Sense Theory

(part 1)

[P-S Standard]

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Abstract.
Cognitive processes of human brain are strongly tied to such a well-known part of the brain as cortex. All psychological, logical or illogical solutions made by a human being are the result of the cortex. Thus, the maximum approximation of mathematical theory to the processes of the cortex can become a good trampoline to the creation of a self-learning intellectual system, a Real Artificial Intelligence.

We propose a new concept of mathematical theory which gives a possibility to form, find and separate senses of two or more objects of different nature. The theory encompasses the knowledge of cybernetics, linguistics, neurobiology, and classical mathematics. The Sense Theory is not a part of traditional mathematics as we know it now, it is a new paradigm of how we can formalize complex cognitive processes of the human brain.

1. Introduction

While the definition of artificial intelligence is unclear so far, we believe that cognitive characteristic is the main and first step anyone who creates AI should start from. This choice has one strong reason. Humans have five traditionally recognized senses, sight (vision), hearing (audition), taste (gustation), smell (olfaction), and touch (somatosensation). All the senses generate data that the brain needs to perceive and comprehend. Thus, mechanisms of data processing are crucial for such an important human act as decision making. That is why we consider the fundamentally different test in comparison with the Turing test.

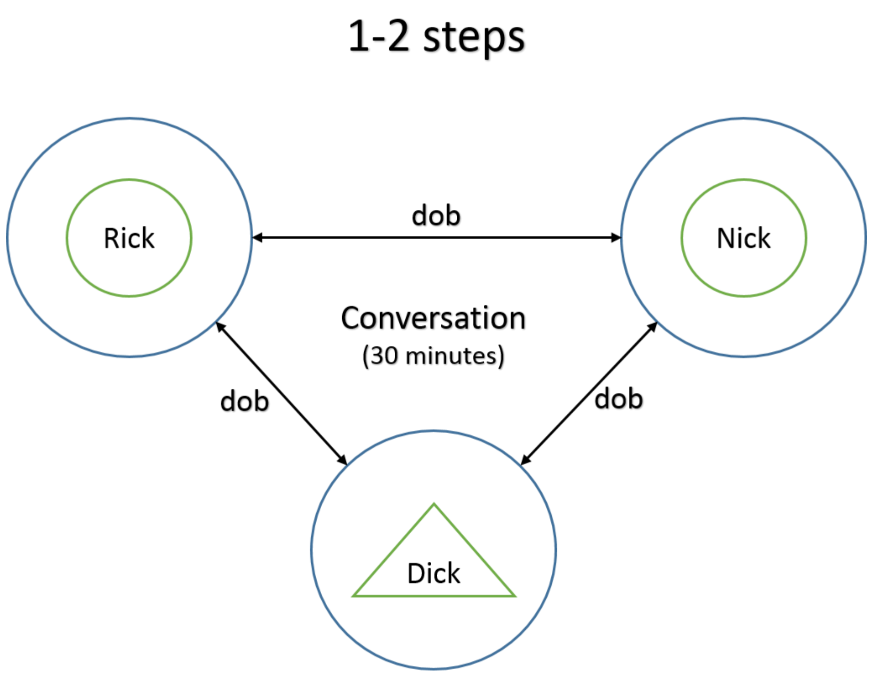
Test of Three Persons.

The test consists of the following steps:

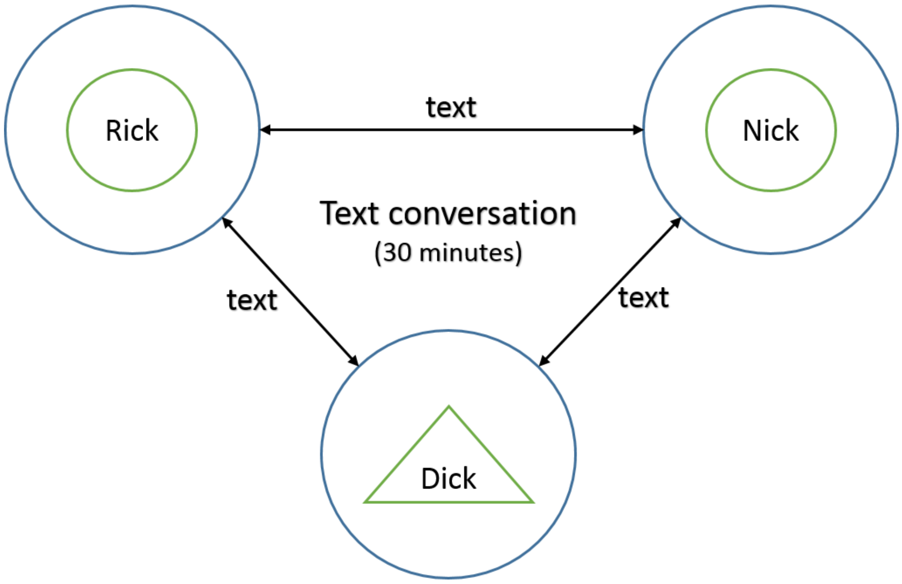
1. Three persons, Rick, Nick and Dick are taken. By arbitrary choice, one of the persons is substituted by a computer program (digital machine).
2. Three persons exchange their dates of birth and start joint conversation during the next 30 minutes.
3. A text with three arbitrary dates and numbers is exposed to the persons for reading.
4. Three persons start a conversation about the text during the next 30 minutes.
5. Each person is asked, “Who is the machine?” with one required sentence of answer explanation.
6. After the exposition of all answers for the persons, they are secondly asked, “Who wants to change the answer?”.
7. The answers of the persons are fixed and calculated. If the machine (Rick, Nick or Dick) was not chosen by other two persons simultaneously, the test is passed.

Remark: During the test, each person (man) can make notices.

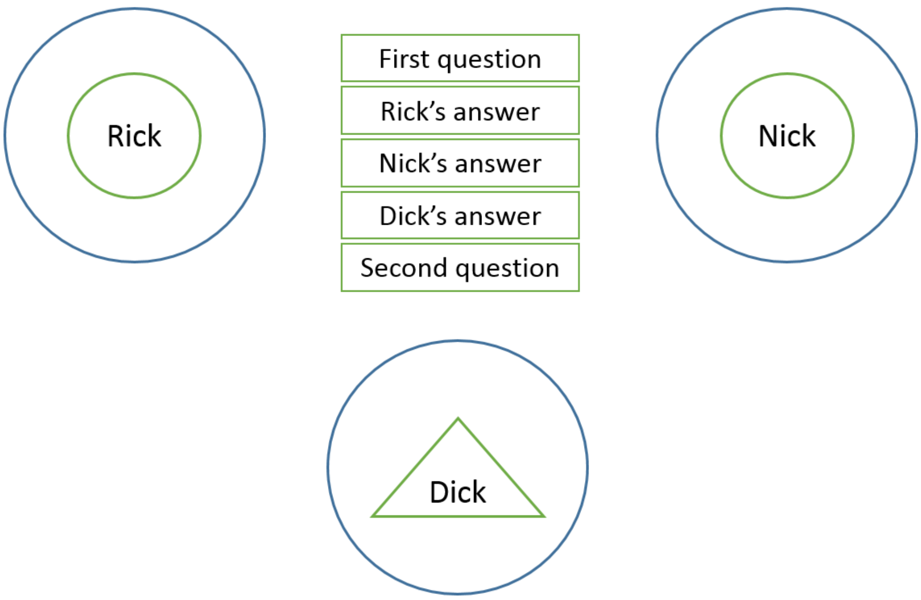
Graphically it can be shown as follows:



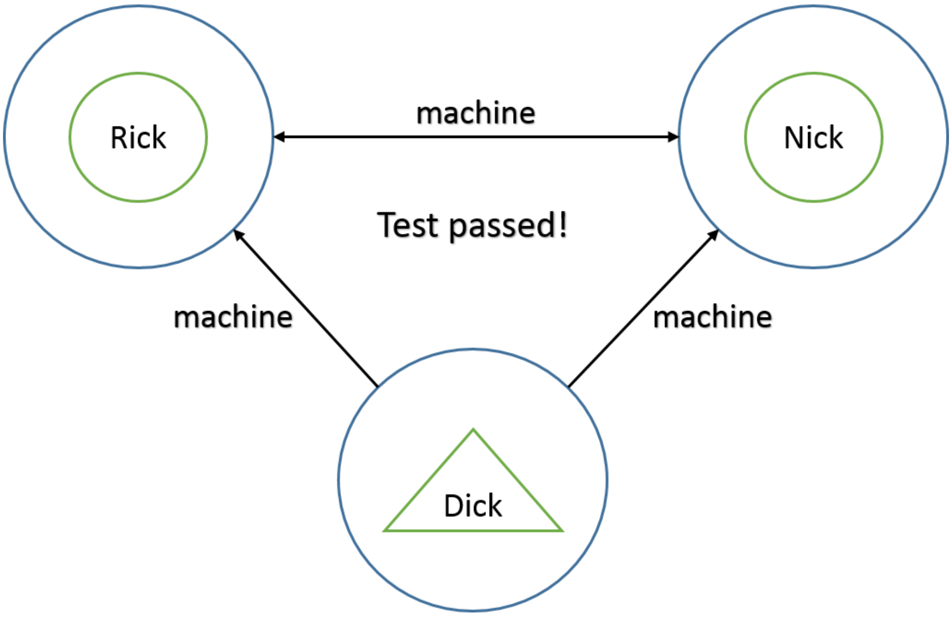
3-4 steps



5-6 steps

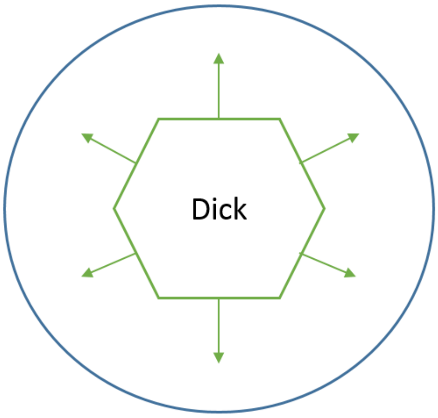


7 step



The passed test means only that the machine is capable of thinking intellectually. For the level of human thinking it needs to pass the test of N persons. We called the test of three persons as a *test of first order*. And the test of N persons as a *test of second order*. Importance of intellectual level for the machine can be compared with a process of approximation. In our context, the figure inside the machine circle has as many sides as the number of persons.

Test of six persons



So, an intellectual excellence of the machine can be reached as soon as the internal machine’s figure becomes the circle with minimum error.

2. Problem

Classical mathematics, namely its basis, classical mathematical logic is not capable of operating with such an object as a sense. In other words, with the qualitative properties of the object. However, it is crucial for building a self-learning system of any kind.

In mathematics, as we know it now, there are a number of direct proofs of the foregoing statement. We will briefly describe here only *two of them* which are the basic ones according to the author of this article.

<u>The law of the excluded middle (third).</u>

By simple words, according to this law, if statement A is true then statement which is opposite to the statement A is always false.

For example:

Statement A - "Bob is a stupid man".

Statement B - "Bob is a smart man".

Statement C - "Bob is a good man".

Statement D - “Bob is a bad man”.

So, if someone states that Bob is a stupid man, according to the law of the excluded middle Bob cannot be simultaneously a smart man. At first sight, it seems logical but as soon as we list the characteristics (properties) of a stupid man as well as a smart one we will necessarily bump into a contradiction. As a matter of fact, part of the properties of the smart (stupid) man can be the same one of any other (not stupid nor smart) man. In terms of mathematical logic, we have:

$$A \lor \neg A$$

or

$$A \lor B$$

where $\neg A = B$ without fail.

Further, if someone states state that Bob is a stupid but good man, we have:

$$A \land C$$

and

$$(A \wedge C) \vee \neg(A \wedge C)$$

or

$$(A \wedge C) \vee (B \wedge D)$$

But in the practical realization of any intellectual system, we frequently meet the situation when a man has several properties simultaneously or property that does not have direct opposite value. For example, the following expression cannot be firmly established or refuted:

$$\neg(B \wedge S \wedge G \wedge \ldots)$$

or

$$(B \wedge S \wedge G \wedge \ldots) \longrightarrow \neg\neg(B \wedge S \wedge G \wedge \ldots)$$

where S – statement “Bob is a shapely man”, G – statement “Bob is an elegant man”. Thus, the classical mathematical logic is good only for homogeneous objects that do not have qualitative properties.

