Stochastic, Granular, Five-dimensional Space-time: a root model for both relativity and quantum mechanics, and a new interpretation of time

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A stochastic model is presented for the Planck-scale nature of space-time. From it, many features of quantum mechanics and relativity are derived. As mathematical points have no extent, the stochastic manifold cannot be tessellated with points and so a granular model is required. For Lorentz invariance, the grains cannot have constant dimensions but instead, constant volumes. We treat both space and time stochastically and thus require a new interpretation of time to prevent an object being in multiple places at the same time. As the grains do have a definite volume, a mechanism is required to create and annihilate grains (without leaving gaps in space-time) as the universe, or parts thereof, expands or contracts. A 'rolled-up' fifth dimension provides the mechanism. As this is a 'root' model, it attempts to explicate phenomena usually taken for granted, such as gravity and the nature of time.

PREFACE

The precursor of this paper appeared years ago in Phys. Rev.[Ap.O]. The paper was highly cited and well regarded, e.g. Siser Roy called it 'profound'[1], Steven Miller 'Remarkable' [2], Luis de la Peña 'Pioneering work' [3]. Since then, there has been much activity in the stochastic approach, some of it spawned by the precursor paper.

However, that earlier paper, being behind a pay-wall, is not easily accessible. And also parts of it needed revisions. Hence the revised and slightly abridged version is included as the appendix to this paper.

OVERVIEW

Much of quantum mechanics may be derived if one adopts a very strong form of Mach's Principle, requiring that in the absence of mass, space-time becomes not flat but fully stochastic, a state of maximum entropy, and thus arguably providing no arrow of time. If one wanted to be philosophical, the idea of no arrow of time might suggest that in the absence of mass (and of photons), space-time should have no properties at all. And that might include the metric signature. In that case, the idea of dimension would seem to have no meaning.

The stochasticity is manifested in the metric tensor which is considered to be a collection of stochastic variables. The stochastic metric assumption is sufficient to generate the spread of the wave packet in empty space. The idea is that vacuum energy fluctuations imply mass fluctuations which imply curvature fluctuations which then imply fluctuations of the metric tensor. The metric fluctuations are then taken as fundamental and a stochastic space-time is theorized. A number of results from quantum mechanics are derived.

If one further notes that all observations of dynamical variables in the laboratory frame are contravariant components of tensors, and if one assumes that, locally, a Lagrangian can be constructed, then one can derive the uncertainty principle. Finally, the superposition of stochastic metrics and the identification of the negative of the determinant of the metric tensor as the indicator of relative probability yields the phenomenon of interference, as will be described for the two-slit experiment. The above is from the precursor paper.

Addressing some of the difficulties of the precurser paper, required an extension of the model: In so far as the fluctuations are not *in* space-time but *of* space-time, and points have no extent, a granular model was deemed necessary. For Lorentz invariance, the grains have constant 4-volumes. Further, as we wish to treat time and space similarly, we propose fluctuations in time. In order that a particle not appear at different points in space at the same time, we find it necessary to introduce a new model for time where time as we know it is emergent from an analogous coordinate, tau-time, τ, where 'τ-Time Leaves No Tracks' (that is to say, in the sub-quantum domain, there is no 'history'). The model provides a 'meaning' of curvature as well as a (loose) derivation of the Schwartzschild metric without need for the General Relativity field equations. In order to tessellate the space-time manifold, it was necessary to introduce a fifth dimension which is 'rolled-up' at the Planck scale. The dimension is associated with mass and energy. Further, the fifth dimension addresses other problems associated with the granular space-time model.

The purpose of the Stochastic, Granular, 5-dimensional model, 'SG5D', is to both fold the seemingly incomprehensible behaviors of quantum mechanics into the (one hopes) less incomprehensible properties of space-time, and also to generate many of the phenomena of relativity. We do this by working with space-time at

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the Planck scale.

I. INTRODUCTION

Although it is a remarkably reliable schema for describing phenomena in the small, quantum mechanics has conceptual problems; e.g. How can entanglement send information faster than light (without violating relativity)? What is happening in the two-slit experiment? How can it be that the wave function can instantaneously collapse? In what medium does the Ψ wave travel? Is the E=hf wave (the Compton wave) the same as the Ψ wave? What is the wave function? What explains superposition? Can the two-slit experiment (at least in theory) be performed with macroscopic masses? Is 'The Cat' alive or dead? (One should say at the outset that this stochastic space-time theory is a DeBroglie-Bohm rather than a Copenhagen model so Schrödinger's cat is not an issue; Waves interfere. Particles do not.) And finally, how can quantum mechanics peacefully coexist with relativity.

The mathematics of quantum mechanics works exceedingly well. What we attempt first in this paper is to provide a conceptual framework for the quantum phenomena described by the mathematical formalism.

Granular space-time theories suffer from the problem that if the grains have a specific size, then the theory cannot be Lorentz invariant. Our grains though (which we call 'venues' to distinguish them from point-like 'events'), have constant 5-volumes (rather than constant dimensions) and volumes *are* Lorentz invariant. [The fifth dimension will be introduced later in this paper. But external to a mass the fifth dimension is usually zero, so for the most part we can consider 4 rather than 5-volumes.] A venue then, is the smallest possible volume. In empty space, with venues in a (average) rest frame, the venues have dimensions of Planck length times Planck length times Planck length times Planck time times c, and with the fifth dimension included, times the Planck length again.

And the stochasticity is exhibited by venues migrating through the space-time manifold (without leaving gaps in the space-time). We required granularity since the (stochastic) space-time must tessellate the manifold. But point-like events have no volume which is to say that multiple events could migrate to the same 'point' in the manifold. (We will see though, that interior to a Schwartzschild singularity gaps could occur if we accept an interpretation of the Kruskal metric for the Schwartzschild solution saying that the 'interior' of the singularity is not in the space-time manifold.) A 'point' then, is an idealization. So is a line. So the shortest possible length is also an idealization without any physical meaning. We maintain, on the other hand, that a volume is 'real'. So the idea of a smallest possible volume is likewise 'real'.

Another problem with stochastic space-time theories is stochasticity in time. For covariance one would like to treat time and space similarly. To do that, we then let the stochasticity apply to both space and time. This leads to an obvious problem: If a venue contains mass, then migrations can position the mass so it appears at multiple positions in space at the same time. E.g. A venue containing mass could migrate one unit backward in time, then one unit forward in, say, x, then one unit forward in time, resulting in the mass being at both (x,y,z,t) and (x+1,y,z,t). Preventing this necessitates a change in how we view time.

First, let's consider the idea of the 'world-line'. Moving forward from the present, we are predicting the future. And with quantum uncertainties (as well as with the intervention of outside forces) that future cannot be certain. And if there is no completely deterministic trajectory going forward, neither is there one going backward in time. The world-line then, seems to have limited utility in quantum mechanics. Instead of a world-line, we consider a 'world-double-cone', with its apex at 'now' and widens as one moves forward or backward in time.

We suggest that for the quantum world, t is not the fourth dimension, and that t is an emergent quantity, if not merely a human construct based on memory. The time coordinate, t, is a defined quantity in the laboratory frame whereas we suggest (below) another quantity, τ (tau-time) is appropriate in the quantum domain.

We'd like to treat the time dimension, t, in the same way as we treat spacial dimensions. But there is a big difference between a space and time coordinate: Consider the graphic below:

x –>

A particle (the black disk) starts at x=0, then moves to x=1, then 2, then 3. (We are considering space-time to be granular, hence the coordinate boxes.) There is a single instance of the particle.

But time is different:

t–>

A particle at rest is at t=0, then moves to t=1, etc. But when it goes from t=0 to t=1, it also remains at t=0. There are now two instances of the particle, etc. In other words, a particle at a particular time is still there as time advances, and the particle is at the advanced time as well.

We define then, a new quantity, τ (tau-time), that acts much like the usual time, but in accord with the first graphic, above. I.e. when the particle advances in time, it erases the previous instance. That is to say, 'τ-Time Leaves No Tracks'. Aside from fixing the problem of the same mass appearing at an enormous number of different locations at the same time, in the section on 'Migrations in Space and Time', τ will be seen to provide a solution to the collapse of the wave-function problem.

The approach taken here considers a granular space-time undergoing Brownian Motion in both space and

time. A Wiener Process is our starting point in modeling a granular, indeterminate space-time.

II. WIENER (AND WIENER-LIKE) PROCESSES

First, we consider Wiener migrations of venues in space.

A Wiener Process W is an idealization of Brownian motion. It is a random walk of n steps where n approaches infinity. (But, as we regard venues not to be point-like but granular, we will not be taking the process to infinity.)

The ith step is defined as

$$W_i = W_i - 1 + \frac{X}{\sqrt{i}}$$

where X is a binary random variable (+ or - 1). As n gets large, the distribution of W_i tends towards the unit normal distribution. As can readily be seen, as i goes to infinity, the W graph is everywhere continuous but nowhere differentiable. The graph is fractal (in that it is scale independent). The graph is a 'space filling' curve with fractal dimension 1.5. Traversing between any two points along the curve requires covering an infinite distance. However, in any finite time interval, there are found all finite values of x. So in the case where a venue can move, it can move to all values of x in an arbitrarily small time interval, e.g. faster than light, which is not a problem for venues not carrying mass. (Again though, in this granular model, we do not let i go to infinity.) One should note here that there are two kinds of motion: motion *in* the space-time, and motion *of* the space-time; a mass can move partially with the venues and partially through the venues and, as we will see, is prevented from traveling faster than c.

100pts t=1.0 x=.5 (textbook image)
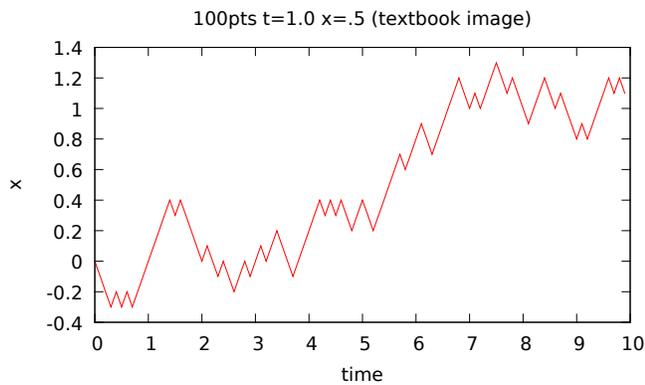
Here is something of a textbook example of a 100 point Wiener Process curve with measure=0.5. Note: 'measure' refers to the probability of a 'coin flip' being heads. E.g. a measure of 0.75 means there is a 75% probability of the coin being heads (or left vs. right, or up vs, down).

40000pts t=1 x=.5
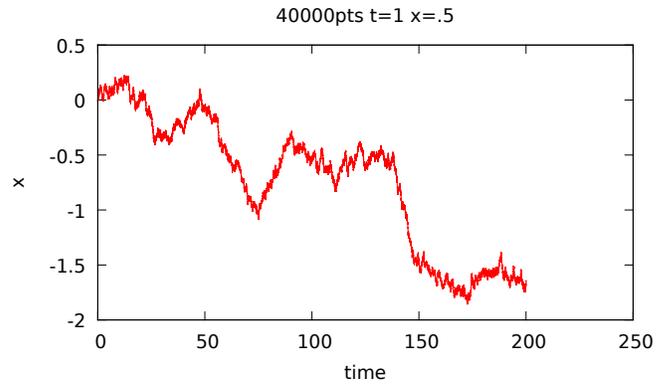
And above is a 40000 point example.

Extended to infinity, the variable i becomes a continuous variable, generally represented as t (time). In our granular model we do not extend to infinity. The above is for a 2-dimensional process (t vs x). To express t, x and y, two coins are flipped, one for x migration and the other for y.

1000 point. t,x,y
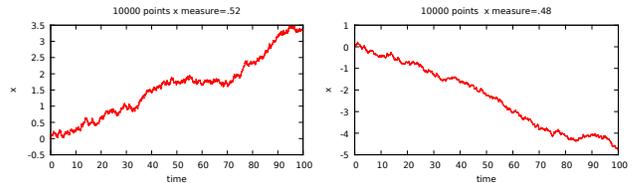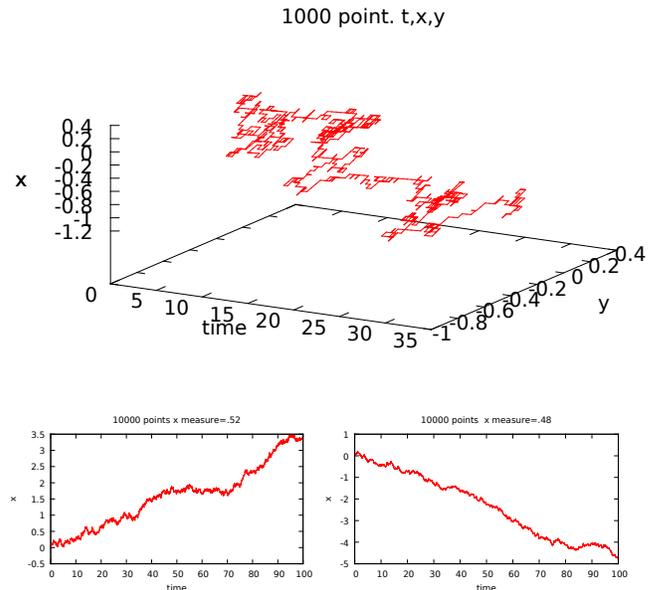
An x measure greater than .5 causes a tendency to drift up. Less than .5 tends downward

III. MIGRATIONS IN BOTH SPACE AND TIME; TIME IN QUANTUM MECHANICS; WORLD-TUBES

For reasons of covariance, we would like to treat time and space similarly. And so we will consider diffusion in space as well as in time.

Consider Graph 'A' (of 1000 points) below. (The vertical and horizontal lines are artifacts of the graphing software.) The graph represents the path of a a single venue migrating in x with a measure of 0.5, and also for a migration in t where the measure of t is 0.501 (meaning that t will slightly tend upward). We can regard the

graph as showing migration in x and also t, where the coordinate axes are laboratory x and laboratory t.


t-measure=.501 x-measure=.5 50000 points
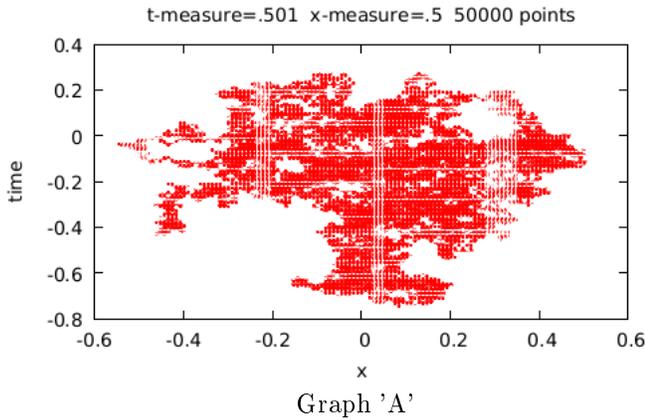
Graph 'A'

There is an immediate problem:

Consider what this graph signifies: At any given laboratory-time t, the same venue will (simultaneously) be at a very large number of x coordinates. If there were mass/energy at the venue, this would be very problematic as causality and conservation of mass would be violated.

This problem has been addressed (in the introduction) by introducing τ (tau-time), and the 'τ-Time Leaves no Tracks' idea.

