Finally, a Unified Theory!
Collision Space-Time: The Missing Piece of Matter
Gravity is Lorentz and Heisenberg Breaks Down at the Planck Scale
Gravity without G

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Abstract

Based on a very simple model where mass, at the deepest level, is colliding indivisible particles and energy is indivisible particles not colliding, we get a new and simple model of matter that seems to be consistent with experiments. Gravity appears to be directly linked to collision time and also the space the collisions take up; we could call it collision space-time. This leads to a completely new quantum gravity theory that is able to explain and predict all major gravity phenomena without any knowledge of Newton's gravitational constant or the mass size in the traditional sense. In addition, the Planck constant is not needed.

Our model, combined with experimental data, strongly indicates that matter is granular and consists of indivisible particles that are colliding. Further, from experiments it is clear that the diameter of the indivisible indivisible particle is the Planck length. Our theory even predicts that there can be no time dilation in quasars, something that is consistent with observations and yet is inconsistent with existing gravity theories.

Several modern quantum gravity models indicate that Lorentz symmetry is broken at the Planck scale, but there have been no signs of this occurring, despite extensive efforts to look for Lorentz symmetry break downs. We show that Lorentz symmetry break downs indeed happen and, to our own surprise, this is actually quite easy to detect. In our model, it is clear that Lorentz symmetry break down is gravity itself. This seems contradictory, as Planck energies are very high energy levels, but we show that this must be seen in a new perspective.

We also introduce a new quantum wave equation that tells us that gravity is both Lorentz symmetry break down and Heisenberg uncertainty break down at the Planck scale. Our wave equation in this sense includes gravity. For masses less than a Planck mass, probability will also dominate gravity, it is then a probability for Heisenberg uncertainty break down. At Planck mass size and up, determinism dominates.

For the first time, we have a simple quantum theory that unifies gravity with the quantum, all derived from a very simple model about the quantum. Our theory is simple, and we show that an indivisible particle is the fundamental unit of all mass and energy – a quantity that has been missing in physics all the time. Newton was one of the last great physicists who thought that such particle was essential, but it was naturally impossible for one man to solve the entire problem. This paper stands on the shoulders of giants like Newton, Einstein, Planck, and Compton to explore these long-standing questions.

The beauty of our theory is that it keeps almost all existing and well-tested equations completely intact (unchanged) all the way to the Planck scale. Anything else would be a big surprise; after all, some areas of physics have been extremely successful in predictions and have been well-tested. Still, in our work, the Planck scale and all equations are united into one simple and powerful theory. Unlike standard physics, there are no inconsistencies in our theory. QM is unified with gravity, and even a simplified version of the Minkowski space-time is consistent with QM and gravity. A long series of mysteries in QM vanish, under our new interpretation.

Key Words: Quantum gravity, granular matter, Lorentz symmetry break down at the Planck scale, Heisenberg uncertainty break down at the Planck scale, indivisible particles, gravity and Lorentz symmetry break down.

1 Introduction to Our New Theory

We are suggesting a new model for understanding matter that leads to a simple quantum gravity theory where all major gravity phenomena can be described and predicted without any knowledge of Newton's gravitational constant. Further, for the first time, we get a quantum wave equation that is consistent and actually indirectly predicts gravity. We postulate that all matter and energy at the deepest level only consist of

- Indivisible particles moving at a constant (unknown) speed in the empty space, but standing absolute still at collision with each other.
- Empty space the indivisible particles can move in.

When such an indivisible particle is moving it is (pure) energy and when it is colliding with another particle it is pure mass. That is, the collision itself is mass. It is a totally binary system. This simple theory seems to explain such things as 'wave-particle" duality in both light and matter. For simplicity's sake, we can think of the indivisible particle as being sphere shaped. This particle is indestructible, something consistent with ancient atomist ideas, and also ideas held by Isaac Newton, who stated:

> *All these things being consider'd it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, movable Particles, of such Sizes and Figures, and in such Proportion to Space, as most conduce to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces; no ordinary Power being able to divide what God himself made one in the first Creation. While the Particles continue entire, they may compose bodies of one and the same Nature and Texture in all Ages; But should they wear away, or break in pieces, the Nature of Things depending on them, would be changed. Those minute rondures, swimming in space, from the stuff of the world: the solid, coloured table I write on, no, less than the thin invisible air I breathe, is constructed out of small colourless corpuscles; the world at close quarters looks like the night sky – a few dots of stuff, scattered sporadically through and empty vastness. Such is modern corpuscularianism.*

The point is that even Newton took such indivisible particles seriously as the constitute of matter and energy, an idea going back at least 2,500 years to the Greek atomists, see [1–7], for example.

The so-called wavelength in light we will claim is simply the empty space distance between indivisible particles traveling after each other in the same direction. Again, mass is constituted by collisions between indivisible particles, and this is the pure mass. This means mass will have two important aspects at the deepest level, namely the number of collisions in an observational time window and the length of each collision. The length of each collision can be measured as time or as length, that is in space or time. In observed elementary particles, such as electrons, we envision a minimum of two indivisible particles traveling back and forth over the reduced Compton wavelength of the particle at the speed of light and then colliding with each other at the reduced Compton time interval. An electron will, therefore, consist of this most fundamental mass that is the collision point between two building blocks of photons at the following times per second

$$f_e = \frac{c}{\bar{\lambda}} \approx 1.23 \times 10^{20} \tag{1}$$

This means the sum of the collision masses, m_x must add up to the experimentally well-known electron mass

$$\begin{aligned} \frac{c}{\lambda_e} m_x &= m_e \approx 9.1 \times 10^{-31} \text{kg} \\ m_x &= m_e \frac{\lambda_e}{c} \approx 7.37 \times 10^{-51} \text{ kg} \end{aligned} \tag{2}$$

We clearly see that the collision mass m_x is observational time window dependent in this model, as it is the electron mass multiplied by a time interval, namely the Compton time of the electron. The electron mass is considered to be observable time-independent. This means the collision mass, m_x must be time dependent, because one can assume we measure the mass of the electron in half a second instead of a second; then the internal frequency must be simply half of what it was in a one second observational time window, that is

$$f_e = \frac{1}{2} \frac{c}{\bar{\lambda}} \approx 1.23 \times 10^{20} \approx 3.88 \times 10^{20} \tag{3}$$

This means that in order to still have the mass as 9.1×10^{-31}kg, the collision mass must now be reduced to twice the size of what it was earlier. That is, the collision mass is observational time dependent. An interesting question is then,"If there exists a shortest possible time interval, what will the mass be then?" This is something we will return to later.

We will discover that standard mass measures such as the kg are far from optimal and we would say even a bit primitive, as they contain a limited amount of information about what exactly mass (matter) is. The reduced Compton [8] wavelength is given by

$$\bar{\lambda} = \frac{\hbar}{mc} \tag{4}$$

solved with respect to m this gives

$$m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{5}$$

This means the mass can be found if we know the Compton wavelength, the Planck constant, and the speed of light (well known). For example, [9] has shown that one can find the rest-mass energy of the electron simply by extracting the Compton wavelength through Compton scattering. This means the Compton wavelength is the only property that changes between different masses (elementary particles), as the Planck constant and the speed of light are constants. Still, the formula does not seem to give us much intuition about exactly what mass is. However, we will suggest that the Planck constant is linked to the number of collisions per second in one kg of matter.

$$f_{1kg} = \frac{c}{\bar{\lambda}} = \frac{c}{\frac{\hbar}{1\text{ kg }\times c}} = \frac{c^2}{\hbar} = 8.52 \times 10^{50} \tag{6}$$

An electron has an internal collision frequency per second of

$$f_e = \frac{c}{\bar{\lambda}_e} \approx 7.76 \times 10^{20} \tag{7}$$

Our mass definition in kg is then simply the number of collisions in a particle relative to the number of collisions in one kg, for an electron this is

$$m_e = \frac{f_e}{f_{1kg}} = \frac{7.76 \times 10^{20}}{8.52 \times 10^{50}} \approx 9.1 \times 10^{-31}\text{kg} \tag{8}$$

So, in our view kg is then simply the collision frequency ratio. In ancient times, a practical quantity of matter (a clump of matter) was chosen to standardize the quantity (weight) of mass. To do this, the mass could not be too small, as this in an age of inaccurate weights would give too much uncertainty for practical purposes. Neither would it be practical to work with too large a clump of matter, as that would be heavy to move around, and in those days, weights in the form of clumps of matter were actively used by merchants. Weight measures were used to standardize trade and to make sure one could compare prices on the same quantity. This led to the establishment of one kg as the standard for a clump of matter (an example of which has even been stored in Paris) and until recently have been the universal standard. Recently, analysis has offered the idea that the Planck constant creates the weight standard, as the Watt balance can be used to measure it very accurately and from this we can find one kg, see [10–12].

For observation time intervals considerably longer than the Compton time, the mass of elementary particles is observational time independent. For example, in half a second the number of collisions in an electron is $\frac{1}{2} \times 7.76 \times 10^{20}$, and the number of collisions in one kg will be reduced in half, so the ratio stays the same. Thus, the collision ratio is observational time independent in this case, as long as the observational time window is much longer than the Compton time. Therefore, it may look like mass is not related to time, but it is. Next let us get close to an observational time of the reduced Compton time of the electron. For example, if we are observing the electron in a time window of $1.5 \times \frac{\bar{\lambda}}{c}$, then the number of observations in the electron will still only be one, since there are only collisions at every whole reduced Compton time. However, the number of collisions in the one kg during this time window is

$$8.52 \times 10^{50} \times 1.5\frac{\lambda_e}{c} \approx 1.64665 \times 10^{30} \tag{9}$$

That is, the observed weight of the electron will now be reduced to

$$m_e = \frac{1}{1.64665 \times 10^{30}} = 6.07 \times 10^{-31} \tag{10}$$

So, all elementary masses are observational time dependent if the observational time-interval is short enough. If we plan to observe the electron in a time interval shorter than the electron Compton time and do not know when the last time it had an internal collision was, then its mass will be probabilistic. A fraction of a collision is a probability, as we only can work with integer numbers of collisions; there cannot be half a collision, there have to be N collisions, where N is an integer, or no collision at all.

