1 \tag{9}$$

Since

$$r_0 = \frac{DM}{\cos\angle OMD} \approx r + \frac{ar_0^2\cos\varphi}{2c^2} \approx r + \frac{ar^2\cos\varphi}{2c^2} \tag{10}$$

The magnitude of the disturbance electric field ***E*** at the point *M* is

$$E \approx E_{r_0} = \frac{q}{4\pi\varepsilon_0 r_0^2} \approx \frac{q}{4\pi\varepsilon_0\left(r+\frac{ar^2\cos\varphi}{2c^2}\right)^2} \approx \frac{q}{4\pi\varepsilon_0 r^2} - \frac{qa\cos\varphi}{4\pi\varepsilon_0 c^2 r} \tag{11}$$

In the figure 2, it can be seen that the direction of ***E*** is not in the direction of the radius vector ***r*** but deflected toward the opposite direction of the acceleration, and the angle between ***E*** and the direction of ***r*** is equal to $\angle QMP$. If we divide ***E*** into two components $\boldsymbol{E}_r$ and $\boldsymbol{E}_\varphi$, here $\boldsymbol{E}_r$ is in the direction of ***r*****,** and $\boldsymbol{E}_\varphi$ is in the direction perpendicular to ***r*** (see figure 2-b), then

$$E_r = E\cos\angle QMP \approx \frac{q}{4\pi\varepsilon_0 r^2} - \frac{qa\cos\varphi}{4\pi\varepsilon_0 c^2 r} \tag{12}$$

$$E_\varphi = E\sin\angle QMP \approx \left(\frac{q}{4\pi\varepsilon_0 r^2} - \frac{qa\cos\varphi}{4\pi\varepsilon_0 c^2 r}\right)\times\frac{v\sin\varphi}{2c} \approx \frac{qa\sin\varphi}{8\pi\varepsilon_0 c^2 r} \tag{13}$$

E can be represented in a vector form:

$$\boldsymbol{E} = \boldsymbol{E}_r + \boldsymbol{E}_\varphi \approx \left(\frac{q}{4\pi\varepsilon_0 r^2} - \frac{qa\cos\varphi}{4\pi\varepsilon_0 c^2 r} \right) \times \boldsymbol{e}_r + \frac{qa\sin\varphi}{8\pi\varepsilon_0 c^2 r} \boldsymbol{e}_\varphi \tag{14}$$

Suppose there is a uniformly accelerating reference frame S' in which the accelerating charge q is always stationary. Then in the S'-frame, the disturbance electric field around the charge q will be constant, and according to the principle of equivalence, it is the same as the electric field of the charge q at rest in the gravitational field equivalent to the S'-frame.

In the S-frame, in the case that $v << c$, the disturbance electric field around the charge q is approximately the same as that in the S'-frame, and as time goes on, the former distribution of the disturbance electric field around the charge q is approximately constant, that is to say, the disturbance electric field at any point around the charge q is an approximate constant; therefore the distribution electric field at any point around the charge q can all be approximated by the equation (14), where r is the distance of this point from the charge q, and φ represents the angle between the radius vector $\boldsymbol{r}$ from the charge q to this point and the direction of the acceleration. Because $v = \alpha r_0/c \approx \alpha r/c$, when $v << c$ we can get that $r << c^2/\alpha$, this is just the range of r in the equation (14). If the accelerating charge is negative, then the disturbance electric field produced by the negative charge is equal and opposite to that produced by the positive charge.

The component of $\boldsymbol{E}$ in the direction of the acceleration is

$$\begin{aligned} E_a &= E\cos\beta = E\cos(\varphi + \angle QMP) \approx E\left(\cos\varphi - \frac{v\sin^2\varphi}{2c} \right) \\ &\approx \left(\frac{q}{4\pi\varepsilon_0 r^2} - \frac{qa\cos\varphi}{4\pi\varepsilon_0 c^2 r} \right) \times \left(\cos\varphi - \frac{ar\sin^2\varphi}{2c^2} \right) \\ &\approx \frac{q\cos\varphi}{4\pi\varepsilon_0 r^2} - \frac{qa}{8\pi\varepsilon_0 c^2 r}(2 - \sin^2\varphi) \end{aligned} \tag{15}$$

3. The interaction mass of charge system

Suppose in the inertial reference frame S, there is a charge system consisting of two positive point charges q_1 and q_2, and the distance between the two charges is r. The total mass M of the charge system includes the mass of the two charges themselves and the interaction mass between the two charges. Here we don't consider the gravitational interaction between the two charges, thus the interaction mass between the two charges is only the mass of the electromagnetic interaction.

Because of external force $\boldsymbol{F}$ on it, the charge system is accelerating at constant acceleration $\boldsymbol{a}$ along a straight line in the direction of the force $\boldsymbol{F}$, and the angle between the radial vector $\boldsymbol{r}$ from q_1 to q_2 and the direction of the acceleration is φ (see figure 3). Let $\boldsymbol{v}$ express the velocity of the charge system, and suppose that $v << c$, and $r << c^2/a$. According to Newton's second law, we can get $\boldsymbol{F} = M\boldsymbol{a}$. Because both of the two charges are accelerating, they can all produce disturbance electric fields, and therefore each of them will produce a disturbance electric field force on another. According to the equation (15), we can get the components of the disturbance electric fields of the charges q_1 and q_2 in the direction of the acceleration, and then we can find the components of the disturbance electric field forces on the charges q_1 and q_2 in the direction of the acceleration are