We can still consider Graph 'A', but we'll interpret it differently: If we take any (horizontal) time (τ) as a 'now', A venue (containing a mass) stochastically flits forward and back in time and space. So that at 'now' there is one and only one particle. But where it is cannot be predicted. However, the likelihood of the particle being at a particular x (+/- dx) position is determined by the relative number of times the particle is at that position. In the case of Graph 'A', if we take as 'now' the τ-time slice at -0.2, for example, we find (by examining the data) the following probability curve:

Graph A probability
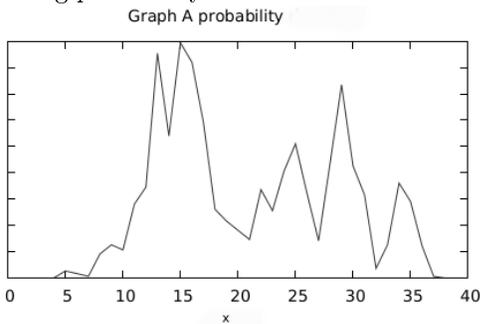
This is analogous to $\Psi^*\Psi$. But the graph is a construct. It represents, but is not actually, the particle. When the particle is measured by, for example, being absorbed in a detector, it freezes (no longer moves stochastically). It no longer flits through time and space so the graph 'collapses' to the measured position. (that position is only determinable by the measurement.) This is analogous to the collapse of the wave function, but here (as the graph was merely a mathematical construct) there is no collapse

problem.

There are a few points/speculations to be made about measurements. First, to be a true measurement, there must be a latch/flip-flop/memory so that the 'film' cannot be run backwards. As an example, consider the two slit experiment with electrons. If a measurement device is placed at a slit, there is no interference pattern. But when an electron goes through a slit, the orbital electrons in atoms of the wall of the slit will be distorted by the passage of the electron. This distortion is *almost* a measurement. But when the electron passes through the slit, the orbital electrons become un-distorted. The interference pattern is still produced because there is no latching of measurement information. A latch could be some mechanical contrivance, or even human (or non-human) memory. A fruit-fly observing at the slit will kill the interference pattern, but only for the fruit-fly. We think the process should be transitive; A human observing the fruit-fly's memory will cause the interference to be killed for the human as well. A measurement forges a connection between the thing being measured and the measurer–forcing them to have the same relative now. In the macro-world, virtually everything observes (via photons) everything else, forcing that macro-world (or a portion thereof) to have the same relative now. And measurements forces time to have tracks. Not that time is frozen, but looking back to a particular time will show uniquely what the world looked like at that time. E.g., if one were to do high-speed filming of particle 'tracks' in a cloud chamber, one would see the time-tracks.

Observation, a crucial part of a measurement, is conducted via photons. We speculate that *all* measurements are via photons?

The time leaves no tracks concept implies that there are multiple futures, and they all 'happen'. (This is somewhat redolent of the Everett many-world interpretation[4].) In SG5D, an observation from the laboratory will select a particular future (making a track).

It should be remembered that the motion of a particle is determined by two factors: the motion of the venues in which the particle is embedded, and the non-quantum dynamics of the particle, the dynamics, for example forbidding super-luminal velocity. So in the above, if the particle were in a potential well with perfectly reflecting walls, the above graph would (after a time) represent the probability density of finding the particle at a particular position in the well.

Again, the particle has always existed at only a single venue, but the venue migrations happen roughly at the rate of the Planck time, making the particle appear (in some sense) to be at multiple positions at a particular time. Further, (because of the properties of Wiener Processes) the particle appears to spread. If the particle were not constrained by the well, (because time is moving forward and back) the graph would evolve (spread) arbitrarily rapidly. In that case the curve would represent the relative probability density of finding the particle at a particular position once the particle (because of its

dynamics) were able to be found there. The curve then would represent DeBroglie's 'ghost waves that guide the particle'.

Note: The jagged lines in the graph (as opposed to a smooth curve) is an artifact of the binning algorithm in the software.

By Statement 1.4 of the precursor paper (the appendix of this paper), the particle location becomes less stochastic as mass increases. There is a point where the stochasticity ceases. At that point, (since it is not migrating back and forth through τ-time), one can use the usual t-time. So, we consider t-time (and also causality) to be an emergent quantity. In the rest of this paper, when we do not reference history, we will simply use t instead of τ. Also, as a result of measurement, when the above graph 'collapses', the time is fixed, so measurement causes time to 'leave tracks' causing τ to become t.

Now we can (briefly) revisit Statement 3 of the precursor paper: The metric probability postulate, $P(x,t) = -kg$. A particle, by its mass, generates a local contribution to the metric tensor at the observer's 'now'. The particle will flit forward or backward in time. The local metric contribution, being an extended field, will not flit with it, and, for the same reason, will not instantaneously decay. When the particle flits back to the observer's now, it will be subject to that extended metric field. (This is somewhat akin to quantum field theory where a particle interacts with the electromagnetic field created by its own charge. Here, the particle interacts with the gravitational field created by its own mass.) So $P(x,t) = -kg$. can fully apply.

As the probability density is not stochastic while the metric components are, that puts constraints on the metric tensor, i.e. the determinant of the metric tensor is constant while the metric components are not. So (stochastic) changes in one or more components are compensated by opposite changes in the others. This implies that a venue is in constant flux, its dimensions continuously and unpredictably changing while the venue maintains a constant volume. This also implies that the metric stochasticity is due to a single (and the same) random variable in each non-zero metric component (That variable will then drop out in the determinant.)

IV. ZITTERBEWEGUNG

The 'Standard Model' roughly says that 'stuff' is made of fermions. And 'stuff' is held together by bosons–the force carriers. SG5D is predominately about fermions. Fermions are described by the Dirac equation. It is a good equation but it predicts that fermions have an instantaneous velocity of plus or minus c. Schrödinger, wanting to believe the equation, suggested that the fermions moved at a very high (and thus undetectable) rate back and forth, the average motion being a drift. He coined the word *zitterbewegung*[5] ('flicker-motion') to describe that motion. SG5D describes mass motion in

much the same way, perhaps thus giving more credence to the zitterbewegung idea.

V. VENUE MIGRATIONS IN EMPTY SPACE

Mach's Principle posits that the local properties of space-time depend on the mass distribution in the universe. We'll adapt the principle to the SG5D model. And we'll introduce another variable: 'Indeterminacy', the probability that migrations will actually happen.

As with 'Measure', Indeterminacy is implemented with a 'coin flip'. And we'll suggest that outside of a mass, the Indeterminacy decreases with decreasing distance from the mass/energy (i.e. space becomes more determinate as one approaches a mass). It will be seen that 'Measure mainly influences quantum effects while Indeterminacy influences relativistic effects. (Indeterminacy is likely related to the concept of inertia.)

The space-time Indeterminacy decreases as one approaches a mass. But this is underspecified; masses can have different densities, so we wouldn't expect the Indeterminacy to necessarily vanish at the surface of a mass. We suggest however, that venues can migrate into a mass until, at some point the Indeterminacy vanishes. Yet we do not want masses to be pulled apart by the space-time so we'll posit that migrations of adjacent venues each containing mass must stay adjacent. And in that case, one could consider each of those venues having zero Indeterminancy compared to the others.

We'd expect that at some distance, Rs, from the center of the mass, the venues, would be trapped, i.e. unable to migrate away. This is highly suggestive of the event horizon of the Schwarzschild solution. We'll assume Rs (the Indeterminacy radius) and the Schwartzschild radius are the same.

The concept of Indeterminacy decreasing with closeness to mass has an interesting *possible* consequence relating to measurement: A measurement requires an exchange of energy between what is being measured and the measurer (an energy that can't be transformed away). But energy of this form (e.g. photons), being equivalent to mass, *possibly* forces determinacy of the photon relative to the object that absorbs and/or emits the photon.

So, for example, if one were to place a measuring apparatus at one slit in the two-slit experiment, activity at that slit (at the time it is measuring if a particle went through it) would be deterministic since the photon 'connects' the slit and the emitter (i.e. the laboratory). And therefore, since Indeterminacy=0 says that the system behaves classically (as opposed to quantummechanically), the interference pattern would not happen.

Insofar as measurements are accompanied by exchanges of photons, it's tempting to consider that photons are the carriers of causality.

Up to this point, we've considered the migration of just a single venue. The model though, assumes space-time

is completely tessellated (tiled) by venues, i.e. there are no regions of space-time that are not fully covered by venues. While we can justify the migration of a single venue, migrations of venues in a completely tiled space-time is more problematic. One might even doubt that there can be any migrations at all in a fully-tiled space-time. We are modeling the stochasticity of space-time as a Wiener-like process on venues (grains). We assume that the space-time completely tessellates the space-time (i.e. there are no holes in the space-time). How then can migrations occur in a fully tiled space-time? We start by modeling any single venue as a Wiener-like process. Other venues must then also migrate to preserve the tiling.

The migration can proceed in one or two ways: The first is like the circulation in a perfect fluid. The 'diffusion' in that case, is via closed loops in the space-time.

The second way is the squishing-interchange of venues, as shown below: The diagrams represent an idealized pair of venues. The black and white venues continuously move to interchange their positions while keeping their volumes constant.

While the SG5D model is of a discrete, granular space-time, the discreteness is expressed in the venue volumes. So local continuous processes (between adjacent venues) as the above are not disallowed.

The migration problem persists though, as can be seen in Indeterminacy: Assume a spherical mass in an otherwise empty space. Indeterminacy is assumed to decrease as a venue migrates towards a mass. Even with Measures $= 0.5$, a venue will at some point approach arbitrarily close to the mass. But (letting R be the radial distance to the mass) as Indeterminacy is the probability that the venue will not migrate at the next coin flip, the venue will spend increasing amounts of 'time' as R decreases. In the case of multiple venues, there will be proportionally more of them in a volume element closer to the mass. This results in the 'piling up' of venues as one gets closer to the mass. How can this be? We don't want to resort to venues 'pushing' against other venues since that would imply that the venues are overlaid onto space-time instead of them being space-time. Nor do we (yet) want to employ higher dimensions. An answer (perhaps the only answer) is curvature. But what is curvature? 't Hooft has theorized[6] that curvature is an artifact of the fact that we live in four dimensions but space-time is actually five dimensional (e.g. a two dimensional being on a sphere can measure curvature, but with the sphere embedded in a flat three dimensions, there is no curvature.) We will take a different approach: Venues are assumed to have constant volume but not constant dimensions. Curvature will be described, below, as the thinning of space dimensions, (particularly the radial distance from a mass

dimension) while the time dimension thickens.

As for the translatory motion of the particle (as opposed to the rotational), the particle doesn't become 'fuzzy', but its location does begin to blur as the mass decreases below the Planck mass. This results in an effectively larger grain size.

Two effects: like a smaller pollen grain in Brownian motion: the smaller the grain, the more it stochastically moves. But as the effective grain radius increases, the movement decreases as there is a larger circumference over which the movements can average.

Note then that the effective radius *rate* of increase decreases as the effective radius increases. To reiterate, this is because, as the particle grows in *effective* size the average effect of the venue migrations against the particle surface begin to average out (analogous to the case of Brownian motion where the jitter of a large pollen grain is less than that of a smaller grain).

We maintain that all physics that uses the radius should use the effective radius. radius= rest-radius + *Radius Quantum Correction:* $r = r_c + r_{qc}$. For an example of the effective radius, see the Schwarzschild metric derivation below.

One might consider the 'actual' radius as the covariant (and hence unobservable radius) whereas the effective radius is the contravariant (in principle, observable) radius.

We explore now whether the model might indeed reproduce the Schwarzschild metric.

A mass generates 'curvature', that is to say, a deformation of venues. While to a distant observer the venues are deformed to be spatially concentrated around the mass, to the venues near the mass there is no observable evidence of such concentration as the space-time itself is 'deformed' (by way of the venues) so any 'observer' in a venue would be unaware of the deformation.

Consider space-time with a single spherical mass m with an Indeterminacy radius Rs. The Wiener graphs are for some undefined unit of time. But as one increases the number of coin flips towards infinity, the time interval decreases to an infinitesimal, dt. For a granular space-time though, the number of coin flips isn't infinite and the time interval, though small, isn't infinitesimal. Once again, Indeterminacy is the probability of, given that the venue is at a position with that Indeterminacy, the venue migrates from that position at the next coin flip.

Since migrations slow as venues approach a mass, indeterminacy then, expresses the slowdown in time and the compression of space as the venue approaches Rs. [As we'll be frequently employing Indeterminacy, we'll represent it by the letter 'u' (from the German word for indeterminacy, Unbestimmtheit)].

As a venue migrates in towards R_s, u decreases. The probability density of the venue being at a particular radial distance, r, therefore, increases. This results in venues piling up as they approach R_s. But as the venues 'tile' space-time, the only way they can pile up is by way of curvature (i.e. squishing in the radial dimension and compensating by lengthening in the time dimension):

To a distant observer, the venues would decrease in size and migrate more slowly which is to say time would slow down.

Recalling (see Statement 2) that the contravariant distance to a lack hole is $\int_0^{\bar{r}} dr = \bar{r}$, while the covariant distance is $\int_0^{\bar{r}} d(\frac{r}{1-2Gm/r}) = \infty$, we can (in Cartesian coordinates) associate the contravariant distance with the number of *Planck lengths* from the observer to the point of observation and the covariant distance with the number of *venues* from the observer to the point of observation.

[This implies that local to the particle, space-time is not stochastic. And there, a deterministic Lagrangian can be defined. That 'local to the particle space-time' coordinate system is covariant (as it is moving with the particle). From another coordinate frame (e.g. the laboratory frame) measurements on that local frame are subject to the intervening stochasticity, and because of that stochasticity, the measurements are also stochastic, and the measurements are contravariant, as can be seen by the raising of the covariant coordinates by the stochastic metric tensor).]

Now, near $r = R_s$, space-time becomes Q-classical (no quantum effects, as opposed here to R-classical: no general relativity effects) so a metric makes some sense. Since the Measures (bias in the coin flips) are presumed not to be a function of location, we take the simplifying assumption that the metric tensor does not depend on the Measures, but only on the Indeterminacy, u. And, for the moment, we'll ignore how a venue migrates *in* a mass (when R_s is less than the mass radius).

Since for a mass, we have spherical symmetry, we can let, $ds^2 = -f(u)dt^2 + g(u)dr^2 + r^2 d\Omega^2$ where f and g are two (to be determined) functions of u, and $d\Omega^2 \equiv d\theta^2 + sin^2(\theta)d\varphi^2$ is the metric of a 2-dimensional sphere. Consider f(u) and g(u). We wish dt to lengthen and dr to shorten as u decreases. ds can be thought of as the time element in the frame of the venue. So, for example, as u goes to zero, a big change in t will result in a small change of s, and a small change in r results in a large change in s. The *simplest* implementation of the above suggests that $f(u)$ is just u itself and $g(u)$ is u^{-1} i.e. $ds^2 = -u dt^2 + u^{-1}dr^2 + r^2 d\Omega^2$.

Now, as to u, note that,

at r = infinity: u = 1,

at r = Rs : u =0, and

for r < Rs : u can become unphysical (u<0).

The *simplest* expression for u satisfying the above is, $u = (1 - \frac{R_s}{r})$ which gives us

$ds^2 = -(1 - \frac{R_s}{r})dt^2 + (1 - \frac{R_s}{r})^{-1}dr^2 + r^2 d\Omega^2$

We have of course, as described earlier, equated the Schwartzschild radius with the Indeterminacy radius.

This is the result Karl Schwartzschild derived from the General Relativity field equations. One can easily go a bit further by noting that R_s can only be a function of the mass, and finding a product of mass with some physical constants to give a quantity with dimensions of length suggests $R_s = \frac{kGm}{c^2}$ where k is a constant. So we now have (setting units so that c=1),

$ds^2 = -(1 - \frac{kGm_s}{r})dt^2 + (1 - \frac{kGm_s}{r})^{-1}dr^2 + r^2 d\Omega^2$.

We still need to determine the value of the constant, k. But this is known territory. R_s was derived (by Karl Schwarzschild and others) by requiring the metric to reproduce the Newtonian result at large values of r and small values of mass, and we need not reproduce the derivation(s) here.

At first glance, there appears to be a problem with Schwarzschild metric and stochastic granular space-time theory in that masses can be arbitrarily smaller than the Planck mass. And that would allow the Schwarzschild radius to be vanishingly small, to the point of exposing the 'naked singularity' at r=0. And that is something we would like not to be possible.

