Neutrino mass, electroweak coupling constant and weak mixing angle

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ABSTRACT

A formula for the magnetic moment of a massive Dirac neutrino was deduced in the context of electroweak interactions at the one-loop level in 1977. A linear dependence on the neutrino mass was found. Alternatively, a magnetic moment for a massive neutrino arising from gravitational origin is predicted by the so-called Wilson-Blackett law. Both formulas for the magnetic moment can be combined, yielding a value of 1.530 meV for the lightest neutrino mass m_1.

The remaining neutrino masses can then be calculated from recent neutrino oscillation experiments. The results are remarkable. First, the so-called geometric mean mass relation between the three neutrino masses m_1, m_2 and m_3 is in good agreement with our results. Moreover, the empirical ratio of m_3 to m_1 is close to 33. This result suggests a value of 32 for the reciprocal value of the electroweak coupling constant α_W at low energy. The latter value for α_W implies an electroweak mixing angle, in reasonable agreement with the value calculated from atomic parity violation experiments on cesium. The obtained result deviates, however, from the weak mixing angle deduced from the standard model.

1. INTRODUCTION

According to the standard model, neutrinos are massless particles. However, neutrino oscillation experiments have shown that neutrinos probably do have mass. In addition, a non-zero neutrino magnetic moment arises at the one-loop level within the minimal extension of the standard model with right-handed neutrinos. For a left-handed Dirac neutrino with a positive mass m_i (i = 1, 2, 3) the following electromagnetic moment $\boldsymbol{\mu}_i$(em) has been deduced [1, 2]

$$\boldsymbol{\mu}_i(\text{em}) = \frac{3|e|G_\text{F} m_i c^4 \hbar}{8\pi^2\sqrt{2}}\boldsymbol{\sigma} = \frac{3G_\text{F} m_i m_e c^4 \mu_B}{4\pi^2\sqrt{2}}\boldsymbol{\sigma} = 3.2026\times10^{-22}\left(\frac{m_i}{\text{meV}}\right)\mu_B\boldsymbol{\sigma}, \qquad (1.1)$$

where $G_\text{F} = 1.16638\times10^{-5}$ GeV^{-2} is the Fermi coupling constant, c is the velocity of light, $\hbar$ is the reduced Planck constant, $\boldsymbol{\sigma}$ is the Pauli matrix and $\mu_B = |e|\hbar/2m_e$ is the Bohr magneton. Note that (1.1) predicts that the neutrino magnetic moment is proportional to the neutrino mass m_i.

At present, no magnetic moment of any neutrino has been measured. The tightest constraint on μ_i comes from studies of a possible delay of helium ignition in the core of red giants in globular clusters. From the lack of observational evidence of this effect a limit of $\mu_i < 3\times10^{-12}\mu_B$ has been extracted [3]. Therefore, the value of m_i cannot yet been calculated from (1.1).

Since 1891 many authors have discussed a gravitational origin of the magnetic field of rotating bodies. Particularly, the so-called Wilson-Blackett formula has often been considered [4–13]

$$\boldsymbol{\mu}(\mathrm{gm}) = -\frac{\beta}{2}\left(\frac{G}{k}\right)^{1/2}\mathbf{S}, \tag{1.2}$$

where $\boldsymbol{\mu}$(gm) is the gravitomagnetic moment of the massive body with angular momentum $\mathbf{S}$, G is the gravitational constant and $k = (4\pi\varepsilon_0)^{-1}$ is Coulomb's constant. For a sphere with a homogeneous mass density the angular momentum $\mathbf{S}$ is given by $\mathbf{S} = 2/5\ mr^2\boldsymbol{\omega}$, where m is the mass of the sphere of radius r and $\boldsymbol{\omega}$ is its angular velocity. Note that $\boldsymbol{\mu}$(gm) is proportional to the mass m. The parameter β is assumed to be a dimensionless constant of order unity. So far, the sign and value of β are unknown, however (see ref. [11] for an ample discussion of this point). It is noted that the Wilson-Blackett relation may also be deduced from a gravitomagnetic interpretation of the Einstein equations [9–13].