The smallest possible mass is one collision. Compared to the number of collisions in one kg, we get the collision ratio

$$m_g = \frac{f_g}{f_{1kg}} = \frac{1}{1.37 \times 10^{50}} \approx 1.17 \times 10^{-51}\text{ kg} \tag{11}$$

However, if we reduce the observational time window but keep the frequency of 1 constant, then its mass will keep increasing. A particle with frequency of only 1 is what we can call the mass gap, and it is indeed 1.17×10^{-51} kg per one second observational time window. If the observational time window is half a second, the mass gap will have twice the collision ratio above. This because only the number of collisions in one kg will be reduced. Assume for a moment the shortest possible time interval is the Planck time, then the collision frequency of the mass gap will be

$$m_g = \frac{1}{.37 \times 10^{50} \times t_p} = \frac{1}{.37 \times 10^{50} \times \frac{l_p}{c}} = 2.17 \times 10^{-8} \text{ kg} \quad (12)$$

which is the Planck mass. However, modern physics disagrees on whether the Planck time is the shortest time-interval and therefore if the Planck length is the shortest distance. Some physicists think there could be a unit smaller than the Planck length, see [13–15], while others maintain that there should be no minimum length at all – that zero is the minimum. Nevertheless, the majority of physicists seem to agree that there is a minimum length and that it likely is the Planck length [16–20]. Still, modern physics claims that the Planck units can only be found using the three so-called universal constants, namely the Newton gravitational constant G, the speed of light c, and the Planck constant. We will show that this is not the case. Since the Planck constant is linked to mass and G is linked to gravity, one already has some hint that the Planck scale is linked to mass and gravity as well as the speed of light, or even the speed of gravity, as they are considered to be the same. Recently, Haug has shown that the Planck units can be extracted from gravity experiments with no knowledge of the gravitational constant, see [21, 22]. After extensive searching through various methods of extracting the Planck length, we find that they all involve matter and gravity. In this paper, we provide deeper insight into why the Planck length can only be found from gravity and why it is, in fact, the very essence of gravity.

In our view, the kg only capture one of two important aspects of mass, namely the number of collisions, and it is, as we have shown, a collision ratio, which is a new perspective on the topic. However, even if we accept the idea that the kg definition (or any weight) definition of mass is actually a ratio of collisions, we are still missing out on the other important aspect of matter, which is how long each collision lasts and how often there are collisions as opposed to there being no collisions in a elementary particle or a given amount of matter. We will suggest that collision time is directly linked to gravity, although this not incorporated into today's mass model, it is there in the physical world. As we continue the analysis, we will see how current gravity theories are indirectly getting this aspect into their models, even the Newton model without being aware of it.

In the next sections, we will show how a quantum theory can be built from the ground up, based on granular matter, that is to say, indivisible particles, and we explain how this is fully consistent with all major gravity phenomena. We can easily calibrate our model to gravity observations without any prior knowledge of the Newton gravitational constant and our model even correctly predicts such things as there being no time dilations in quasars, which has been observed but does not appear to have given a good explanation in current gravity theories. We will also show how Lorentz symmetry break down at the Planck scale is actually gravity. Naturally, extraordinary claims require extraordinarily good evidence, but we will offer strong support for this theory, and encourage rigorous investigation by the physics community.

2 Mass as Collision Time per Shortest Time Interval

Assume the indivisible particle has an unknown diameter of x. Since, we will claim, all matter and even the particle aspects of photons are made up of such a unit, then this diameter must be incredibly small. Also, we assume that the indivisible particle, when not colliding, moves at a speed of c. Or, more precisely, we assume that when it is not colliding, it will move at the distance of its own diameter in the same period of time that two indivisibles will spend in collision. This would explain the deeper aspects of why there is a maximum speed limit. The idea is that there is ultimately only one particle and it can move its own diameter during the period two indivisible particles spend in collision. Since this is the ultimate particle that all other particles consist of, then it must also be the fastest particle. Again, one should not confuse this with the conventional thinking about such particles. This is a massless particle, but because collisions between such particles are what we call mass, then such a particle has mass when it is colliding and is massless when it is not. It can either move or stand absolutely still, but these are the only choices. This will appear to be in conflict with well-established ideas including the relativity of simultaneity, but it is important to study the whole framework carefully without rejecting the theory prematurely.

In our model, mass has two important properties. Mass is a collision, and the number of collisions per time unit will depend on the mass size. How long the collisions last, we will claim, is directly linked to gravity. As modern mass is a collision ratio only, it misses out of the central part of gravity, namely collision time. Newton and Einstein gravity must incorporate this into their gravity model using a gravity constant, which is a calibrated constant needed to get the models to fit observations, even if it in unclear what it truly represents. Further, we find some mysterious units, which are $m^3 \cdot kg^{-1} \cdot s^{-2}$. The universe did not invent such constants; it much more likely consists of fundamental building blocks, and the chance that any of these building blocks are $m^3 \cdot kg^{-1} \cdot s^{-2}$ is unlikely. Still, the gravitational constant is "clearly" universal and important in the existing gravity models, but the fact that even with all of the work completed over the past century, we have not been able to unite ideas about the quantum world and gravity. Modern gravity theory does include collision time in the theoretical model in an unnecessarily complex and perhaps unaware way; this is something we soon will get back to in this paper. Returning to our analysis, the collision time per shortest time interval is given by

$$\tilde{m}_t = \frac{\frac{x}{c}}{\frac{\bar{\lambda}}{c}}\frac{x}{c} = \frac{x}{c}\frac{c}{\bar{\lambda}}\frac{x}{c} = \frac{x}{\bar{\lambda}}\frac{x}{c} \tag{13}$$

This will be one of our our two mass measures. The part $\frac{\frac{x}{c}}{\frac{\bar{\lambda}}{c}} = \frac{x}{\bar{\lambda}}$ is simply the percentage of the time the mass is in collision, and we have to multiply this with the time it takes to collide $\frac{x}{c}$ to find the collision time over the shortest possible time interval. Again, this is based on the assumption that the indivisible particle can travel its own diameter in the period during which two indivisible particles are in collision. Alternatively, we can express the mass

$$\tilde{m}_L = \frac{\frac{x}{c}}{\frac{\bar{\lambda}}{c}}x = \frac{x}{c}\frac{c}{\bar{\lambda}}x = \frac{x}{\bar{\lambda}}x \tag{14}$$

This is the collision time divided by the non-collision time, so it is the collision time ratio multiplied by the length of the indivisible particle. This is what we can call "collision space." Further, the mass in form of length, divided by the mass in form of collision time should give us the speed of light (collision space-time)

$$c = \frac{\tilde{m}_L}{\tilde{m}_t} \tag{15}$$

Our model is quite interesting because it means that mass is directly linked to space and time and therefore also to speed. We could even call it the collision space-time model of matter. Still, it is important to understand that it is always quantized, and that pure mass comes in integer units.

However, this theory will only hold true if one indivisible particle travels its own diameter during the period two other indivisible particles spend in collision. If, for example, an indivisible particle that is not in collision only travels half of its own diameter during the period in which two indivisible particles are colliding, then our model will be off, and will not fit observations. Whether or not our model fits observations, we will examine a bit later. In addition, our model provides insight about the speed of light; the speed of light is simply the length that an indivisible particle can travel (when not colliding) during the period two indivisible particles are spending time in collision. Here we are indirectly suggesting that this distance is the particle's own diameter. If this holds true, it must lead to a theory that fits observations.

3 Gravity Is Directly Linked to Collision Time (Length)

If our theory is right, in order to find the mass in kg we need to multiply our mass with $\frac{\hbar}{x^2}$. However, by doing this we actually lose important information; we are removing information about the diameter of the indivisible particle that we claim is essential to find the collision time (length) in any mass. And the Planck constant is needed only to make the mass into something relative to an arbitrary quantity of matter (like one kg or one pound). For example, the mass of an electron in kg must, from our model, be

$$m = \tilde{m}\frac{\hbar}{x^2} = \frac{x}{c}\frac{c}{\lambda_e}\frac{x}{c} \times \frac{h}{x^2} = \frac{h}{\lambda_e}\frac{1}{c} \approx 9.1 \text{ kg} \tag{16}$$

However, the mass as kg (or pound or any such) we will claim only takes into account the number of internal collisions in a particle; as we have seen p that kg simply is a collision ratio, and it does not say anything about how long the collisions last. How long the collisions last is the key to gravity, in fact, it is gravity. With mass as collision length, we will claim the gravity force formula should be

$$F = c^3\frac{\tilde{M}_t\tilde{m}_t}{R^2} \tag{17}$$

That is, the gravity constant is now simply the speed of light squared and the masses are the collision length per shortest time interval – the masses are now unknown since we do not know the diameter of the indivisible particle. Since the mass at a deeper level is $\tilde{m}_t = \frac{x}{\bar{\lambda}}\frac{x}{c}$, then we can rewrite the gravity formula as

$$F = c\frac{\frac{x^2}{\bar{\lambda}_M}\frac{x^2}{\bar{\lambda}_m}}{R^2} \tag{18}$$

Thus, x is the unknown diameter of the particle, c is the speed of light (and gravity), and $\bar{\lambda}_M$ and $\bar{\lambda}_m$ are the reduced Compton wavelength of the small and the large masses. We have a simple quantum gravity model, so far in the non-relativistic world, in a later section we will add relativistic effects to it. We will claim all the parameters in the model can be found without knowledge of G or $\hbar$, and as we will show later, even without knowledge of c.

4 Finding the Parameters for our Gravity Model

In our gravity formula described in the last section, we need to find the unknown diameter of the indivisible particle and the reduced Compton wavelength of the small and large masses. To do this, we first measure the Compton length of an electron by Compton scattering and find it is $\bar{\lambda}_e$. We are not going to measure gravity only on an electron, but this helps us finding the reduced Compton wavelength for large masses. Further, the cyclotron frequency is linearly proportional to the reduced Compton frequency. Conducting a cyclotron experiment, one can find the reduced Compton frequency ratio between the proton and the electron. For example, [23] measured it to be about

$$\frac{\frac{c}{\bar{\lambda}_P}}{\frac{c}{\bar{\lambda}_e}} = \frac{f_P}{f_e} = 1836.152470(76) \tag{19}$$

.

In fact, they measured the proton-electron mass ratio this way and not the mass in kg. Interestingly, the reduced Compton frequency is only a deeper aspect of mass that has recently been more or less confirmed by experimental research, see [24, 25]. Theoretically, it is no surprise that $\frac{f_P}{f_e} = \frac{m_P}{m_e}$. This also holds true in our mass definition

$$\begin{aligned} \frac{f_P}{f_e} &= \frac{\tilde{m}_P}{\tilde{m}_e} \\ \frac{f_P}{f_e} &= \frac{\frac{l_p^2}{\bar{\lambda}_P}\frac{1}{c}}{\frac{l_p^2}{\bar{\lambda}_e}\frac{1}{c}} = \frac{\bar{\lambda}_e}{\bar{\lambda}_P} \end{aligned} \tag{20}$$

That is, we can find the Compton length of an electron and also a proton without any knowledge of $\hbar$, or traditional mass measures such as kg. Now, to find the Compton frequency and the reduced Compton length in larger amounts of matter we just need to count the amounts of protons and electrons in them. Twice the mass has twice the Compton frequency.

We will claim that the diameter of the indivisible particle is directly linked to the time it takes for collisions and that the collision space-time is what we call gravity. We must therefore perform a gravity measure to calibrate our model. After we have calibrated the model once, it should give us the one and unknown diameter of the indivisible particle x. We should then be able to predict all other known gravity phenomena based on the model.

