On the Origin of 1/f Noise due to Generated Entropy - Version 2

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Abstract

Noise measurements analysis has been associated with degradation. In particular one such noise type called 1/f noise is likely the leading indicator. It is important to identify the noise frequency region of the spectrum associated with degradation occurring in materials to aid in noise type of reliability tests. The literature on 1/f noise appears to have a broad commonality in explanations and models related to degradation between materials and the measurement environment. It is reasonable in this regard to look at 1/f noise aspects in a broad sense in terms of disorder and the associated spectral content. This lends itself to a thermodynamic entropy framework for analysis. We review some of the key aspects of 1/f noise in the literature and discuss how observations relate to entropy. Once describe, we then suggest two thermodynamics models that one might use to model 1/f noise. Results help to provide a broader understanding of 1/f noise by using a different framework in terms of thermodynamic degradation processes. Such interpretation suggests that 1/f noise is a good tool for measuring certain aspects of disorder in materials and likely the associated spectral signatures. Experiments are suggested to demonstrate the importance of 1/f noise as a prognostic tool for 1/f noise reliability testing to identify and predict degradation in materials over time. We also suggest using standardized spectral identification methods.

1.0 Introduction – Resistor flicker Noise

In this article we slowly build a case that *1/f* noise, widely studied in the literature, is a sensitive measure of irreversibility. As well we suggest experiments to investigate the possibility of using flicker noise as a prognostic tool for making reliability predictions of degradation in materials. We will initially look at the literature and illustrate why it likely makes sense to explain flicker noise in terms of generated entropy. But first, consider a fundamental situation of a current flowing through a resistor to illustrate how noise in general appears to be a fundamental measure of entropy production. The typical argument of entropy change occurring due to current flow is

$$\Delta s = \Delta s_{resistor} + \Delta s_{Environment} \geq 0 \tag{1}$$

The entropy change to the environment is the heat dissipated

$$\Delta s_{Environemnt} = \frac{\Delta Q}{T} = \frac{i^2 R \Delta t}{T} \tag{2}$$

The entropy change to the resistor is often considered to be negligible since the average current is considered constant (*$\Delta i=0$*) as is the average temperature. Therefore, no net change to ΔQ and $\Delta s_{resistor} = 0$

However, in noise measurements, a much more sensitive measure, we observe voltage fluctuations across a resistor. Therefore, current is not constant, a temperature gradient exist in order to dissipate heat and current fluctuation must generate complex entropy at the microscopic level. Clearly, this is not a reversible process. We have the possibility that the resistor entropy could increase $\Delta s_R \neq 0$, and the current itself becomes disorganized when looked at using sensitive measurement 1/f noise methods, so $\Delta s_{Current} \neq 0$. We suggest a possible model for the complex entropy generated

$$\Delta s = \Delta s_{resistor}(W) + \Delta s_{current}(s_{resistor}) + \Delta s_{Environment}(i^2 R) \geq 0 \tag{3}$$

The $\Delta s_{Environment}$ in general represent entropy flow of heat to the environment and is not of immediate interest. Here we will focus on the resistor and current generate entropy change. Often in thermodynamics entropy flow (heat for example) is distinguished from generated entropy which causes damage or disorder to the material and in this case also to the current flow..

Many of the features of flicker noise in resistors are illustrated by the phenomenological equation due to Hooge [1]

$$S(f) = \gamma \frac{V_{DC}^2}{f} \tag{4}$$

Here capital S, is the noise spectral density, γ is the Hogg constant, and V_{DC} is the applied voltage. We see that noise power $S(f) \sim \langle V^2 \rangle = \langle (IR)^2 \rangle$, where *I* is the driving current and *R* is the sample resistance. In terms of an entropy interpretation, *R* is a direct measure of internal friction, which when interacting with current flow, generates entropy.

The concept that 1/f noise related to internal friction is not new as it has been described in metals by Kogan and Nagaev (1982) [2], Their argument was that 1/f noise low frequency fluctuations could occur in mechanical strain and then electrical resistance would depends on the strain displacements. Their detailed model is a type of mechanical approach. Here we are providing an interpretation from an energy approach.

