Calculation of the mass of the Universe based on propagation history of the Cosmic Background Radiation and finite speed of gravity for a Λ= 0 relativistic model

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The standard cosmological model calculates the gravitational mass-energy contribution of the Cosmic Background Radiation (CBR) to the mass of the Universe from the single energy density currently observed. The model assumes a homogeneous energy distribution at zero red shift and applies the energy density across the Universe using a $\Lambda > 0$ relativistic model. After reviewing the Friedmann equations for a matter dominated universe and Lematre extension of the space time metric to relativistic energy, we offer an alternative mathematical calculation of the gravitational mass-energy of the CBR component using a $\Lambda = 0$ relativistic model. A complete propagation history of the photons comprising the CBR is used rather than only the current energy density. Because the effects of gravity travel at the speed of light (according to general relativity), and using hot big bang cosmology, we suggest that the higher energy states of the CBR photons in the past also contribute to the currently observed gravitational effects. In our alternative calculation, the CBR energy density is integrated over a range of red shifts in order to account for the gravitational effects of the radiation energy density as it was in the past. By accounting for propagation effects, the resultant gravitational mass-energy calculated for the CBR radiation component almost exactly equals the amount attributed to dark energy. The calculation suggests an extension of the standard cosmological model in which co-moving distance at higher red shift increases with red shift more slowly than it does in the standard model. When compared with Type 1A Supernova data (in an available range of red shift from $z = 0.4$ to $z = 1.5$), distance predictions from the extended model have reasonable agreement with observation. The predictions also compare closely with the standard model (up to $z = 1.0$). Further observations are needed for $z > 1.5$ to make a final comparison between the standard model and the proposed extended model.

I. INTRODUCTION

Cosmologists calculate the mass of the Universe using a model comprised of five physical components: baryonic matter, dark matter, photons, neutrinos, and vacuum energy. In this paper, the neutrino component will be neglected due to the lack of consensus regarding the mass of the neutrino. In the standard cosmological model, the total mass of the Universe is deduced primarily from gravitational observations and Friedmann's equations. Observational measurements of the Universe's spatial curvature indicate that it is very close to zero and therefore the Universe is at critical density. The gravity required to accomplish this is far greater than the mass currently attributed to the first three of these components. Ordinary baryonic matter is found to be only about 4.8% of the critical density and dark matter is estimated to make up roughly 25.6% [1]. In the standard cosmological model, the fraction of critical density attributed to photons is considered negligible, a mere 0.005%. This is based on current observations of the energy at red shift $z = 0$ of the photons that comprise the Cosmic Background Radiation (CBR). Dark energy is attributed to the vacuum energy to make up the remaining 69.6% [1].

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In Ref. [2], gravitational aberration in general relativity is seen to be almost exactly canceled by velocity-dependent interactions, establishing that effect of gravity propagates at (or almost exactly at) the speed of light. This statement is reinforced by references [3–5], which assert that no gravitational effect propagates faster than light. Furthermore, gravitational waves produced by a binary back hole merger were detected in September of 2015 by the Laser Interferometer Gravitational-Wave Observatory with speeds consistent with the speed of light [5]. Based on references [2–5], the speed and effects of gravity is therefore assumed to be equal to the speed of light.

Big bang cosmology describes an expanding Universe, with a uniform bath of cosmic background radiation that decreases in temperature over time. The Universe was denser and hotter in the past, which is the basis of the hot big bang theory [6]. Consequently, photons at earlier times in the background radiation had more energy than they do today. We believe that the gravity produced from these higher energy states is just now affecting us from distant locations. This is the basis of the alternative calculation of the mass of the Universe offered in this paper. Conceptual and foundational details are provided in the next four subsections.

A. Review of the Friedmann model and Lematre extension

The standard cosmological model is based on the FriedmannLematreRobertsonWalker (FLRW) metric, which is an exact solution of Einstein's field equations of general relativity:

$$ds^2 = c^2 dt^2 - a^2(t)\gamma_{ij} dx^i dx^j, \tag{1}$$

where γ_{ij} is the metric of a 3-dimensional homogeneous and isotropic manifold. For the spatially flat Universe, the geometry and metric are those of a 3-dimensional Euclidean space. The term $a(t)$ is called the scale factor, which has a geometrical meaning for the spatially flat Universe. There is a physical meaning in this case for the ratio of scale factors at different times, i.e. $a(t_1)/a(t_2)$, and for the Hubble parameter $H(t) = (da/dt)/a(t)$, which is the rate of cosmological expansion.

