On the Accelerating Universal Expansion (revised)

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Abstract Repulsive gravity at large distances has been included in the universal solution of Einstein's equations by introducing a cosmological constant, excluding the dark energy interpretation. For a new cosmological model, the big-bang singularity has been replaced by a granular primeval particle and expansion is controlled by the velocity of light. Subsequently, well known problems inherent in the standard model of cosmology do not arise.

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1. Introduction

Various observations indicate that the expansion of the Universe is not slowing with time as previously expected, but is accelerating: see Riess et al. [1], Perlmutter et al., [2], Riess et al.[3], Tonry et al., [4], Kirshner, [5], Kirshner et al. [6]. Thus, gravity has apparently become anti-gravity at very large distances, yet remains normal within observed clusters of galaxies. In the literature, this phenomenon has been explained by adding dark energy of negative pressure to Einstein's theory of spacetime curvature. Herein, a cosmological constant will be incorporated into the universal solution of Einstein's equations of general relativity which describes energetic graviton fields; see Wayte, [7]. Then graviton-graviton repulsion at large distances is due to their inner mechanisms.

It is instructive to study static systems containing the cosmological constant before developing a model of the accelerating universe. Correct choice of line element appears to be essential for compatibility with Newtonian gravitation.

2. Exterior field of a spherically-symmetric static body

Einstein's equations describing the spherically-symmetric static gravitational field in polar coordinates will be used, (see Tolman, [8], for clear notation). For the line element:

$$ds^2 = -e^{\lambda}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2 + e^{\nu}dt^2, \tag{2.1}$$

the surviving components of the energy-momentum tensor are:

$$8\pi\left(G/c^4\right)T_1^1 = -e^{-\lambda}\left(\nu'/r + 1/r^2\right) + 1/r^2, \tag{2.2a}$$

$$8\pi\left(G/c^4\right)T_2^2 = 8\pi\left(G/c^4\right)T_3^3 = -e^{-\lambda}\left\{\nu''/2 - \lambda'\nu'/4 + \nu'^2/4 + \left(\nu' - \lambda'\right)/2r\right\}, \tag{2.2b}$$

$$8\pi\left(G/c^4\right)T_4^4 = e^{-\lambda}\left(\lambda'/r - 1/r^2\right) + 1/r^2 \quad . \tag{2.2c}$$

From [7, Section 2], we have $(e^{-\lambda} = e^{\nu} = \gamma^2)$, $(T_1^1 = -T_2^2 = -T_3^3 = T_4^4 = GM^2/8\pi r^4)$, that is field properties analogous to electromagnetic theory. Then normal gravitational potential is described by the metric tensor component:

$$\gamma = \left(1 - GM/c^2 r\right) = \left(1 - r_0/r\right) \quad , \tag{2.3a}$$

which ranges from zero at gravitational radius (r_0 = GM/c^2), to unity at $(r = \infty)$. The gravitational field strength is then:

$$F = -c^2\left(d\gamma/dr\right) = -GM/r^2 \quad . \tag{2.3b}$$

Evidently, anti-gravity could be produced by adding a term which would change the negative sign in Eq.(2.3a) to positive over a range of *r*, however, such variation in γ would incur un-realistic negative energy in T_4^4. Alternatively, Einstein's cosmological constant Λ can be introduced by adding it to the right side of Eqs.(2.2a,b,c), see [8, p 242]. Then partial solution gives

$$8\pi\left(\frac{G}{c^4}\right)T_1^1 = 8\pi\left(\frac{G}{c^4}\right)T_4^4 = -\frac{1}{r}\left(\frac{d\gamma^2}{dr}\right) + \left(\frac{1-\gamma^2}{r^2}\right) - \Lambda \quad , \tag{2.4a}$$

$$8\pi\left(\frac{G}{c^4}\right)T_2^2 = 8\pi\left(\frac{G}{c^4}\right)T_3^3 = -\frac{1}{2r^2}\frac{d}{dr}\left\{r^2\frac{d}{dr}\left(\gamma^2\right)\right\} - \Lambda \quad . \tag{2.4b}$$

Now, T_4^4 will keep the same positive value given for zero cosmological constant:

$$T_4^4 = T_1^1 = -T_2^2 = -T_3^3 = +GM^2/8\pi r^4 , \tag{2.5}$$

and the metric tensor component has a new form, remaining less than unity:

$$\gamma^2 = \left[\left(1-\frac{r_0}{r}\right)^2 - \left(1-\frac{r_0{}^3}{r^3}\right)\frac{\Lambda r^2}{3}\right] . \tag{2.6}$$

Coefficient Λ is understood here to represent the *inherent* capacity for repulsion, by a modification of graviton behaviour at large radii, without changing the field energy/momentum density. It is remarkable that Einstein's equations should include long-range repulsion so efficiently. Recall reference [7, Eqs.(8) – (13)] wherein gravitons are shown to be circularly polarised and to have tangential momentum. In contrast, the interpretation of Λ as being due to dark energy growing spontaneously throughout an infinite open universe appears excessive.

