Three Numbers

Edgar Valdebenito

April 15 , 2019

abstract
This note presents some formulas for Pi.

1. Three Numbers : s,u,v

Entry 1.

$$s=\frac{1}{3}+\frac{1}{6}\left(8+6\sqrt{78}\right)^{1/3}-\frac{7}{3}\left(8+6\sqrt{78}\right)^{-1/3} \tag{1}$$

$$u=\frac{1}{12}+\frac{1}{12}\left(181+24\sqrt{78}\right)^{1/3}-\frac{23}{12}\left(181+24\sqrt{78}\right)^{-1/3} \tag{2}$$

$$v=\sqrt{\frac{5}{6}-\frac{1}{6}\left(181+24\sqrt{78}\right)^{1/3}+\frac{23}{6}\left(181+24\sqrt{78}\right)^{-1/3}} \tag{3}$$

Entry 2.

$$2s^3-2s^2+3s-1=0 \tag{4}$$

$$4u^3-u^2+2u-1=0 \tag{5}$$

$$2v^6-5v^4+8v^2-1=0 \tag{6}$$

Entry 3.

$$\pi=4\tan^{-1}s+4\sin^{-1}s \tag{7}$$

$$\pi=4\sum_{n=0}^{\infty}\frac{s^{2n+1}2^{-2n}}{2n+1}\left(\binom{2n}{n}+(-4)^n\right) \tag{8}$$

$$\pi=4\tan^{-1}s+4\tan^{-1}u \tag{9}$$

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left(s^{2n+1}+u^{2n+1}\right) \tag{10}$$

$$\pi=4\sin^{-1}s+4\sin^{-1}v \tag{11}$$

$$\pi=4\sum_{n=0}^{\infty}\binom{2n}{n}\frac{2^{-2n}}{2n+1}\left(s^{2n+1}+v^{2n+1}\right) \tag{12}$$

References

1. Arndt, J., and Haenel, C.: π unleashed. Springer – Verlag , 2001.
2. Beckmann, P.: A History of π . 2nd ed., Golem Press, Boulder, CO, 1971.
3. Blatner, D.: The Joy of π . Walker Publ., 1999.