Pi Formula

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Abstract. In this note we recall a formula for Pi.

1. Formula

Entry 1. If $\alpha = 1+\cos\left(3\cos\left(3\cos\left(3...\right)\right)\right) = 0.020633202...$, then

$$\pi = 3-3\alpha+2\tan^{-1}\sqrt{\frac{\alpha}{2-\alpha}} \tag{1}$$

Entry 2.

$$\pi = -3\cos\left(3\cos\left(3\cos\left(3...\right)\right)\right)+2\tan^{-1}\sqrt{\frac{1+\cos\left(3\cos\left(3\cos\left(3...\right)\right)\right)}{1-\cos\left(3\cos\left(3\cos\left(3...\right)\right)\right)}} \tag{2}$$

2. Roots of : $x = \cos\left(3x\right), x \in \mathbb{R}$

Entry 3. If $x = \cos\left(3x\right), x \in \mathbb{R}$ then

$$x = \begin{cases} a = -0.9793667979902706... \\ b = -0.8877262944545929... \\ c = 0.3900403166675420... \end{cases} \tag{3}$$

Entry 4. Graphics:

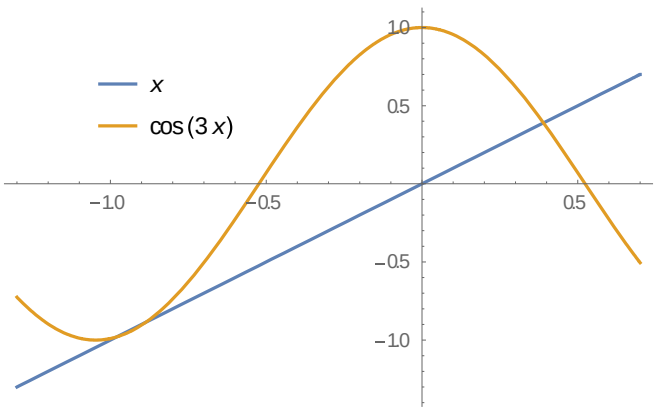

Figure 1.

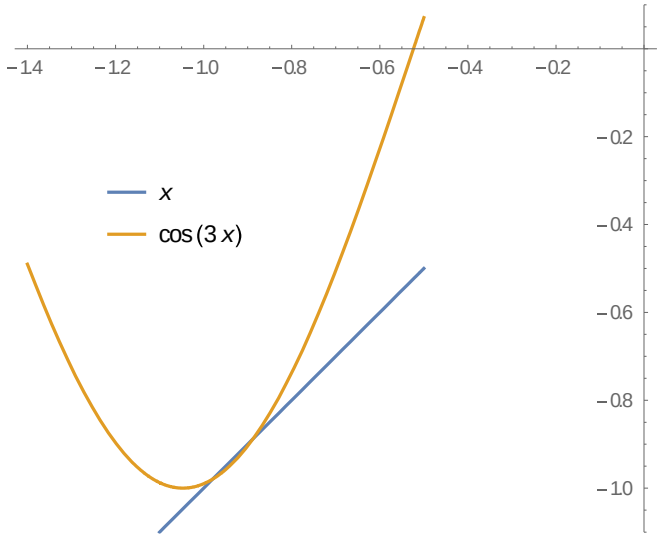

Figure 2.

3. Iteration

Entry 5.

$$x_{n+1} = \cos\left(3x_n\right) \quad , x_0 = 0 \Rightarrow x_n \to a = -0.979366... \tag{4}$$

Entry 6. Graphics:

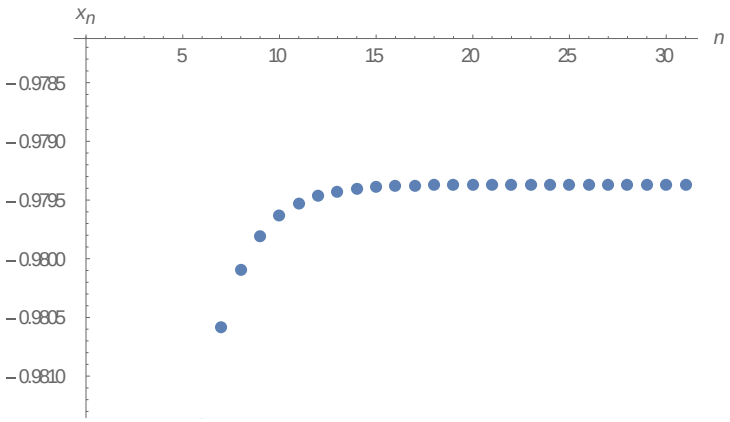

Figure 3.

References

1. Arndt, J. , and Haenel, C. : π unleashed. Springer-Verlag , 2001.