1.1 Current Fluctuations Entropy

If we consider the random current fluctuations observable in sensitive 1/f noise measurements, we can consider this as a type of disorganization occurring in the current; therefore quantifiable by generaed entropy current change. Above, we have assumed this disorganization is a function of the entropy state of the resistor. This is theoretically supported by the fact that if current instead were to flow through a material with zero resistance; the process would be reversible with no generated entropy.

1.2 Resistance Entropy Change

Theoretically, any thermodynamic process creates entropy due to work. Thermodynamic stress in the material creates strain and this work is denoted as *W* likely created by current interactions in the material resulting in current fluctuations, other neighboring thermodynamic process in the material may also occur. For example, the resistor may not be in complete thermodynamic equilibrium, even in the absence of current flow? This is likely due to manufacturing stresses built into any fabricated material. Furthermore, the state of a system's entropy can itself create stress due to lack of structural internal integrity. This might suggest that the entropy change in the material in the absence of current flow goes as the entropy to some power m in the material

$$\frac{d\Delta s_R}{dt} = k\, s_R^m \tag{5}$$

Here we simply are providing a possible entropy argument in keeping with the intent of the paper. At this point, we will need to look further into modeling to determine an entropy approach which can lead to a 1/f noise dependence.

1.3 Wire Wound vs. Carbon Resistor Entropy Comparison

From our above discussion, we assumed that the current noise entropy was a function of the entropy state of the resistor and certainly 1/f noise measurement such as the Hooge model, confirms that noise increases with resistance. It also depends on the resistance type. For example, 1/f noise observations indicate that wire wound (w) resistors have less noise than carbon (c) resistors [3]. In terms of entropy, a comparison of entropy created in the current i has

$$\Delta s_{i-c}(s_c) - \Delta s_{i-w}(s_w) \ge 0 \tag{6}$$

so that $s_c > s_w$. Furthermore, a comparison of any damage entropy contribution in the two materials indicates

$$\Delta s_{R-c}(W_i) - \Delta s_{R-w}(W_i) \ge 0 \tag{7}$$

so damage entropy occurring in a carbon resistor is higher compared to a wire wound resistor for the same amount of work, or alternately, wire would resistors are likely manufactured with higher stability.

1.4 Oscillator Phase Noise Entropy

The phase noise of an oscillator is perhaps one of the most important parameters. Here 1/f noise is known to dominate. Phase noise is important as it affects the purity of the carrier frequency in transmission. It is known that the unloaded Q in flicker noisem(1/f) goes as the inverse of Q to the forth power observed [4,5,6] in the low flicker frequency area (i.e. near the carrier frequency) as noted in oscillator power noise spectral density. Here again, damping (an inverse function of Q), a characteristic of another form of internal friction, can be strongly associated with entropy generation. A higher Q also indicates a more stable material and less susceptible to generated entropy damage.

1.5 MOS Observation and Entropy

Flicker model vary widely for MOSFET type devices. One basic theory that flicker noise results due to fluctuations in bulk mobility based on Hooge's [1] empirical relation is

$$S(f) = \gamma \frac{I^2}{N\ f} \tag{8}$$

S is the noise spectral density, γ is the Hooge constant, I is the current, N is the number of charge carriers, A simply entropy interpretation indicates that charge carriers reduce internal friction or alternately resistance in the bulk varies inversely with N. I is the current flow increases the amplitude of the flicker noise as it is perturbed in the channel [7].

1.6 Effect of Temperature

Note the spectral density in the Hooge Equation 8, is independent of temperature. This reinforces the fact of generated entropy damage compared with entropy flow (heat added) is the issue in 1/f noise. However, 1/f noise shows some atypical temperature dependent characteristic (Eberhard and Horn, 1978 [8]) where they noted an adhoc function of γ(T). We would view this due to damage created by heat as part of the thermodynamic entropy damage process.

1.7 Noise Measurement as a tool to Observe Entropy Damage

In a simpler view, if there is no internal friction during the system-environment interaction, we would have a reversible process and an absence of flicker noise. In such a case we would have no measurement process since all such process are irreversible Noise in operating systems is starting to be recognized as important for prognostics of failure [9, 10]. One of the more telling signs of the association of noise with failure is related to the human heart degradation where congestive heart patients compared with healthy hearts had a distinctly different noise spectrum [11].

1.8 The Measurement Process Itself Creates Irreversibilities

Although no process is truly reversible, a common thermodynamic argument, it is worthy of comment. We could state that if a system process is in thermal equilibrium, then the process is reversible. However in thermal equilibrium there is no measurement process!