To calibrate the model, we will use a Cavendish apparatus first developed by Henry Cavendish, [26]. Assume we count 3×10^{26} number of protons and add them in a clump of matter. This clump of matter we will divide in two and use as two large balls in the Cavendish apparatus. We now know that the Compton frequency in the large balls in the Cavendish apparatus are approximately $1836.15 \times 1.5 \times 10^{26} = 2.13 \times 10^{50}$ per second. The reduced Compton length must then be $\bar{\lambda}_M = \frac{f}{c} = \frac{2.13 \times 10^{50}}{c} \approx 1.4 \times 10^{-42}$ m. This Compton wavelength is even smaller than the Planck length, something that we soon will understand is physically impossible. But it is important to be aware we are working with a composite mass consisting of many elementary particles. Even though a composite mass does not have one physical Compton wavelength (it has many), they can mathematically be aggregated in the following way

$$\bar{\lambda} = \frac{\hbar}{\sum_{i=1}^{N} m_i c} = \frac{1}{\frac{1}{\bar{\lambda}_i} + \frac{1}{\bar{\lambda}_{i+1}} + \frac{1}{\bar{\lambda}_n}} \tag{21}$$

So, the reduced Compton length of any mass we can find by direct measurements of elementary particles and then count the number of such particles in a larger mass. However, there is still an unknown parameter, namely the diameter of our suggested indivisible particles. Combining our new theory of matter and gravity with a torsion balance (Cavendish apparatus), we can measure the unknown diameter of the indivisible particle. We have that

$$\kappa\theta \tag{22}$$

where κ is the torsion coefficient of the suspending wire and θ is the deflection angle of the balance. We then have the following well-known relationship

$$\kappa\theta = LF \tag{23}$$

where L is the length between the two small balls in the apparatus. Further, F can be set equal to our gravity force formula, but with a Compton view of matter and therefore no need for Newton's gravitational constant, this is important to help us bypass the need for the Planck constant as well. Our Newton-equivalent gravity formula is equal to

$$F = c^3\frac{\tilde{M}_t\tilde{m}_t}{R^2} = c^3\frac{\frac{x^2}{\lambda_M}\frac{1}{c}\frac{x^2}{\lambda_m}\frac{1}{c}}{R^2} \tag{24}$$

where x is unknown. This means we must have

$$\kappa\theta = Lc^3\frac{\tilde{M}\tilde{m}}{R^2} \tag{25}$$

We also have that the natural resonant oscillation period of a torsion balance is given by

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \tag{26}$$

Further, the moment of inertia I of the balance is given by

$$I = \tilde{m}\left(\frac{L}{2}\right)^2 + \tilde{m}\left(\frac{L}{2}\right)^2 = 2\tilde{m}\left(\frac{L}{2}\right)^2 = \frac{\tilde{m}L^2}{2} \tag{27}$$

this means we have

$$T = 2\pi\sqrt{\frac{\tilde{m}L^2}{2\kappa}} \tag{28}$$

and when solved with respect to κ, this gives

$$\begin{aligned}
\frac{T^2}{2^2\pi^2} &= \frac{\tilde{m}L^2}{2\kappa} \\
\kappa &= \frac{\tilde{m}L^2}{2\frac{T^2}{2^2\pi^2}} \\
\kappa &= \frac{\tilde{m}L^2 2\pi^2}{T^2}
\end{aligned} \tag{29}$$

Next, in equation 25 we are replacing κ with this expression

$$\begin{aligned}
\frac{\tilde{m}_t L^2 2\pi^2}{T^2}\theta &= Lc^3\frac{\tilde{M}_t\tilde{m}_t}{R^2} \\
\frac{L^2 2\pi^2}{T^2}\theta &= Lc^3\frac{\tilde{M}_t}{R^2}
\end{aligned} \tag{30}$$

Next remember our mass definition is $\tilde{M}_t = \frac{x^2}{\lambda}\frac{1}{c}$, which we now replace in the equation above and solving with respect to the unknown diameter of the particle, we get

$$\begin{aligned}
\frac{L^2 2\pi^2}{T^2}\theta &= Lc^3\frac{\frac{x^2}{\lambda}\frac{1}{c}}{R^2} \\
\frac{L^2 2\pi^2}{T^2}\theta &= Lx^2\frac{\frac{c^2}{\lambda}}{R^2} \\
\frac{L2\pi^2 R^2}{T^2\frac{c^2}{\lambda}}\theta &= x^2 \\
x &= \sqrt{\frac{L2\pi^2 R^2}{T^2\frac{c^2}{\lambda}}\theta} \\
x &= \sqrt{\frac{L2\pi^2 R^2\theta}{T^2 f_C c}}
\end{aligned} \tag{31}$$

where f_C is the reduced Compton frequency of the mass in question, that we earlier have shown how to find. Experimentally, one will find that x must be the Planck length and that the standard error in measurements is half of that of using Newtonian theory in combination with Cavendish.

5 History and Deeper Insight into the Newton Gravity Constant

Before returning to our new quantum gravity theory, we will take a closer look at the Newtonian theory. Despite the fact that Newton himself actually never introduced or used a gravitational constant, today the so-called Newton gravity constant, also known as big G, seems almost holy and untouchable. Newton's [27] gravity formula was simply

$$F = \frac{Mm}{R^2} \tag{32}$$

That is, that the gravity force is proportional to the masses multiplied divided by the square root of the center to center distance. Other physicists have had similar ideas, including Hooke. The gravity constant was first indirectly measured in 1798 by Cavendish using a torsion balance apparatus, also known as Cavendish apparatus [26]. Cavendish used this to measure the weight of the Earth. And in 1873, the Newton gravity formula as it is known today was first formally described by Cornu and Baille [28] using the Newton constant, namely

$$F = f\frac{Mm}{R^2} \tag{33}$$

In the 1890s, the gravity constant was first called G, but many physicists still called it f in the early 1900s, see, for example, [29]. The gravity constant is, in modern physics, actually a constant that is found by calibrating the Newton model to fit observations. However, the gravity constant is heavily dependent on the definition of mass and our understanding (or we could even say our lack of understanding) of the nature of mass. It is a parameter that captures what one missed and this is fully understandable, as one has to start someplace. Still, in our view, little progress has been achieved since the time of Newton in understanding gravity at a deeper level. General relativity simply adapted the gravitational constant from Newtonian gravity.

Besides being a parameter needed to calibrate the Newtonian formula (and GR) to fit data the Newton gravity constant gives little intuition. That the constant does not seem to vary naturally indicates that it is related to something at a deeper level that is unchangeable. But could it really be $m^3 \cdot kg^{-1} \cdot s^{-2}$?

In several papers, [30–32] we have suggested that the Newton gravity constant is a composite constant of the form

$$G = \frac{l_p^2 c^3}{\hbar} \tag{34}$$

This can be found simply by solving the Planck length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ of Planck with respect to G. It is then easy to think this is just creating a circular problem, as from the Planck formula we need G in order to find the Planck length. However, as we have shown, the Planck length plays an essential role in matter and energy, and it can be found without any knowledge of G and the Planck constant [21, 22, 33]. In gravity we can do without both the gravity constant and the Planck constant. The gravity constant is only needed when one wants to go from gravity, which is a property of mass, namely collison time (length) between indivisible particles.

The standard mass definition model is incomplete, the gravity constant that is embedded contains the Planck constant, the Planck length, and the speed of light. The Planck constant is actually needed to get rid of the Planck constant embedded in the mass to perform gravity calculations, the Planck length needs to be introduced, and the speed of gravity c, which is the speed of the indivisible particle.

6 Performing other Gravity Predictions

Now that we have found that x is the Planck length, we can use our gravity model to predict other gravity phenomena. In our new quantum gravity theory, for example, the gravity acceleration field is found by

$$\begin{aligned}
\tilde{m}a &= c^3\frac{\tilde{M}\tilde{m}}{R^2} \\
a &= \frac{c^3}{R^2}\tilde{M} \\
g = a &= c^3\frac{\frac{l_p^2}{\bar{\lambda}c}}{} = \frac{c^2}{R^2}l_p\frac{l_p}{\bar{\lambda}}
\end{aligned} \tag{35}$$

Similarly, we can derive all other major gravity phenomena. Table 1 shows our gravity theory for our two different mass definitions and its prediction side by side with Newton's gravity theory. As we can see, the models are identical at a deeper level. Again, in the Newton theory one must understand that G is a composite constant needed to get the Planck constant out of the mass and to get the Planck length into the mass. Our new theory, on the other hand, has a much deeper understanding of mass and can do this directly when building our theory up from a quantum understanding of mass. This is essential when we later will address issues in quantum mechanics. It is important to keep in mind that so far, we have not taken relativistic effects into account. The results below are, therefore, only weak field approximations in several cases, such as for escape velocity.

	Modern "Newton"	**Alternative-1**	**Alternative-2**
Mass seen as	Compton frequency relative to Compton frequency kg	Collision time per shortest time interval	Collision space per shortest time interval
Mass mathematically	$M = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$	$\tilde{M}_t = \frac{l_p}{c}\frac{l_p}{\bar{\lambda}}$	$\bar{M}_L = l_p\frac{l_p}{\bar{\lambda}}$
Gravity constant	$G = \frac{l_p^2 c^3}{\hbar}$	$\tilde{G}_t = c^3$	$\bar{G}_L = c^2$
Non "observable" predictions:			
Gravity force	$F = G\frac{Mm}{R^2}$	$\tilde{F} = c^3\frac{\tilde{M}_t\tilde{m}_t}{R^2}$	$\bar{F} = c^2\frac{\bar{M}_L\bar{m}_L}{R^2}$
Gravity force	$F = \frac{\hbar c}{R^2}\frac{l_p}{\bar{\lambda}_M}\frac{l_p}{\bar{\lambda}_m}$	$\tilde{F} = \frac{c}{R^2}\frac{l_p^2}{\bar{\lambda}_M}\frac{l_p^2}{\bar{\lambda}_m}$	$\bar{F} = \frac{c^2}{R^2}\frac{l_p^2}{\bar{\lambda}_M}\frac{l_p^2}{\bar{\lambda}_m}$
Observable predictions:			
Gravity acceleration	$g = c^2\frac{l_p}{R^2}\frac{l_p}{\bar{\lambda}}$	$g = c^2\frac{l_p}{R^2}\frac{l_p}{\bar{\lambda}}$	$g = c^2\frac{l_p}{R^2}\frac{l_p}{\bar{\lambda}}$
Orbital velocity	$v_o = c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}}}$	$v_o = c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}}}$	$v_o = c\sqrt{\frac{l_p}{R}\frac{l_p}{\bar{\lambda}}}$
Escape velocity	$v_e = c\sqrt{2\frac{l_p}{R}\frac{l_p}{\bar{\lambda}}}$	$v_e = c\sqrt{2\frac{l_p}{R}\frac{l_p}{\bar{\lambda}}}$	$v_e = c\sqrt{2\frac{l_p}{R}\frac{l_p}{\bar{\lambda}}}$
Time dilation	$T_R = T_f\sqrt{1-\frac{v_e^2}{c^2}}$	$T_R = T_f\sqrt{1-\frac{v_e^2}{c^2}}$	$T_R = T_f\sqrt{1-\frac{v_e^2}{c^2}}$
Gravitational red-shift	$z(r) = \frac{l_p}{r}\frac{l_p}{\bar{\lambda}}$	$z(r) = \frac{l_p}{r}\frac{l_p}{\bar{\lambda}}$	$z(r) = \frac{l_p}{r}\frac{l_p}{\bar{\lambda}}$
Schwarzschild radius	$r_s = 2l_p\frac{l_p}{\bar{\lambda}}$	$r_s = 2l_p\frac{l_p}{\bar{\lambda}} = 2\tilde{M}_t$	$r_s = 2l_p\frac{l_p}{\bar{\lambda}} = 2\bar{M}_L c$
Energy	$E = Mc^2$	$E = \tilde{M}_t c^2$	$\bar{E} = \bar{M}_L c^2$

Table 1: The table shows the Newton gravitational force in addition to two alternative Newton-type theories, but with different gravitational constants that all predict the same results.

