About the congruent number

Hajime Mashima

April 23, 2019

Abstract

The three sides of the right triangle are rational numbers, and those with natural numbers are congruent numbers.

Theorem 1 *Pythagorean theorem*

$$\left(m^2+n^2\right)^2 = (2mn)^2 + \left(m^2-n^2\right)^2$$

Definition 2

$$\begin{aligned} k'(m^2+n^2) &= k' \cdot \frac{f}{e} = acf \\ k'(2mn) &= k' \cdot \frac{b}{a} = bce \\ k'(m^2-n^2) &= k' \cdot \frac{d}{c} = ade \\ k' &= ace \end{aligned}$$

Definition 3 *S is a congruent number.(m , $n = \mathbb{N}$)*

$$S' = mn\left(m^2-n^2\right) = k^2 S \quad (k \geqq 1 \quad , \ m \neq n)$$

Proposition 4

The multiplication of the hypotenuse and one side of a right triangle is a congruent number.

Proof 5

$$\begin{aligned} &m = M^2+N^2 \qquad n = 2MN \\ &S' = 2MN(M^2+N^2)(M^2-N^2)^2 \qquad M \neq N \\ &S'' = 2MN(M^2+N^2) \end{aligned}$$

□

Corollary 6

$$\begin{aligned} S' &= M^2N^2(M^4-N^4) \Rightarrow M^4-N^4 \\ S'' &= 2 \cdot 2m^2n^2(2^2m^4+n^4) \Rightarrow 2^2m^4+n^4 \end{aligned}$$