For an elementary particle like a neutrino with mass m_i (i = 1, 2, 3) and angular momentum $\mathbf{S} = (\hbar/2)\boldsymbol{\sigma}$ the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) may be written as [12]

$$\boldsymbol{\mu}_i(\mathrm{gm}) = -\frac{g_i\beta}{2}\left(\frac{G}{k}\right)^{1/2}\mathbf{S} = -\frac{g_i\beta}{4}\left(\frac{G}{k}\right)^{1/2}\hbar\boldsymbol{\sigma}, \tag{1.3}$$

where the parameter g_i (i = 1, 2, 3) is a dimensionless quantity of order unity, related to the g_l -factor for charged leptons ($l = e, \mu, \tau$). In addition, the unknown dimensionless constant β has also been added to $\boldsymbol{\mu}_i$(gm). Note that $\boldsymbol{\mu}_i$(gm) does not explicitly depend on neutrino mass.

The gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) of (1.3) for a neutrino with mass m_i may be distinguished by different g_i-factors. Starting from the Dirac equation, however, in first order the same factor $g_i = +2$ is deduced (see ref. [12]) for all neutrinos m_i, analogously to the factor $g_l = +2$ for all charged leptons.

It is assumed in the deduction of the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) of (1.3) that this moment generates a magnetic induction field, equivalent to the electromagnetic field of $\boldsymbol{\mu}_i$(em) from (1.1). When the magnetic moments $\boldsymbol{\mu}_i$(em) from (1.1) and $\boldsymbol{\mu}_i$(gm) from (1.3) are taken equal, the following expression for mass m_i results

$$m_i = -\frac{2\pi^2\sqrt{2}}{3|e|G_F c^4}\, g_i\beta\left(\frac{G}{k}\right)^{1/2}. \tag{1.4}$$

Note that $\boldsymbol{\mu}_i$(em) from (1.1) and $\boldsymbol{\mu}_i$(gm) from (1.3) have the same direction for a negative value of the product $g_i\beta$. Since a positive value $g_i = +2$ has been deduced from the Dirac equation, β must be negative. This result is important, for the sign of the β-factor was unknown, so far. Insertion of the value $g_1 = +2$ and a value $\beta = -1$ into (1.4) yields a value of 1.530 meV/c^2 = 2.727×10^{-39} kg for the neutrino mass m_1, the main result of ref. [12].

According to the neutrino oscillation theory [14, 15], the masses of the three neutrino flavour states ν_α ($\alpha = e, \mu, \tau$) can be expressed as a superposition of three massive eigenstates ν_i with masses m_i (i = 1, 2, 3). In addition, mass-squared differences $\Delta m_{21}{}^2 \equiv m_2{}^2 - m_1{}^2$ and $\Delta m_{32}{}^2 \equiv m_3{}^2 - m_2{}^2$ follow from observations. So, two relations between the masses m_1, m_2, and m_3 are available. Consequently, when the neutrino mass m_1 is known, the remaining masses m_2 and m_3 can be calculated. In section 2 such a calculation has been performed. Subsequently, the so-called "geometric mean neutrino mass relation" is tested. Finally, in section 3 a possible dependence for the masses m_i on the weak coupling constant α_W is considered. Furthermore, the value of the weak mixing angle following from the obtained value of α_W is discussed.

2. CALCULATION OF THE NEUTRINO MASSES

In table 1 the latest three-neutrino oscillation data are summarized, assuming normal hierarchy. A recent value of the squared-mass difference $\Delta m_{21}^2 = 74.9$ meV^2 from solar neutrino data and KamLAND is taken from Abe *et al.* [16]. The same value is given by Pocar *et al.* [17]. This value can also be compared with the $\Delta m_{21}^2 = 74.6$ meV^2 from a global analysis from solar experiments, KamLAND, and short baseline experiments given by Bellerive *et al.* [18]. The value of $\Delta m_{32}^2 = 2471$ meV^2 from latest electron antineutrino oscillation measurements is taken from Adey *et al.* [19]. Recently, a review of the results for Δm_{32}^2 is given by Roskovic [20].

Insertion of the value $g_1 = +2$ and a value $\beta = -1$ into (1.4) yields a value of 1.530 meV/c^2 for neutrino mass m_1. The accuracy of this result is constrained by the relative inaccuracy of the gravitational constant $G = 6.674\times10^{-11}$ kg^{-1}m^{3}s^{-2}. Subsequently, the masses m_2 and m_3 have been calculated from the values of $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ and are also given in table 1.

Table 1. Calculated neutrino masses m_2 and m_3 from data of refs. [16–17, 19–20] for the normal hierarchy. All masses are given in units of meV. Mass ratios m_2/m_1, m_3/m_2 and m_3/m_1 are also shown. For comment, see text.