<u>Gödel’s incompleteness theorems.</u>

Gödel’s theorems are a good example of the absence of a clear and single definition of what "negation operation" is all about. In classical logic, it is primarily used in the context of two possible values, "true" or "false". In this way, only propositions that can be evaluated by two states are possible for operation and analysis. Therefore, the negation operation is good if and only if the outcome of any proposition can take two opposite forms.

One of the Gödel’s theorem says that *we cannot derive two formulas*

$f(x)\ and\ \neg f(x)$ *simultaneously, where* $x \in N$.

But what exactly does “$\neg f(x)$” mean? Suppose we have the following series of values:

$$f(1), f(2), f(3), \ldots f(n)$$

Thus, all the above values are true. Then, “$\neg f(x)$” should mean situation when the values are false. In other words, $f(x)$ is undefined for $x \in N$. For example, if we take the following simple formula $f(x) = x,\ where\ x \in N$, then $\neg f(x) = x\ where\ x \in (Z \backslash N)$. In case of “$\neg$” means opposite value, $\neg f(x) \neq x$. Thus, we have two formulas and two sets. It clearly shows that

Gödel’s theorems as well as classical logic (its operators) are primarily focused on Boolean domain. In other words, it works only when a bijective function is defined.

In the context of the Sense Theory as well as any practical realization of a semantical live algorithm, there are more than two states for an object. For example, the object "device" can have more than one qualitative properties such as "plastic", "thin", etc. But in the context of sense, it is undefined if it does not have a single property. In practice, the Sense Theory operates multivalued functions.

Resuming above-said and what can be derived from it, all logical operators of the classical logic are primarily suiting to bijective sets. But it is absolutely not suited to the nature of cognitive processes as well as the Sense Theory.

3. Solution

At the core of the theory lies an object which has a qualitative property ('s). The object can be the nature of any kind. For example, a word "device" presents a template of some element with no relationship to any categorical context. As soon as we prefix the word "medicine" to the word "device", the corresponding context becomes evident. In this case, the word "medicine" is the qualitative property of the word "device".

$$O_1 = \text{"device"}\{\text{"}medicine\text{"}\},$$

where O – object,

index – quantity of object properties.

In case of prefixing more different-in-sense-words, we get the following notation:

$$O_7 = \text{"device"}\{\text{"}medicine\text{"}, \text{"}digit\text{"}, \text{"}accurate\text{"}, \text{"}blue\text{"}, \text{"}aluminum\text{"}, \text{"}waterproof\text{"}, \text{"}universe\text{"}\}$$

or

$$O_N,$$

where N – total quantity of object properties.

In terms of linguistics, the property of any object can be any part of speech.

The object that does not have any properties is called *zero object*:

$$O_0 = \text{"device"}\{\}$$

or

$$◎_{\text{device}}$$

A Sense Set (SS) of the objects is a plurality thought of as a sense unit.

Let us consider the following several objects:

$$O_N = \text{"frame"}\{"air", "aluminum", "plastic", \dots\},$$

$$O_N = \text{"chassis"}\{"titanium", "rings", "rubber", \dots\},$$

$$O_N = \text{"engine"}\{"reactive", "fuel", "power", \dots\},$$

$$O_N = \text{"cockpit"}\{"dashboard", "chairs", "parameters", \dots\},$$

$$O_N = \text{"..."}\{\dots\}$$

In the next step, we will consider the all above-mentioned objects as properties:

$$\{"frame", "chassis", "engine", "cockpit", \dots\}$$

This set of properties forms a *No-Sense Set* (NS):

$$\not{S} = \{"frame", "chassis", "engine", "cockpit", \dots\}$$

As it was said earlier, an object has a qualitative property (‘s). But some of them can be a zero ones with no properties at all.

Now, if we start to select a zero object iteratively, with high probability we will end up with the object “aircraft” or “airplane”.

$$O_0 = \text{"airplane"}\{\},$$

$$O_0 \Subset \not{S} \longrightarrow S$$

and

$$S = \text{(airplane)}\{"frame", "chassis", "engine", "cockpit", \dots\}$$

or

$$S = \text{“airplane”}\{\{"frame", "chassis", "engine", "cockpit", \dots\}\}$$

where S – Sense Set.