The propagation of light in an expanding Universe is subject to redshift. In terms of cosmological parameters, the wavelength λ_o of a photon of light observed at the Earth is related to its wavelength λ_e at time of emission t_e by the equation,

$$\lambda_o = \lambda_e \left[a_o/a(t_e)\right] = \lambda_e \left[1 + z(t)\right], \tag{2}$$

where a_o is the scale factor of the present Universe, $a(t_e)$ is the scale factor at time of emission. The redshift of the wavelength is given by unitless variable $z(t) = [a(t)/a(t_e)] - 1$.

The FLRW metric describes a homogeneous, isotropic, expanding universe. The very fact that the universe expands implies that it was denser and hotter in the past, which is the basis of the hot big bang theory [6]. The energy densities of the components for baryonic matter, dark matter and photons are based on (local) current observables in the Universe. In the standard model, a homogeneous energy distribution is then applied to each of these components to extend the densities homogenously across the whole of the Universe. The z-dependency (redshift) of the energy density for the matter components scales with $(1 + z)^3$ and the photonic (radiation) component scales with $(1 + z)^4$.

These dependencies will be used in Sec. II C and Sec. II D to formulate a cosmological model for the alternative z-dependent calculations of the photonic mass - energy of the Universe. Due to the finite speed of gravity and the diminishing energy of CBR photons, this will include an adaptation of the standard model homogeneous energy distribution to apply to red-shifted energy.

When Lematre extended Friedmann's equations to include the effects of radiation, pressure was now considered in the total energy calculation. Lematre modeled the Universe after a perfect fluid, and now the equations governing the expansion of the Universe could be reversed to a time known as the radiation - dominated epoch. This was a time when the mass energy of photons dwarfed that of matter because the Universe was so hot. This more complete model was a great achievement in theoretical cosmology, but the Lematre's model, like Friedmann's, only considers the current mass-energy of the radiation in the Universe. It does not include the complete propagation history of all photons in our Universe. If the complete propagation history of all photons in the Universe is considered, then the higher energy states of CBR photons would also be seen to contribute to the current gravitational effects. The authors understand that the critical density from Friedmann's equation is only valid for uniform proper time. Although these higher energy states are from the past, they are arriving at our current proper time, and therefore should be considered as part of the current critical density, even though they came from a time when the critical density was higher than it is today.

B. $\Lambda = 0$ relativistic model and the hypothesis of the alternative calculation

The two substantial differences between our alternative calculation and the conventional calculations using the standard model are: (i) the use of a $\Lambda = 0$ relativistic model and (ii) how the FLRW metric is integrated. The cosmological constant Λ was introduced by Einstein into his field equations for General Relativity to create a relativistic model that allowed for a static Universe (i.e. one that was neither expanding nor contracting). Introducing Λ into the field equations in their tensor form created a simple relationship between space-time curvature and the mass and energy content of the Universe:

$$G_{\mu v} + \Lambda g_{\mu v} = 8\pi G c^{-4} T_{\mu v}. \tag{3}$$

The left hand side of the field equations in this form determines space-time curvature and the right hand side determines the mass and energy content. Einstein's choice of the value of cosmological constant (for a static universe) led to an unstable model of the Universe. A decade after the introduction of Λ, the astronomical observations of distant galaxies by Hubble confirmed that the Universe is expanding. Einstein subsequently abandoned the cosmological constant. For several years afterwards, the cosmological constant was almost universally agreed to be zero.

More recently though, cosmologists have algebraically manipulated the field equations by moving Λ to the right hand side and including it as part of the stress-energy tensor $T_{\mu v}$. This leads to a vacuum energy term, $\rho_{\text{vac}} = \Lambda c^2/8\pi G$. Consequently, the existence of a non-zero cosmological constant is equivalent to the existence of a non-zero vacuum energy. Choosing a value $\rho_{\text{vac}} > 0$ leads to a cosmological model of an accelerating expansion of the Universe.