Note that our mass definition is closely linked to the Schwarzschild radius. This is no coincidence. However, we will claim that the Schwarzschild radius is grossly misunderstood in standard physics. It is said represent a radius of a black hole, but it actually represents the collision time ratio multiplied by the Planck length. The Schwarzschild radius is a very key component of mass and gravity; it is the essence of all mass, and even if the collision point has mathematical properties identical to a black hole, it has little to do with the standard interpretation of black holes.

If our theory is right, then the Schwarzschild radius should easily be extracted by observing gravity with no knowledge off G and the Planck constant. We will return to this idea later.

7 Finding the Mass (Schwarzschild radius) of Any Particle without Knowing G or $\hbar$ (and thereby also being able to predict all gravity phenomena without G)

The Schwarzschild radius can be found from the Schwarzschild metric of Einstein's field equation, see [34–36] and is normally given as

$$r_s = \frac{2GM}{c^2} \tag{36}$$

Newton weak field gravity theory gives an identical radius, as pointed out by Michell in 1784, see [37]. However, the standard derivation from Newton of a radius where the escape velocity is c must be wrong, as it is derived from a kinetic energy of the form $\frac{1}{2}mv^2$, which can only hold when $v << c$. We will show that our new quantum theory gives a relativistic Schwarzschild radius identical to GR, but with a very different interpretation.

As we know, GR adopted G from Newton, but, as we have claimed, G is only needed to get the Planck constant out of the mass, and to get the Planck length into the mass. This means that G is not always required to perform gravity calculations; it is only needed when we are working with a definition of mass that does not incorporate the collision time between indivisible particles. The Schwarzschild radius can be found directly for cosmological-sized objects simply by

$$r_s = 2g\frac{R^2}{c^2} \tag{37}$$

where the gravitational acceleration g, the radius from the center of the gravitational object (for example from Earth), and the speed of light can be found totally independent of any knowledge of G, or even any knowledge of our traditional mass measure M. When the Schwarzschild radius is found, all known gravity phenomena can be found from it. Only when we want to return to our incomplete mass measure do we need G. Otherwise, G is never needed. It is the Planck length that is essential for gravity and it is inside the mass, as mass consists of indivisible particles with a diameter equal to the Planck length that are in a collision with each other. Many physicists will likely say this way of finding the Schwarzschild radius experimentally without knowledge of G is obvious, but examining it in greater detail leads to many interesting insights in other areas.

We can even extract the mass (the Schwarzschild radius) of the Earth directly from a beam of light in the gravitational field with no knowledge of G, the Planck constant, or the traditional mass measure, as recently published by [38]

$$\tilde{m} = \frac{r_s}{2} = \frac{l_p}{\lambda}\frac{l_p}{c} = \frac{1}{2c}\frac{R_L R_h(\lambda_h - \lambda_L)}{(\lambda_h R_h - \lambda_L R_L)} \tag{38}$$

That is the collision time, that is our mass definition is simply its Schwarzschild radius divided by $2c$. This is no coincidence.

8 Our Gravity Model Is a Deterministic and Probabilistic Model

For example, half the Schwarzschild radius for any mass is given by

$$\frac{1}{2}r_s = l_p\frac{l_p}{\bar{\lambda}} \tag{39}$$

when we are considering a Planck mass particle, then $\bar{\lambda} = l_p$, and half the Schwarzschild radius is then the Planck length. This, we claim, is the diameter of the indivisible particle, and it is the radius of the Planck mass particle that consists of two colliding indivisible particles. The collision lasts for one Planck second and the two indivisible particles colliding must be observed inside that time period. Its half Schwarzschild radius is, therefore, the Planck length, and it is something physical and real, even if it is not possible to measure directly with current technology. Indirectly we measure it all the time, as it is what we call gravity. It is mostly likely a shielding effect, so gravity could indeed be a push shielding gravity. That is, the push is the collision, but the collision also hinders any indivisible particle to go through the collision moment, so it is also a shielding gravity.

For an electron, we have $\bar{\lambda} = \bar{\lambda}_e$, the reduced Compton wavelength of the electron is enormous compared to the Planck length, and we claim the electron must consist of at a minimum two indivisible particles moving back and forth over the reduced Compton wavelength of the electron and colliding every Compton time periodicity; the collision itself lasts for one Planck second. This means that particles with mass lower than a Planck mass also have half a Schwarzschild radius equal to the Planck length, but that this half a Schwarzschild radius comes in and out of existence. If we observe an electron in any given Planck second, we do not know if it is in a collision state or not before we complete the observation. That is, particles with mass less than a Planck mass will have a probabilistic Schwarzschild radius. When $\frac{l_p}{\bar{\lambda}} < 1$, this should be interpreted a probability for having a Schwarzschild radius inside one Planck second. All known observable elementary particles have a mass much smaller than a Planck mass and therefore a probabilistic Schwarzschild radius and a probabilistic gravity. When $\frac{l_p}{\bar{\lambda}} > 1$, this means we have to work with a composite mass. The integer part of $\frac{l_p}{\bar{\lambda}}$ is the number of full Planck masses we have, something that leads to deterministic gravity, while the remaining fraction (if any) is a probabilistic part. For any mass considerably larger than a Planck mass, gravity will be deterministic, and for a mass close to a Planck mass, it will be partly deterministic and partly probabilistic, while for a mass much smaller than a Planck mass such as a proton or an electron, probability will dominate. These are not mystical quantum mechanics probabilities, instead these are simple and logical frequency probabilities.

The special case of $\frac{l_p}{\bar{\lambda}} = 1$ is for a Planck mass particle, but then it only lasts for one Planck second, as well as for aggregations of elementary particles where the mass reaches the Planck mass. Again, the reduced Compton wavelength of a composite mass is given by $\bar{\lambda} = \frac{\hbar}{\sum_{i=1}^{N} m_i c} = \frac{1}{\frac{1}{\bar{\lambda}_i} + \frac{1}{\bar{\lambda}_{i+1}} + \frac{1}{\bar{\lambda}_n}}$.

This gives us quite useful information. Since we do not know the exact Planck mass, we do not know if any sizable mass $m >> m_p$ is an exact integer, or an integer plus a fraction. It is likely to be an integer plus a fraction, but the larger the mass is, the smaller the fraction part will be relative to the integer number. This means that to measure the Planck length (Planck mass) accurately, one way is to use a very large mass, something we can partly do with a Cavendish apparatus. A one kg lead ball is indeed a very large mass compared to the Planck mass.

9 Relativistic Model

To explain a series of gravity phenomena, we need to extend our gravity model to take relativistic effects into account. Our atomist model follows the standard relativistic model, with a few exceptions that we will come to soon. For example, the relativistic energy mass relation is given by

$$\begin{aligned} E &= \frac{\tilde{m}_t c^2}{\sqrt{1-\frac{v^2}{c^2}}} \\ E &= \frac{\frac{l_p^2}{\lambda}\frac{1}{c}c^2}{\sqrt{1-\frac{v^2}{c^2}}} \\ E &= \frac{l_p^2}{\lambda\sqrt{1-\frac{v^2}{c^2}}}c \end{aligned} \tag{40}$$

That is, energy is now m^2 per second. This is not too different than the modern physics' mass energy relation that is

$$\begin{aligned} E &= \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \\ E &= \frac{\frac{\hbar}{\lambda}\frac{1}{c}c^2}{\sqrt{1-\frac{v^2}{c^2}}} \\ E &= \frac{\hbar}{\lambda\sqrt{1-\frac{v^2}{c^2}}}c \end{aligned} \tag{41}$$

However, back to our equation

$$E = \frac{l_p^2}{\lambda\sqrt{1-\frac{v^2}{c^2}}}c \tag{42}$$

As the indivisible particles cannot contract, but the distance between them can, namely $\bar{\lambda}$, this means the maximum length contraction is (until the Compton wavelength) the Planck length. This means we must have

$$l_p \leq \bar{\lambda}\sqrt{1-\frac{v^2}{c^2}} \tag{43}$$

solved with respect to v this gives

$$v \leq c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \tag{44}$$

This is the same maximum velocity of matter that has been suggested by Haug [21, 30, 39–41]. We basically get the same maximum velocity for escape velocity, however surprising this may be.

10 Relativistic Gravity Model

The full relativistic gravity model is given by

$$F = c^3\frac{\frac{\tilde{M}_t}{\sqrt{1-\frac{v_M^2}{c^2}}}\frac{\tilde{m}_t}{\sqrt{1-\frac{v^2}{c^2}}}}{r^2\left(1-\frac{v_M^2}{c^2}\right)} = c^3\frac{\tilde{M}_t\frac{\tilde{m}_t}{\sqrt{1-\frac{v^2}{c^2}}}}{r^2\left(1-\frac{v_M^2}{c^2}\right)^{3/2}} \tag{45}$$

The formula 45 seems to predict the correct precession of Mercury. Relativistic extensions of Newton have been completed before, but these solutions have only predicted half off Mercury's precession. When we observe (or predict a gravity phenomenon) from the gravity object M, then the formula above can be simplified to

$$F = c^3\frac{\tilde{M}_t\frac{\tilde{m}_t}{\sqrt{1-\frac{v^2}{c^2}}}}{r^2} \tag{46}$$

Formula 46 only holds if the gravitation observations is observed from one of the gravity objects. In case we observe two masses from a third reference frame, such as observing the Sun's effect on Mercury as observed from the Earth, then formula 45 must be used. And this actually (after lengthy calculations) seems to give the correct precession of mercury, that is $\delta = \frac{6\pi l_p^2}{\lambda c^2 a(1-e^2)}$. What earlier papers have not taken into account is that the distance from center to center between the two gravity objects is contracted as seen from a third reference frame. Other researchers have, in the past, used a similar formula to 46 and shown that it only predicts half of the precession of Mercury.