[16–17] Δm_{21}^2 (meV^2)	[19–20] Δm_{32}^2 (meV^2)	[12] m_1 (meV)	m_2 (meV)	m_3 (meV)	$\Sigma_i\, m_i$ (meV)	m_2/m_1	m_3/m_2	m_3/m_1	$(m_2)^2/(m_1 m_3)$
$74.9^{+1.9}_{-1.8}$	2471^{+68}_{-70}	1.530	8.79	50.5	60.8	5.74	5.74	33.0	1.00

From calculated values m_i (i = 1, 2, 3) in this table a sum $\Sigma_i\, m_i = 60.8$ meV can be calculated. This value can be compared with cosmological constraints. So far, the tightest constraint of the sum $\Sigma_i\, m_i < 92.6$ meV at 90% C. L. has been given by Di Valentino *et al*. [21]. They extracted this bound by combining the full Planck measurements, Baryon Acoustic Oscillation and Planck clusters data. In addition, they imposed a low reionization redshift prior. The obtained value for $\Sigma_i\, m_i$ illustrates that a cosmological measurement of the neutrino mass may be at reach.

In addition, it appears that the ratios m_2/m_1 and m_3/m_2 in table 1 are equal within observational accuracy. From these two ratios the so-called “geometric mean neutrino mass relation” R_ν can be calculated

$$R_\nu \equiv \frac{m_2^2}{m_1 m_3} = 1.00. \tag{2.1}$$

This result is in good agreement with the value $R_\nu = 1$, first proposed and discussed by He and Zee [22], and later on by Sazdović [23]. The former authors obtained a value of m_1 = 1.58 meV from the mass differences used by them, whereas Sazdović calculated a value of 1.55 meV from his choice for Δm_{21}^2 and Δm_{32}^2.

Furthermore, the empirical ratio $m_3/m_1 = 33.0$ is close to 33. Combination of this integer and relation $R_\nu = 1$ leads to the values $m_2/m_1 = m_3/m_2 = 33^{½} = 5.7446$. Of course, these mass ratios depend on the input values for Δm_{21}^2 and Δm_{32}^2, but comparison with other mass-squared differences in, e.g., ref. [12] show small deviations from this value.

3. ELECTROWEAK COUPLING CONSTANT AND WEAK MIXING ANGLE

One can also try to express the masses m_2 and m_3 in units of the electroweak coupling constant $\alpha_W = kg^2/\hbar c$, a basic constant for electroweak interactions. Combination of the values for the ratios of m_2/m_1 and m_3/m_1 from table 1, and a chosen value $\alpha_W^{-1} = 32.00$, yields

$$m_2 = 5.74 m_1 = (0.148\,\alpha_W^{-1} + 1)\,m_1, \tag{3.1}$$

$$m_3 = 33.0\,m_1 = (1.00\,\alpha_W^{-1} + 1)\,m_1. \tag{3.2}$$

A simple dependence on α_W^{-1} is obtained on the right hand side of (3.2). Future measurements of Δm_{21}^2 and Δm_{32}^2 may affect the result of equation (3.2), however.

For comparison, the masses m_l ($l = e, \mu, \tau$) of the charged leptons can be expressed in terms of the fine-structure constant $\alpha = ke^2/\hbar c = 1/(137.036)$ (see, e.g., discussion in ref. [12])

$$m_\mu = \left(\frac{3}{2}\alpha^{-1} + 1\right) m_e \quad \text{and} \quad m_\tau = 17\left(\frac{3}{2}\alpha^{-1} - 1\right) m_e. \tag{3.3}$$

Using the observed electron mass $m_e = 0.5109989$ MeV, the calculated muon mass $m_\mu = 105.54887$ MeV from (3.3) differs -0.104 % from the observed mass $m_\mu = 105.65837$ MeV, whereas the calculated tauon mass $m_\tau = 1776.96$ MeV differs $+0.0054$ % from the observed mass $m_\tau = 1776.86$ MeV (see for the observed data ref. [24]). Compared to the dependence of the muon mass m_μ in (3.3) on the factor ($3/2\ \alpha^{-1} + 1$) on the right hand side, the dependence of the neutrino mass m_3 in (3.2) on the factor ($\alpha_W^{-1} + 1$) is remarkable.