But, as described earlier, any physical radius must be the *effective* radius (effective radius= rest-radius + Radius Quantum Correction). As a mass decreases to below the Planck mass, quantum effects occur which increase the effective radius. So a Schwarzschild radius of one Planck length is the minimum possible Schwarzschild radius. Masses less than one mass then increases the (effective) Schwarzschild radius (until the rate of increase decreases to zero). That the Schwarzschild radius of a Planck mass is the Planck length is then consistent with the granular hypothesis.

VI. STOCHASTIC GRANULAR SPACE-TIME AND THE LORENTZ AETHER THEORY

We consider that our Stochastic Granular Space-time (SG5D) theory is (or can be made to be) a super-set of the Lorentz Aether Theory (LAT) where the aether is space-time itself (specifically, the 'grains'/venues making up the space-time). By doing so, we can appropriate the LAT derivation of the constancy of the speed of light. (We feel that any theory of space-time should contain an explanation of that constancy.)

As is widely known[7], the Michelson-Morley experiment failed to find the Lorentz aether, thus seemingly invalidating the Lorentz Theory[8]. Less widely known perhaps, is that the second version of Lorentz's theory (with H. Poincaré as second author) reproduced Einstein's Special Relativity (ESR) so well that there is no experimental way to decide between the two theories[8]. The second LAT theory differs from the first in that it posits that the aether is partially dragged along with a moving body in the aether. This is akin to frame dragging (e.g. the Lense-Thirring effect) in the Kerr Metric[9]. We will posit frame dragging in SG5D as well, i.e. the dragging along of venues by a moving object. (Note that the Kerr metric itself 'breaks' the continuity space-time. If it didn't, the frame dragging would 'wind-up' space-time, and it doesn't[10]. One might take this as an argument for a discrete space-time such as in SG5D.)

Although LAT derives the constancy of the speed of

light whereas ESR takes it as a given, there are objections to LAT:

1. There is an 'aether', the makeup of which is not specified.

2. There is a privileged, albeit unobservable, reference frame where the aether is at rest (isotropic).

3. The (constant) velocity of light results from electromagnetic interactions with waves (and matter), and not from properties of space-time.

SG5D can address these issues: As for 1, the makeup of the aether, SG5D says the aether is the space-time itself. And in 1922, Einstein himself said essentially the same thing.

[Note: Einstein (translation)-"Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable media, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it"]

2. A privileged reference frame, is also not an issue in SG5D. The stochastic nature of space-time makes it impossible to define a *global* rest frame. But we can consider a *local* privileged reference frame where the correlation region (the region where we can consider a background privileged frame) is large compared to the region where we are doing experiments.

3. The constancy of the speed of light not a result of the properties of space-time, can be addressed as well. While there is nothing wrong with the LAT derivation of the constancy, we can give a qualitative geometrical model as an alternate way of thinking about the constancy:

We suggest (and this is highly speculative) that frame-draging occurs whenever a mass (non-zero rest mass) moves through space-time. Photons, as their rest mass is zero, moves without frame-draging. This (as we will see) allows an argument showing the constancy of c.

Consider an object (here, the black circle) moving at high speed in the direction of the arrow. The object moves through the venues (here represented by the white rectangles). But due to venue frame dragging at high velocities, the venues are pushed ahead of the moving object. But venues are constant in volume, and the only way that they can 'pile up' is by contracting in the direction of motion (and expanding in other dimensions). The

object must move through these venues. As the object's speed increases, the contraction increases (rather in the way a 'curvature well' becomes ever deeper). To an external observer (making contravariant observations), the objects increase in velocity slows until it stops completely where the venue dimension in the direction of motion approaches zero. To that observer (as can be seen in the diagram above) the object is accelerating (which because of the Equivalence Principle, is under the influence of gravity). This establishes that a mass has a limiting velocity.

We have postulated that a particle with non-zero rest mass drags along (empty) venues as it moves, Photons, having zero rest mass, do not drag venues.

So, if a particle moving with respect to the local privileged reference frame emits a photon, the photon does initially travel with a velocity of c plus the velocity of the particle. But the particle is dragging venues. As the venue contracts in the direction of motion, since its volume is constant, it expands in the time dimension. And this makes the time a photon takes to pass through the venue constant. The photon has more venues to pass through than it would have if the particle were not moving. Because of the additional distance (i.e. number of venues) the photon needs to travel, its speed at the detector, would be a constant, which is to say c.

If the detector were extremely close to the emitter (on the order of Planck lengths) one would measure a value of the velocity greater than c.

This length scale is too small to measure so the velocity greater than c is unobservable. But other phenomena related to frame dragging might be large enough to detect. A comet in an extremely elliptical orbit or a space-craft 'slingshotting' around a planet might exhibit a detectable motion anomaly.

The SG5D model violates Galilean Relativity in that motion is not (in this model) relative. LAT violates it as well. This is allowed (in both cases) by having a privileged reference frame.

With SG5D then, there is a new phenomenon at play: 'Velocity Induced Frame-draging'. So, in addition to frame-draging being generated by mass (or acceleration), it is also generated by an object's linear motion in the space-time aether. One way of perhaps justifying this is to consider the conservation of energy, as the sum of potential and kinetic energy. The former is gravity dependent while the other is motion dependent. Since gravity yields curvature, perhaps velocity does as well. Potential then, could be considered a result of Mach's Principle.

Frame-draging has much in common with curvature, specifically Schwarzschild curvature. We might therefore expect the metric tensors to be similar. Indeed, without doing any calculations, we can guess at a metric for the moving object. Consider the g_{11}(the radial component of the Schwarzschild metric) $(1 - \frac{2Gm}{rc^2})^{-1}$. The velocity induced model is not a function of mass, so m and G are unlikely to be in g_{11}. However, note that Gm/rc^2 have units of v^2/c^2, so we might expect g_{11} to be $(1 - k\frac{v^2}{c^2})^{-1}$

where k is a constant. We would expect a (coordinate) singularity to occur when $v = c$, so that would make $k = 1$. A similar argument can be made for g_{00}(the time component).

VII. BRIEF SUMMARY OF THE MODEL SO FAR

The aim of 'Stochastic space-time' is to introduce stochasticity into the structure of space-time itself, rather than into the properties of the particles in the space-time. This is a similar, geometrodynamic, approach to Nelson's groundbreaking model[11] that indeed has matter moving stochastically *in* the space-time.

Mass reduces stochasticity as one approaches the mass. And, insofar as stochasticity correlates to entropy which establishes the arrow of time, the 'length' of that arrow is not constant throughout space-time.

Because points have no extent, there seemed to be no way to prevent events (points) migrating to the same point. Therefore tessellating space-time would be problematic. So a granular model of space-time seemed necessary. Further, whereas the only geometrical property of an event is its coordinate location, grains, having extent, can have different values of Δx, Δy, Δz, and Δt. And that allows an explanation of curvature within four dimensions (as opposed to explaining it by embedding the four dimensional space-time manifold in a five dimensional Euclidean space). And as long as the 4-volume of the grains (which we call 'venues') is constant, we do not violate Lorentz invariance.

In order that we treat time in the same way as we treat space (and not to have particles appear at different places at the same time), we needed a new version of time, τ-time. The implication is that our usual t-time is just a human construct, not actually intrinsic to space-time.

VIII. THE FIFTH-DIMENSION

The are problems with the granular model: What happens when the universe (or a local region within it) expands or shrinks? How can venues increase or decrease in number? Related to this is the question of the constancy of the infinitesimal volume element. The field equations of General Relativity state that interior to a mass ($T_{\mu\nu} \neq 0$) the Ricci tensor is not zero, and so the volume element, the determinant of the metric tensor, is not conserved. This creates a problem with the tessellation of space-time.

To handle these (and other problems) we postulate a fifth dimension. It is 'rolled-up' at the Planck scale.

A. Time-like or Space-like

We postulate a fifth dimension associated with mass. But is it space-like or time-like? The Space-Time-Matter consortium asserts it is space-like and the dimension *is* mass[12] (and the dimension is not rolled-up).

Were the dimension time-like there would be difficulties. A characteristic of time is that it progresses. An identification of mass as the coordinate wouldn't work as the coordinate couldn't progress (unless the mass were continuously increasing). And a time-like fifth dimension wouldn't allow the annihilation/creation of venues mechanism shown below in B4.

But there are advantages to a time-like, mass-related dimension. It could provide a mechanism for waves/periodicity and an explanation of the 720 degree symmetry for quantum mechanical rotations (see section B3 below). We (tentatively) suggest that the 5th dimension, which we call u, is both space-like or time-like depending on whether a venue is hosting mass/energy or not. If not u is space-like, otherwise it is time-like. And when time-like, it also leaves no tracks.

The notion of a both time and space-like dimension is not entirely new; for example, see the Braneworld model[13].

Instead of a both time and space-like dimension, we could obtain the same results by suggesting a space-like 5th dimension, and a time-like 6th dimension, also 'rolled up'. For the moment we'll keep to just the five dimensions.

B. The Utility of a Fifth Dimension

So then, a fifth dimension helps the model in many ways. In particular, it gives us:

1- a way of achieving consistency with quantum mechanics and relativity,

2- an argument for the constancy of the volume element,

3(highly speculative)- a mechanism for waves (without the need for forces) (and the 720 degree symmetry for Fermions),

4- a way of creating and annihilating venues,

5- a geometric interpretation of mass,

6- the Kaluza-Klein[14] formalism for bringing electromagnetism into the stochastic space-time model,

7- uniqueness of tessellation of the space-time by a regular honeycomb,

Below, we briefly expand on these topics.

1. Consistency of the model with Relativity and Quantum Mechanics

In differential geometry, Loveridge[15] has pointed out that the Ricci tensor governs the evolution of a small volume element (i.e. $\sqrt{-g}$) as it travels along a geodesic.

Following Loveridge, assume a very small spherical volume of dust o centered on point $x^\mu(0)$ moving along a direction T^μ. ($T^\mu \equiv \frac{dx^\mu}{d\tau}$). One has that $\frac{D^2}{d\tau^2}o - \frac{D^2_{flat}}{d\tau^2}o = -oR_{\mu\nu}T^\mu T^\nu$, where D is the covariant derivative along the path. The equation applies for both three and four dimensional volumes. The reason for subtracting the second term is that the choice of coordinates could give an apparent (not intrinsic) change of volume.

But in Special Relativity, the Ricci tensor is zero. Which means that the volume element, $\sqrt{-g}$, is invariant. (For Special Relativity, this is easy to see: In a Lorentz transformation, as the length shrinks, time expands to leave the volume unchanged.) In General Relativity, in empty space-time, while the Riemann tensor is not zero, the Ricci tensor is. So, in empty space (i.e. exterior to a mass), the volume element is also invariant.

But for quantum mechanics, our stochastic space-time model postulates that the volume element, $\sqrt{-g}$, is *non*-constant and proportional to $\Psi^*\Psi$. This would seem to be a contradiction.

To address this, what we would like is a mechanism where, in a region of space-time where the wave function Ψ does not vanish, the volume element is not constant. And, ideally, that mechanism would be a function of the space-time geometry.

We propose then, that there is a fifth dimension and that the five-dimensional volume element is *everywhere* constant (see *2* below). We take it that the fifth dimension coordinate is zero except when in a mass or where the wave function Ψ is non-zero or where an electro-magnetic field is present.

The wave function itself is postulated to be proportional to the *4*-dimensional volume element. And external to a mass, the General Relativity field equations, i.e. $R_{\mu\nu} = 0$, still hold (for both 4 and 5 dimensions, i.e. µ and ν range from 0 to **3**, or from 0 to **4**).

In order that the 5-dimensional line element not be observably different from the 4-dimensional line element, we adopt the Kaluza-Klein idea that the fifth dimension is 'rolled-up'. (See *7* below.)

2. Constancy of the Volume Element

While the global invariance of the five-dimensional volume could be seen as a simplifying and/or unifying concept, the idea seems important for being able to tessellate space-time without leaving holes in the space-time fabric.

It is difficult to fully cover space-time with venues where the venues do not have a constant volume; if a venue's volume were less than a neighbor's, the neigboring volumes would need to become larger. But, while this is possible, this is a stochastic space time model; i.e. the venues migrate throughout the space-time (and indeed they *are* the space-time). It is difficult to see how venues of different volumes could migrate while still tiling space-time.

In order that venues continue to be able to tessellate the space-time manifold, we postulate a fifth dimension where the volume element is *always* conserved, but in five dimensions. As in **1** above, that fifth dimension is not zero only where mass or energy (or Ψ) is present. This is similar to the idea that the fifth dimension *is* mass, as proposed by Mashhoon & Wesson[16] and the Space-Time-Matter consortium[12]. SG5D differs in that the fifth dimension is an *indicator* of mass (and energy), where their's *is* mass.

We need the fifth dimension to hold the 'overflow' from a single venue's 4-dimensional contraction. And as a venue's volume is at the Planck scale, the dimension needs to hold very little. It can be well represented by a rolled up dimension at a Planck length 'circumference'. E.g., if, say, the x coordinate were to go from 1 to zero, the fifth dimension would go from 0 to 1.

We could not find any other method for venues to tessellate space-time other than the idea of an invariant volume element made possible by a fifth dimension. In that sense, it was forced on us.

The invariant volume element has a value of 1 Planck length (for the x coordinate) times 1 Planck length (for the y coordinate) times 1 Planck length (for the z coordinate) times 1 Planck time times c (for the t [or τ] coordinate) times 0 Planck time (for the u coordinate). It is 0 for the u coordinate because it is a rolled up dimension with a circumference of 1 Planck time, and a coordinate value of 1 Planck time is the same as 0 Planck time.

3. A Mechanism for Waves (without the need of forces), and the 720 Degree Fermion Symmetry (without recourse to spinors)

In the stochastic space-time model, the motion of an object can be broken down into two parts: motion *through* the space-time, and motion due to the migrations *of* the space-time. Analogously to the way a Brownian pollen grain moves under the collective collisions with water molecules, the more the object is at the quantum scale the more the motion is due to motions *of* the space-time. We expect then, that the *rotational* motion of an elementary particle is due mainly to the rotations of venues in the space-time manifold.

We begin by seeking a mechanism for generating a particle's rotational frequency (as a function of mass) with respect to u-time (the fifth dimension as a time) and then seek a mechanism for τ-time (the fourth dimension time).

We postulate that mass sets the relative rate of progression between the 4th and the 5th dimensional time. So, for example, if a venue contains no mass or energy, the 5th dimensional time is (and remains at) zero. Increasing amounts of mass in a venue increases the rate of 5th dimensional time with respect to the 4th. Since we regard the 5th dimension, u, as rolled-up, one might visualize the situation by imagining a clock face. The

more mass in a venue, the more rapidly the minute hand goes around.

The different rates of the two times is analogous to Special Relativity where the rates of time between two inertial frames in relative motion are dependent on their relative speed V.

Mass seems to come in chunks. In any case, mass does not seem to have a continuous range of values. And if mass is a discrete quantity, then so too is u-time. And it is not unreasonable to assume τ-time is discrete as well. This time discreteness could be the reason an orbital electron can transition 'instantaneously' between orbits.

From the laboratory frame, consider observing a distant venue containing a small mass. u-time at a venue is, by hypothesis, associated with the mass in a venue. In particular, the *increment* of u is so associated.

Masreliez[17] and Mukhopadhyay[18] among others have suggested that a mass oscillates at its Compton frequency, (and without such oscillation, there would be no DeBroglie wave, or indeed a Ψ). We accept that suggestion. The Compton frequency ,f_c, is defined as $f_c = \frac{mc^2}{h} Hz$.

We first convert Hz to cycles/Planck time.

$\frac{f_c}{\sqrt{\frac{khG}{c^5}}} = \frac{mc^2}{h}$ where k is a constant that equals either $\frac{1}{2\pi}$ for reduced Planck units, or 1 for *un*reduced Planck units (The reason we do this will be seen soon.)

Now we'll convert m from kilograms to Planck mass, m_p.

$\frac{f_c}{\sqrt{\frac{khG}{c^5}}} = \frac{mc^2}{h} \sqrt{\frac{khc}{G}}$

Simplifying, we have $f_c = km_p$.

Depending on k, this gives either $f_c = \frac{m}{2\pi}$ or $f_c = m_p$.