There could be additional adjustments, but our model seems to be consistent with the actual precession of Mercury. We will also show how this model is consistent with the observed bending of light.

11 Escape Velocity

The Newton escape velocity is normally derived from the following equation

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0 \tag{47}$$

solved with respect to v this gives

$$v = \sqrt{\frac{2GM}{r}} \tag{48}$$

This is even the same escape velocity one gets from general relativity theory, see [42]. Still, when deriving the Newton escape velocity, one is using a kinetic energy approximation $\frac{1}{2}mv^2$ that only holds when the speed of the small mass is much less than that of light $v << c$. As the speed v will approach c in a very strong gravitational acceleration field, this formula cannot hold for a strong gravitational acceleration field when derived from the Newton formula. To do that, we need to look at the kinetic energy for high velocities, and then we need to take into account special relativity in the kinetic energy formula, something we will look at in the next section.

Since $E_k \approx \frac{1}{2}v^2$ is an approximate kinetic energy that only holds when $v << c$ and is used to derive the Newton escape velocity, we can conclude this must be an approximate escape velocity that only holds in the weak field limit. In other words, the Newton escape velocity formula should always be written as

$$v \approx \sqrt{\frac{2GM}{r}} \tag{49}$$

It is interesting that GR supposedly gives exactly the same escape velocity as the Newton weak field approximation. Is it not a bit strange that GR is not even slightly different than a weak field approximation when it supposedly also holds for very strong gravitational fields? We will not answer this here, but we will show how we may be able to properly derive the Newton escape velocity to hold for a strong gravitational field.

Newton Escape velocity that also holds for a strong gravitational field

To get a Newton escape velocity to hold for strong gravitational fields, we must use a kinetic energy formula that is valid when v is close to c; this can naturally only happen if we use a relativistic theory. The exact Einstein kinetic energy formula [43] is given by

$$E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 \tag{50}$$

The question is whether or not this can be combined with the Newton gravity theory and also if the Newton gravity formula then needs to be modified from a relativistic point of view. In 1981 and 1986, Bagge [44] and Phillips [45] each suggested a relativistic Newton formula simply replacing the smaller mass in the formula with a relativistic mass

$$F = G\frac{M\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}{r^2} \tag{51}$$

This formula was soon forgotten, as it only predicted half of Mercury's precession, see also [46–51]. Recently, Haug [52] has shown that the reason for being off with regard to Mercury's precession is likely due to the fact that we are not observing Mercury from the Sun, but naturally from Earth and that we then need to complete one more relativistic adjustment that we will come back to later in this paper. However, we will first start out with the case where the escape velocity is observed from the object from which the other object is escaping, that is we are observing from M or m and not from a third object outside the realm of these two objects.

In a recent working paper, Haug attempted to derive a Newton relativistic escape velocity simply by setting $\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 - G\frac{Mm}{r} = 0$, and solving with respect to v. We are now convinced that this particular method was not sufficient, as we think the small mass in the gravitational formula also must be made relativistic, which we have done in this paper. Thus we get the following equation

$$\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 - G\frac{M\frac{m}{\sqrt{1-v^2/c^2}}}{r} = 0 \quad (52)$$

Solved with respect to v, this gives

$$v = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}} \quad (53)$$

(See the full derivation in the Appendix.)
This can also be rewritten as

$$v = \sqrt{\frac{2GM}{r} - \frac{\left(\frac{1}{2}r_s\right)^2 c^2}{r^2}} = \sqrt{\frac{2GM}{r} - \frac{r_s^2c^2}{4r^2}} \quad (54)$$

This formula is structurally different from the standard weak field Newton escape velocity, and it is also clearly different from the GR escape velocity. The difference is the term $-\frac{\left(\frac{1}{2}r_s\right)^2c^2}{r^2}$, which is very small as long as $r >> r_s$. In other words, in weak gravitational fields, the standard Newton (or GR formulation) will naturally do. However, when setting $r = \frac{1}{2}r_s$, we have an escape velocity of c.

$$v = \sqrt{\frac{2GM}{\frac{GM}{c^2}} - \frac{c^2r_s^2}{4\left(\frac{1}{2}r_s\right)^2}} = c \quad (55)$$

But here we have one more constraint that is easy to forget, namely that the formula is derived from equation 52. The mass will become infinite in two places if $v = c$, so it cannot be allowed that $r = \frac{1}{2}r_s$; we must have $r > \frac{1}{2}r_s$ for any mass. This means that no traditional mass can be at half of the Schwarzschild radius. This means we must have masses only at radius $r > \frac{1}{2}r_s$, but, we may ask,"How close can a mass be relative to half of this radius?"

This brings us back to another debate, namely how far can v approach c?

$$v = \sqrt{\frac{4GM}{r} - \frac{c^2r_s^2}{r^2}} \quad (56)$$

We will suggest the maximum relativistic mass any elementary particle can take is the Planck mass, which gives us

$$\begin{aligned}
m_p &= \frac{m}{\sqrt{1-\frac{v_e^2}{c^2}}} \\
m_p &= \frac{m}{\sqrt{1-\frac{\sqrt{\frac{4GM}{r}-\frac{c^2r_s^2}{r^2}}^2}{c^2}}} \\
m_p &= \frac{m}{\sqrt{1-\frac{\frac{4GM}{r}-\frac{c^2r_s^2}{r^2}}{c^2}}} \\
\frac{\hbar}{l_p}\frac{1}{c} &= \frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}}{\sqrt{1-\frac{\frac{4GM}{r}-\frac{c^2r_s^2}{r^2}}{c^2}}} \\
\frac{1}{l_p} &= \frac{\frac{1}{\bar{\lambda}}}{\sqrt{1-\frac{\frac{4GM}{r}-\frac{c^2r_s^2}{r^2}}{c^2}}} \\
\sqrt{1-\frac{\frac{4GM}{r}-\frac{c^2r_s^2}{r^2}}{c^2}} &= \frac{l_p}{\bar{\lambda}} \\
1-\frac{\frac{4GM}{r}-\frac{c^2r_s^2}{r^2}}{c^2} &= \frac{l_p^2}{\bar{\lambda}^2} \\
\frac{\frac{4GM}{r}+\frac{c^2r_s^2}{r^2}}{c^2} &= 1-\frac{l_p^2}{\bar{\lambda}^2} \quad (57)
\end{aligned}$$

assume also $M = m_p$; this gives $r = \frac{l_p\bar{\lambda}}{\bar{\lambda}+l_p}$. Now replacing this r back in the escape velocity formula and having $M = m_p$, we get

$$v = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \tag{58}$$

This is the same maximum velocity of matter that has been suggested by Haug in a series of papers [21, 30, 40, 53]. Further, in the special case of a Planck mass particle, we have $\bar{\lambda} = l_p$, which gives a maximum velocity for Planck mass particles of zero. Haug has suggested that the Planck mass particle is the collision point between photons. In addition, we will argue that the value of G should have twice its normal value when dealing with photons. One can argue for this in several ways. Traditional Newton calculations on light bending are off by a factor of 2 relative to experiments (and relative to GR). However, Newton never suggested a gravitational constant. If we calibrated the Newton formula first to mass light experiments rather than matter experiments, we would have twice the value of G as we do today. So, for a moment assume G has twice the value of the G given today, this gives the solution

$$v = \sqrt{\frac{4GM}{r} - \frac{4c^2 r_s^2}{r^2}} = 0 \tag{59}$$

Now, when $r = \frac{1}{2}r_s$, we get $v = 0$. This means that we have two important solutions for so-called black holes. At half the Schwarzschild radius, there are only Planck mass particles and they cannot move. They are standing absolutely still. There is also a solution of c.

On a particle level, we claim we must have that the gravitational mass is a Planck mass, as only collision time gives gravity. This means we have

$$c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} = \sqrt{\frac{2Gm_p}{r} - \frac{c^2\left(\frac{1}{2}r_s\right)^2}{r^2}} = \sqrt{\frac{2Gm_p}{r} - \frac{c^2 r_s^2}{4r^2}} \tag{60}$$

Solved with respect to r this gives

$$r = \frac{l_p\bar{\lambda}}{l_p+\bar{\lambda}} > l_p \tag{61}$$

That is, nonPlanck mass particles can never be closer to a Planck mass than this radius, and they then have an escape velocity equal to our earlier suggested maximum velocity for matter, which is below the speed of light. Further, in the special case where the particle escaping is a Planck mass particle, then we have the following equation

$$c\sqrt{1-\frac{l_p^2}{l_p^2}} = \sqrt{\frac{2G\frac{1}{2}m_p}{r} - \frac{c^2\left(\frac{1}{2}r_s\right)^2}{r^2}} \tag{62}$$

Solved with respect to r, this gives $r = 1/2l_p$. But if the gravity model is calibrated to light matter, then $G = 2G$ (light), and in this case the solution to the formula above is $r = l_p$. So, the speed very close to the Schwarzschild radius is very close to c and at the Schwarzschild radius it is zero. This explains why no time dilation exists in quasars. Quasars are, in our view, not holes – they are spheres where matter is maximally packed as Planck masses. The indivisible particles in a Planck mass particle have no distance between them and when such particles are packed together, the indivisible spheres stand still. At the surface of a quasar the mass is standing still and cannot undergo time dilation, but also it is likely that mass is converted into energy all the time, therefore, so-called black holes should be very bright objects.

This should also be seen in light of the Planck acceleration, which is given by

$$a = \frac{c^2}{l_p} \approx 5.56\times 10^{51}\ m/s^2 \tag{63}$$

In 1984, Scarpetta predicted this as the maximum acceleration possible, [54], something also suggested by [55]:

> *"the 'Planck acceleration' is both the maximum acceleration for an elementary particle in free space and also the surface gravity of a black hole with minimum mass m_p" – Falla and Landsberg, 1994*

However, as pointed out by [56], this enormous acceleration means that one will reach the speed of light after one Planck second, $a_p t_p = c$. Yet nothing with mass can travel at the speed of light, so nothing that still has mass after acceleration can undergo such rapid acceleration. In general, modern physics is even compatible with such an acceleration without modifications. No mass can move at the speed of light, as it would give infinite relativistic mass that would require infinitely much energy to get there. On the other hand, a Planck mass particle that is consistent with being two light particles in our calculations (that is indivisible particles in collision for one Planck second before dissolving into light) is fully consistent with this view.

Interestingly, the minimum Rindler horizon is approximately the same as the minimum distance a standard particle can have to a Planck mass particle (without itself becoming a Planck mass particle), see [57].