The relation between the charges g and e is given by $g \sin\theta_W = e$, where θ_W is the electroweak mixing angle. This weak mixing angle can then be calculated from

$$\sin^2\theta_W = \frac{\alpha}{\alpha_W} = \frac{32.00}{137.036} = 0.2335, \tag{3.4}$$

where α and α_W are the values at low energy. This value of $\sin^2\theta_W$ is lower than the standard model prediction $\sin^2\theta_W = 0.23857 \pm 0.00005$, at near zero momentum transfer, calculated from the modified minimal subtraction ($\overline{\text{MS}}$) scheme by Erler and Freitas [25]. In 1997 Wood *et al.* [26] reported the first accurate measurements of the atomic parity violation in cesium and deduced a first more precise value for the weak mixing angle. Using results from later ^{133}Cs experiments from the Boulder group [27], Dzuba, Berengut, Flambaum and Roberts [28] reanalysed the parity non conservation in cesium and calculated a more accurate value of $\sin^2\theta_W = 0.2356 \pm 0.0020$. The latter value is about 1.5 σ lower than the value 0.23857 from the standard model. Recently, Cadeddu and Dordei [29]), however, reinterpreted the ^{133}Cs experiment at about 2.4 MeV and removed the 1.5 σ tension. They obtained an updated value of $\sin^2\theta_W = 0.239^{+0.006}_{-0.007}$, to be compared with the value 0.23857 from the standard model at low momentum transfer. However, the uncertainty in the result of ref. [29] is significantly enlarged compared to that from ref. [28]. Comparison of the value for $\sin^2\theta_W$ from (3.4) shows the best agreement with the result $\sin^2\theta_W = 0.2356 \pm 0.0020$ from ref. [28], but many uncertainties remain. A more definite value for $\sin^2\theta_W$ at low momentum transfer would be welcome.

Summing up, combination of the proposed value for the lightest neutrino of mass $m_1 = 1.530$ meV/c^2 = 2.727×10^{-39} kg with recent neutrino oscillation data is in good agreement with the so-called geometric mean mass relation $R_\nu = 1$ (compare to (2.1)). In addition, the empirical ratio m_3/m_1 is close to the integer value 33. When a value $\alpha_W^{-1} = 32$ for the reciprocal electroweak coupling constant is adopted, one obtains the remarkable relation $m_3 = (\alpha_W^{-1}+1)\ m_1 = 33\ m_1$. As a consequence, a value $\sin^2\theta_W = 0.2335$ for the electroweak mixing angle is found. This value is compared with other theoretical and observational results.

REFERENCES

[1] Lee, B. W. and Shrock, R. E., "Natural suppression of symmetry violation in gauge theories: muon- and electron-lepton-number nonconservation", *Phys. Rev. D* **16**, 1444-1473 (1977).

[2] Fujikawa, K. and Shrock, R. E., "Magnetic moment of a massive neutrino and neutrino-spin rotation", *Phys. Rev. Lett.* **45**, 963-966 (1980) and references therein.

[3] Raffelt, G. G., "New bound on neutrino dipole moments from globular-cluster stars", *Phys. Rev. Lett.*, **64**, 2856-2858 (1990).

[4] Schuster, A., "A critical examination of the possible causes of terrestrial magnetism", *Proc. Phys. Soc. Lond.* **24**, 121-137 (1912).

[5] Wilson, H. A., "An experiment on the origin of the Earth's magnetic field", *Proc. R. Soc. Lond.* **A 104**, 451-455 (1923).

[6] Blackett, P. M. S., "The magnetic field of massive rotating bodies", *Nature* **159**, 658-666 (1947).

[7] Bennett, J. G., Brown, R. L. and Thring, M. W., "Unified field theory in a curvature-free five-dimensional manifold", *Proc. R. Soc. Lond.* **A 198**, 39-61 (1949).

[8] Sirag, S-P., "Gravitational magnetism", *Nature* **278**, 535-538 (1979).

[9] Biemond, J., *Gravi-magnetism*, 1st ed. (1984). Postal address: Sansovinostraat 28, 5624 JX Eindhoven, The Netherlands. E-mail: j.biemond@gravito.nl Website: http://www.gravito.nl

[10] Biemond, J., *Gravito-magnetism*, 2nd ed. (1999). See ref. [9] for address and website.

[11] Biemond, J., "Which gravitomagnetic precession rate will be measured by Gravity Probe B?", arXiv:physics/0411129v1 [physics.gen-ph], 13 Nov 2004.