Clarke and Voss [12] found that 1/f noise was present if there was no driving current at equilibrium but they could not guarantee true thermal equilibrium. In this view there could not be a measurement process, if thermodynamic equilibrium had been reached. Therefore, thermal equilibrium could not have been achieved during their measurement process. This goes to the point of Eq. 3 if $\Delta s_{current}=0$ that we can still have an entropy change. Recall that the thermodynamic work W is defined to be due to the measurement current or any other neighboring thermodynamic work process such as internal fabricated material stresses.

2.0 Overview of Entropy Models

Here we suggest two possible models to illustrate how an entropy approach can be modeled with a 1/f noise dependence. The first is a time domain model that is transformed to the frequency domain, leading to the *1/f* dependence. The second is a free energy flicker model that is compared to Schottky's original model [13].

2.2 Time Domain Entropy Model

The definitions of entropy, s, for discrete and continuous variable X are,

Discrete *X, p(x):*

$$s(X) = -\sum p(x)\log_2 P(x) \tag{9}$$

and Continuous *X, f(x), Differential Entropy [14,15]:*

$$s(X) = -\int f(x)\log\,(f(x))\,dx \tag{10}$$

Here we are concerned with the continuous variations in time t distributed by *f(t).* Consider a Gaussian spectral density due to a process that create generated entropy current fluctuations with distribution

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(\frac{-(t-\mu)^2}{2\sigma^2}\right) \tag{11}$$

Gaussian spectra density for 1/f noise time domain processes is often described as logical in the literature [such as 16]. Milotti [17] summarized the question noting:

> Voss [18] produced experimental plots of the quantity <*V*(*t*)|*V*0>/ *V*0 in several conductors and was able to show that the noise processes observed were reasonably Gaussian. Further, it was noted that the superposition of many non Gaussian microscopic processes can results as Gaussian at the macroscopic level (demonstrated via the central limit theorem). J. B. Johnson in his 1925 experiment [19] in vacuum tubes asserted that the spectral density characterizes a noise process completely only if the process is stationary, ergodic and Gaussian: does the observed *1/f* noise satisfy all these constraints.

When Eq. 11 is inserted into the differential entropy Eq. 10, the result for a temporal process is [15]

$$s(t) = \frac{1}{2}\log(2\pi\, e\,\sigma(t)^2) \tag{12}$$

Note that the entropy does not depend on the mean only on the variance, a more sensitive measure. Solving in terms of the variance and expanding terms by assuming small change to the entropy and looking at the temporal part

$$\sigma(t)^2 = \frac{1}{2\pi\, e}Exp\{4.606\, s(t)\} \approx \frac{1}{2\pi\, e}(1+4.6s(t) \approx C + 4.6\{s(0)+s'(0)t\} \approx 4.6s'(0)t \tag{13}$$

A common noise measure related to the variance in the time domain is the Allan Variance

$$\text{Allan Variance}: \sigma^2(\tau) = \frac{1}{2(n-1)}\sum_i (\bar{y}(\tau)_{i+1} - \bar{y}(\tau)_i)^2 \tag{14}$$

The Allan variance transforms to the frequency domain are well established and the frequency domain spectral density of Equation 13 is [20]

$$S(f) = \frac{2.3}{\ln 2}\frac{1}{f}s'(0) = \frac{1}{f}\chi\, s'(0) \tag{15}$$

Here the noise shows a 1/f dependence and is a function of the rate of change of the entropy as anticipated. We suggest flicker noise be simplified by two variables, a calibration variable χ (similar to the Hooge constant) and an entropy variable related to the measurement material s'(0) which is explained in Section 3.

A second alternate approach perhaps more insightful is to look at the rate of change of the entropy in Equation 12

$$\frac{ds(t)}{dt} = \frac{1}{2}\frac{1}{\sigma(t)}\frac{\sigma(t)}{dt} \tag{16}$$

We write this as

$$\sigma^2(t) = \frac{1}{4}\left(\frac{\sigma(t)}{dt}\bigg/\frac{ds(t)}{dt}\right)^2 = k\,t^u \tag{17}$$