Escape velocity as seen from third observer

The escape velocity as seen from a third reference frame moving relative to m and M must be different than the escape velocity as seen from m or M, and it must be

$$v = \frac{\sqrt{G}\sqrt{M}\sqrt{-\frac{GM}{R(c-\mathrm{vM})^2} - \frac{2GM}{R(c-\mathrm{vM})(c+\mathrm{vM})} - \frac{GM}{R(c+\mathrm{vM})^2} + \frac{4c}{c-\mathrm{vM}} + \frac{4c}{c+\mathrm{vM}}}}{2\sqrt{R}} \approx \sqrt{\frac{2GM}{r}}\frac{1}{\sqrt{1-\frac{v_M^2}{c^2}}} \tag{64}$$

As the orbital velocity is $v_o \approx \frac{v_e}{\sqrt{2}}$, this means the galaxy arm orbital velocity will be considerably larger than predicted by standard theory when the whole galaxy is moving relative fast relative to the Earth. In other words, when v_M is significant.

12 Gravity is Lorentz Symmetry Break Down at the Planck Scale

Several quantum gravity theories predict Lorentz symmetry break down at the Planck scale, but they have not been able to give observable predictions. One of the reasons for this is that the Planck scale is assumed to be an extremely high energy level at which we are not even close to performing experiments right now. As a recent review article [58] on the possibility for Lorentz symmetry breaking in relation to quantum gravity predictions and experiments noted:

> *In conclusion, though no violation of Lorentz symmetry has been observed so far, an incredible number of opportunities still exist for additional investigations.*

Modern physics has incorporated collision time in their definition of mass. It has been added externally and without specific awareness of it through Newton's gravitational constant, which is calibrated to gravity experiments. In this way, collision time is indirectly incorporated into the mass model, but not in a deliberate or conscious way. As we have shown, gravity is directly linked to the Planck scale. It is also linked to collision time (length) in the mass over the shortest time interval. Modern physics has not understood that gravity itself is Lorentz symmetry break down at the Planck scale and interpretations have missed out on several important aspects of the Planck scale. In addition, it has not been noted that gravity could actually be the Lorentz symmetry break down in matter. The Planck mass particle in terms of kg is observational time dependent, this means we should be looking for a very low energy (which is gravity) and not very high energy. In fact, the very high energies can only be observed at the Planck time scale. The Planck second, the Planck mass, and the Planck energy are invariant across reference frames. This because the Planck mass always stands still and only can be directly observed inside one Planck second.

Further, quantum mechanics has not incorporated the Planck scale in any way. As we soon will show, in our new quantum mechanics the Heisenberg uncertainty principle breaks down inside the Planck scale. That is, inside a Planck time interval there is not particle wave duality; at this time interval it can only be a collision between two indivisible particles, which are collision time, and which are also gravity. We will show this in our derivations in a section below. We have unified gravity with our quantum model and shown how, based on only a simple model of matter that takes collision time into account, we are able to get the correct gravity predictions at any level, from elementary particles to the cosmological scale. In the next section, we show how this is consistent with a new quantum mechanics.

13 New Quantum Mechanics

Here we will introduce a new quantum wave equation that also gives gravity without understanding the importance of collision time and taking into account that one ultimately has a collision time.

The Klein–Gordon equation is often better known in the form (dividing by $\hbar^2$ and c^2 on both sides):

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Psi - \nabla^2\Psi + \frac{m^2c^2}{\hbar^2}\Psi = 0 \tag{65}$$

The Klein–Gordon equation has strange properties, such as energy squared, which is one of several reasons that Schröedinger did not like it that much. We have argued length for that one should make a wave equation from the Compton wavelength rather than the de Broglie wavelength [59, 60]. Today, matter has two wavelengths, the de Broglie version, which is a hypothetical wavelength and the Compton wavelength. The Compton wavelength has been measured in many experiments and we can find the traditional kg mass from that plus the Planck length and the speed of light. We cannot find the rest-mass from the de Broglie wavelength, as this length is infinite for a rest-mass. The relation between these two waves, even in a relativistic model, is simply $\bar{\lambda}_B = \bar{\lambda}_c \frac{c}{v}$. To switch from de Broglie to Compton leads to a new momentum definition, where we have rest-mass momentum, kinetic momentum, and total momentum. The traditional relativistic momentum definition is rooted in the de Broglie wavelength (actually the de Broglie wavelength is rooted in an old, non-optimal definition of momentum), that is

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{66}$$

while our momentum rooted in the measured Compton wavelength is given by

$$p_t = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{67}$$

and the rest-mass momentum is given by $p_r = mc$ and the kinetic momentum by

$$p_k = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} - mc \tag{68}$$

This gives us a new and simpler relativistic energy momentum relation, that both gives the same correct output, but one is much simpler mathematically, which is key to obtaining a simpler and fully correct wave equation. The old energy momentum relation rooted in de Broglie wavelength is given by

$$E = \sqrt{p^2c^2 - m^2c^4} \tag{69}$$

while our new energy momentum relation is given by

$$E = p_kc - mc^2 \tag{70}$$

They are identical, except that a standard physics version goes through the de Broglie wavelength (i.e., a nonexistent wavelength that is a derivative of the physical Compton wavelength). The math, therefore, gets unnecessarily complex and lacks intuition, which has led to many different interpretations in standard QM of the same equations. Our theory is much more straightforward and is fully consistent with our gravity theory.

This in turn leads to a simpler relativistic energy momentum relation than the standard one and also to a new wave equation, see [61] for details. In fact, this gives the same wave equation that we have derived before, but now we show that the Heisenberg collapse at the Planck scale that we found before is directly linked to gravity.

If we use our new momentum definition and its corresponding relativistic energy–momentum relation, we get

$$\begin{aligned}
E &= \mathbf{p}_k c + \tilde{m}c^2 \\
E &= \left(\frac{\tilde{m}c}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}} - \tilde{m}c \right) c + \tilde{m}c^2 \\
E &= \left(\frac{\tilde{m}c}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}} \right) c \\
E &= \mathbf{p}_t \mathbf{c}
\end{aligned} \tag{71}$$

where $\mathbf{v}$ is the particle's three-velocity. Now we can substitute E and $\mathbf{p}_t$ with corresponding energy and momentum operators and get a new relativistic quantum mechanical wave equation

$$-l_p^2 \frac{\partial \Psi}{\partial t} = -l_p^2 \nabla \cdot (\Psi \mathbf{c}) \tag{72}$$

where $\mathbf{c} = (c_x, c_y, c_z)$ would be the light velocity field. Interestingly, the equation has the same structural form as the advection equation, but here for quantum wave mechanics. Dividing both sides by $\hbar$, we can rewrite this as

$$-\frac{\partial \Psi}{\partial t} = -\nabla \cdot (\Psi \mathbf{c}) \tag{73}$$

The light velocity field should satisfy (since the velocity of light is constant and incompressible)

$$\nabla \cdot \mathbf{c} = 0 \tag{74}$$

that is[1]. The light velocity field is a solenoidal, which means we can rewrite our wave equation as

[1]For people not familiar or rusty in their vector calculus, we naturally have $\nabla \cdot (\Psi \mathbf{c}) = \Psi \nabla_x c_x + \Psi \nabla_y c_y + \Psi \nabla_z c_z + c_x \nabla_x \Psi + c_y \nabla_y \Psi + c_z \nabla_z \Psi = \Psi \nabla \cdot \mathbf{c} + \mathbf{c} \cdot \nabla \Psi$. For an incompressible flow such as we have, the first term is zero because $\nabla \cdot \mathbf{c} = 0$. In other words, we end up with $\nabla \cdot (\Psi \mathbf{c}) = \mathbf{c} \cdot \nabla \Psi$

$$\frac{\partial \Psi}{\partial t} - \mathbf{c} \cdot \nabla \Psi = 0 \tag{75}$$

So, in the expanded form, we have

$$\frac{\partial \Psi}{\partial t} - c_x \frac{\partial \Psi}{\partial x} - c_y \frac{\partial \Psi}{\partial y} - c_z \frac{\partial \Psi}{\partial z} = 0 \tag{76}$$

The equation above is only for a single particle. In the more general case, we have

$$i l_p^2 \frac{\partial}{\partial t} |\Psi\rangle = \hat{H}_H |\Psi\rangle \tag{77}$$

where $\hat{H}_H$ basically is the Hamilton operator, but with one big difference compared to the Schrödinger solution: In our model, we cannot use the standard momentum to get to the kinetic energy in the way Schrödinger does, which is why we have marked our Hamilton operator with a different notation (with H as subscript).

Our new relativistic quantum equation has quite a different plane wave solution than the Klein–Gordon and Schrödinger equations, but at first glance it looks exactly the same

$$\psi = e^{i(kx - \omega t)} \tag{78}$$

However, in our theory $k = \frac{2\pi}{\lambda_c}$, where λ_c is the relativistic Compton wavelength and not the de Broglie wavelength, as in standard wave mechanics. Due to this, we have

$$k = \frac{p_t}{l_p^2} = \frac{\frac{\tilde{m}c}{\sqrt{1-\frac{v^2}{c^2}}}}{l_p^2} = \frac{2\pi}{\lambda_c} \tag{79}$$

So, we can also write the plane wave solution as

$$e^{i\left(\frac{p_t}{l_p^2}x - \frac{E}{l_p^2}t\right)} \tag{80}$$

where p_t is the total relativistic momentum as defined earlier. Our quantum wave function is rooted in the Compton wavelength instead of the de Broglie wavelength. For formality's sake, we can look at the momentum and energy operators and see that they are correctly specified

$$\frac{\partial \psi}{\partial x} = \frac{i p_t}{l_p^2} e^{i\left(\frac{p_t}{l_p^2}x - \frac{E}{l_p^2}t\right)} \tag{81}$$

This means the momentum operator must be

$$\hat{\mathbf{p}}_t = -i l_p^2 \nabla \tag{82}$$

and for energy we have

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{l_p^2} e^{i\left(\frac{p_t}{l_p^2}x - \frac{E}{l_p^2}t\right)} \tag{83}$$

and this gives us a time operator of

$$\hat{E} = -i l_p^2 \frac{\partial}{\partial t} \tag{84}$$

The momentum and energy operator are the same as under standard quantum mechanics. The only difference between the non-relativistic and relativistic wave equations is that in a non-relativistic equation we can use

$$k = \frac{p_t}{l_p^2} = \frac{\tilde{m}c}{l_p^2} = \frac{2\pi}{\lambda_c} \tag{85}$$

instead of the relativistic form $p_t = \frac{\tilde{m}c}{\sqrt{1-\frac{v^2}{c^2}}}$. This is because the first term of a Taylor series expansion is $p_t \approx mc$ when $v << c$.