The $k = 1$ case, $f_c = m_p$, is compelling. It implies that if the mass in a venue is zero, (from the viewpoint of the laboratory observer) the u time does not advance. The more mass in a venue, the more 'rapidly' u advances until at a maximum venue mass of one Planck mass, the frequency has increased to one cycle per Planck time. This also argues that the Planck units should be defined using Planck's constant rather than (as is usually done) the reduced Planck's constant, $h/2\pi$. Henceforth, we'll then use the unreduced Planck units.

'Time' then can be considered made up of two characteristics: a coordinate (t) going from minus to plus infinity, and u, the fifth dimensional time, representing an ordering schema as described by H. Reichenbach[19]. Feasible time-like extra dimensions (in the context of Kaluza-Klein theory) have been discussed by Aref'eva[20] and Quiros[21] (among others).

The rolled-up fifth dimension has proven quite useful. As mentioned in Section II, the 5th dimension was invoked for the idea that the volume element is constant in five dimensions.

[Note: The idea of a constant volume element has an interesting corollary: In the presence of mass, external to the mass, the 4-dimensional volume element is also constant. The evolution of the 4-dimensional volume element is governed by the Ricci tensor. So, if the volume element is constant, the Ricci tensor is zero, i.e. $R_{\mu\nu} = 0$, which are the mass-free General Relativity field equations. If then, Einstein had never achieved his theory of relativity, we would still have the empty-space field equations. Karl Schwarzschild could have taken it from there.

To recapitulate, the chain of logic is as follows: We need constant venue volumes to tessellate space-time; We need a rolled up fifth-dimension for creation and annihilation of venues; *If* the fifth dimension is (on average) non-zero only in a mass, then (on average) the 4-volume outside of a mass is constant, which says the Ricci tensor is zero.]

(We are used to the concept of complex phase. Perhaps that fifth dimension is represented by the complex phase in quantum wave equations.)

We consider a space-time occupied by a single (indivisible) mass. We can impose a coordinate system centered on the mass. We consider its rotation. We take migrations (coin flips) for rotations about the three coordinate axes and (optionally) for time. Note that the particle is (in this model) rotating in all directions (reminiscent of spin).

In a mass, migrations in time allow an easy interpretation: A 'flip' from forward to backward in τ time is evidenced by the space 'measures' ($m_{x,y,z}$) (probabilities of moving clockwise) going from m to 1-m.

Consider the following idealized graph:
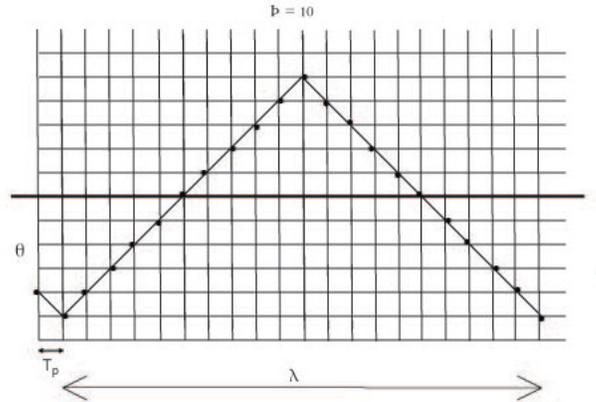
There is no reason the rate of space flips (here angle flips) and time flips must be the same. We define Þ (Icelandic/Old-English 'thorn') as the ratio of space vs. (τ) time flips. In the graph, we, for example, have set the time measure m_t =0.5, Þ equal to 10 and the angle measure equal 1. The graph shows a typical cycle produced as follows: Assuming that initially, τ time is progressing forward, at successive time steps t_p (unit of Planck time), the angle increases. When ten (Þ) steps have occurred, time has a 50 percent chance of flipping backwards. Here we assume it has done so. At that point, the angle measure goes to 0 (meaning that the angle continually decreases) until another 10 steps occur. This is an idealized case which we'll generalize further below.

In the above graph, note that the wavelength is 2Þ. (1 cycle = 2Þ) the frequency f is then $1/(2Þ)$ cycles/Planck time.

In the graph below, we've let the angle measure be less than unity. This, of course has resulted in a noisier graph, but the above dominant wavelength is still evident.
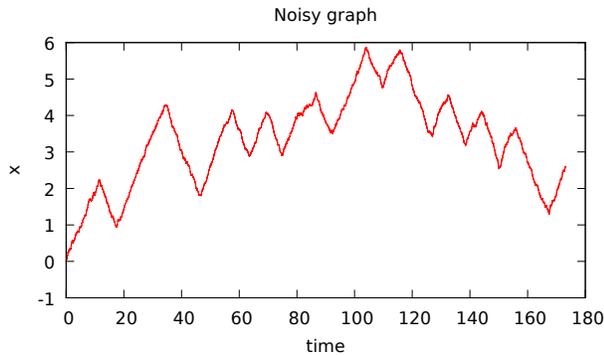
As Þ increases, the frequency of this (torsional) oscillation decreases until, as the mass goes to zero, the oscillations cease, leaving only the migrations of the center of mass. As Þ decreases, the frequency increases and the graph gets noisier. We ask now when will the dominant frequency get lost in the noise. We expect that will happen when Þ is at or slightly above 2 (because wavelength=2Þ).

Below are some graphs showing how the frequency gets 'lost in the noise' as Þ decreases:
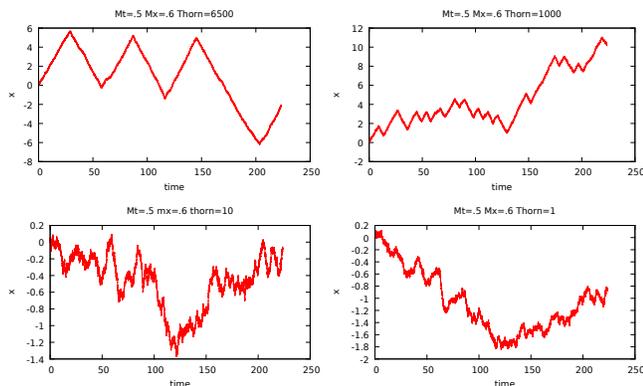
As in the case of u time, we also assume that with τ time the frequency is proportional to the mass.

The u time case had $f_c = m_p$. And that fits very well to the stochastic space-time model.

But the u time case is very different than that of τ time: the u time argument is for rotations in a one degree of freedom plane whereas with τ time, one has oscillations of a sphere, which is two degrees of freedom (latitude and longitude). And as rotations don't commute, it is hard to see how the rotations in the two degrees of freedom can occur at the same time. This might suggest that the u rotations go at twice the rate of the τ oscillations, i.e. there are two cycles for u for every one of τ. In this scenario, the Planck mass is the limiting mass for quantum mechanics i.e. particles heavier than the Planck mass behave classically, and, the u frequency is half the τ frequency which says that the particle must rotate through 720 degrees before returning to its initial state.

This allows us to claim:

the Planck length is the smallest possible length,

the Planck time is the smallest possible time, and

the Planck mass is the smallest possible purely classical (i.e. not subject to quantum mechanics) mass. Further, it supports the argument that waves are intrinsic to this model.

As the mass increases, the 'waves' sub-harmonics grow in intensity until the composite wave is indistinguishable from the foam—resulting in a vanishing of the wave. So in contrast to conventional QM where a massive particle is presumed to have a wave function (that is perhaps beyond the current measurement threshold), if Ψ is intimately related to the Compton wave, the SG5D model predicts that there isn't a wave function for a sufficiently large, spherical (non-interacting) mass. And that limiting mass is (very close to) the Planck mass. So, even in principle, the two slit experiment cannot be done with bee-bees, or marbles, or cannonballs.

The above was a simplification where time changed direction every Þ Planck times. SG5D though, says that every Þ Planck times, a coin flip determines if time changes direction. The argument still holds as the signal still gets lost in the noise, but less smoothly.

The above takes the Planck length and time as the smallest possible in free space, i.e. the quantum of space and time. But what is the smallest possible mass, the quantum of mass? The Planck mass is the upper bound for quantum masses. What is the lower bound? SG5D can't say. But that mass must be smaller than anything in nature. The mass of a neutrino isn't known, but it is in the order of 10^{-36}kg. The mass difference between the types of neutrinos will be smaller still. We can say then, that the quantum of mass is less than 10^{-28} Planck masses.

If, reflective of venues, masses can oscillate and rotate, it is reasonable to expect that volume-preserving pulsations (like squeezing jelly-babies) also occur. But whereas rotations of a mass do not necessarily disturb nearby venues, pulsations propagate through space-time. Each mass in the space-time manifold can generate pulsations, causing a stochastic/chaotic foam in the space-time. This may well be the genesis of the vacuum energy.

As to forces, this model doesn't currently consider forces. It is concerned with 'stuff', which is composed of Fermions. The force carriers, i.e. Bosons, will be addressed in a future paper.

4. Creating and Annihilating Venues

A problem with a space-time of granules (with constant volumes) rather than of points is how to handle an expanding or contracting space-time or region of space-time. We need a mechanism to create and annihilating empty venues (venues not containing mass) without leaving gaps in the space-time manifold. Either the constants

c, G, and/or h depend on the size of the universe and so change the Planck units in such a way as to preserve the number of venues, or the Susskind landscape model[22] is applicable and in addition the overall volume of the multiverse is constant and venues can migrate between universes, or there is a mechanism for the creation and annihilation of venues. We suggest the following:

The 'rolled-up' (here, space-like since we're considering venues thst do no hold mass) 5th dimension provides that mechanism as follows: Consider a space-time contraction in one direction (x).

Initially the 5th dimension is zero as there is no mass in the venue. And as a result, there is no 5th dimension frequency.

The following diagrams show coordinates x and υ, and a rectangular solid representing a venue.

$$1 \leftarrow \quad \upsilon \quad 0 \quad x \rightarrow 1 \qquad 1 \leftarrow \quad \upsilon \quad 0 \quad x \rightarrow 1$$

As the x coordinate of the venue contracts, to preserve the 5-volume, the 5th dimension must expand.

At some point, the contraction coordinate, x, approaches zero while the 5th dimension, u, approaches its circumference.

$$1 \leftarrow \quad \upsilon \quad 0 \quad x \rightarrow 1 \qquad 1 \leftarrow \quad \upsilon \quad 0 \quad x \rightarrow 1$$
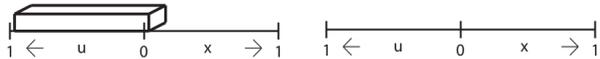
At the point where the contraction reaches zero, the 5th dimension component 'rolls over' to zero. The 5-volume is then zero and the venue blinks out of existence.

Creation of venues is similar: When a venue (outside a mass) expands, it subtracts from the fifth dimension (which due to vacuum fluctuations can be slightly greater than zero).

When it so subtracts, the fifth dimension 'rolls over' to its maximal value. This pushes the venue's 5-volume volume to an even larger value which violates the constant 5-volume. The venue then splits in two across one coordinate. Each half contains half the maximal 4-volume and half the maximal dimension five value, again preserving the constant 5-volume of each new venue. The new venues equilibrate by a reduction of the fifth dimension and increase of the 4-volume thus accommodating the expansion (that required a new venue).

At no point then, is the space-time manifold not fully tessellated.

However, expansion/contraction would occur not in a single coordinate direction, but in all four. But because venues are in a constant state of volume-preserving pulsations, at any instant, the expansion/contraction would be effectively in a single (albeit random) direction, so the above, single direction argument would still hold.

Although the mechanism allows for both venue creation and annihilation, perhaps only creation is relevant; Since venue migration is a diffusion process, if the universe is not infinite, the diffusion will increase the size of the universe. Which is to say, the granular, stochastic space-time model predicts an ever expanding universe.

5. Geometric Properties of Mass

The SG5D model doesn't attempt to say what mass is, but instead examine the geometric properties pertaining to the mass.

The principal function of mass is (in the model) the stabilization of space-time, i.e, one would like the fluctuations in/of space-time not to rip apart masses. In particular, a mass causes adjacent mass-containing venues, because of stabilization, to act as a single larger venue (which is why E(mass)=hf works).

In empty space, in particular, the venues' dimension coordinates fluctuate (and this is required for the creation and annihilation of empty venues). The fluctuating mass can be associated with vacuum energy fluctuations and metric tensor fluctuations. The idea of metric tensor fluctuations was the initial idea behind our stochastic space-time theory.

Since the fifth dimension is associated with mass and energy, venue fluctuations imply mass(energy) fluctuations. That is to say that energy is not strictly conserved. But since the fluctuations occur at Planck-time time scales, on short-time average energy effectively *is* conserved. However, because of the venue creation/annihilation mechanism (**4** above) during space-time expansion or contraction (either of the universe or a region thereof), energy is not conserved. It is conceivable that some of this energy of expansion/contraction can be extracted.

In (the usual interpretation of) General Relativity, mass causes 'curvature'. But what is curvature? Arguably, it is merely an artifact of describing space-time with one too few dimensions. For example, if a (two dimensional) ant were wandering on the surface of a sphere, he could measure curvature and determine that his environment was non-Euclidean. A three-dimensional (ignoring time) being would say the space(-time) *was* Euclidean and the ant was not able to see that third dimension. G. 't. Hooft has made a similar argument, as has J. Beichler. (And, of course, 'Campbell's Embedding Theorem'[23] states that any n-dimensional Riemannian manifold can be embedded locally in an n+1-dimensional Ricci-flat manifold.) But for us, rather than using a full extra dimension to explain curvature, we describe curvature as an artifact of the four-dimensional contraction or expansion of venue volumes. A feature of this granular model is that some phenomena attributed to the large-scale structure of space-time (e.g curvature) can be explained by extremely small scale phenomena (e.g. compression and expansion of venue dimensions).

Graphically, the left image below represents the traditional 'gravity well' curvature representation, and the right image is the left image but looking directly down from above.
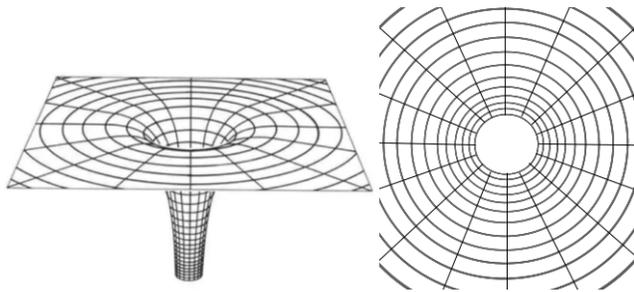
The right image is still a three-dimensional representation. But, in the SG5D interpretation, it is a two-dimensional image, the 'depth' being due to the compression of venues. We have reduced space-time by one dimension (without losing information) and since there is no curvature, the space-time (arguably) is flat. (This is a kind of 'holographic ptinciple'.)

The idea that in a mass dimension 5 is time-like and associated with that mass is an attractive notion. If, as with τ-time, u-time, goes forward, this gives a frequency proportional to the mass in the venue. This would explain e=hf.

The problem is that it works only with an isolated venue (and a venue is about 20 orders of magnitude smaller than the size of a proton). If adjacent venues have mass, each venue would have a frequency proportional to the mass in that venue. What we need is for the frequency to be the sum of the frequencies in each venue. We postulate then, that adjacent venues containing mass act as one larger venue (except that the venues may still be able to migrate). This encapsulates the idea that mass stabilizes space-time. The postulate might be a result of forces (the strong force, in particular) between the mass-containing venues. (Forces are outside the scope of this paper.)

Note: As for the translatory motion of a mass (as opposed to the rotational), the mass doesn't become 'fuzzy', but (because of migration external to the mass) its location does begin to blur as the mass decreases below the Planck mass. This results in an effectively larger mass diameter.

Two effects: like a smaller pollen grain in Brownian motion: the smaller the grain, the more it stochastcally moves. But as the effective grain radius increases, the movement decreases as there is a larger circumference over which the movements can average.

Note then that the effective radius *rate* of increase decreases as the effective radius increases. To reiterate, this is because, as the particle grows in *effective* size the average effect of the venue migrations against the particle surface begin to average out (analogous to the case of Brownian motion where the jitter of a large pollen grain is less than that of a smaller grain).