In conventional calculations using the standard model, a surface of co-moving distance in a spatially flat Universe is defined by fixing the scale factor to the time of current observation. This is used to capture the complete distribution of matter of the Universe on the basis of homogeneity. However, the use of homogeneity for photonic energy (radiation) in conventional calculations does not account for propagation.

To observe all the light in the universe in a single snapshot (which cannot be done using a co-moving surface of constant scale factor as is done with matter), light must be considered on a null geodesic in relativistic 4-dimensional space-time, i.e. $ds^2 = 0$, which gives

$$dt^2 = c^2 a^2(t) \gamma_{ij} dx^i dx^j. \quad (4)$$

It is along this geodesic that the complete propagation history of photonic radiation is to be found; and the wavelengths of the light will be distributed over a range of redshifts based on time of emission.

Furthermore, it is well known that in the limit of small perturbations, gravitational disturbances travel along null geodesics. Integration along the null geodesic is therefore physically meaningful.

The Null Geodesic Hypothesis: the gravitational effect of each photon in the CBR (to include those emitted from distant locations) propagates along the path of the null geodesic of the photon without loss due to redshift; and contributes to the currently observed gravitational effect of the photonic relativistic mass component of the present Universe. I Unlike light, the propagation of gravitational effects is not affected by redshift. Therefore, due to the finite speed of gravity and the diminishing energy of CBR photons, the alternative calculation is an adaptation of the standard model of homogeneous energy distribution but applied to redshifted energy and not just the current state of observation (i.e. $z = 0$).

If current observers truly are experiencing gravity from the past, then the difference in relativistic mass effects of gravity and of the CBR's redshifted photons must be accounted for, which was first put proposed in Ref. [7] and has been further built upon. The gravitational effects of redshifted photons will be accounted for by considering the relativistic mass of the photons over their complete propagation history. These will be integrated over a null geodesic to account for all (redshifted) photons currently being observed from the CBR.

C. An alternative approach to the calculation of red shifted mass-energy

In the standard cosmological model, the equivalent mass energy of all CBR photons is calculated by multiplying the current energy density by the current proper volume of the entire Universe. This calculation of the mass energy of the photonic radiation will be reviewed in Sec. II A. It carries an assumption that the photonic energy density is constant throughout the span of the Universe. However, when considering the propagation history of a cosmic photon over a large distance, its wavelength cannot remain constant because of the red shift. Consequently the energy of the photon must also vary along the path of propagation, because quantum mechanically photons at higher frequency (shorter wavelength) have higher energy. So, rather than doing calculations that relate distance to look-back time (and energy of the CBR), the red-shift parameter z will be used instead.

In Sec. II C, an alternative calculation for the mass - energy of the CBR will be investigated in a radiation dominated Universe. The gravitational effects of the CBR are then considered. The additional mass - energy of the CBR attributed to the time of its emission (rather than the time of its observation) will be accounted for. This model of the energy density is then extended to a model that includes both matter and radiation. Our hypothesis is then that the total energy of cosmic photons should be based on integrating a z-dependent energy density $\rho_r = \rho_r(z)$ and proper volume $V = V(z)$ throughout the appropriate range of values taken on by z.

In this alternative approach, the gravitational effects observers currently experience have therefore been generated by non-monochromatic cosmic photons. Because a single photon has an energy density, integrating over all past energy densities (as a function of distance) would account for the gravitational effects produced by the photon along it's entire path of propagation towards us. This argument is then extended to the entire CBR because it too has an energy density, just like a single photon.

The only remaining foundational detail is the range of values taken on by z. The lower limit clearly must be the current state of observation of the CBR at $z = 0$. Choosing a plausible upper limit admits more than one alternative, the details of which will be discussed in the next section.

D. Upper limit to red shift calculation

This section determines a plausible upper limit of the red shift in order to avoid integrating over an infinite range of red shifts. It is widely regarded that the currently observed CBR photons were emitted from the surface of last scattering, when the temperature of the Universe was around $3,000$K [8]. The expansion of the Universe has actually been extrapolated back to temperatures that would have generated nucleosynthesis. This is based on experimental observations of the Universe's atomic mass abundances: approximately 75% hydrogen-1 and 25% helium-4, with trace amounts of deuterium, lithium, and beryllium [9]. Big bang nucleosynthesis is believed to have occurred when the Universe's temperature was around 10^{10}K [9]. Earlier times such as the electro-weak epoch have been theoretically explored, when temperatures of at least 10^{15}K were attained [10]. It has been speculated that the Universe can be extrapolated back to a time when the Planck Temperature was attained [11].