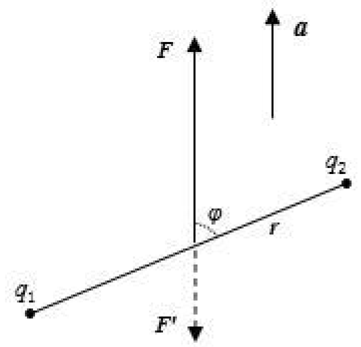

Figure 3. The self-action force produced by an accelerating charge system. The charge system which consists of two positive charges q_1 and q_2 accelerates at constant acceleration $\boldsymbol{a}$ along a straight line because of external force $\boldsymbol{F}$ on it, and produces a self-action force $\boldsymbol{F'}$ opposite to the external force $\boldsymbol{F}$.

$$F_1' = \frac{q_1 q_2 \cos(\pi - \varphi)}{4\pi\varepsilon_0 r^2} - \frac{q_1 q_2 a}{8\pi\varepsilon_0 c^2 r}[2 - \sin^2(\pi - \varphi)] \tag{16}$$

and

$$F_2' = \frac{q_1 q_2 \cos\varphi}{4\pi\varepsilon_0 r^2} - \frac{q_1 q_2 a}{8\pi\varepsilon_0 c^2 r}(2 - \sin^2\varphi) \tag{17}$$

respectively. Adding F'_1 and F'_2, we can get the resultant force of $\boldsymbol{F}'_1$ and $\boldsymbol{F}'_2$ is

$$\boldsymbol{F}' = \boldsymbol{F}_1' + \boldsymbol{F}_2' = -\frac{q_1 q_2 \boldsymbol{a}}{4\pi\varepsilon_0 c^2 r}(2 - \sin^2\varphi) \tag{18}$$

The value of the equation (18) is negative, indicating that $\boldsymbol{F'}$ is opposite to the acceleration $\boldsymbol{a}$. This shows that the interaction forces between q_1 and q_2 cannot cancel each other out in the direction of the acceleration; therefore the charge system will be subjected to the force $\boldsymbol{F'}$ opposite to the acceleration, which is not an external force but the self-action force of the charge system. The force $\boldsymbol{F'}$, which is opposite to the external force $\boldsymbol{F}$, will resist the change of the motion of the charge system caused by the external force, and it reflects the inertia of the charge system; therefore $\boldsymbol{F'}$ belongs to the inertial force produced by the charge system. Because inertial mass is the measure of the inertia of object, and the inertia force $\boldsymbol{F'}$ is equivalent to inertial mass, and therefore $\boldsymbol{F'}$ must correspond to a certain inertial mass. If the inertia mass corresponding to $\boldsymbol{F'}$ is expressed in m', then $\boldsymbol{F'} = -m'\boldsymbol{a}$. Substituting this formula into the equation (18), we get that

$$m' = \frac{q_1 q_2}{4\pi\varepsilon_0 c^2 r}(2 - \sin^2\varphi) \tag{19}$$

4. Discussions

The mass m' is owned by the charge system, and it is just the interaction mass of the charge system. Since m' varies with φ proportionally as $2 - \sin^2\varphi$, the values of m' can also be different for the different values of φ. When $\varphi = 0$, m' takes a maximum value $q_1q_2/(2\pi\varepsilon_0c^2r)$. When $\varphi = \pi/2$, m' takes a minimum value $q_1q_2/(4\pi\varepsilon_0c^2r)$. This shows that the mass m' varies with the direction of the acceleration with respect to the charge system; therefore the mass m' has directivity, which is caused by the directivity of the structure of the charge system. When q_1 and q_2 are opposite in the equation (19), the value of m' is negative, indicating that the inertial mass of the charge system decreases.

Because the interaction mass of the charge system has directivity, while the mass of the charges q_1 and q_2 is constant, so that the total mass of the charge system also has directivity.

According to the equivalence principle, gravity is equivalent to the inertia force produced by accelerated object. If the charge system is at rest in a gravitational field, then because of the interaction between the charges q_1 and q_2, the charge system will produce a force on itself in the direction of the gravitational field, which is a part of the gravity on the charge system. The mass corresponding to this force is just the interaction gravitational mass of the charge system. According to the equivalence principle, the interaction gravitational mass of the charge system should be equal to its interaction inertial mass; therefore the interaction gravitational mass of the charge system also has directivity.

Above we find the interaction mass m' of the charge system. Because the interaction energy (electrostatic energy) of the charge system is $U = q_1q_2/(4\pi\varepsilon_0 r)$, according to the mass-energy relation, we get that the mass corresponding to U is $m = U/c^2 = q_1q_2/(4\pi\varepsilon_0 c^2 r)$. Comparing this formula with the equation (19), we can see that $m' \neq m$. This shows that the inertia mass determined by Newton's second law is not equal to the energy mass determined by the mass-energy relation.

5. Conclusions

To sum up the foregoing, we can draw the following conclusions: (1) An accelerated object in vacuum can produce a real inertial force that comes from the self-action of the object. (2) Gravity like the inertial force also comes from the self-action of object. (3) The mass (inertia) of object is related to its self-action, and the interaction mass originates from the self-action of the object. (4) The mass of object is not constant but has directivity. (5) The inertial mass of object does not necessarily equal its energy mass that corresponds to its energy. The problem of the origin of mass remains to be further researched and explored. We would hope the conclusions of this paper can be enlightening for future research.

References

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