Unlike zero object, Sense Set cannot be empty as it is a result of “inclusion” of two elements, zero object and No-Sense Set.

Definition 1: S is a Sense Set if and only if the following expression is true:

$$S = \{\circledcirc_N \Subset \not{S}_K\}$$

where N,K = {1,2,3,…n}, K ≥ N, K,N – finite numbers.

Definition 2:

$\not{S}$ is a No-Sense Set if and only if the following expression is true:

$$\not{S}_N = \{qualitative\ properties\}_N$$

where N – finite number.

Definition 3: O_0 is a zero or empty object if and only if the following expression is true:

$$O_0 = \circledcirc \Subset \not{S}$$

Definition 4:

$\not{S}_O$ is an Object No-Sense Set if and only if the following expression is true:

$$\not{S}_{O(N)} = \{O_N\}$$

where N – finite number of objects.

Definition 5:

S_C is a Complete Sense Set if and only if the following expression is true:

$$S_C = \circledcirc \Subset \not{S}_{O(N)}$$

Definition 6:

$S_{\not\subset}$ or S is an Incomplete Sense Set if and only if the following expression is true:

$$S_{\not\subset} = \circledcirc \subset \not{S}_N$$

where

$$\not{S}_N \overset{\not{E}}{|\!<=>\!|} \not{S}_{O(N)}$$

Definition 7:

Subset of S_C is each object of $\not{S}_{O(N)}$.

$S_{\not\subset}$ can have only one subset of first order.

Definition 8:

Power of S_C is equal to number of objects of $\not{S}_O$.

$$P_S = N \; (for \; \not{S}_{O(N)}),$$

$$P_S = N + 1 \; (for \; \not{S}_{O(N+1)}),$$

$$P_S = N + 2 \; (for \; \not{S}_{O(N+2)}), etc.$$

Definition 9: Object A semantically connects to Object B if the following expression is true:

$$\not{S}(A)_N \overset{E}{|\!<=>\!|} \not{S}(B)_N$$

"Semantic connection" (SC) – is measured by percent. The following formula is used:

$$SC_{\%} = \frac{N_S * 100}{N_M}$$

where N_S – number of similar properties of both objects,

N_M – number of properties of largest object.

In order to formulate the first Axiom of the Sense Theory, we need to enter such definitions as "sense sequence" and "sense limit".

Definition 10: The set A of $a_1, a_2, a_3, \dots a_n$ elements is a *sense sequence* if and only if there is at least a single zero object O_0 that satisfies the following expression:

$$◎_A ⋐ A = S$$

Definition 11: The *sense limit* of the set A is the zero object of a Sense Set.

In other words, an object the properties of which are the elements of the set A is a sense limit of that set.

It has the following notation:

$$\lim_S A_N = ◎_A$$

where $A_N = \{a_1, a_2, a_3, \dots a_n\}$.

For the subset of the set A, we can have two outcomes:

$$\lim_S sub(A_N)_M = ◎_A$$

or

$$\lim_S sub(A_N)_M \neq ◎_A$$

Moreover, after a single application of either ⩌ or ⊖ operator to the set, its sense limit can drastically be changed.

The Axiom of Object Limit:

"Every object of Sense Set consists of two parts, *zero object* and *sense sequence*, where the first one is a sense limit of the second one."

The following two expressions are equivalent:

$$\lim_{S} A_N \Subset A_N,$$

$$A_N \Subset \lim_{S} A_N$$

The Axiom of Object Equality:

Object O_M is equal to Object O_L if the following expression is true:

$$\not{S}_M \overset{E}{|\!<\!=\!>\!|} \not{S}_L$$

or

$$\not{S}_{O(M)} \overset{E}{|\!<\!=\!>\!|} \not{S}_{O(L)}$$

The Axiom of Set Equality:

The Sense Set S_M is equal to the Sense Set S_L if the following expression is true:

$$S_M(\circledcirc) \overset{E}{|\!<\!=\!>\!|} S_L(\circledcirc)$$

or

$$S_{O(M)}(\circledcirc) \overset{E}{|\!<\!=\!>\!|} S_{O(L)}(\circledcirc)$$

The Axiom of Semantic Union (left-to-right):

1. For any two No-Sense Sets, $\not{S}_M$ and $\not{S}_L$ there is such $\not{S}_K$ the properties of which are both, the properties of $\not{S}_M$ and $\not{S}_L$:

$$\not{S}_K = \not{S}_M \Cup \not{S}_L$$

where K = M + L.