In principle there is a quantum gravity limit as the scale factor approaches zero and the temperature approaches infinity at the moment of the big bang. Known physics breaks down where the thermal energy of particles is such that their de Broglie wavelength is smaller than their Schwarzschild radius. Quantum black holes cause extreme difficulty within the usual concept of background spacetime [12]. Calculations using general relativity in combination with quantum mechanics break down above the Planck Temperature [13]. This provides a natural cut-off temperature. Therefore, the red shift that results in the Planck Temperature will be used to determine the upper limit in the alternative calculation.

$$\max z = \frac{1.417 \times 10^{32}\text{K}}{2.725\text{K}} = 5.1 \times 10^{31}. \tag{5}$$

This is based on a Planck Temperature of 1.417×10^{32}K, a CBR temperature of 2.725K, and the relation of temperature to red shift ratio in the standard cosmological model.

II. CALCULATION OF THE ENERGY DENSITY OF THE OBSERVABLE CBR

This section provides the basis for the alternative calculation performed in Sec. III. In Sec. II A, the energy density for the radiation component is derived for the standard cosmological model. This is the basis for the traditional calculation of the mass energy of the radiation component. The first step in extending the traditional calculation is the z-dependent proper volume element, which is introduced in Sec. II B. The consequences of integrating a z-dependent proper volume element against the energy density derived for the standard cosmological model are then considered in Sec. II C for a radiation dominated Universe. The insights gained point the way as to how to formulate a cosmological model for z-dependent calculations, which is the subject of Sec. II D.

A. Photonic energy density in the standard cosmological model

The density term ρ in Friedmann's equation is commonly expressed in terms of unitless fractional densities. The terms commonly used for the radiation and mass fractional densities are:

$$\Omega_r = \frac{\rho(\text{photons})}{\rho(\text{critical})};\ \Omega_m = \frac{\rho(\text{matter})}{\rho(\text{critical})}. \tag{6}$$

Here, ρ(photons) is the current photonic energy density in the Universe, and ρ(matter) is the current energy density in ordinary matter as well as dark matter. In the standard model for a spatially flat Universe, the total fractional density is $= 1$. According to Friedmann's equation, the critical density for a spatially flat universe is then:

$$\rho(\text{critical}) = \frac{3H_0^2}{8\pi G} = 8.72 \times 10^{-27}\text{kgm}^{-3}, \tag{7}$$

where G is Newton's constant and H_0 is Hubble's constant as measured today. The value $68.14\text{kms}^{-1}\text{Mpc}^{-1}$ has been used for Hubble's constant [1].

The photonic energy density of the currently observed CBR can be calculated using the radiation constant $a = 7.566\times10^{-16}\text{JK}^{-4}\text{m}^{-3}$ and the current CBR temperature $T = 2.725$K, where

$$\rho(\text{photons}) \equiv \rho_r = aT^4 = 4.172 \times 10^{-14}\text{Jm}^{-3} \tag{8}$$

Dividing the photonic energy density by the speed of light squared, c^2, gives an equivalent mass density for the currently observed CBR,

$$\begin{aligned} \rho_r &= 4.636 \times 10^{-31}\text{kgm}^{-3} \\ \Omega_r &= 5.317 \times 10^{-5}. \end{aligned} \tag{9}$$

which will be used in Sec. III for the alternative calculation. Note that this fraction of critical density attributed to photons corresponds to the 0.005% mentioned in the introduction.

The total mass-energy equivalent of all CBR photons is traditionally calculated by multiplying the current mass density in Eq. 9 by the entire current proper volume of the Universe. This calculation carries with it the assumption of uniform energy density of the CBR photons. Our hypothesis is that the energy of cosmic photons needs to be integrated over proper volume $V = V(z)$ and energy density $\rho_r = \rho_r(z)$ throughout the range of values taken on by z.