We maintain that where quantum mechanics uses the radius, it should use the *effective* radius. radius= rest-radius + *Radius Quantum Correction*: $r = r_{c} + r_{qc}$.

One might consider the 'actual' radius as the covariant (and hence unobservable radius) whereas the effective radius is the contravariant (in principle, observable) radius.

A quantum particle moving with venues effectively spreads. So, in some sense, the mass is spread through the space-time. And the field equations act on the spread mass. And (since inside a mass, the Ricci tensor is not zero) the space-time near a *quantum* particle has a nonconstant 4-volume element.

In short then, there is relationship between a particle's mass and its radius; the higher the mass, the shorter the radius.

6. Mass and Gravity

Whilst Newton and Einstein described the action of gravity, a mechanism for gravity was not explained. SG5D, on the other hand, does suggest a mechanism: When venues are near a large mass (from outside the mass), their 3-volumes are compressed. The constancy of the volume is maintained by a corresponding expansion of the time coordinate. The space compression continues when a venue is interior to the mass. Here the fifth dimension is important and is non-zero. As one approaches The Schwarzschild radius, (at least one of) the venue's dimensions approaches zero while at the same time, the fifth dimension (which has expanded to maintain a constant 5-volume) rolls over to zero. The venue thus annihilates. A venue then comes in to take its place. So there is a continuous stream of venues approaching the Schwarzschild radius and annihilating. Venue creation in the space-time at large makes up for the loss of venues. If a venue has a test mass (small compared to the 'large' mass), it will fall in to the surface of the large mass. The speed (as a function of the radial distance from the center from the large mass) can easily be calculated (and the result of the calculation compared with the Newtonian result):

Consider a spherical shell at some distance from a mass. As the venues at the shell migrate in toward the mass, the number of venues at the shell do not change, so as the shell's radius changes the venues must compress in the two dimensions perpendicular to the radial direction. To keep the volume constant, the time component must expand as the square of a coordinate perpendicular to r (because of the *two* space coordinate compressions). Venues are being annihilated at a constant rate. So if one uses a stopwatch to monitor how fast a venue (containing a test particle) is falling toward the surface of the mass, it will appear to go faster as it approaches because of the slowing of the stopwatch. So the distance covered by the falling venue will go as the square of the rate of the slowing of time. This is to say that, $v^2 = \frac{constant}{r}$. There is also a contraction of the venue in the r direction, but that is a relativistic effect that we will ignore for the moment. The velocity equation from Newtonian physics is, $v^2 = \frac{2GM}{r}$. The *constant* is (related to) the rate at which venues are annihilated, so we can associate that rate with 2GM which gives a connection of SG5D to 'physics'. Fur-

ther, since the Newtonian description of an object falling under the influence of gravity is a conversion of potential to kinetic energy, the SG5D derivation of the fall of gravity provides the link to kinetic and potential energy. More importantly though, it might explain the concept of energy in terms of SG5D

Note: Carlo Rovelli states that relativity's the slowing of time is the source of gravity[24]. SG5D says the annihilation of venues is the source of gravity and the rate of fall due to gravity is determined by the slowing of time.

7. A Note on the Geometric Interpretation of Electromagnitism (Kaluza-Klein)

Theodor Kaluza and Oskar Klein's not entirely unsuccessful unification of General Relativity and electromagnetism is well-trod territory. Kaluza-Klein theory is not, per se, part of our Stochastic, Granular Space-time model. But the Kaluza-Klein model and our Stochastic Space-time model both employ a fifth dimension that is rolled-up at the Planck scale. So, for free so to speak, our Stochastic Space-time model can take in electromagnetism.

There is a difference, however. The (usual) Kaluza-Klein fifth dimension is space-like whereas ours is sometimes time-like. Researchers have considered a time-like rolled up fifth dimension, but rejected it due to the prediction of tachyons and (bad-)ghosts (unphysical solutions, e.g. negative probability states).

Other more recent work (e.g Aref'eva & Volovich[20], Quiros[21], Kociński & Wierzbicki[25]) suggests that a time-like dimension can work after all, if one has constraints imposed on the dimension.

We have also had the tachyon problem when we allowed both space and time migrations. The 'time leaves no tracks' interpretation of time was postulated to prevent particles to be at different places at the same time (and to prevent tachyons). The time-like Kaluza-Klein ghosting problem also seems resolved with the 'time leaves no tracks' hypothesis. So we postulate that both τ-time and u-time each leave no tracks.

8. Uniqueness of Tessellation by a Regular Honeycomb

Far from any masses, we would expect the venues tessellating 5-dimensional space-time, on average (temporal and positional), to be identical.

In two dimensions (i.e. the plane) regular tessellation can be accomplished with squares, equilateral triangles, or regular hexagons. In four dimensions, there are three regular tessellations (honeycombs). But in five dimensions, there is only one: the 5-cubic honeycomb[26] (the order-4 penteractic honeycomb).

It would be awkward were there more then a single regular honeycomb possibility. Different types of regular honeycombs can not be mixed (for complete tessellation).

And the stochastic space-time model would be hard put to assure that different regions of space-time would use the same type of honeycomb. Five-dimensional space-time then, can be tessellated only with five-dimensional cubes. This is an argument (albeit a weak one) for the existence of five rather than four dimensions.

IX. ROTATIONS *IN* AND *OF* THE SPACE-TIME MANIFOLD (HIGHLY SPECULATIVE)

In general, with a stochastic space-time, there are two ways in which a particle can move: it can move through the space-time manifold, or it can move as a result of the venues at the particle's position migrating.

Rotations in a discrete space-time has a possible problem: A 'point' near the axis of rotation could rotate to another point within the same venue. And that seems to conflict with the idea that a venue cannot have any internal structure.

As an alternative, we'll consider that for an elementary particle, its rotations are entirely the result of venue migration. This is somewhat akin to frame-dragging in the Kerr metric, except rather than the rotating particle dragging the space-time, the rotating space-time is dragging the particle. And, if the particle were charged, it would not radiate since not the particle but the space-time is rotating.

There is however a particular issue with (rigid) particle rotations in a granular manifold. We are mindful however, that in Special Relativity, there are no rigid objects. We are interested in whether granular space-time theory also does not allow rigid objects.

The diagram below shows a mass with the dark circle indicating its circumference. It is sitting on a background of venues (the squares). It also shows an arc length of one Planck length.
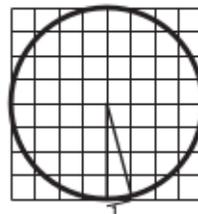
If the particle, rather than the space-time were rotating, then when the particle rotates through the one Planck length arc, a point close to the origin would rotate far less than a Planck length, leaving the point in the same venue. But, by hypothesis, a venue has no internal structure; e.g. there can't be two distinct points in a venue.

So with the above diagram, rigid rotations of the particle cannot happen.

In the case of the space-time itself rotating, it is also difficult to see how the grid of venues could rotate.

We'll address this by assuming the particle is subject to General Relativity.

Earlier in this paper curvature was described as not a property of space-time, but merely an expression of the compression of venues. (E.g. a venue could compress in space dimensions while expanding in the conventional time dimension).

Consider the diagram below.
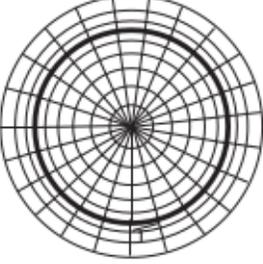
Here the particle (circle) is set against a grid of venues distorted by the particle's mass.

We consider that the space-time exterior to the particle is discribed by the usual Schwartzschild metric and the interior by the Schwartzschild, perfect fluid interior solution:

$$c^2 ds^2 \; = \; \frac{1}{4}\left(\left(1 - \frac{2Gm}{c^2 r_m}\right)^{\frac{1}{3}} - \left(1 - \frac{r^2 2Gm}{c^2 r_m^3}\right)\right)^2 c^2 dt^2 \; - \; \left(1 - \frac{2Gmr^2}{c^2 r_m^3}\right)^{-1} dr^2 - r^2\left(d\Theta^2 + sin^2\Theta d\phi^2\right)$$ where r_m is the radius of the mass.

Notice that the 'curvature' increases as one approaches the surface from the exterior, and decreases as one proceeds from the radius towards the center. And, as long as the Schwartzschild radius is less than the particle radius, there is no Swartzschild singularity. Further, because venues are not points, there is no singularity at the center either. The rotation now is like a 'pizza slice' or wedge, no point on any venue rotates into the same venue.

(Note that if we truly consider rotations, the Kerr metric would be more appropriate. But as we consider the rotations as going in all directions at the same time, Kerr would also not be the appropriate metric.)

There are however (at least) two reasons why the above schema doesn't work: First, while the argument seems reasonable in two dimensions, it does not work in three. For a spherical particle rotating as above, a point on a venue on the surface on the axis of rotation would rotate in the same venue. Second, if the Swartzschild interior metric is roughly appropriate, then at the center of the particle, the 'curvature' would vanish. That is to say that the venues near the center would be minimally distorted. And, especially as in the schema, the number of venues circling the particle at any radius is the same, the venues near the center would be extremely distorted. We conclude then, that rotations are not rigid. Can we explain how even non-rigid rotations can be explained?

We can modify the above schema. First, we let the circling venues at any radius can circle the particle independently of the circling venues at any other radius. This,

incidentally, would allow zero total angular momentum if the various rotating circles of venues were rotating in opposite directions. And second, we can allow rotations to occur simultaneously in any plane containing the center of the particle. This gives a geometric interpretation of particles rotating simultaneously in all directions and is suggestive of spin.

X. QUIDDITY, ENTANGLEMENT, THE TWO-SLIT EXPERIMENT

A. Information & Quiddity : Pilot-waves & Entanglement

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There are two forms of information at play: one of which is restricted to travel at no greater than the speed of light and the other (e.g. collapse of the wave function, entanglement and the like) not so restricted. These are very different processes, and so using the word 'information' for the both of them is confusing. We'll reserve 'information' for the first case, and 'quiddity' for quantum information. (Quiddity means the inherent nature or essence of something. And the first three letters, qui, make it easy to remember QUantum Information.)

Information is carried by photons or mass (energy). Quiddity, as it travels faster than light (and therefore can also travel backward in time), can not be carried by energy. In Stochastic Granular Space-time theory then, what can carry quiddity? The only thing left is empty venues. While a venue has an invariant 5-dimensional volume, it can vary in its individual dimensions. As described earlier, the 4-dimensional volume is related to the probability density, $\Psi^*\Psi$. So that probability density is a type of quiddity.

The wave function acts as a 'pilot wave' (as proposed by Louis de Broglie), moving well in advance of a quantum particle. When the particle 'catches up' to a place where the pilot wave is, that wave then determines the particle's probability density.

Entanglement seems to work the same way: by the superluminal propagation of probability densities. Entanglement then is not an extremely strange peripheral property of quantum theory, but a necessary and central component of the theory. An entangled set of particles then could interract superluminaly, but an observer in the laboratory frame could not observe the result of the interaction until a time later, when a classical (subluminal) signal could have reached the interaction.

Arguably, Entanglement is a process requiring superposition plus faster than light quiddity, and SG5D provides for both.

Our aim in the following is not to provide a theory/mechanism for entanglement, but to argue that Stochastic Granular space-time Theory allows for it, within the confines of five dimensions.

Bell's theorem[27] requires that to have entanglement,

we must abandon 'objective reality' and/or 'locality'. Dropping locality means that things separated in space can influence each other instantaneously. Dropping objective reality means that a physical state isn't defined until it is measured (e.g. is the cat dead or alive?).

Weak measurement experiments[28–30] building on the work of Yakir Aharonov and Lev Vaidman[31] imply that there *is* objective reality in quantum mechanics[31, 32] (in contradiction to the Copenhagen interpretation of quantum mechanics). By objective reality, we mean a particle does have a path (blurred somewhat by space-time fluctuations) regardless of whether it is being observed or not.

We're left then, with non-locality. SG5D is non-local. The issue, of course, is how to have non-locality whilst not violating Einstein's prohibition of information traveling faster than light. We slightly re-interpret that prohibition by positing that it is energy (as opposed to information) that can't travel faster than light.

Empty venues carry no energy, and so (as we have seen) can migrate through space-time arbitrarily rapidly. The hope then is that we can find a way that empty venues can carry quiddity. (The Time Leaves No Tracks idea will help with that.)

A single empty venue seems not to fulfill that hope as a single empty venue's only quiddity is the fact of its existence, and since number of (empty) venues is not conserved, that fact doesn't seem to be able to explain entanglement.

We suggest though, that through some *unknown* mechanism (which is why this is a suggestion and not a theory) that a number of empty venues can be bound together can migrate collectively through space-time (e.g. spiraling through space-time) they would then carry a more complex quiddity. So, for instance, two created entangled particle would carry this quiddity with them as they spread out (as a link between them). And through another unknown mechanism, a measurement of one particle forces the state of the other and then dissolves the link.

Again, this discussion is not an *explanation* of entanglement, but just an attempt to show that SG5D can contain such entangled states.

B. The Delayed-choice Two-slit Experiment
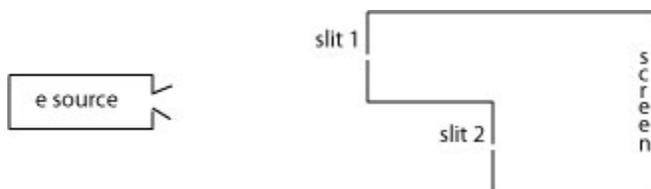
The diagram shows the 'delayed choice two-slit experiment': A low-intensity source directs electrons to a box containing two slits (slit 1 and slit 2). The beam intensity is such that there is only one electron traveling in the box at any time. As expected, an interference pattern is gradually produced on the screen at the back of the box. If a particle detector is introduced at slit 2 to determine which slit an electron passed through, then there will be no interference produced. One can arrange that the detector is optionally turned on only when the electron has passed by slit 1. If the detector is on at that point, then again, there will be no interference pattern produced. So it seems that when the electron gets to slit 2 and finds that the detector is on, it goes back in time to tell the electron to go through, or not go through slit-1.

How does the Granular Stochastic Space-time model explain this?

First, we introduce the concept of an 'ephemeral' measurement: An electron has an associated electro-magnetic field. As it goes through a slit, that field will interact with the electrons in the wall of the box at the slit. The box electrons then can tell if an electron has passed through a slit. And this could be considered a measurement; the box electrons could be considered a particle detector. But the interference pattern still occurs in this case. The difference is that the box electrons measurements are ephemeral; After the moving electron passes through the slit, the box electrons return to their undisturbed state, retaining no 'memory' of the measurement. The measurement is not preserved. The film can be run backward and it would be a valid physical situation. For there to be a true measurement then, there must be a mechanism to 'remember' the measurement – a latch or flip-flop of sorts. And that would mean the film could *not* be run backward. We regard measurement then, as a breaking of time-reversal symmetry. In the macro-world, everything is a measurement of sorts (viewing a scene gives an estimate of positions, etc.) and hence we can't run macro-world scenes backwards.

With quiddity (in this case, the pilot wave) able to move superluminaly as well as to move backward in time, there isn't much to explain. The pilot wave precedes the electron going into the box. The pilot wave determines the probability of the electron being found at any point in the box at any time. If (at any time) the detector is switched on, that would change the geometry and hence the wave (at all points, future and past). The electron would continue its motion, catching up with the revised pilot wave and then moving accordingly. (This is much like the mechanism of entanglement).

XI. DISCUSSION

General relativity is a theory relating the large scale structure of space-time to the masses in it. Similarly, the stochastic space-time model relates the micro-structure of space-time to the behavior of masses at the quantum level. One says for general relativity, mass tells space how to bend. Space tells mass how to move. And in SG5D, we say mass tells space how to jell. Space tells mass how to jiggle. The model is neither one of quantum mechanics nor General Relativity. It requires both theories in its

development.