Here we assume some possible time dependence power with constant k for the moment. In this model we assume the variance rate of change likely goes as the entropy rate in Eq. 17, so

$$u=0 \tag{18}$$

When this is the case, we get a *stationary process* and the equivalent frequency domain spectrum S is transformed for $\sigma^2(t) \propto t^0$ to [20] to the frequency domain, so Eq. 17 becomes

$$S(f) = \frac{k}{2\ln 2}\frac{1}{f} \tag{19}$$

Again showing the 1/f noise dependence. Note if we had some temporal dependence (non stochastic process) in the ratio $\sigma^2(t) \propto t$ in Eq. 17 where

$$u=1 \tag{20}$$

we would get Brownian noise as it transform to [17,20]

$$S(f) \propto 1/f^2 \tag{21}$$

We note that the temporal model, u=0, indicates that the variance and entropy rates change together. Therefore, we anticipate 1/f noise provides more fundamental significance to generated entropy damage sensitivity then say brown noise (u=1).

2.1 Free Energy Flicker Model

It can prove helpful to have a second supportive entropy model to have a clearer understanding of flicker noise. As a measurable quantity, entropy damage, prior to macroscopic observations, can likely be observed with flicker noise measurements. The interaction of the system (current typically) with the environment (material, or visa versa), causes entropy damage as described in Eq 3. In the case of a *1/f* noise measurement, the current and the material could both be degraded. We note an increase in entropy damage, Eq. 3, corresponds to a decrease in the free energy ϕ, i.e.

$$\frac{ds_{Damage}}{dt} > 0 \rightarrow \frac{d\phi}{dt} < 0 \tag{22}$$

Given a system interacting with the environment at temperature T, the thermodynamic probability P of a microstate L is

$$P_L \, \alpha \, Exp(-\phi_L/K_B T) \tag{23}$$

with free energy ϕ_L. For low frequencies fluctuations from a thermodynamic process, the microstate probability P is time dependent function of the free energy change

$$P_L(t) = \frac{1}{Z}\exp\left(-\frac{\Delta\phi_L(t)}{K_B T}\right) \text{ and Z is } \sum_i P_i = \frac{1}{Z}\sum_i \exp\left(-\frac{\phi_i}{K_B T}\right) = 1 \tag{24}$$

That is, Z is the normalized partition function. We then model the free energy ϕ_L via a Taylor expansion

$$\phi(t) = \phi(0) + t y_1 + \frac{t^2}{2} y_2 + \ldots \tag{25}$$

where y_1 and y_2 are given by

$$y_1 = \frac{\partial\phi(0)}{\partial t} \quad and \quad y_2 = \frac{\partial^2\phi(0)}{\partial t^2} \tag{26}$$

Microscopic damage corresponds to free energy change which is assumed small, then taking the first term in the expansion

$$\Delta\phi_L = \phi(t) - \phi(0) \cong t y_L \tag{27}$$

Therefore the damage probability is

$$P_L(t) = \frac{1}{Z}\exp(-\lambda_L t) \tag{28}$$

where $\lambda = \dfrac{y_1}{K_B T}$. Note that $\Delta\phi_L$ itself is not temperature dependent and the systems free energy goes as the thermodynamic work of damage, for an isothermal process

$$\delta W = \sum_a Y_a dX_a \propto \Delta\phi \tag{29}$$

where Y and dX are conjugate work variable like stress and strain, voltage and charge, etc. [1].

We are now in a position to compare Equations 28 with 30 below, which relates to Schottky's (1926) [13] original model. In his model, contribution to the vacuum tube current from cathode surface trapping sites, released electrons according to a simple exponential relaxation

$$N(t) = N_o \exp(-\lambda t) \tag{30}$$

In the entropy viewpoint, the origin would be due to damage fluctuations in the free energy observed from the cathode current and related to the surface trapping. The results lead to the Schottky's [13,17] spectrum model

$$S(\omega) = \frac{N_0^2 n}{(\lambda^2 + f^2)} \tag{31}$$

Bernamont [16] later pointed out that only a superposition of processes with a variety of relaxation rates λ would yield *1/f* noise for a reasonable range of frequencies. He showed that if λ is uniformly distributed between λ_1 and λ_2, and the amplitudes remain constant, the spectrum can be interpreted in the pink *1/f* noise region

$$S(\omega) = \frac{N_0^2 n\pi}{2\omega(\lambda_2 - \lambda_1)}, \quad \lambda_1 << \omega << \lambda_2 \tag{32}$$

and Brown noise for example

$$S(\omega) = \frac{N_0^2 n}{\omega^2}, \quad \lambda_1 << \lambda_2 << \omega \tag{33}$$

3.0 Suggested Experiments to Illustrate the Usefulness of the Entropy Approach

Accelerated testing of materials and products is often done in industry. Since entropy increase with aging time, and we have illustrated how flicker noise is a likely a sensitive measure of entropy change, it is important to work with standardize test so degradation can be quantified through noise. Below are some suggestions on this measurement process.

