About the congruent number

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Abstract

The three sides of the right triangle are rational numbers, and those with natural numbers are congruent numbers.

Theorem 1 *Pythagorean theorem*

$$\left(m^2+n^2\right)^2=(2mn)^2+\left(m^2-n^2\right)^2$$

Definition 2 S *is a congruent number.*

$$S'=k^2S \quad (k>0)$$

$$S'=mn\left(m^2-n^2\right) \qquad\qquad m\neq n$$

Proposition 3

If mn is a rational number, it is a natural number.

Proof 4

$$m=\frac{b}{a} \qquad\qquad n=\frac{d}{c}$$

$$S'=\frac{b}{a}\cdot\frac{d}{c}\left(\frac{b^2}{a^2}-\frac{d^2}{c^2}\right)$$

$$S'=\frac{bd}{ac}\left(\frac{b^2c^2-a^2d^2}{a^2c^2}\right)$$

$$S'=\frac{bd\left((bc)^2-(ad)^2\right)}{(ac)^3}$$

$$
\begin{aligned}
S' &= \frac{y}{z} \\
1 &= \frac{y}{S'z} \quad \Rightarrow \quad 1 = \frac{S'z}{y} \\
2 &= \frac{y}{S'z} + \frac{S'z}{y} \\
2 &= \frac{y^2 + S'^2 z^2}{S'zy} \\
2S' &= \frac{y^2 + S'^2 z^2}{zy}
\end{aligned}
$$

$y \perp z$ と置けるので $z = 1$。よって $ac = 1$ なので $a = 1, c = 1$。 □

Proposition 5

The multiplication of the hypotenuse and one side of a right triangle is a congruent number.

Proof 6

$$
S' = mn(m^2 - n^2) \qquad m \neq n
$$

$$
\begin{aligned}
m &= M^2 + N^2 \qquad n = 2MN \\
S' &= 2MN(M^2 + N^2)(M^2 - N^2)^2 \qquad M \neq N \\
S'' &= 2MN(M^2 + N^2)
\end{aligned}
$$

□

Example 7

$$
3^2 + 4^2 = 5^2
$$

$$
\begin{aligned}
S &= 3 \cdot 5 = 15 \\
S'' &= 4 \cdot 5 = 2^2 \cdot 5
\end{aligned}
$$