In the model, particles move (in an indeterminate manner) due to the space-time fluctuations exterior to the particle (similar to the way a Brownian Motion pollen grain moves). But unlike with Brownian motion, time (as well as space) fluctuates.

In free-space, there is no meaning in retracing a trajectory as, because of the space fluctuations, there is no well-defined 'place'. Inside a mass, it is different. In particular, going backwards in time doesn't mean a system goes to a well-defined earlier place in time. If there is a time reversal, it is evidenced only by the reversal of global quantities (such as direction of rotation).

A previous paper[33] suggested that quantum oscillations of particles could be described as torsional vibrations occurring simultaneously in all directions It a model whereby such oscillating particles could pass through a polarizer admitting fifty percent throughput rather than just those particles aligned perfectly with the polarizer.

The object of the present model is to provide a conceptual basis for quantum mechanics–to show that the 'quantum weirdness' can be explained in terms of the behavior of space-time. And indeed, SG5D has managed to replicate some of the fundamental processes in conventional quantum theory.

But the model is incomplete: It doesn't explain the electro-weak force nor the strong force. But it should. A description of the world in terms of venues might be expected to provide those explanations. And there is room in the model to provide those; We've shown many attributed of quantum mechanics and relativity in terms of venue placement. But we've not considered an individual venue's orientation and/or rotation, or soap-bubble-like vibrational modes. We expect these to provide an explanation, at least, for electromagnetism. And finally, we've not provided an overarching mathematical formalism for the SG5D model.

We recognize that this paper contains some unusual ideas. But they were forged in a chain (a long chain) of logic from the initial conjecture that space-time is stochastic. In addition, this paper jumps around between topics. For that, I apologize. The problem was that the concepts are so interconnected that I could not find a single, compelling, logical order in which to present them.

POSTSCRIPT

For the better part of a century, researchers have sought an understanding of the physical nature of quantum mechanics. Perhaps then, a mathematical solution to the problem is impossible and it is an example of the Gödel incompleteness theorem[34]. Does Gödel imply that there is no mathematical solution, or perhaps there is one but it can not be derived? Roger Penrose, Herman, Weyl, among others, felt that the theorem shows that we can never know many, if not most, of the physical laws of the universe.

The SG5D model is (at the moment) more phenomenological than mathematical; it is an assemblage of interconnected (hopefully self-consistent) phenomena-–a scaffold onto which quantum phenomena can be attached. Each phenomenon attached to the scaffold might well be describable by mathematics, but the entire populated scaffold, because of Gödel, might not be. (and including a mathematical treatment of the accessible phenomena described would have made this paper far to big.)

While the mathematical description is thus far from complete, the model is specified sufficiently to allow (super)computer simulations. And perhaps, because of Gödel, computer simulations are the best we can do.

ACKNOWLEDGMENTS

APPENDIX

THE ORIGINAL *STOCHASTIC SPACE-TIME AND QUANTUM THEORY* PAPER (REVISED)

INTRODUCTION

When considering the quantum and relativity theories, it is clear that only one of them, namely relativity, can be considered, in the strict sense, a theory. Quantum mechanics, eminently successful as it is, is an operational description of physical phenomena. It is composed of several principles, equations, and a set of interpretive postulates[Ap.1]. These elements of quantum mechanics are justifiable only in that they work. Attempts[Ap.2, Ap.3] to create a complete, self-contained theory for quantum mechanics are largely unconvincing. There are, in addition, a number of points where quantum mechanics yields troubling results. Problems arise when considering the collapse of the wave function, as in the Einstein-Podolsky-Rosen paradox[Ap.4]. Problems also arise when treating macroscopic systems, as in the Schrödinger cat paradox[Ap.5] and the Wigner paradox[Ap.6]. And quantum mechanics is not overly compatible with general relativity[Ap.6].

One way of imposing some quantum behavior on general relativity is the following: The uncertainty relation for time and energy implies that one can "borrow" any amount of energy from the vacuum if it is borrowed for a sufficiently short period of time. This energy fluctuation of the vacuum is equivalent to mass fluctuations which then gives rise to metric fluctuations via the general-relativity field equations.

An alternative approach is to impose, *ab initio,* an uncertainty on the metric tensor, and to see if by that, the results of quantum mechanics can be deduced. As

this paper will shoe, with a few not particularly unreasonable assumptions, a large segment of the formalism of quantum theory can be derived and, more importantly, understood.

Mathematical spaces with stochastic metrics have been investigated earlier by Schweizer[Ap.7] for Euclidian spaces, and by March[Ap.9, Ap.10] for Minkowski space. In a paper by Blokhintsev[Ap.11], the effects on the physics of a space with a small stochastic component are considered. It is our goal, however, not to show the effects on physical laws of a stochastic space, but to show that the body of quantum mechanics can be deduced from simply imposing stochasticity on the space-time. Our method will be to write down (in Section II) a number of statements (theorems, postulates, etc.). We will then (in Section III) describe the statements and indicate proofs where the statements are theorems rather than postulates. Finally (in Section IV) we will derive some physical results, namely, the spread of the free particle (in empty space), the uncertainty principle, and the phenomenon of interference. The paper concludes (Sec. V) with a general discussion of the approach and a summary of results.

THE STATEMENTS

Statement 1. Mach's principle (Frederick's version).

1.1. In the absence of mass, space-time becomes not flat, but stochastic.

1.2. The stochasticity is manifested in a stochastic metric $g_{\mu\nu}$.

1.3. The mass distribution determines not only the space-time geometry, but also the space-time stochasticity.

1.4. The more mass in the space-time, the less stochastic the space-time becomes.

1.5 At the position of a mass "point", the space-time is not stochastic.

Statement 2, the contravariant observable theorem.

All measurements of dynamical variables correspond to contravariant components of tensors.

Statement 3. The metric probability postulate.

$P(x,t) = f\sqrt{-g}$, where for a one particle system $P(x,t)$ is the particle probability distribution. f is a real-valued function and g is the determinant of the metric. [But see ‡ in the description of the statements, below, for a revised interpretation.]

Statement 4. the metric superposition postulate.

If at the position of a particle the metric due to a specific physical situation is $g_{\mu\nu}(1)$ and the metric due to a different physical situation is $g_{\mu\nu}(2)$ then the metric at the position of the particle due to the presence of both of the physical situations is $g_{\mu\nu}(3)$,

$g_{\mu\nu}(3) = \frac{1}{2}[g_{\mu\nu}(1) + g_{\mu\nu}(2)]$.

This is the case where the probabilities, P_1 and P_2, of the two metrics are the same. In general though, Statement 4 becomes,

$g_{\mu\nu}(3) = P_1 g_{\mu\nu}(1) + P_2 g_{\mu\nu}(2)$.

Statement 5. The metric Ψ postulate.

There exists a local complex diagonal coordinate system in which a component of the metric at the location of the particle is the wave function Ψ.

DESCRIPTION OF THE STATEMENTS

Statement 1, Mach's principle, is the basic postulate of the model. It should be noted that requirement 1.5, that at the position of a mass point the space-time be not stochastic, is to insure that an elementary mass particle (proton, quark, etc.) is bound.

In our interpretation, a charged particle in stochastic motion does not radiate because it is the space–time rather than the particle which is stochastic. (This is in contrast to Nelson's formulation where the particle [when it's position is time-dependent] is simply posited not to radiate.)

Similarly, local to the particle, space-time is not stochastic. And there, a deterministic Lagrangian can be defined. That 'local to the particle space-time' coordinate system is covariant (as it is moving with the particle). From another coordinate frame (e.g. the laboratory frame) measurements on that local frame are subject to the intervening stochasticity (due to the stochasticity of the metric tensor), and because of that stochasticity, the measurements are also stochastic (and the measurements are contravariant [as can be seen by the raising of the covariant coordinates by the stochastic of the metric tensor]).

Statement 2, the contravariant observable theorem, is also basic. It is contended, and the contention will be weakly proved, that measurements of dynamical variables are contravariant components of tensors. By this we mean that whenever a measurement can be reduced to a displacement in a coordinate system, it can be related to contravariant components of the coordinate system. Of course, if the metric $g_{\mu\nu}$ is well known, one can calculate both covariant and contravariant quantities. In our model however, the quantum uncertainties in the mass distribution imply that the metric cannot be accurately known, so that measurements can only be reduced to contravariant quantities. Also, in our picture, the metric is stochastic, so again we can only use contravariant quantities. We will verify the theorem for Minkowski space by considering an idealized measurement. Before we do, consider as an example the case of measuring the distance to a Schwarzschild singularity (a black hole) in the Galaxy. Let the astronomical distance to the object be $\bar{r}(\equiv \bar{\xi}^1)$. The covariant equivalent of the radial coordinate r is ξ_1, and

$\xi_1 = g_{1\nu}\xi^\nu = g_{11}\xi^1 = \frac{r}{1-2Gm/r}$,

so that the contravariant distance to the object is

distance$= \int_0^{\bar{r}} dr = \bar{r}$,

while the covariant distance is

$\bar{\xi}_1 = \int_0^{\bar{r}} d(\frac{r}{1-2Gm/r}) = \infty$.
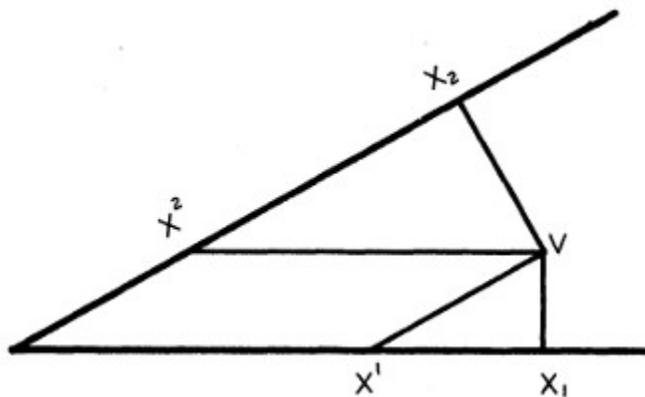
Figure 1. Covariant and contravariant components in oblique coordinates.

It is clear that only the contravariant distance is observable.

Returning to the theorem, note that when one makes an observation of a dynamical variable (e.g. position, momentum, etc.), the measurement is usually in the form of a reading of a meter (or meter-stick). It is only through a series of calculations that one can reduce the datum to, say, a displacement in a coordinate system. For this reduction to actually represent a measurement (in the sense of Margenau[Ap.12]) it must satisfy two requirements. It must be instantaneously repeatable with the same results, and it must be a quantity which can be used in expressions to derive physical results (i.e., it must be a physically "useful" quantity). It will be shown that for Minkowski space, the derived "useful" quantity is contravariant.

Note first (Fig. 1) that for an oblique coordinate system, the contravariant coordinates of a point V are given by the parallelogram law of vector addition, while the covariant components are obtained by orthogonal projection onto the axes[Ap.13].

We shall now consider an idealized measurement in special relativity, i.e., Minkowski space. Consider the space-time diagram of Fig. 2.

We are given that in the coordinate system x',t', an object (m,n) is at rest. If one considers the situation from a coordinate system x,t traveling with velocity v along the x' axis, one has the usual Minkowski diagram[Ap.14] with coordinate axes Ox and Ot and velocity $v = tan(\alpha)$ (where the units are chosen such that the velocity of light is unity). OC is part of the light cone.

Noting that that the unprimed system is a suitable coordinate system in which to work, we now drop from consideration the original x',t' coordinate system.

We wish to determine the "length" of the object in the x,t coordinate system. Let it be arranged that at time $t(0)$ a photon shall be emitted from each end of the object (i.e., from points F and B). The emitted photons will intercept the t axis at times $t(1)$ and $t(2)$.
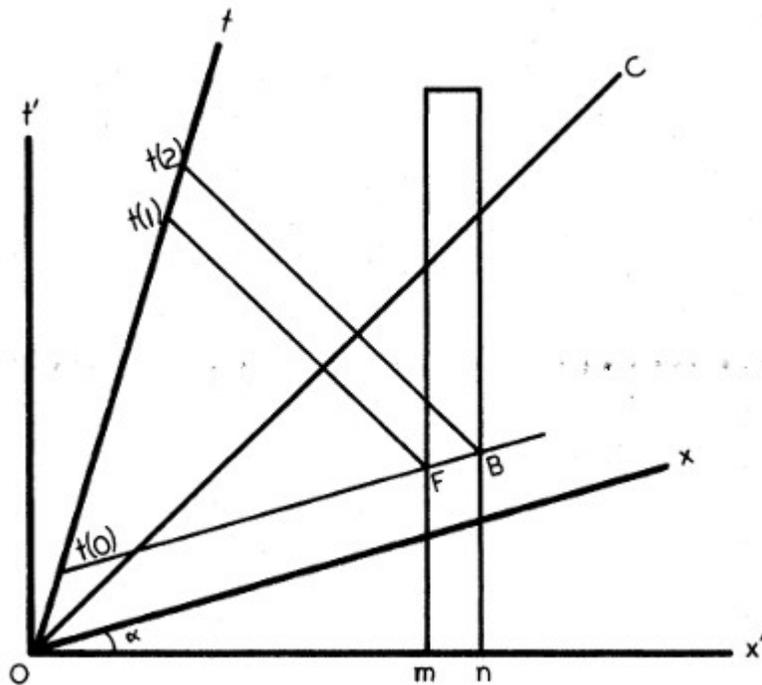
Figure 2. An idealized measurement.

The observer then deduces that the length of the object is $t(2) - t(1)$ (where $c = 1$). The question is: What increment on the x axis is represented by the time interval $t(2) - t(1)$? One should note that the arrangement that the photons be emitted at time $t(0)$ is nontrivial, but that it can be done in principle. For the present, let it simply be assumed that there is a person on the object who knows special relativity and who knows how fast the object is moving with respect to the coordinate system. This person then calculates when to emit the photons so that they will be emitted simultaneously with respect to the x,t coordinate system.

Consider now Fig. 3, which is an analysis of the measurement. Figure 3 is just figure 2 with a few additions: the contravariant coordinates of F and B, x^1 and x^2 respectively. We assert, and it is easily shown, that $t(2) - t(1) = x^2 - x^1$. This is seen by noticing that $x^2 - x^1 =$ line segment B, F, and that triangle $t(2), t(0), Z$ is congruent to triangle $B, t(0), Z$. However, if we consider the covariant coordinates, we notice that $x_2 - x_1 = x^2 - x^1$. This is not surprising since coordinate differences (such as $x_2 - x_1$) are by definition (in flat space) contravariant quantities. To verify our hypothesis we must consider not coordinate differences which automatically satisfy the hypothesis, but the coordinates themselves. Consider a measurement not of the length of the object, but of the position (of the trailing edge m) of the object. Assume again that at time $t(0)$ a photon is emitted at F and is received at $t(1)$. The observer would then determine the position of m at $t(0)$ by simply measuring off the distance $t(1) - t(0)$ on the x axis. Notice that this
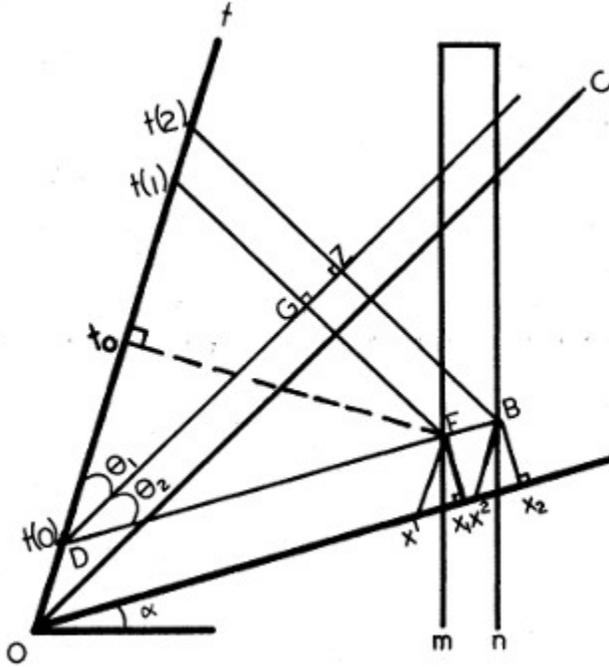
Figure 3. Analysis of the idealized measurement.

would coincide with the contravariant quantity x^1. To determine the corresponding covariant quantity x_1, one would need to know the angle α (which is determined by the metric).