From Eq.(2.6), the field strength in the weak field case is derivable as:

$$F = -c^2\left(\frac{d\gamma}{dr}\right) \approx -c^2\left(\frac{r_0}{r^2} - \frac{\Lambda r}{3}\right). \tag{2.7}$$

Clearly, this field changes from attractive to repulsive at a particular radius,

$$r_a \approx (3r_0/\Lambda)^{(1/3)} \quad . \tag{2.8}$$

However, as radius r increases to infinity, there is no theoretical limit to the repulsive force in Eq.(2.7) even though the field energy density T_4^4 falls towards zero. A precise reach of gravitons to a maximum radius r_m would be more realistic. For overall consistency, this will be chosen so as to set the total gravitational field energy at (½Mc^2), as in [7, Eq.(13)] but now by only integrating T_4^4 from r_0 to r_m . Such field energy conservation and limitation could be achieved by steadily strengthening each graviton prior to r_m. To implement this, γ^2 will be modified to:

$$\gamma^2 = \left[\left(1-\frac{r_0}{r}\right)^2 - \left(1-\frac{r_0{}^3}{r^3}\right)\left(\frac{\Lambda r^2}{3} + \frac{r_0{}^2 r}{(r_m{}^3 - r_0{}^3)}\right)\right]. \tag{2.9}$$

Intuitively, r_a in Eq.(2.8) should be related to maximum radius r_m ; for example, ($r_a = r_m/2\pi$) in Eq.(6.9). Upon introducing this latest expression for γ^2 into Eq.(2.4a), we find that the field *energy density* is more complicated than the simple form of Eq.(2.5), but remains independent of Λ, namely:

$$8\pi\left(\frac{G}{c^4}\right)T_4^4 = \frac{r_0{}^2}{r^4} + \frac{r_0{}^2}{(r_m{}^3 - r_0{}^3)}\left(\frac{2}{r} + \frac{r_0{}^3}{r^4}\right) . \tag{2.10a}$$

At the maximum radius ($r = r_m$) this will approximate to:

$$8\pi\left(\frac{G}{c^4}\right)T_4^4 \approx \frac{3r_0{}^2}{r_m{}^4} . \tag{2.10b}$$

The exact field strength is derived from Eq.(2.9) as:

$$F = -c^2\left(\frac{d\gamma}{dr}\right) = -\frac{c^2}{\gamma}\left[\frac{r_0}{r^2} - \frac{r_0{}^2}{r^3} - \frac{\Lambda r}{3}\left(1 + \frac{r_0{}^3}{2r^3}\right) - \frac{r_0{}^2}{2(r_m{}^3 - r_0{}^3)}\left(1 + \frac{2r_0{}^3}{r^3}\right)\right], \tag{2.11}$$

which will approximate to Eq.(2.7).

Thus, Λ describes how the graviton field inherently changes its force character smoothly from attractive to repulsive, as revealed by introducing Eq.(2.8) into Eq.(2.7), with ($r_0 = GM/c^2$):

$$F \approx -\frac{GM}{r^2}\left[1 - \left(\frac{r}{r_a}\right)^3\right] . \tag{2.12}$$

The first term on the right represents the flux density of gravitons through a spherical surface. Then the negative term suggests that there is an internal mechanism for each individual graviton, which determines the strength of repulsion at any radius from r_o through r_a to r_m . Graviton propagation velocity is maintained at the velocity of light throughout, since ($T_1^1 = T_4^4$) always. And for compatibility, we will presume that Λ is always proportional to r_o , then r_a is constant and the gravitational force is proportional to mass. If this were not so, then the force in Eq.(2.7) could change sign simply by making r_o very small. The viable choice is ($\Lambda = 3r_0 / r_a{}^3$) from Eq.(2.8), see Eq.(6.12).

3. Interior field of a static spherically-symmetric body

3.1 Solid static spherical body

Einstein's equations (2.4) may be solved to get the interior gravitational field for a solid sphere of uniform material density ρ and zero pressure. Given the essential requirement of compatibility with Newtonian gravitation, then Eq.(2.4a) has to yield the metric tensor component:

$$\gamma^2 = 1 + \frac{8\pi G}{3c^2}\rho\frac{r^2}{2} - \frac{\Lambda}{3}r^2 \ , \tag{3.1}$$

so that field strength is given by:

$$F = -c^2\left(\frac{d\gamma}{dr}\right) = -\frac{1}{\gamma}\left(\frac{4\pi G}{3}\rho - \frac{\Lambda c^2}{3}\right) r \ . \tag{3.2}$$