2. For any two Objects, $O_{1(M)}$ and $O_{1(L)}$, there is such $O_{2(K)}$ that the following two expressions are true:

$$◎_L \xrightarrow{S}\!| \; ◎_M$$

$$\not{S}_K = \not{S}_M \uplus \not{S}_L$$

3. For any two Sense Sets, $S_{1(M)}$ and $S_{1(L)}$ there is such $S_{2(K)}$ that the following two expressions are true:

$$S_{1(L)}(◎) \xrightarrow{S}\!| \; S_{1(M)}(◎)$$

$$\not{S}_K = \not{S}_M \uplus \not{S}_L$$

<u>The Axiom of Semantic Subset:</u>

Any Object O_K can be only one of two types of subsets for any Sense Set S_N:

<u>Subset of first order:</u>

$$O_K \xrightarrow{S}\!| \; S_N(◎)$$

where K = N.

<u>Subset of second order:</u>

$$\lim_{S} \not{S}_K \xrightarrow{S}\!| \; O_K(◎)$$

where K = N or K ≠ N.

<u>The Axiom of Set of Subsets:</u>

"There are at least N+1 subsets for any $S_{O(N)}$."

Theorem (Existence of Set)**.**

"The Sense Set S_N ($S_{O(N)}$) is defined if and only if there is a sense limit of

$$\not{S}_N \quad (\not{S}_{O(N)})$$

."

Proof.

For any given S_K there are two elements, $◎_S$ and $\not{S}_K$ by definition. Now, presume that there is no sense limit of $\not{S}_K$. In symbolic notation, it presents the following:

$$\lim_{S} S_K \neq ◎_S$$

and

$$◎_S \Subset \not{S}_K \neq S_K$$

The latter expression contradicts the definition of Sense Set. The theorem is proven.

Theorem (Existence of Subsets)**.**

"There is at least one subset for S_N and N+1 subsets for $S_{O(N)}$."

Proof.

$$S_N\!: ◎_S \Subset \not{S}_N \Rightarrow \not{S}_N \neq \{O_N\} \Rightarrow \lim_{S} \not{S}_N \overset{S}{\longrightarrow}\!| \; O_K = ◎_S$$

$$S_{O(N)}\!: ◎_S \Subset \not{S}_{O(N)} \Rightarrow \not{S}_{O(N)} = \{O_N\} \Rightarrow \lim_{S} \{O_N\} = O_K = S_{O(N)}(◎) = ◎_S$$

Further, $O_N = \{O_1', O_2', O_3', \ldots O_n'\}_{n=N}$ =>

$\lim_{S} \not{S}_1' = O_1'(◎)$

$\lim_{S} \not{S}_2' = O_2'(◎)$

$\lim_{S} \not{S}_3' = O_3'(◎)$

… … … … … …

$\lim_{S} \not{S}_n' = O_n'(◎) \;\; \Rightarrow sub\left(S_{O(N)}\right) = \{◎_S,\ O_1',\ O_2',\ O_3',\ \ldots O_n'\}_{N+1}$

The theorem is proven.

4. Conclusion

In this article, we presented the new “mathematical” theory with own signature. Unlike classical mathematical or intuitionistic logic, the Sense Logic which is the basis for the Sense Theory can drastically improve understanding methods and possible algorithms in the creation of human-like AI.

We hope that our decent work will help other AI researchers in their life endeavors.

To be continued.