B. The z-dependent proper volume element

The z-dependency of volume will be introduced through the differential of the proper volume v. Based on the FLRW metric, the equation for the proper volume element at the time of current observation is [12]:

$$dv = \frac{4\pi r^2 c^3}{H_0^3} dr. \tag{10}$$

Note that as a function of r, the volume element would therefor integrate to yield a current proper volume of $4/3\pi(c/H_0)^3 r^3$. Whereas the standard model considers the volume at time of current observation, the alternative calculation will be based on the differential proper volume at time of emission which is related to dv by the equation:

$$dV = \left(\frac{1}{1+z}\right)^3 dv. \tag{11}$$

In the next section, the z-dependency of r and dr will be expressed first for a radiation-dominated model (Sec. II C) and then for a cosmological model using z-dependent calculations (Sec. II D).

C. Alternative calculations in a radiation dominated Universe

It is useful to first consider a radiation dominated expanding Universe, i.e. the matter density is small enough that the simplification $\Omega_r = 1$ is reasonable. Unlike the traditional calculation in Sec. II A, the energy state of all photons will vary throughout their propagation history because a photon's energy depends on its state of red shift z. This calculation can be thought of using the propagation history of one photon or many, because the energy density (not number density) is considered.

The z-dependency of r and dr must also be accounted for. For the simplified radiation dominated model (i.e. $\Omega_r = 1$), the differential of r and differential proper volume element in the FLRW metric are respectively given by:

$$dr = \frac{1}{\Omega_r^{1/2}(1+z)^2} dz \tag{12}$$

$$dv = \frac{4\pi r^2}{\Omega_r^{1/2}(1+z)^2} \left(\frac{c}{H_0}\right)^3 dz. \tag{13}$$

The z-dependency of r in the FLRW radiation dominated model is based on [12]

$$r = 1 - \frac{1}{1+z} \tag{14}$$

$$r^2 = 1 - \frac{2}{1+z} + \frac{1}{(1+z)^2}. \tag{15}$$

The complete z-dependency of the proper volume element at time of observation is then:

$$dv(z) = 4\pi \left(\frac{c}{H_0}\right)^3 \left[\frac{1}{(1+z)^2} - \frac{2}{(1+z)^3} + \frac{1}{(1+z)^4}\right] dz \tag{16}$$

When the factor $(1+z)^3$ is accounted for between time of emission and observation, the differential proper volume at the time of emission is then given by:

$$dV(z) = 4\pi \left(\frac{c}{H_0}\right)^3 \left[\frac{1}{(1+z)^5} - \frac{2}{(1+z)^6} + \frac{1}{(1+z)^7}\right] dz \tag{17}$$

Given the apparent steady state of the emissions from the surface of last scattering, it is reasonable to assume that the photons currently being observed from the CBR (and hence absorbed) are constantly being refreshed by more CBR photons. Therefore, it is true that we will continue to be bombarded by CBR photons from the surface of last scattering as the cosmic horizon recedes, and will always be bombarded by CBR photons from the surface of last scattering if indeed the Universe is infinite (and is not accelerating).

The assumption of homogeneity in the standard cosmological model (at $z = 0$) is extended to the spatial distribution of photons in each element $dV(z)$. Specifically, every volume element contributes to the currently observed energy state of the CBR and consequently the energy state of the photons in each volume element is determined by z and the currently observed energy state of the CBR. It is therefore straight forward to calculate an energy density $\rho_r = \rho_r(z)$ for each volume element. However, across the volume elements (i.e. different values of z), the photons are distributed non-monochromatically due to their different states of red shift.

Therefore, multiplying Eq. 17 by the photonic z-dependent energy density $\rho_r = \rho_r(z)$ will then determine the mass of each differential volume throughout the propagation history. Using the mass equivalent energy density ρ_r derived in Eq. 9 as the currently observed mass density, i.e. $\rho_r(z) = \rho_0 = \rho_r$, at $z = 0$, and multiplying by $(1+z)^4$in order to account for the radiation density dependency on $(1+z)$, gives $\rho_r(z) = \rho_0(1+z)^4$. The differential mass for the volume element $dV(z)$ is then:

$$\begin{aligned} dM(z) &= \rho_o(1+z)^4 dV(z) \\ &= 4\pi\rho_0 \left(\frac{c}{H_0}\right)^3 \left[\frac{1}{(1+z)} - \frac{2}{(1+z)^2} + \frac{1}{(1+z)^3}\right] dz. \end{aligned} \tag{18}$$