The metric $g_{\mu\nu}$ is defined as $\bar{e}_\mu \cdot \bar{e}_\nu$, where $\bar{e}_\mu$ and $\bar{e}_\nu$ are the unit vectors in the directions of the coordinate axes x^μ and x^ν. Therefore, in order to consider an uncertain metric, we can simply consider that the angle α is uncertain. In this case measurement x^1 is is still well defined $[x^1 = t(1) - t(0)]$, but now there is no way to determine x_1 because it is a function of the angle α. In this case then, only the contravariant components of position are measurable. [It is also easy to see from the geometry that if one were to use the covariant representation of $t(0)$, t_0, one could not obtain a metric-free position measure of m.]

Statement 3, the metric probability postulate, can be justified by the following: Consider that there is given a sandy beach with one black grain among the white grains on the beach. If a number of observers on the beach had buckets of various sizes, and each of the observers filled one bucket with sand, one could ask the following: What is the probability that a particular bucket contained the black grain? The probability would be proportional to the volume of the bucket.

Consider now the invariant volume element dV_I in Riemann geometry. One has that[Ap.15]

$dV_I = \sqrt{-|g|}dx^1dx^2dx^3dx^4$.

It is reasonable then, to take $\sqrt{-|g|}$ as proportional to the probability density ($\Psi^*\Psi$) for free space.

See ‡ below for major revisions to Statement 3 (not in the original paper).

Note that the metric $g_{\mu\nu}$ is stochastic while the determinant of the metric is not. This implies that the metric components are not independent.

Consider again, the sandy beach. Let the black grain of sand be dropped onto the beach by an aircraft as it flies over the center of the beach. Now the location of the grain is not random. The probability of finding the grain increases as one proceeds toward the center, so that in addition to the volume of the bucket there is also a term in the probability function which depends on the distance to the beach center. In general then, we expect the probability function $P(x,t)$ to be $P(x,t) = A\sqrt{-|g|}$ where A is a function whose value is proportional to the distance from the center of the beach.

‡ Major changes from the original paper regarding Statement 3

Again, the metric probability postulate, can be justified by the buckets on a beach argument. And again, the probability that a particular bucket contained the black grain would be proportional to the volume of the bucket.

Consider the invariant volume element dV_I in Riemann geometry. One has that

$dV_I = \sqrt{-|g|}dx^1dx^2dx^3dx^4$.

(From here on, we'll represent the determinant of $g_{\mu\nu}$ by g rather then by $|g|$.)

At first sight then, it might seem reasonable to take $\sqrt{-|g|}$ as proportional to the probability density for free space.

The arguments above apply to the three-dimensional volume element. But we left out the other determinant of the probability density, the speed of the particle (the faster the particle moves in a venue, the less likely it is to be there.) And therefore, the larger the Δt the more likely the particle is to be found in the venue. So indeed (it seems as if) it is the four-dimensional volume element that should be used.

The metric probability statement above, as it stands, has problems:

First, if one considers the 'particle in a box' solution, one has places in the box where the particle has zero probability of being. And if $P(x,t) = k\sqrt{-g} = 0$, that means the determinant of the metric tensor is zero and there is a space-time singularity at that point. We address this problem by noting that the metric tensor is composed of the average, non-stochastic, background (Machian) metric $g_{\mu\nu}^M$ and the metric due to the Particle itself $g_{\mu\nu}^P$. We say then that the probability density is actually $P(x,t) = k(\sqrt{-g^T} - \sqrt{-g^M})$ where g^T is the determinant of the composite metric. In this case, $P(x,t)$ can be zero without either $g_{\mu\nu}^T$ or $g_{\mu\nu}^P$ being singular.

A second problem is that $P(x,t) = k\sqrt{-g}$ describes the probability density for a test particle placed in a space-time with a given (average) metric due to a mass, with determinant g. What we want, however, is the probability of the particle (not the test-particle) due to the metric contribution of the particle itself. Related to this is that

$P(x,t) = k\sqrt{-g}$ doesn't seem to replicate the probability distributions in quantum mechanics in that the probability distribution, $\Psi^*\Psi$, is the square of a quantity (assuring that the distribution is always positive). But the differential volume element, $dV = \sqrt{-g}\,dxdydzdt$ is not the square of any obvious quantity. Further, $P(x,t) = k\sqrt{-g}$ is something of a dead end, as it gives $\Psi^*\Psi$ but no hint of what Ψ itself might represent. It would be nice if the probability density were proportional to the square of the volume element rather than to the volume element itself. With that in mind we'll again look at the probability density. (Multiple researchers[Ap.16, Ap.17] have agreed with Part A's $P(x,t) = k\sqrt{-g}$ and it is therefore with some trepidation that we consider that the probability density might be subject to revision.)

The initial idea was that, given a single particle, if space-time were filled with 3-dimensional boxes (venues), then the probability of finding a particle in a box would be proportional to the relative volume of the box. That was extended to consider the case where the particle was in motion. The probability density would then also depend on the relative speed of the particle. We will however, now argue that $P(x,t) \neq k\sqrt{-g}$, but instead $P(x,t) = -kg$ (essentially the square of the previous). But this will apply only when the quantum particle is measured (a contravariant measurement) in the laboratory frame. If however, one considers the situation co-temporally (i.e. co-variantly) with the quantum particle, then $P(x,t)$ does equal $k\sqrt{-g}$, which is to say that the probability density is [co- or contra-variant] frame dependent.

There is another argument, but it assumes the main body of this paper, relating to a time-like fifth dimension we call tau.

Consider a quantum particle at a τ-time slice at, say, τ=now. And also consider a static quantum probability function (e.g. a particle in a well) at τ=now+1. (That function is a result of the quantum particle's migrations in time and space,) Then if we take a negligible mass test particle at τ=now, it will have a probability of being found at a particular location at τ=now+1 equal to that static probability function. And that function is proportional to the volume element (the square root of minus the determinant of the metric tensor). But what we're interested in is the probability function of the quantum particle as τ goes from now to now+1. We are considering the probability function at τ+1 as static. But it is the result of the migrations of the particle. At tau=now, it would then be the same probability function. So, as we go from now to now plus one, we would need to multiply the two (equal) probability functions. This results in the function being proportional to the determinant of the metric tensor (not its square root). This is rather nice as it allows us to suggest that the volume element is proportional to Ψ while the probability density is proportional to $\Psi^*\Psi$. Note that this result is due to a mass interacting with the gravitational field it itself has generated. (This is analogous to the quantum field theory case of a charge interacting with the electromagnetic field it itself has created.)

As yet another approach, consider the spread of probability due to the migration of venues. In the absence of a potential, the spread (due to Brownian-like motion) will be a binomial distribution in space (think of it at the moment, in a single dimension and time). But there is also the same binomial distribution in time. This, for example, expresses that the distant wings of the space distribution require a lot of time to get to them. The distribution then seems to require that we multiply the space distribution by the time distribution. The two distributions are the same so the result is the square of the binomial distribution. (The argument can be extended to the three spacial dimensions.) In the laboratory frame, time advances smoothly, which is to say that the time probability density distribution is a constant, so we do not get the square of the binomial distribution..

It seems then that there are both the distribution and its square in play. It might be that the covariant representation, i.e. the distribution 'at' the particle, is the binomial while a distant observer where time advances smoothly (not in the quantum system being observed) observes (i.e contravariant measurements) the square of the binomial distribution.

So now we have $P(x,t) = -kg$, which is to say that the probability density is proportional to the square of the volume element. This is rather nice as it allows us to suggest that the volume element is proportional to Ψ while the probability density is proportional to $\Psi^*\Psi$. (We will in a later paper suggest that the imaginary component of Ψ represents an oscillation of space-time.)

Statement 4, the metric superposition postulate, is adopted on the grounds of simplicity. Consider the metric (for a given set of coordinates) $g_{\mu\nu}^{s1}(x)$ due to a given physical situation $s1$ as a function of position x. Also let there be the metric $g_{\mu\nu}^{s2}(x)$ due to a different physical situation $s2$ (and let the probabilities of the two metrics be the same). What is the metric due to the simultaneous presence of situations $s1$ and $s2$? We are, of course, looking for a representation to correspond to quantum mechanical linear superposition. The most simple assumption is that

$g_{\mu\nu}^{s3}(x) = \frac{1}{2}[g_{\mu\nu}^{s1}(x) + g_{\mu\nu}^{s2}(x)]$.

However, this assumption is in contradiction with general relativity, a theory which is nonlinear in $g_{\mu\nu}$. The linearized theory is still applicable. Therefore, the metric superposition postulate is to be considered as an approximation to an as yet full theory, valid over small distances in empty or almost empty space. We expect, therefore, that the quantum-mechanical principle will break down at some range. (This may eventually be the solution to linear-superposition-type paradoxes in quantum mechanics.

Statement 5, the metric Ψ postulate, is not basic to the theory. It exists simply as an expression of the following: There are at present two separate concepts, the metric $g_{\mu\nu}$ and the wave function Ψ. It is the aim of this geometrical approach to be able to express one of these quantities in terms of the other. The statement that in

some arbitrary coordinate transformation, the wave function is a component of the metric, is just a statement of this aim.

PHYSICAL RESULTS

We derive first the motion of a test particle in an otherwise empty space-time. The requirement that the space is empty implies that the points in this space are indistinguishable. Also, we expect that, on the average, the space (since it is mass-free) is (in the average) Minkowski space.

Consider the metric tensor at point Θ_1. Let the metric tensor at Θ_1 be $\tilde{g}_{\mu\nu}$ (a tilde over a symbol indicates that it is stochastic). Since $\tilde{g}_{\mu\nu}$ is stochastic, the metric components, do not have well-defined values. We cannot then know $\tilde{g}_{\mu\nu}$ but we can ask for $P(g_{\mu\nu})$ which is the probability of a particular metric $g_{\mu\nu}$. Note then that for the case of empty space, we have $P_{\Theta1}(g_{\mu\nu}) = P_{\Theta2}(g_{\mu\nu})$ where $P_{\Theta1}(g_{\mu\nu})$ is to be interpreted as the probability of metric $g_{\mu\nu}$ at point $\Theta1$.

If one inserts a test particle into the space-time, with a definite position and (ignoring quantum mechanics for the moment) momentum, the particle motion is given by the Euler-Lagrange equations,

$$\ddot{x}^i + \left\{{i \atop jk}\right\} \dot{x}^j \dot{x}^k = 0,$$

where $\left\{{i \atop jk}\right\}$ are the Christoffel symbols of the second kind, and where $\dot{x}^j \equiv dx^j/ds$ where s can be either proper time or any single geodesic parameter. Since $\tilde{g}_{\mu\nu}$ is stochastic, these equations generate not a path, but an infinite collection of paths, each with a distinct probability of occurrence from $P(g_{\mu\nu})$. (That is to say that $\left\{{i \atop jk}\right\}$ is stochastic; $\left\{{\tilde{i} \atop jk}\right\}$.)

In the absence of mass, the test particle motion is easily soluble. Let the particle initially be at (space) point Θ_0. After time dt, the Euler Lagrange equations yield some distribution of position $D_1(x)$. [$D_1(x)$ represents the probability of the particle being in the region bounded by x and $x + dx$.] After another interval dt, the resulting distribution is $D_{1+2}(x)$. From probability theory[?], this is the convolution,

$$D_{1+2}(x) = \int_{-\infty}^{\infty} D_1(y) D_1(x-y) dy.$$

but in this case, $D_1(x) = D_2(x)$. This is so because the Euler-Lagrange equation will give the same distribution $D_1(x)$ regardless of at which point one propagates the solution. This is because
$g_{\mu\nu}(x) \equiv \{g_{\mu\nu}(x_1), g_{\mu\nu}(x_2), g_{\mu\nu}(x_3)....\}$
are identically distributed random variables.
Thus,

$D_i(x) \equiv \{D_1(x), D_2(x),...\}$
are also identically distributed random variables. The motion of the test particle(the free particle wave functions) is the repeated convolution $D_{1+2+....}(x)$, which by the central limit theorem is a normal distribution. Thus the position spread of the test particle at any time $t > 0$ is a Gaussian. The spreading velocity is found as follows: After N convolutions (N large), one obtains a normal distribution with variance σ^2 which, again by the central limit theorem, is N times the variance of $D_1(x)$. Call the variance of $D_1(x)$, a.
$Var(D_1) = a$.
The distribution D_1 is obtained after time dt. After N convolutions then,

$$\Delta x = Var\left(D_{\sum_0^n i}\right) = Na.$$

This is obtained after N time intervals dt. One then has,

$$\frac{\Delta x}{\Delta t} = \frac{Na}{N},$$

which is to say that the initially localized test particle spreads with a constant velocity a. In order that the result be frame independent, $a = c$, and one has the results of quantum mechanics. At the beginning of this derivation it was given that the particle had an initial well-defined position and also momentum. If for the benefit of quantum mechanics we had specified a particle with a definite position, but with a momentum distribution, one would have obtained the same result but with the difference of having a different distribution D_1 due to the uncertainty of the direction of propagation of the particle.

In the preceding, we have made use of various equations. It is then appropriate to say a few words about what equations mean in a stochastic space-time.

Since in our model the actual points of the space-time are of a stochastic nature, these points cannot be used as a basis for a coordinate system, nor, for that reason, can derivatives be formed. However, the space-time of common experience (i.e., the laboratory frame) is non-stochastic in the large. It is only in the micro world that the stochasticity is manifested. One can then take this large-scale non-stochastic space-time and mathematically continue it into the micro region. This mathematical construct provides a non-stochastic space to which the stochastic physical space can be referred.

The (physical) stochastic coordinates $\tilde{x}^\mu$ then are stochastic only in that the equations transforming from the laboratory coordinates x^μ to the physical coordinates $\tilde{x}^\mu$ are stochastic.

For the derivation of the motion of a free particle we used Statement 1, Mach's principle. We will now use also Statement 2, the contravariant observable theorem, and derive the uncertainty principle for position and momentum. Similar arguments can be used to derive the

uncertainty relations for other pairs of conjugate variables. It will also be shown that there is an isomorphism between a variable and its conjugate, and covariant and contravariant tensors.

We assume that we're able to define a Lagrangian, L. One defines a pair of conjugate variables as usual,

$$p_j = \frac{\partial L}{series \partial \tilde{q}^j}.$$

Note that this defines p_j a covariant quantity. So that a pair of conjugate variables so defined contains a covariant and a contravariant member (e.g. p_j and q^j). But since p_j is covariant, it is not observable in the laboratory frame. The observable quantity is just,

$$\tilde{p}^j = \tilde{g}^{j\nu} p_\nu,$$

but $\tilde{g}^{j\nu}$ is stochastic so that $\tilde{p}^j$ is a distribution. Thus if one member of an observable conjugate variable pair is well defined, the other member is stochastic. By observable conjugate variables we mean not, say, p_j, q^j derived from the Lagrangian, but the observable quantities $\tilde{p}^j, q^j$ where $\tilde{p}^j = \tilde{g}^{j\nu} p_\nu$; i.e. both members of the pair must be contravariant.

However, we can say more. Indeed, we can derive an uncertainty relation. Consider

$$\triangle q^1 \triangle p^1 = \triangle q^1 \triangle \left(p_\nu \tilde{g}^{\nu 1} \right).$$

What is the minimum value of this product, given an initial uncertainty $\triangle q^1$? Since p_j is an independent variable, we may take $\triangle p_j = 0$ so that

$$\triangle p^1 = \triangle \left(p_\nu \tilde{g}^{\nu 1} \right) = p_\nu \triangle \tilde{g}^{\nu 1}.$$

In order to determine $\triangle \tilde{g}^{\nu 1}$ we will argue that the variance of the distribution of the average of the metric over a region of space-time is inversely proportional to the volume,

$$Var \left(\frac{1}{V} \int_v \tilde{g}_{\mu\nu} dv \right) = \frac{k}{V}$$

In other words, we wish to show that if we are given a volume and if we consider the average values of the metric components over this volume, then these average values, which of course are stochastic, are less stochastic than the metric component values at any given point in the volume. Further, we wish to show that the stochasticity, which we can represent by the variances of the distributions of the metric components, is inversely proportional to the volume. This allows that over macroscopic volumes, the metric tensor behaves classically (i.e. according to general relativity).