14 Gravity is Breakdown of the Heisenberg Uncertainty Principle at the Planck Scale

This is the most important missing part of modern wave mechanics, that the wave equation breaks down is the only place where the Planck length can enter quantum mechanics, and it is where the Heisenberg uncertainty principle breaks down and also where Lorentz symmetry breaks down. As we have shown earlier in this paper, gravity is directly linked to the Planck length, which is the collision space-time of mass. This means gravity is the Heisenberg break down and the Lorentz symmetry break down.

In the first part of our paper, we have shown that gravity is directly linked to a minimum length, and experimentally this length is the Planck length. The Planck length in relation to mass is essential for the collision length and collision time of indivisible particles. So, gravity in a wave equation must be the Planck mass particles in the wave equation. So, then something special should happen at the Planck scale. We have already from our previous analysis claimed that the Planck length, the Planck time, and the Planck mass must be invariant, because it is the only particle that stands absolutely still. We can only observe a Planck mass particle from the Planck mass particle itself. That is, it can only be observed when it is at rest relative to itself. But what does this lead to in our wave equation?

Our plane wave function is given by

$$\Psi = e^{i\left(\frac{p_t}{l_p^2}x - \frac{E}{l_p^2}t\right)} \tag{86}$$

the total momentum p_t is given by

$$p_t = \frac{\tilde{m}c}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{l_p^2}{\lambda}\frac{1}{c}c}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{l_p^2}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}} \tag{87}$$

Then we can rewrite the wave function as

$$\Psi = e^{i\left(\frac{\frac{l_p^2}{\lambda\sqrt{1-\frac{v^2}{c^2}}}}{l_p^2}x - \frac{\frac{l_p^2 c}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}}{l_p^2}t\right)} = e^{i\left(\frac{1}{\lambda\sqrt{1-\frac{v^2}{c^2}}}x - \frac{c}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}t\right)} \tag{88}$$

Next we have $v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$, and in the case of a Planck mass particle, we have $v_{max} = c\sqrt{1-\frac{l_p^2}{l_p^2}} = 0$. Further, as explained earlier, the Planck mass particle (a photon–photon collision) only lasts for one Planck second, and has a fixed "size" (reduced Compton wavelength) equal to the Planck length. This means that in order to observe a Planck mass particle, we must have $x = l_p$ and $t = \frac{l_p}{c}$. This gives

$$\Psi = e^{i\left(\frac{1}{l_p}l_p - \frac{c}{l_p}\frac{l_p}{c}\right)} = e^{i\times 0} = 1 \tag{89}$$

That is, the Ψ is always equal to one in the special case of the Planck mass particle, see also [62]. This means if we derive the Heisenberg uncertainty principle from this wave function, in the special case of a Planck mass particle it breaks down and we get a certainty instead of an uncertainty. This certainty lasts the whole of the Planck particle's life time, which is one Planck second.

This is fully consistent with our wave equation; when $\Psi = 1$, we must have

$$\begin{aligned}\frac{\partial \Psi}{\partial t} &= c_x\frac{\partial \Psi}{\partial x} + c_y\frac{\partial \Psi}{\partial y} + c_z\frac{\partial \Psi}{\partial z}\\ \frac{\partial 1}{\partial t} &= c_x\frac{\partial 1}{\partial x} + c_y\frac{\partial 1}{\partial y} + c_z\frac{\partial 1}{\partial z}\end{aligned} \tag{90}$$

which means there can be no change in the wave equation (in relation to the Planck mass particle), which would also mean no uncertainty. Basically particle-wave duality breaks down inside the Planck scale. The Planck mass particle is the collision between two photons and it only lasts for one Planck second. While all other particles are vibrating between energy and Planck mass at their Compton frequency, the Planck mass is just Planck mass, it is actually the building block of all other masses. This is a revolutionary view, but a conceptually simpler one that removes a series of strange interpretations in quantum mechanics, such as spooky action at a distance.

We can also derive this more formally. Since $\Psi = 1$, for a Planck mass particle we must have

$$\frac{\partial \Psi}{\partial x} = 0 \tag{91}$$

Thus, the momentum operator must be zero for the Planck mass particle. Therefore, we must have

$$\begin{aligned}[\hat{p},\hat{x}]\Psi &= [\hat{p}\hat{x}-\hat{x}\hat{p}]\Psi \\ &= \left(-0\times\frac{\partial}{\partial x}\right)(x)\Psi-(x)\left(-0\times\frac{\partial}{\partial x}\right)\Psi \\ &= 0\end{aligned} \tag{92}$$

That is, $\hat{p}$ and $\hat{x}$ commute for the Planck particle, but do not commute for any other particle. For formality's sake, the uncertainty in the special case of the Planck particle must be

$$\begin{aligned}\sigma_p\sigma_x &\geq \frac{1}{2}\left|\int\Psi^*[\hat{p},\hat{x}]\Psi dx\right| \\ &\geq \frac{1}{2}\left|\int\Psi^*(0)\Psi dx\right| \\ &\geq \frac{1}{2}\left|-0\times\int\Psi^*\Psi dx\right|=0\end{aligned} \tag{93}$$

In the special case of the Planck mass particle, the uncertainty principle collapses to zero. In more technical terms, this implies that the quantum state of a Planck mass particle can simultaneously be a position and a momentum eigenstate. That is, for the special case of the Planck mass particle we have certainty. In addition, the probability amplitude of the Planck mass particle will be one $\Psi_p = e^0 = 1$. However, we have claimed the Planck mass particle only lasts for one Planck second. We think the correct interpretation is that if one observes a Planck mass particle, then one automatically also knows it's momentum, since the particle (according to our maximum velocity formula) must stand still, so it only has rest-mass momentum. In other words, for this and only this particle, one knows the position and momentum at the same time. All particles other than the Planck mass particle will have a wide range of possible velocities for v, which leads to the uncertainty in the uncertainty principle.

Again, the breakdown of the Heisenberg uncertainty principle at the Planck scale is easily to detect, from our analyses in this paper we know that it must be gravity. Modern physics have totally missed out of this. They have their gravity theory on one hand, and they have their quantum theory on the other hand, and they have been thinking break down at the Planck scale is something special happening outside this system. They have for 100 years almost tried to unify QM with gravity but with basically no success. In our theory we see gravity is the break down at the Planck scale. We have derived our whole theory from the Planck scale, naturally combined with some key concepts from giants like Newton, Einstein, Compton, and many more. Still, for the first time in history we have a unified theory.

15 Minkowski Space-Time is Unnecessarily Complex at the Quantum Level

Our 4-dimensional wave equation is invariant. It should be consistent with relativity theory, since it is a relativistic wave equation. As pointed out by Unruh [63], for example, time in standard quantum mechanics plays a role in the interpretation distinct from space, in contrast with the apparent unity of space and time encapsulated in Minkowski space-time [64]. This has been a challenge in standard QM: why is it not fully consistent with Minkowski space-time? According to Unruh, whether or not Minkowski space-time is compatible with quantum theory is still an open question. From our new relativistic wave equation, we have good reason to think this may provide the missing bridge to the solution. This is something we will investigate further here. Minkowski space-time is given by

$$dt^2c^2-dx^2-dy^2-dz^2=ds^2 \tag{94}$$

where the space-time interval ds^2 is invariant. Or, if we are only dealing with one space dimension, we have

$$dt^2c^2-dx^2=ds^2 \tag{95}$$

This is directly linked to the Lorentz transformation (space-time interval) by

$$t'^2c^2-x'^2=\left(\frac{t-\frac{L}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2c^2-\left(\frac{L-tv}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2=s^2 \tag{96}$$

Assume we are working with only two events that are linked by causality. Each event takes place in each end of a distance L. Then for the events to be linked, a signal must travel between the two events. This signal moves at velocity v_2 relative to the rest frame of L, as observed in the rest frame. This means $t = \frac{L}{v_2}$. In addition, we have the speed v, which is the velocity of the frame where L is at rest with respect to another reference frame. That is, we have

$$t'^2c^2 - x'^2 = \left(\frac{\frac{L}{v_2} - \frac{L}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}} \right)^2 c^2 - \left(\frac{L - \frac{L}{v_2}v}{\sqrt{1-\frac{v^2}{c^2}}} \right)^2 \tag{97}$$

The Minkowski space-time interval is invariant. This means it is the same, no matter what reference frame it is observed from. To look more closely at why this is so, we can do the following calculation

$$\begin{aligned} t'^2c^2 - x'^2 &= \left(\frac{\frac{L}{v_2} - \frac{L}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}} \right)^2 c^2 - \left(\frac{L - \frac{L}{v_2}v}{\sqrt{1-\frac{v^2}{c^2}}} \right)^2 \\ & \left(\frac{L\frac{c}{v_2} - L\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \right)^2 - \left(\frac{L - L\frac{v}{v_2}}{\sqrt{1-\frac{v^2}{c^2}}} \right)^2 \\ & \frac{L^2 - 2L^2\frac{v}{v_2} + L^2\frac{v^2}{v_2^2}}{1-\frac{v^2}{c^2}} - \frac{L^2\frac{c^2}{v_2^2} - 2L^2\frac{v}{v_2} + L^2\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} \\ & \frac{L^2 + L^2\frac{v^2}{v_2^2} - L^2\frac{c^2}{v_2^2} - L^2\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} \\ & \frac{L^2\left(1 - \frac{v^2}{c^2} + \frac{v^2}{v_2^2} - \frac{c^2}{v_2^2}\right)}{1-\frac{v^2}{c^2}} \\ & \frac{L^2\left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{c^2}{v_2^2}\right)}{1-\frac{v^2}{c^2}} \\ & L^2\left(1 - \frac{c^2}{v_2^2}\right) \end{aligned} \tag{98}$$

We can clearly see that v is falling out of the equation, and that the Minkowski interval therefore is invariant. For a given signal speed v_2 between two events, the space-time interval is the same from every reference frame. We can also see that it is necessary to square the time and space intervals to get rid of the v and get an invariant interval. If we did not square the time and space intervals, we would get

$$\begin{aligned} t'c - x' &= \left(\frac{\frac{L}{v_2} - \frac{L}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}} \right) c - \left(\frac{L - \frac{L}{v_2}v}{\sqrt{1-\frac{v^2}{c^2}}} \right) \\ &= \frac{L\frac{c}{v_2} - L\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{L - L\frac{v}{v_2}}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{L\frac{c}{v_2} - L\frac{v}{c} - L + L\frac{v}{v_2}}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned} \tag{99}$$

The v will not go away if we do not square the time transformation and length transformation. That is $ds = dtc - dx$ is in general not invariant. However, the squaring is not needed in the special case where the causality between two events is linked to the speed of light; that is, a signal goes with the speed of light from one side of a distance L to cause an event at the other side of L. In this case, we have

$$\begin{aligned} t'c - x' &= \frac{\frac{L}{c} - \frac{L}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}} c - \frac{L - \frac{L}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{L - \frac{L}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{L - \frac{L}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} = 0 \end{aligned} \tag{100}$$

In other words, we do not need to square the space interval and the time interval to have an invariant space-time interval when the two events follow causality and where the events are caused by signals traveling at the speed of light. We are not talking about the velocity of the reference frames relative each other to be c (which would cause the model to blow up in infinity), but the velocity that causes one event at each side of the distance L to communicate. And in our Compton model of matter, every elementary particle is a Planck mass event that happens at the Compton length distance apart at the Compton time. Each Planck mass event is linked to the speed of light and the Compton wavelength of the elementary particle in question. This means in terms of space-time (only considering one dimension), for elementary particles we must always have

$$\begin{aligned} t'c - x' &= \frac{\frac{\bar{\lambda}}{c} - \frac{\bar{\lambda}}{c^2}v}{\sqrt{1-\frac{v^2}{c^2}}}c - \frac{\bar{\lambda} - \frac{\bar{\lambda}}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{\bar{\lambda} - \frac{\bar{\lambda}}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\bar{\lambda} - \frac{\bar{\lambda}}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} = 0 \end{aligned} \tag{101}$$

That is, inside elementary particles there are Planck mass events every Compton time, and these events, we can say, follow causality; they cannot happen at the same time. Two light particles must each travel over a distance equal to the Compton length between each event. The Planck mass events inside an elementary particle follows causality and are linked to the speed of light, which is why we always have $v_2 = c$ at the deepest quantum level. However, two electrons can, at the same time, travel at velocity $v \leq c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$ relative to each other.