The indefinite integral of $dM(z)$ is:

$$M(z) = 2\pi\rho_0 \left(\frac{c}{H_0}\right)^3 \left[2\ln(1+z) + \frac{4}{(1+z)} - \frac{1}{(1+z)^2}\right]. \tag{19}$$

When evaluating Eq. 19 over the limits of integration established in Sec. I C, i.e. from $z = 0$ to $z = 5.2 \times 10^{31}$, it is clear that the dominant contribution to the value of the integral comes from the term:

$$M_z = 4\pi\rho_0 \left(\frac{c}{H_0}\right)^3 \ln(1+z). \tag{20}$$

This z-component of photonic energy in an expanding, radiation-dominated universe is dependent on and scales with the natural logarithm of $1+z$.

D. Alternative model for r and dr as a function of z

The cosmological model for the complete alternative calculation will consider a Universe with the FLRW metric dominated only by matter and radiation. No vacuum energy will be assumed. The model for the radiation dominated universe in the previous section can be extended to include matter. In the standard cosmological model with no spatial curvature [12], the differential proper distance as a function of z is then,

$$dr = [\Omega_r(1+z)^4 + \Omega_m(1+z)^3]^{-1/2}dz. \quad (21)$$

Because this model is for a Universe at critical density, it follows that $\Omega_r + \Omega_m = 1$. The radiation term $\Omega_r = 5.317 \times 10^{-5}$ is the same as in the standard model. However, the mass term will not be from the standard model rather it will be $\Omega_m = 0.9994683$ by the critical density equation and the arguments made at the end of Sec. II C.

The term Ω_m in Eq. 21 will be replaced with $1 - \Omega_r$ so as to maintain its expression in terms of the observable quantity Ω_r. Eq. 21 is then re-written as:

$$dr = [(1+z)^{3/2}(\Omega_r z + 1)^{-1/2}]dz. \quad (22)$$

Because $\Omega_m \approx 1$, it is reasonable to use Mattig's formula [14] to define r (and consequently r^2) in terms of z for a critical density matter dominated FLRW model:

$$r = 2 - \frac{2}{(1+z)^{1/2}}. \quad (23)$$

Eq. 22 and Eq. 23 will be used in the next section for the alternative mass-energy calculation that will extend the results of the z-dependent radiation component of the Universe calculated in the previous subsection.

III. DETAILS OF THE ALTERNATIVE CALCULATION

This section extends the calculations for a radiation dominated Universe in Sec. II C to the model of the matter and radiation dominated Universe using the alternative model for r and dr offered in Sec. II D. In Sec. III A the z-dependent proper volume element is extended to one appropriate for the matter and radiation dominated Universe model. This is integrated in Sec. III B to make the alternative mass calculation and the results are compared to the critical mass in Sec. IV.

A. Proper volume at time of observation and time of emission

Applying Eq. 22 to Eq. 10, based on the FLRW metric, the differential proper volume for the current time of observation for the matter dominated Universe of Sec. II D is:

$$dv = \frac{4\pi r^2}{(1+z)^{3/2}(\Omega_r z + 1)^{1/2}} \left(\frac{c}{H_0}\right)^3 dz. \quad (24)$$

Next, using Eq. 23, Eq. 24 becomes,

$$dv = \frac{16\pi}{(\Omega_r z + 1)^{1/2}} \left(\frac{c}{H_0}\right)^3 \left[\frac{1}{(1+z)^{3/2}} - \frac{2}{(1+z)^2} + \frac{1}{(1+z)^{5/2}}\right] dz. \quad (25)$$

Expressing Eq. 26 as a differential volume using proper distances at the time of emission requires dividing the above equation by a factor of $(1+z)^3$. The differential proper volume at the time of emission is then:

$$\begin{aligned} dV &= \frac{1}{(1+z)^3} dv \\ &= \frac{16\pi}{(\Omega_r z + 1)^{1/2}} \left(\frac{c}{H_0}\right)^3 \left[\frac{1}{(1+z)^{9/2}} - \frac{2}{(1+z)^5} + \frac{1}{(1+z)^{11/2}}\right] dz. \end{aligned} \quad (26)$$

