An Interpretation of The Identity $0.999999...... = 1$

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Abstract: In this short paper, we will give a very simple and important interpretation for the identity: $0.999999...... = 1$, because we have many questions for the identity from general people. Furthermore, even mathematicians and mathematics teachers will see an interesting interpretation in this paper.

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1 Introduction

On January 8, 2008, Yuusuke Maede, 8 years old boy, asked the question, at Gunma University, that (Announcement 9(2007/9/1): Education for genius boys and girls): What does it mean by the identity:

$$0.999999...... = 1?$$

At the same time, he said: I am most interesting in the structure of large prime numbers. Then, a teacher answered for the question by the popular reason based on the convergence of the series: $0.9, 0.99, 0.999, ...$ Its answer seems to be not suitable for the 8 years old boy with his parents (not mathematicians). Our answer seems to have a general interest, and after then, such our answer has not been heard from many mathematicians, indeed. This is why writing this paper.

2 An interpretation

In order to see the essence of the question, we will consider the simplest case:

$$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...=1. \tag{2.1}$$

Imagine a tape of one meter length, we will give you its half tape: that is,

$$\frac{1}{2}. \tag{2.2}$$

Next, we will give you its (the rest's half) half tape; that is, $\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{2^2}$, then you have, altogether

$$\frac{1}{2}+\frac{1}{2^2}. \tag{2.3}$$

Next, we will give you the last one's half (the rest's half); that is, $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{2^3}$, then, you have, altogether

$$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}. \tag{2.4}$$

By this procedure, you will be able to obtain the small tapes endressly. Imagine all the sum as in the left hand side of (2.1). However, we will see that this sum is just the division of the one meter tape. Therefore, we will be able to confim the identity (2.1), clearly. The question proposed by Y. Maede is just the small change the ratio $\frac{1}{2}$ by $\frac{1}{10}$ as in

$$\frac{9}{10}+\frac{9}{10}\frac{1}{10}+\frac{9}{10}\frac{1}{10^2}+.....$$

.

3 Conclusion

Y. Maede asked the true sense of the limit in the series:

$$0.999999.....$$

that is, this series is approaching to 1; however, is it equal or not ? The above interpretation means that the infinite series equals to one and it is just the infinite division of one. By this inverse approarch, the question will make clear.

4 Remarks

Y. Maede stated a conjecture that for any prime number p ($p \geqq 7$), for 1 of $p-1$

$$11111111111 \tag{4.1}$$

may be divided by p (2011.2.6.12:00 at University of Aveiro, by skype) (No.81, May 2012(pdf 432kb) www.jams.or.jp/kaiho/kaiho-81.pdf) ([3]).

This conjecture was proved by Professors L. Castro and Y. Sawano, independently. Y. Maede gave later an interesting interpretation for his conjecture (2015.2.26).

5 In addition from division by zero

We will state an interesting fact from the viewpoint of the division by zero in connection with this paper problem. Of course, the summation

$$1 + 2 + 3 + ...$$

diverges to infinity in the usual sense. However, we can consider that

$$1 + 2 + 3 + ... = 0$$

in some reasonable sense, because the point at infinity may be represented by zero. Look [2, 5] for the details. See also [1, 4].

References

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