For simplicity, let the distribution of each metric component at any point Θ be normal.

$$f_{\tilde{g}_{\mu\nu}}(g_{\mu\nu}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{g_{\mu\nu}}{\sigma} \right)^2}.$$

Note also that if $f(y)$ is normal, the scale transformation $y \longrightarrow y/m$ results in $f(y/m)$ which is normal with

$$\sigma^2_{(y/m)} = \frac{\sigma^2_y}{m^2}$$

Also, for convenience, let
$f_{g_{\mu\nu}}$ at $e_1(g_{\mu\nu}) \equiv f_{\Theta_1}(g_{\mu\nu})$.
We now require

$$Var(f_{((\Theta_1 + \Theta_2 + ... + \Theta_m)/m)} \equiv \sigma^2_{((\Theta_1 + \Theta_2 + ... + \Theta_m)/m)},$$

where $f_{(\Theta)}$ is normally distributed. Now again, the convolute $f_{(\Theta_1 + \Theta_2)}(g_{\mu\nu})$ is the distribution of the sum of $g_{\mu\nu}$ at Θ_1 and $g_{\mu\nu}$ at Θ_2,

$$f_{(\Theta_1 + \Theta_2)} = \int_{-\infty}^{\infty} f_{\Theta_1}(g_{\mu\nu}^1) f_{\Theta_2}(g_{\mu\nu}^1 - g_{\mu\nu}^2) dg_{\mu\nu}^2,$$

where $g_{\mu\nu}^1$ is defined to be $g_{\mu\nu}$ at Θ_1. Here, of course, $f_{\Theta_1} = f_{\Theta_2}$ as the space is empty so that,

$$f_{(\Theta_1 + \Theta_2)/2} = f_{(g_{\mu\nu}/2 \, at \, \Theta_1 + g_{\mu\nu}/2 \, at \, \Theta_2)}$$

is the distribution of the average of $g_{\mu\nu}$ at Θ_1 and $g_{\mu\nu}$ at Θ_2. $\sigma^2_{(\Theta_1 + \Theta_2 + ... \Theta_m)}$ is easily shown from the theory of normal distributions to be,

$$\sigma^2_{(\Theta_1 + \Theta_2 + ... + \Theta_m)} = m\sigma^2_\Theta.$$

Also, $f_{(\Theta_1 + \Theta_2 .... + \Theta_m)}$ is normal. Hence,

$$\sigma^2_{((\Theta_1 + \Theta_2 + ... + \Theta_m)/m)} = \frac{m\sigma^2_\Theta}{m^2} = \frac{\sigma^2_\Theta}{m},$$

or the variance is inversely proportional to the number of elements in the average, which in our case is proportional to the volume. For the case where the distribution $f_{(g_{\mu\nu})}$ is not normal, but also not 'pathological', the central limit theorem gives the same result as those obtained for the case where $f_{(g_{\mu\nu})}$ is normal. Further, if the function $f_{(g_{\mu\nu})}$ is not normal, the distribution $f_{((\Theta_1 + \Theta_2 + ... + \Theta_m)/m)}$ in the limit of large m is normal,

$$f_{((\Theta_1 + \Theta_2 + ... + \Theta_m)/m)} \longrightarrow f_{((\int_V g_{\mu\nu} d\tilde{V})/V)}.$$

In other words, over any finite (i.e. non-infinitesimal) region of space, the distribution of the average of the metric over the region is normal. Therefore, (anticipating Part B) in so far as we do not consider particles to be 'point' sources, we may take the metric fluctuations at the location of a particle as normally distributed for for each of the metric components $\tilde{g}_{\mu\nu}$. Note that this does

not imply that the distributions for any of the metric tensor components are the same for there is no restriction on the value of the variances σ^2 (e.g., in general, $f_{(\tilde{g}_{11})} \neq f_{(\tilde{g}_{22})}$. Note also that the condition of normally distributed metric components does not restrict the possible particle probability distributions, save that they be single-valued and non-negative. This is equivalent to the easily proved statement that the functions

$$f_{(x,\alpha,\sigma)} = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{1}{2}\left(\frac{x-\alpha}{\sigma}\right)^2\right)}$$

are complete for non-negative functions.

Having established that,

$$Var\left(\frac{\Theta_1 + \Theta_2 + ... + \Theta_m}{m}\right) = \frac{\sigma_\Theta^2}{m},$$

consider again the uncertainty product,

$\triangle q^1 \triangle p^1 = p_\nu \triangle q^1 \triangle \tilde{g}^{\nu 1}.$

$\triangle q^1$ goes as the volume [volume here is V^1 the one-dimensional volume]. $\triangle \tilde{g}^{\nu 1}$ goes inversely as the volume, so that $p_\nu \triangle q^1 \triangle \tilde{g}^{\nu 1}$ is independent of the volume; i.e. as one takes q^1 to be more localized, p^1 becomes less localized by the same amount, so that for a given covariant momentum p_j (which we will call the proper momentum),$p_\nu \triangle q^1 \triangle g^{\nu 1} = $ a constant k. If also p_ν is also uncertain,

$p_\nu \triangle q^1 \triangle g^{\nu 1} \geq k$.

The fact that we have earlier shown that a free particle spreads indicates the presence of a minimum proper momentum. If the covariant momentum were zero, then the observable contravariant momentum $p^\nu = g^{\nu\mu}p_\mu$ would also be zero and the particle would not spread. Hence,

$p_{\nu min} \triangle q^1 \triangle g^{1\nu} = k_{min}.$

or in general,

$\triangle q^1 \triangle (p_{\nu min} g^{\nu 1}) = \triangle q^1 \triangle p^1 > k_{min},$

which is the uncertainty principle.

With the usual methods of quantum mechanics, one treats as fundamental, not the probability density $P(x,t)$, but the wave function Ψ [$\Psi^*\Psi = P(x,t)$ for the Schrödinger equation]. The utility of using Ψ is that Ψ contains phase information. Hence, by using Ψ the phenomenon of interference is possible. It might be thought that our stochastic space-time approach, as it works directly with $P(x,t)$, might have considerable difficulty in producing interference. In the following, it will be shown that Statements 3 and 4 can produce interference in a particularly simple way.

Consider again the free particle in empty space. By considering the metric only at the location of the particle, we can suppress the stochasticity by means of Statement 1.5. Let the metric at the location of the particle be $g_{\mu\nu}$. We assume, at present, no localization, so that the probability distribution $P(x,t)$=constant. $P(x,t) = -Ag$ by Statement 3. Here A is just a normalization constant so that $-g = constant$. We can take the constant to be unity.

Once again, the condition of empty space implies that the average value of the metric over a region of space-time approaches the Minkowski metric as the volume of the region increases.

Now consider, for example, a two-slit experiment in this space-time. Let the situation $s1$ where only one slit is open result in a metric $g_{\mu\nu}^{s1}$. Let the situation $s2$ where only slit two is open result in a metric $g_{\mu\nu}^{s2}$. The case where both slits are open is then by statement 4,

$g_{\mu\nu}^{s3} = \frac{1}{2}\left(g_{\mu\nu}^{s1} + g_{\mu\nu}^{s2}\right).$

Let us also assume that the screen in the experiment is placed far from the slits so that the individual probabilities $-|g^{s1}|$ and $-|g^{s2}|$ can be taken as constant over the screen.

Finally, let us assume that the presence of the two-slit experiment in the space-time does not appreciably alter the situation that the metrics $g_{\mu\nu}^{s1}$ and $g_{\mu\nu}^{s2}$ are in the average $\eta_{\mu\nu}$ (that is to say that the insertion of the two-slit experiment does not appreciably change the geometry of space-time).

Now we will introduce an unphysical situation, a 'toy' model, the utility of which will be seen shortly

It is of interest to ask what one can say about the metric $g_{\mu\nu}^{s1}$. Around any small region of space-time, one can always diagonalize the metric, so we'll consider a diagonal metric. If the particle is propagated in, say, the x^3 direction and, of course, the x^4 (time) direction. We might expect the metric to be equal to the Minkowski metric, $\eta_{\mu\nu}$ save for g_{33} and g_{44}. (Here, we'll suppress the metric stochasticity for the moment, by, for example, averaging the metric components over a small region of space-time.) We will then, for the moment, take the following:

$$\tilde{g}_{\mu\nu}^{s1} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & -t \end{Vmatrix}, \text{ and so } |\tilde{g}_{\mu\nu}^{s1}| = -st,$$

where s and t are as yet undefined functions of position. In order that $|\tilde{g}_{\mu\nu}^{s1}|$ be constant, let $s = t^{-1}$ so that $|g_{\mu\nu}^{s1}| = |\eta_{\mu\nu}| = -1$.

Now we will introduce an unphysical situation, the utility of which will be seen shortly.

Let $s = e^{i\alpha}$ where α is some unspecified function of position. Consider the following metrics:

$$g_{\mu\nu}^{s1} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \end{Vmatrix},$$

$$g_{\mu\nu}^{s2} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\beta} & 0 \\ 0 & 0 & 0 & -e^{-i\beta} \end{Vmatrix},$$

where β is again some unspecified functions of position;

$(-|\tilde{g}_{\mu\nu}^{s1}|)^{1/2} = (-|\tilde{g}_{\mu\nu}^{s2}|)^{1/2} = 1$

(Note $|\frac{1}{2}A_{\mu\nu}| = \frac{1}{16}|A_{\mu\nu}|$),

$(-|\tilde{g}_{\mu\nu}^{s3}|)^{1/2} = (-\frac{1}{16}|\tilde{g}_{\mu\nu}^{s1} + \tilde{g}_{\mu\nu}^{s2}|)^{1/2}$

$= [\frac{-1}{16}(2 + e^{i(\alpha-\beta)} + e^{-i(\alpha-\beta)})]^{1/2}$

$(-|\tilde{g}_{\mu\nu}^{s3}|)^{1/2} = \frac{1}{2}abs[cos\frac{1}{2}(\alpha-\beta)]$.

This is, of course, the phenomenon of interference. The metrics $g_{\mu\nu}^{s1}$, $g_{\mu\nu}^{s2}$, and $g_{\mu\nu}^{s3}$ describe, for example, the two-slit experiment described previously. The analogy of the function $e^{i\alpha}$ and $e^{-i\alpha}$ with Ψ and Ψ^* (the free particle wave functions) is obvious. The use of complex functions in the metric, however, is unphysical. The resultant line element $ds^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu$ would be complex and hence unphysical. The following question arises: Can we reproduce the previous scheme, but with real functions? The answer is yes, but first we must briefly discuss quadratic-form matrix transformations[Ap.18].

Let,

$$X = \begin{Vmatrix} dx^1 \\ dx^2 \\ dx^3 \\ dx^4 \end{Vmatrix}, \text{ and again let } G = |g_{\mu\nu}|.$$

Then $X^t GX = ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where X^t is the transpose of X. Consider transformations which leave the line element ds^2 invariant. Given a transformation matrix W,

$$X' = WX$$

and

$$X^t GX = X'^t G'X' = (X^t(W^t)^{-1})G'(W^{-1}X).$$

[Note: $(WX')^t = X'^t W'^t$.] However, $X^t GX \equiv (X^t(W^t)^{-1})(W^t GW)(W^{-1}X)$ so that $G' = W^t GW$.

In other words, the transformation W takes G into $W^t GW$. Now in the transformed coordinates, a metric $\tilde{g}_{\mu\nu}^{s1} \equiv G^{s1}$ goes to $W^t G^{s1}W$. Therefore,

$$\Psi_1^*\Psi_1 = -|W^t G^{s1}W|$$
$$= -|W^t|\,|g_{\mu\nu}^{s1}|\,|W|,$$
$$\Psi_3^*\Psi_3 = -\frac{1}{16}|W^t G^{s1}W + W^t G^{s21}W|.$$
$$= -\frac{1}{16}|W^t(G^{s1}+G^{s2})\,W|.$$
$$= -\frac{1}{16}|W^t|\,|G^{s1}+G^{s2}|\,|W|.$$

If we can find a transformation matrix W with the properties,

(i) $|W| = 1$,

(ii) W is not a function of α or β,

(iii) $W^t GW$ is a matrix with only real components,

then we will again have the interference phenomenon with $g_{\mu\nu}'$ real, and again $\Psi_1^*\Psi_1 = \Psi_2^*\Psi_2 = 1$, and

$\Psi_3^*\Psi_3 = \frac{1}{2}abs\left(cos\frac{\alpha-\beta}{2}\right)$. The appropriate matrix W is,

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{Vmatrix} = W.$$

If, as previously,

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & -e^{-i\alpha} \end{Vmatrix} \equiv \|\tilde{g}_{\mu\nu}^{s1}\|,$$

then,

$$\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -cos\alpha & sin\alpha \\ 0 & 0 & sin\alpha & cos\alpha \end{Vmatrix} = \|\tilde{g}_{\mu\nu}'^{s1}\| = W^t \tilde{g}_{\mu\nu}^{s1}W,$$

so that in order to reproduce the phenomenon of interference, the stochastic metric $\tilde{g}_{\mu\nu}$ will have off-diagonal terms. Incidentally, coordinates appropriate to $\tilde{g}_{\mu\nu}^{s1}$ are,

$x^{1'} = x^1$,

$x^{2'} = x^2$,

$x^{3'} = \frac{-i}{\sqrt{2}}x^3 + \frac{1}{\sqrt{2}}x^4$,

$x^{4'} = \frac{1}{\sqrt{2}}x^3 - \frac{i}{\sqrt{2}}x^4$,

which is to say that with an appropriate coordinate transformation (which is complex), we can treat the free space probability distribution $\Psi^*\Psi$ in a particularly simple way. Since the components x^μ do not appear in predictions (such as $\Psi^*\Psi$), we may simply, as an operational convenience, take $\tilde{g}_{\mu\nu}$ to be diagonal, but with complex components.

DISCUSSION

Having recognized that quantum mechanics is merely an operational calculus, and also having observed that general relativity is a true theory of nature with both an operational calculus and a *Weltanschauung*, we have attempted to generate quantum mechanics from the structure of space-time. As a starting point we have used a version of Mach's principle where in the absence of mass, space-time is not flat, but undefined (or more exactly, not well defined) such that $P_\Theta(g_{\mu\nu}) = -k|g_{\mu\nu}|$ (where k is a constant) is, at a given point Θ, the probability distribution for $g_{\mu\nu}$ (in the Copenhagen sense[Ap.19]) .

From this, the motion of a free (test) particle was derived. This is a global approach to quantum theory. It should be noted that there are two logically distinct approaches to conventional quantum mechanics: a local, and a global formulation. The local formalism relies on the existence of a differential equation (such as the Schrödinger equation) describing the physical situation (e.g. the wave function of the particle) at each point in space-time. The existence of this equation is operationally very convenient. On the other hand, the global formulation (or path formulation, if you will) is rather like the Feynman path formalism for quantum mechanics[Ap.20], which requires the enumeration of the "action" over these paths. This formalism is logically very simple, but operationally it is exceptionally complex. Our approach is a local formalism. Statement 3, $P(x,t) = -Ag$, is local and provides the basis for the further development of stochastic space-time quantum theory. Statements 1 and 3 are then logically related. The remaining Statements 2, 4, and 5 are secondary in importance.

The conclusion is that with the acceptance of the statements, the following can be deduced:

(i) the motion of a free particle, and the spread of the wave packet,

(ii) the uncertainty principle,

(iii) the nature of conjugate variables,

(iv) interference phenomena,

(v) an indication of where conventional quantum mechanics might break down (i.e. the limited validity of linear superposition).

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