3.1 Calibration Standard Measurement Process

Prior to any measurement, a calibration standard of material like a 1K, 10K, and 100K wire wound resistor of a specified reasonably stable material. Each measurement made at room temperature conditions at a given voltage (10 Volt). We see in the literature that it is hard to compare measurements from different researcher. A calibration standard is suggested so measurements can be compared more easily. The noise level should be provided in a reasonable frequency region such as 1 Hz to 1000 Hz.

$$S_{Cal}(f) = \frac{1}{f} \chi s_{Cal}'(0) \tag{34}$$

Here we use a simple flicker degradation reliability equation, were χ is the calibration factor (similar to Hooge's constant) and s'(0) is the entropy of the calibration sample. To separate out χ from s'(0) would likely require two or three standards (the 1K, 10K, 100K) at the specified voltage (1V). Here we assume χ is independent of stress where s'(0) is dependent on stress, which must be specified in assessing material prognostics.

3.2 Suggested Flicker Aging Experiments

Once calibration is obtained and χ is assessed, we can model s'(0) for different materials in a degradation reliability type test with the suggested flicker entropy model

$$S_{Material-current}(f) = \frac{1}{f} \chi s'(0) = \frac{1}{f} \chi F(\kappa, t) \tag{35}$$

Here *F* is the entropy function of k-stress or stresses with dependence on time in reliability testing. Flicker noise measurements could be done at room temperature after periodic temperature exposure in an oven so the measurement itself is stationary even though reliability aging is a non stationary process. $S_{material-current}(f)$ then becomes a spectral identication characteristic of the material-current interaction and can be used as a library signature similar to FTIR analysis.

Thin Film Resistors
Thin film resistors are known to age as a power law in time, for example possibly $F(kt)=kt^n$. Since it is known that thin film resistance increase over temperature over time, so too will the flicker noise PSD level and entropy function. However, now the option is available to look at aging rates at lower temperatures to observe the flicker aging law and if needed transfer it to the time domain and compare it to gross measurements (i.e. higher temperatures and longer macroscopic gross measurements).

Biological Aging Experiment in living systems
The human heart is known to have different noise characteristics for CHF compared to healthy heart. However, now the option in this views is available to study aging in normal healthy hearts using flicker noise measurement over a person's lifetime. Here we might suggest both long term tracking of a group of people and also looking ar different aging groups. All measurements should be first done with a calibration standard as suggested.

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Appendix: Particle Analogy "PhoDons" and Measurement Uncertainty

We briefly can be a bit creative and assign a word "Pho*D*on" for a fundamental *D*amage particle created with energy change $\Delta\phi_i$. The "phodon" associated energy of creation would then be somewhat analogous to phonon energy, but unlike phonons with modes of vibration in say a crystal structure, phodon wave nature becomes associated with random current fluctuation in the measurement process.

Degradation can then be thought of as creating phodon damage particles created with energies that depend on the material properties and interaction with the neighboring environment so that according to the second law and Eq. 2

$$Tds = \delta Q + Tds_{Damage} = \delta Q - d\phi + \delta W$$

Here we see the phodon creation causes a change to the free energy via the current work (consistent with Eq 2).

In Eq. 2, when the current flow is zero (i.e. $\Delta s_{current}=0$), it is apparent that phodons can be created. This is because we described W_i typically due to measurement current of any other thermodynamic process. This is possibly due to thermal fluctuations. As well, we might wonder if phodon creation can also be due to zero point fluctuation. When we talk about the difficulty of obtaining thermodynamic equilibrium, we cannot be sure how microscopic irreversibilities occur in materials.

Finally, we should be mindful of the difficulty of taking very low frequencies measurement (near 0), the observation time must be long enough to be certain of the frequency value $\Delta t \geq 1/ \Delta \nu$ due to the uncertainty principle for measurement accuracy.