B. Mass calculation

As with the differential and integrated mass calculations in Eq. 18 and Eq. 19 for the radiation dominated model, the differential proper volume at time of emission is integrated against the z-dependent mass-energy density $\rho_r(z) = \rho_r(1+z)^4$, where ρ_r is the equivalent mass density for the current observation of the CBR given by Eq. 9. The result is:

$$\begin{aligned} dM &= \rho_r(1+z)^4 dV \\ &= \frac{16\pi\rho_r}{(\Omega_r z + 1)^{1/2}} \left(\frac{c}{H_0}\right)^3 \left[\frac{1}{(1+z)^{1/2}} - \frac{2}{(1+z)} + \frac{1}{(1+z)^{3/2}}\right] dz. \end{aligned} \quad (27)$$

The indefinite integral of $dM(z)$ is:

$$\begin{aligned} M &= 32\pi\rho_r \left(\frac{c}{H_0}\right)^3 \left\{ \frac{\ln\left[(\Omega_r z + 1)^{1/2} + (\Omega_r z + \Omega_r)^{1/2}\right]}{\Omega_r^{1/2}} \right. \\ &\quad - \frac{1}{(1-\Omega_r)^{1/2}} \ln\left[\frac{(\Omega_r z + 1)^{1/2} - (1-\Omega_r)^{1/2}}{(\Omega_r z + 1)^{1/2} + (1-\Omega_r)^{1/2}}\right] \\ &\quad \left. + \frac{(\Omega_r z + 1)^{1/2}}{(\Omega_r - 1)(z+1)^{1/2}} \right\}. \end{aligned} \quad (28)$$

Using the value 5.317×10^{-5} for Ω_r and the value of ρ_r from Eq. 9, and evaluating the above expression at the upper limit of $z = 5.2 \times 10^{31}$) and the lower limit of $z = 0$ yields the mass of the CBR z-component, M_z:

$$M_z = 5.194 \times 10^{53} \text{kg}. \quad (29)$$

C. Fraction of critical mass

The critical mass in the standard cosmological model is equivalent to calculating the product of the critical mass density and the total volume of the Universe. In Eq. (2), the critical density for a spatially flat Universe was given as $\rho(\text{critical}) = 8.72 \times 10^{-27} \text{kgm}^{-3}$. The volume of the Universe depends on the choice of cosmological model. As noted after Eq. (6), the current proper volume of the

Universe based on the FLRW metric is $4/3\pi r^3(c/H_0)^3$, which depends on r and hence the choice of cosmological model. Because the alternative model is matter dominated (even though it involves both matter and radiation), taking $r = 2$ gives the following critical mass, M_c:

$$M_c = 7.33 \times 10^{53}\text{kg}. \tag{30}$$

The fraction of the critical mass that the z-dependent component of the CBR radiation accounts for is then:

$$\frac{M_z}{M_c} = 70.86\%. \tag{31}$$

This resultant mass-energy almost exactly equals the amount currently attributed to dark energy.

IV. COMPARATIVE ANALYSES

The calculation of the z-component M_z in the previous section is based on the dominant contribution noted in Eq. 20 suggest an extension of the standard cosmological model to include a new component associated with the CBR of the form $\Omega_z(1+z)^3\ln(1+z)$. The resulting equation for the derivative of r with respect to z is:

$$\begin{aligned} dr &= \left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4\right. \\ &\quad \left.+\Omega_v + \Omega_z(1+z)^3\ln(1+z)\right]^{-1/2} dz. \end{aligned} \tag{32}$$

Based on the previous calculations, the term Ω_z is expected to be near 0.7, which would make the vacuum energy term Ω_v small. In order to simplify the comparisons below, only two cases will be considered: (i) $\Omega_z = 0$ and $\Omega_v = 0.7$; and $\Omega_z = 0.7$ and $\Omega_v = 0$. For simplicity, these will be referred to as the standard model and the extended model. The mass and radiation fractions are fixed in both cases at $\Omega_r = 0.00005$ and $\Omega_m = 0.29995$.