[12] Biemond, J., "Neutrino magnetic moment and mass: an update", viXra:quantum physics/1507.0208v2, 25 Apr 2018.

[13] Widom, A. and Ahluwalia, D. V., "Magnetic moments of astrophysical objects as a consequence of general relativity", *Chin. J. Phys.* **25**, 23-26 (1987).

[14] Maki, Z., Nakagawa, M. and Sakata, S., "Remarks on the unified model of elementary particles", *Prog. Theor. Phys.* **28**, 870-880 (1962).

[15] Kobayashi, M. and Maskawa, T., "*CP-Violation in the renormalizable theory of weak interaction.*", *Prog. Theor. Phys.* **49**, 652-657 (1973).

[16] Abe, K. *et al.* (Super-Kamiokande Collaboration), "Solar neutrino measurements in Super-Kamiokande-IV", *Phys. Rev. D* **94**, 052010 (2016); arXiv:1606.07538v1 [hep-ex], 24 Jun 2016.

[17] Pocar, A. *et al.* (Borexino Collaboration), "Solar neutrino measurements", arXiv:1812.02326v1 [nucl-ex], 5 Dec 2018.

[18] Bellerive, A., Klein, J. R., McDonald, A. B., Noble A. J. and Poon, A. W. P. (SNO Collaboration), "The Sudbury Neutrino Observatory", *Nucl. Phys. B* **908**, 30-51 (2016); arXiv:1602.02469v1 [nucl-ex], 8 Feb 2016.

[19] Adey, D. *et al.* (Daya Bay Collaboration), "Measurement of electron antineutrino oscillation with 1958 days of operation at Daya Bay", *Phys. Rev. Lett.* **121**, 241805 (2018); arXiv:1809.02261v5 [hep-ex], 19 Dec 2018.

[20] Roskovec, B., "Neutrino physics with reactors"; arXiv:1812.03206v1 [hep-ex], 7 Dec 2018.

[21] Di Valentino, E., Giusarma, E., Mena O., Melchiorri, A. and Silk, J., "Cosmological limits on neutrino unknowns versus low redshift priors" *Phys. Rev. D* **93**, 083527 (2016); arXiv:1511.00975v2 [astro-ph.CO], 17 Sep 2016.

[22] He, Xiao-Gang and Zee, A. "Geometric mean neutrino mass relation", *Mod. Phys. Lett. A* **22**, 2107-2112 (2007); arXiv:hep-ph/0702133v2, 27 Feb 2007. See also ref. [7] therein.

[23] Sazdović, B. "Charge dependent relation between the masses of different generations and neutrino masses", arXiv:1501.07617v1 [hep-ph], 29 Jan 2015.

[24] Tanabashi M. *et al.* (Particle Data Group), *Phys Rev D* **98**, 030001 (2018). Website: http://pdg.lbl.gov/

[25] Erler, J. and Freitas, A. "Electroweak model and constraints on new physics", chapter 10. In: Tanabashi, M. *et al.* (Particle Data Group), *Phys Rev D* **98**, 030001 (2018).

[26] Wood, C. S., Bennett, S. C., Cho, D., Masterson, B. P., Roberts, J. L., Tanner, C. E. and Wieman, C. E. "Measurement of parity nonconservation and an anapole moment in cesium", *Science* **275**, 1759-1763 (1997).

[27] Bennett, S. C. and Wieman, C. E., "Measurement of the 6S→ 7S transition polarizability in atomic cesium and an improved test of the standard model", *Phys. Rev. Lett.* **82**, 2484-2487 (1999). Errata: *Phys. Rev. Lett.* **82**, 4153 (1999) and *Phys. Rev. Lett.* **83**, 889 (1999); arXiv:hep-ex/9903022v1, 13 Mar 1999.

[28] Dzuba, V. A., Berengut, J. C., Flambaum, V. V. and Roberts, B. "Revisiting parity nonconservation in cesium", *Phys. Rev. Lett.* **109**, 203003 (2012); arXiv:1207.5864v1 [hep-ph], 25 Jul 2012 and references therein.

[29] Cadeddu, M. and Dordei, F. "Reinterpreting the weak mixing angle from atomic parity violation in view of the Cs neutron rms radius measurement from COHERENT", arXiv:1808.10202v1 [hep-ph], 30 Aug 2018 and references therein.