Appendix

$\Subset$ - “inclusion”, binary operation

1. $O_{f(1)} \Subset O_{p(1)} = O_{p(1)} \Subset O_{f(1)}$ (“O” commutativity)
2. $\circledcirc \Subset \not{S} = \not{S} \Subset \circledcirc$ (“O/NS” commutativity)
3. $S_{f(N)} \Subset O_p \neq O_p \Subset S_{f(N)}$ (“S/O” inequality)

Operation that does not make a sense:

1. $\not{S}_M \Subset \not{S}_L$
2. $\not{S} \Subset O_N$ **or** $O_N \Subset \not{S}$
3. $\circledcirc \Subset S$ **or** $S \Subset \circledcirc$
4. $\not{S} \Subset S$ **or** $S \Subset \not{S}$
5. $\circledcirc \Subset O_N$ **or** $O_N \Subset \circledcirc$
6. $\circledcirc \Subset \circledcirc$

$\uplus$ - “semantic union”, binary operation

1. $\not{S}_M \uplus \not{S}_L = \not{S}_L \uplus \not{S}_M$ (“NS” commutativity)
2. $\not{S} \uplus O_N = O_N \uplus \not{S}$ (“O/NS” commutativity)
3. $\circledcirc_A \uplus \circledcirc_B \neq \circledcirc_B \uplus \circledcirc_A$
4. $O_M \uplus O_L \neq O_L \uplus O_M$
5. $S_M \uplus O_L \neq O_L \uplus S_M$

Operation that does not make a sense:

1. $\circledcirc \mathbin{\cup\kern-0.6em\circ} \not{S}$ **or** $\not{S} \mathbin{\cup\kern-0.6em\circ} \circledcirc$

$\mathbin{\Subset\kern-0.8em\ominus}$ - “exclusion”, binary operation

1. $O_{fp(2)} \mathbin{\Subset\kern-0.8em\ominus} O_{p(1)} \neq O_{pf(2)} \mathbin{\Subset\kern-0.8em\ominus} O_{f(1)}$
2. $\circledcirc \mathbin{\Subset\kern-0.8em\ominus} \not{S} \neq \not{S} \mathbin{\Subset\kern-0.8em\ominus} \circledcirc$
3. $S_{fp(N+1)} \mathbin{\Subset\kern-0.8em\ominus} O_p \neq O_p \mathbin{\Subset\kern-0.8em\circ} S_{fp(N+1)}$

Operation that does not make a sense:

1. $\not{S}_{2(M)} \mathbin{\Subset\kern-0.8em\ominus} \not{S}_{1(L)}$
2. $\not{S} \mathbin{\Subset\kern-0.8em\ominus} O_N$ **or** $O_N \mathbin{\Subset\kern-0.8em\ominus} \not{S}$
3. $\circledcirc \mathbin{\Subset\kern-0.8em\ominus} S$ **or** $S \mathbin{\Subset\kern-0.8em\ominus} \circledcirc$
4. $\not{S} \mathbin{\Subset\kern-0.8em\ominus} S$ **or** $S \mathbin{\Subset\kern-0.8em\ominus} \not{S}$
5. $\circledcirc \mathbin{\Subset\kern-0.8em\ominus} O_N$ **or** $O_N \mathbin{\Subset\kern-0.8em\ominus} \circledcirc$
6. $\circledcirc \mathbin{\Subset\kern-0.8em\ominus} \circledcirc$

$\mathbin{\cup\kern-0.7em\ominus}$ - “semantic disunion”, binary operation

1. $\not{S}_N \mathbin{\cup\kern-0.7em\ominus} \not{S}_L \neq \not{S}_L \mathbin{\cup\kern-0.7em\ominus} \not{S}_N$
2. $\not{S}_N \mathbin{\cup\kern-0.7em\ominus} O_K \neq O_K \mathbin{\cup\kern-0.7em\ominus} \not{S}_N$
3. $\circledcirc_{1(2)} \mathbin{\cup\kern-0.7em\ominus} \circledcirc_1 \neq \circledcirc_1 \mathbin{\cup\kern-0.7em\ominus} \circledcirc_{1(2)}$
4. $O_N \mathbin{\cup\kern-0.7em\ominus} O_L \neq O_L \mathbin{\cup\kern-0.7em\ominus} O_N$
5. $S_N \mathbin{\cup\kern-0.7em\ominus} O_K \neq O_K \mathbin{\cup\kern-0.7em\ominus} S_N$

Operation that does not make a sense:

1. $\circledcirc \ \overset{\ominus}{\cup} \ \not{S}$ **or** $\not{S} \ \overset{\ominus}{\cup} \ \circledcirc$