A. Comparison with dark energy

In the standard cosmological model, dark energy is attributed to the vacuum energy, and is said to make up 69.6% of the critical density. Dark energy has been speculated to exist on the basis of two physical observations. The first observation is that there is not enough baryonic matter and dark matter to account for the critical density required (i.e. $\Omega_{\text{total}} = 1$) for the Universe to be spatially flat. The spatial curvature has been measured to be very close to zero and the critical density has been derived from Friedmann's equations [1]. By way of comparison, using the alternative calculation with no dark energy attributed to the vacuum energy and a model that has a volume equal to that of a matter dominated Universe; the integrated energy density of the CBR Eq. 29 and Eq. 31 also account for a fraction of missing mass-energy that complements baryonic and dark matter, i.e. 70.86%.

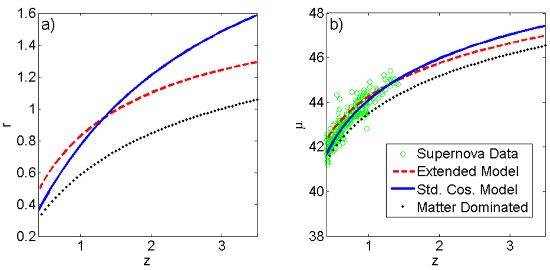

FIG. 1: a) comparison of co-moving distance predictions (r on the vertical axis) for the standard cosmological model (blue) and the extended cosmological model (red dash) as defined in Eq. 32; b) comparison of standard and extended cosmological models with Type 1A Supernova data (green circle) for $z = 0.4$ to 1.5; μ is distance modulus defined as in Eq. 33. Discrete values of the matter dominated model are provided for reference. Distance predictions from the extended model have good agreement with observation and compare closely with the standard model (up to $z = 1.0$).

B. Comparison to Type 1A Supernova Data

The extended model proposed in this paper can be tested against observable data provided by Type 1A Supernova's distance modulus provided in [15–18]. The distance D is related to brightness by the equation for the apparent magnitude of a distant light source and μ by:

$$\mu = 5\log D + 25 + K. \tag{33}$$

The distance D is measured in (1/10) parsecs (3.09×10^{16} meters) and with the K correction, $K = 5\log(1+z)$. The distance is calculated from the equation:

$$D = \frac{c}{H_0} r. \tag{34}$$

Fig. 1 a) provides a comparison of co-moving distance predictions (r on the vertical axis) for the standard cosmological and the extended cosmological models as defined in Eq. 32. Fig. 1 b) rescales distance to the distance modulus μ and includes Type 1A Supernova data from red shift values from z near 0.4 up to the furthest observation ($z = 1.4$) [15–18].

The distance calculations for the matter dominated model, has been calculated by the conventional practice, using the exact solution for distance based on the equation of state formula:

$$r = \frac{2(1-(1+z)^{-\frac{1+3w}{2}})}{1+3w}. \tag{35}$$

The equation of state for matter has $w = 0$. The distance calculations for the standard and the extended model used numerical integration of Eq. 32. The extended model for $z > 1$ is seen to remain close to the Supernova data but tends towards the matter dominated model.

V. CONCLUSION

In this preliminary report, an alternative cosmological model using only matter and radiation components, and an alternative calculation of the mass energy of the Universe has been offered based on a complete propagation history of photons from the CBR. The energy states of these photons are distributed from and integrated over the energy state associated with the temperature of the CBR currently observed and up to the past energy state associated with a redshift of $z = 5.2 \times 10^{31}$. Because the effect of gravity travels at the speed of light, according to general relativity [2–4], the higher energy states of the CBR in the past are seen to generate most of the gravity being experienced in the Universe today. This result suggests a new z-dependent radiation component with fractional density Ω_z that could account for a substantial fraction of the gravitational mass of the Universe.

Distance calculations have been presented in Sec. IV A, and have been compared to high redshift Type 1A Supernova data from references [15–18]. The calculations are in reasonable agreement with the data, and make a prediction that should be decidable or falsifiable at slightly higher redshift data points than the authors have access to. The calculation suggests an extension of the standard cosmological model in which co-moving distance at higher red shift increases with red shift more slowly than it does in the standard model. When compared with Type 1A Supernova data (in an available range of red shift from $z = 0.4$ to $z = 1.5$), distance predictions from the extended model have reasonable agreement with observation. The predictions also compare closely with the standard model (up to $z = 1.0$). Further observations are needed for $z > 1.5$ to make a final comparison between the standard model and the proposed extended model.

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