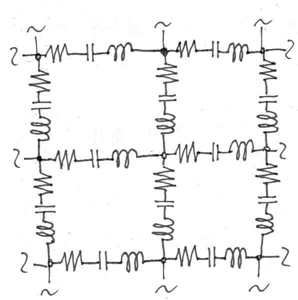
I try to write the differential equation for a continuous electric circuit (an electric circuit with the meshses that tend to zero dimension), so that I can write the continuous Kirchhoff's laws like a differential equation for the currents and voltages: a wave equation must be exist for the continuous circuit (similarly to the electromagnetic wave for Maxwell's equations).



An optimal circuit can be built (for example an optimal band-pass filter), changing the complex impedances in the space: the continuous RLC circuit. It should be possible to study lattices with inhomogeneous ions, or metal coating (for example superconductors) using the mechanical-electrical analogy (rlc circuit like an ion interaction analogy)

The continuous Kirchhoff's laws are:

$$\begin{cases} \nabla \cdot \mathbf{I} = 0 & Kirchhoff's\ current\ law \\ \nabla \times \mathbf{V} = 0 & Kirchhoff's\ voltage\ law \end{cases}$$

if there is a continuous variation of the impedances:

$$\mathbf{V}(\mathbf{x},\omega) = Z(\mathbf{x},\omega)\ \mathbf{I}(\mathbf{x},\omega)$$

$$\begin{cases} \nabla \cdot \mathbf{I} = 0 \\ \nabla \times (Z\mathbf{I}) = 0 \end{cases}$$

if the voltages are irrotational, then

$$\mathbf{V} = Z\ \mathbf{I} = \nabla\phi$$

$$\mathbf{I} = Y\nabla\phi$$

$$\nabla \cdot (Y\ \nabla\phi) = 0$$

$$\nabla Y \cdot \nabla\phi + Y\Delta\phi = 0$$

$$\boxed{\nabla\phi \cdot \nabla \ln Y + \Delta\phi = 0}$$

this is the Kirchhoff's law for a continuous circuit.

There is a solution in a neighbourhood of $\mathbf{x}$:

$$\phi(\mathbf{x} + \epsilon) = e^{i\mathbf{k}\cdot\epsilon}$$

$$\mathbf{k} \cdot \nabla \ln Y(\mathbf{x}, \omega) + \mathbf{k} \cdot \mathbf{k} \simeq 0$$

$$\mathbf{k} \cdot [\nabla \ln Y(\mathbf{x}, \omega) + \mathbf{k}] = 0$$

there are two solutions for $\mathbf{k}$:

$$\mathbf{k} = \mathbf{0}$$

$$\mathbf{k} = -\nabla \ln Y(\mathbf{x}, \omega)$$

then the local wave solution is:

$$\phi(\mathbf{x} + \epsilon) \simeq e^{-i\epsilon\cdot\nabla \ln Y(\mathbf{x}_0,\omega)+i\omega t}$$

the velocity of the local wave equation is:

$$c(\mathbf{x}, \omega) = \frac{\omega}{|\nabla \ln Y(\mathbf{x}, \omega)\ |}$$

and the wave equation for the local current is:

$$\Delta\phi - \frac{|\nabla \ln Y|^2}{\omega^2}\partial_{tt}\phi = 0$$

the current value, for local wave equation is:

$$\mathbf{I} = Y\ \nabla\phi = \phi\ Y\ \nabla \ln Y = \phi\ \nabla Y$$

$$\mathbf{I} = \nabla\phi = \phi\nabla \ln Y$$

There is a solution for constant current, and constant voltages:

$$\phi = \mathbf{k} \cdot \mathbf{x}$$

$$\mathbf{V} = \nabla\phi = \mathbf{k}$$

$$\mathbf{I} = Y\ \nabla\mathbf{V} = Y\ \mathbf{k}$$

for stationary currents, the current divergence is null: this is a solution for coaxial cable, or telegraphic cable.

If there is a periodic variability of the impedance

$$Y(\mathbf{x}) = e^{i\omega\mathbf{p}\cdot\mathbf{x}}$$

then there is a periodicity of the currents for the Kirchhoff's continuous law (with not zero gradient, and progressive and regressive waves): the phonons.