Or, in three space dimensions (four dimensional space-time), we should have

$$dtc - dx - dy - dz = 0 \tag{102}$$

The Minkowski space-time is unnecessarily complex for the quantum world. Space-time in the quantum world is a simplified special case of Minkowski space-time, where no squaring is needed and where the space-time interval always is zero. What does this mean? This means time, which is equivalent to mass, is linked to the ultimate building block of light, that in an elementary particle (mass) keeps traveling back and forth at the speed of light, but when it is colliding with another light particle, both light particles are standing still for one Planck second. This also means that mass can be seen as a Compton clock.

In the special case of a Planck mass particle, we have $\bar{\lambda} = l_p$ and also $v = 0$ because v_{max} for a Planck mass particle is zero. Again, this is simply because two light particles stand absolutely still for one Planck second during their collision, which gives

$$\begin{aligned} t'c - x' &= 0 \\ \frac{\frac{\bar{l_p}}{c} - \frac{l_p}{c^2} \times v}{\sqrt{1-\frac{v^2}{c^2}}}c - \frac{l_p - \frac{l_p}{c}v}{\sqrt{1-\frac{v^2}{c^2}}} &= 0 \\ \frac{l_p - \frac{l_p}{c} \times 0}{\sqrt{1-\frac{0^2}{c^2}}} - \frac{l_p - \frac{l_p}{c} \times 0}{\sqrt{1-\frac{0^2}{c^2}}} &= 0 \\ t_p c - l_p &= 0 \end{aligned} \tag{103}$$

This means our theory is consistent with the Planck scale. It simply means that time at the most fundamental level is a Planck mass event. As we have claimed before, the Planck mass event has a radius equal to the Planck length and it only lasts for one Planck second.

New space-time operator and space-time wave equation

Further, we can define the following space-time operators (instead of the d'Alembert's operator used in Minkowski space-time) that should be fully consistent with our simplified space-time geometry:

$$\frac{1}{c}\frac{\partial}{\partial t} - \frac{\partial}{\partial x} - \frac{\partial}{\partial y} - \frac{\partial}{\partial z} \tag{104}$$

That is, we have

$$\frac{1}{c}\frac{\partial}{\partial t} - \nabla \tag{105}$$

where ∇ is the 3-dimensional Laplacian. This gives us a wave equation for the wave $u(x,t)$ of the form

$$\frac{1}{c}\frac{\partial}{\partial t}u - \nabla u = 0 \tag{106}$$

16 Unified Summary

- Mass has two important properties, the number of internal collisions and the length such a collision lasts. The standard mass measure such as kg only has incorporated the number of collisions, and this is even hidden in a collision ratio that modern physics is not aware of. The number of collisions is closely linked to energy, and the missing collision time (space-time) is closely linked to gravity.
- Our theory predicts that the building blocks of photons, that we claim are indivisible particles stand still for one Planck second when colliding. This gives a breakdown of Lorentz symmetry at the Planck scale. This break in Lorentz symmetry is easy to detect, as it is what is known as gravity. Modern physics has misunderstood the Planck scale and has not been able to make this connection. There have been extensive searches for breakdown of the Planck scale even at low energies, but it has not been proposed (to our knowledge) that gravity itself is the Lorentz symmetry break down.
- Heisenberg's uncertainty principle also breaks down at the Planck scale, and this is directly linked to Lorentz symmetry break down. It is easily detectable, as this is gravity. All gravity observations are Heisenberg uncertainty break downs at the Planck scale. Our new quantum wave equation framework is fully consistent with this. Standard quantum mechanics is incompatible with gravity as it misses both Lorentz symmetry and Heisenberg's uncertainty principle break downs.
- A long series of known gravity phenomena can be predicted without any knowledge of the Newton gravity constant or any knowledge of traditional mass sizes. We do not need Newton's gravitational constant or the Planck constant. The gravitational constant is a composite constant that contains the Planck length, the Planck constant, and the speed of light. The Planck constant embedded in the Newton constant is needed to get rid of the Planck constant in the standard incomplete mass definition. The fact that the gravitational constant has the Planck length to incorporate is needed to get collision time back into the mass. While the convention in physics has been to work with a constant that is empirically calibrated, there is another way to see things. It is when one first understands there is an indivisible particle at the depth of reality that one fully realizes this.
- Modern physics assumes incorrectly that Newton gravity moves instantaneously. Even if Newton suggested this on some occasions, it is not what the Newton formula says. The Newton formula in modern use has the Newton gravity constant that contains embedded the speed of light (gravity). Recently, we have shown how the speed of light (gravity) easily can be found only from a series of standard gravity observations [65].
- We have presented a fully unified theory, which unites the deepest quantum aspects of mass with gravity. It gives us a quantum wave equation that is consistent with gravity and shows that observed gravity is a breakdown of Lorentz symmetry and Heisenberg uncertainty principle at the Planck scale. We have been observing Planck scale break down all the time. Modern physics has mistakenly thought that gravity is one thing and the quantum world is something separate and that one should find Planck scale break down in some other "place/way" – instead we maintain that gravity itself is Lorentz symmetry break down, and, as we have shown, it is also a break down in Heisenberg uncertainty.
- We now have a theory where our quantum mechanics are consistent with gravity; we have also, for the first time, a theory where a simplification of Minkowski space time is consistent with QM. And further, we have an extended Newtonian gravity. To what degree it is consistent with every aspect of GR we are not sure at this time, but our theory seems to predict precession of Mercury correctly, it predicts the bending of light correctly, and it predicts that there is no time dilation for quasars, also correctly.

17 Conclusion

We have, based on a new theory of mass, shown how the quantum scale and cosmological scale are connected. Gravity is the collision space-time of indivisible particles. There exists an indivisible particle with diameter equal to the Planck length. The mass gap is observational time dependent. Modern physics' definition of mass only indirectly has the numbers of collisions in the mass, and there seems to be a lack of awareness about of this, even if recent research clearly points in this direction. Modern physics only get this indirectly by having to rely on a gravitational constant that is almost mystical and not understood at deeper level by its theorists and practitioners. This mode of thinking actually masks what is really going on in reality. From our new and deeper understanding of mass, it is clear that gravity itself is the Lorentz symmetry break down at the Planck scale.

Lorentz symmetry break down at the Planck scale is therefore everywhere and has been observed, because it is gravity.

We have also derived a new quantum wave equation rooted in our model of matter. This model shows that the Lorentz symmetry as well as the Heisenberg's uncertainty principle breaks down at the Planck scale. This break down at the Planck scale is gravity. The Lorentz symmetry break down at the Planck scale actually happens at every Compton time in elementary particles, but this break down only lasts for one Planck second. Our model is built from the quantum and gives exactly the same same gravity predictions as standard gravity theory, with the exception that Planck mass particles and quasars cannot have time-dilation. No time dilation in quasars has been observed, but this is not consistent with existing gravity theories. Finally, based on our insight into matter, we have shown that at the quantum scale, Minkowski space-time can be simplified (but not replaced); this makes our new quantum mechanics fully consistent with a simplified Minkowski space-time.

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Appendix

Deriving the escape velocity in relativistic Newton mechanics

$$\begin{aligned}
\frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 - G\frac{M\frac{m}{\sqrt{1-v^2/c^2}}}{r} &= 0 \\
1 - \sqrt{1-v^2/c^2} - \frac{GM}{c^2 r} &= 0 \\
\sqrt{1-v^2/c^2} &= 1 - \frac{GM}{c^2 r} \\
1 - v^2/c^2 &= \left(1 - \frac{GM}{c^2 r}\right)^2 \\
1 - v^2/c^2 &= 1 - 2\frac{GM}{c^2 r} + \frac{G^2 M^2}{c^4 r^2} \\
v^2/c^2 &= \frac{2GM}{c^2 r} - \frac{G^2 M^2}{c^4 r^2} \\
v^2 &= \frac{2GM}{r} - \frac{G^2 M^2}{c^2 r^2} \\
v &= \sqrt{\frac{2GM}{r} - \frac{G^2 M^2}{c^2 r^2}} \qquad (107)
\end{aligned}$$

Deriving the escape velocity in our quantum mass model is basically the same as the one above, but then without G

$$\begin{aligned}
\frac{\tilde{m}c^2}{\sqrt{1-v^2/c^2}} - \tilde{m}c^2 - c^2\frac{\tilde{M}\frac{\tilde{m}}{\sqrt{1-v^2/c^2}}}{r} &= 0 \\
1 - \sqrt{1-v^2/c^2} - \frac{c^2 \tilde{M}}{c^2 r} &= 0 \\
\sqrt{1-v^2/c^2} &= 1 - \frac{\tilde{M}}{r} \\
1 - v^2/c^2 &= \left(1 - \frac{\tilde{M}}{r}\right)^2 \\
1 - v^2/c^2 &= 1 - 2\frac{\tilde{M}}{r} + \frac{\tilde{M}^2}{r^2} \\
v^2/c^2 &= \frac{2\tilde{M}}{r} - \frac{\tilde{M}^2}{r^2} \\
v^2 &= \frac{2c^2\tilde{M}}{r} - \frac{c^2\tilde{M}^2}{r^2} \\
v &= c\sqrt{\frac{2\tilde{M}}{r} - \frac{\tilde{M}^2}{r^2}} \\
v &= c\sqrt{\frac{r_s}{r} - \frac{r_s^2}{4r^2}} \qquad (108)
\end{aligned}$$