$\overset{\circ}{\cap}$ - “semantic intersection”, binary operation

1. $\not{S}_N \ \overset{\circ}{\cap} \ S_N \ \ (it.2)$
2. $\not{S}_N \overset{\circ}{\cap} O_N$ *if and only if* $\not{S}_N \ |\overset{E}{\Leftrightarrow}| \ \not{S}_N(O_N)$
3. $\circledcirc_{1(M)} \ \overset{\circ}{\cap} \ \circledcirc_{1(L)}$ *if and only if* $\circledcirc_{1(M)} \ |\overset{E}{\Leftrightarrow}| \ \circledcirc_{1(L)}$
4. $O_N \ \overset{\circ}{\cap} \ \circledcirc$ *if and only if* $\circledcirc \ |\overset{E}{\Leftrightarrow}| \ O_N(\circledcirc)$
5. $S_N \ \overset{\circ}{\cap} \ \circledcirc$ *if and only if* $S_N(\circledcirc) \ |\overset{E}{\Leftrightarrow}| \ \circledcirc$
6. $O_N \ \overset{\circ}{\cap} \ O_K$ *if and only if* $O_N(\circledcirc) \ |\overset{E}{\Leftrightarrow}| \ O_K(\circledcirc)$
7. $O_N \ \overset{\circ}{\cap} \ S_K \ \ (it.6)$
8. $\not{S}_N \overset{\circ}{\cap} \not{S}_K$ *if and only if* $\not{S}_N \ |\overset{E}{\Leftrightarrow}| \ \not{S}_K$

Operation that does not make a sense:

1. $\circledcirc \ \overset{\circ}{\cap} \ \not{S}$ **or** $\not{S} \ \overset{\circ}{\cap} \ \circledcirc$

“Semantic Intersection” is commutative for all operands.

$\overset{S}{\subset}$ - “semantic subset”, binary operation

1. $S_K \ \overset{S}{\subset} \ S_N$ *if* $O_K \in \not{S}_{O(N)}$
2. $O_M \overset{S}{\subset} O_L$ *if* $O_M \in \not{S}_{O(L)}$

$\overset{S}{\not\subset}$ - “no semantic subset”, binary operation

1. $S_K \overset{S}{\not\subset} S_N$ *if* $O_K \notin \not{S}_{O(N)}$
2. $O_M \quad O_L$ *if* $O_M \notin \not{S}_{O(L)}$

$\overset{E}{|{<}{=}{>}|}$ - “equivalence”, binary operation

Object O_N is equivalent to Object O_K if the following two conditions are met:

1. length($\not{S}_{(N)}$) = length($\not{S}_{(K)}$).
2. $O_N(\circledcirc) \in \not{S}_K$ *and* $O_K(\circledcirc) \in \not{S}_N$ *(can be)*

$\overset{S}{\rightarrow|}$ - “semantic connection”, binary operation

Definition 9.

$\overset{S}{\neg}$ - “semantic negation”, unary operation

$\overset{S}{\neg} O_N$ means any Object O_K that satisfies the following condition:

$$O_K \ \overset{\not{E}}{|{<}{=}{>}|} \ O_N$$

<u>Associativity (“inclusion”):</u>

$$(O_{A(1)} \Subset O_{B(1)}) \Subset O_{C(1)} = O_{A(1)} \Subset (O_{B(1)} \Subset O_{C(1)})$$

<u>Associativity (“semantic union”):</u>

$$(O_A \mathbin{\cup\!\!\!\!\!\;\circ}\ O_B)\ \mathbin{\cup\!\!\!\!\!\;\circ}\ O_C = O_A \mathbin{\cup\!\!\!\!\!\;\circ} (O_B \mathbin{\cup\!\!\!\!\!\;\circ}\ O_C)$$

Associativity (“semantic disunion”):

$$(O_A \mathbin{\cup\!\!\!\!\!\;\ominus}\ O_B)\ \mathbin{\cup\!\!\!\!\!\;\ominus}\ O_C = O_A \mathbin{\cup\!\!\!\!\!\;\ominus} (O_B \mathbin{\cup\!\!\!\!\!\;\ominus}\ O_C)$$

References

More than 100 books and articles.