These require $(T_4^4 = -\rho c^2/2)$, which represents the *energy density* of an attractive field. According to Eq.(3.1), gravitational potential increases outwards from the centre, and density ρ could apparently be decreased to make the field repulsive for an arbitrary Λ. However, the constant Λ represents a repulsive modification to the existing attractive field as in Eq.(2.8) and will be proposed to depend on the *total* body mass M_x ,thus:

$$\left(\frac{\Lambda}{3}\right) = \frac{r_o}{{r_a}^3} = \frac{GM_x}{{r_a}^3 c^2} \ , \tag{3.3}$$

using ($r_0 = GM_x/c^2$). Then, for $(M_x = (4\pi/3)\rho {r_x}^3)$ in general, we have from Eq.(3.2) the field at any radius r within maximum radius r_x :

$$F \approx -\left(\frac{GM_x}{{r_x}^3} - \frac{GM_x}{{r_a}^3}\right) r \ , \tag{3.4}$$

which is compatible with Eq.(2.12) for ($r = r_x$). Zero field occurs *everywhere* in the bulk when the sphere radius r_x is increased to r_a . If r_x is increased further, the whole field becomes repulsive, with strength dependent on position *r* within the body.

Now, compatibility with Newtonian gravitation only resulted from the use of line element Eq.(2.1) in Eq.(2.4) by putting T_μ^μ in terms of the field energy/momentum density ($-\rho c^2/2$). Had we used the mechanical density and pressure expressions, ($T_1^1 = T_2^2 = T_3^3 = -p_o$, and $T_4^4 = \rho_{oo} c^2$) with Eq.(2.1) in Einstein's equations, then we would have had the incompatible result [8, p246]:

$$e^{-\lambda} = 1 - \frac{8\pi G}{3c^2}\rho_{oo} r^2 - \frac{\Lambda}{3}r^2 \ , \tag{3.5}$$

which describes *increase in potential* upon climbing towards the centre, that is anti-gravity. A cosmological model cannot be built upon this foundation.

3.2 Fluid static spherical body

When pressure is not negligible, the material needs to be considered as a "perfect fluid". Then in view of the isotropic nature of hydrostatic pressure, the line element for a spherically-symmetric body is expressed in isotropic form [8, p 244]:

$$ds^2 = -e^{\mu}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right) + e^{\nu} dt^2 . \qquad (3.6a)$$

The previous line element Eq.(2.1) will not lead to sensible physical results compatible with Newtonian theory, nor approximate to Eq.(3.1) in the weak field. For the energy-momentum tensor components we take the local hydrostatic pressure and constant local mass density,

$$T_1^1 = T_2^2 = T_3^3 = -p_o \ , \quad \text{and} \quad T_4^4 = \rho_{oo} c^2 \ . \qquad (3.6b)$$

Then Einstein's equations yield the surviving components:

$$8\pi\left(\frac{G}{c^4}\right)p_o = e^{-\mu}\left(\frac{\mu'^2}{4} + \frac{\mu'\nu'}{2} + \frac{\mu' + \nu'}{r}\right) + \Lambda \ , \qquad (3.7a)$$

$$8\pi\left(\frac{G}{c^4}\right)p_o = e^{-\mu}\left(\frac{\mu''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{4} + \frac{\mu' + \nu'}{2r}\right) + \Lambda \ , \qquad (3.7b)$$

$$8\pi\left(\frac{G}{c^4}\right)\rho_{oo} c^2 = -e^{-\mu}\left(\mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{r}\right) - \Lambda \quad . \qquad (3.7c)$$

Solution of Eq.(3.7c) produces the metric tensor component:

$$e^{-\mu} = \left(1 + \frac{8\pi G}{3c^2}\rho_{oo}\frac{r^2}{4} + \frac{\Lambda}{3}\frac{r^2}{4}\right)^2 , \qquad (3.8)$$

which is compatible with Eq.(3.1) in the weak field if the *arbitrary* Λ-term here is negative and doubled in size. The field strength is also compatible with Eq.(3.2), when given by:

$$F = -c^2 \frac{d}{dr}\left(e^{-\mu/2}\right) = -\left(\frac{4\pi G}{3}\rho_{oo} + \frac{\Lambda c^2}{6}\right) r \ . \qquad (3.9)$$

Therefore, this isotropic form of solution could be suitable for describing an isotropic universe with effective pressure.

4. Standard cosmology model

Now that the phenomenon of gravitational repulsion has been explained as an inherent property of the gravitons from all mass particles, it is possible to manage the observed universal acceleration. First consider the Standard Model in order to identify its numerous problems prior to developing an improved model in Section 5. Commonly, the Robertson-Walker metric is employed:

$$ds^2 = -R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right\} + dt^2 \quad , \tag{4.1}$$

yet according to the analysis above leading to Eq.(3.5), problems could arise regarding compatibility with Newtonian gravitation. We shall therefore use the metric proposed by Tolman [8, Eq.(150.2)], explicitly for isotropic coordinates. In more practical units this can be written:

$$ds^2 = -\frac{a^2(t)}{(1+kr^2/4)^2}\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right\} + dt^2 \quad , \tag{4.2}$$

where a*(t)* is a universal scale factor, and *t* is local/cosmic time. Constant *k* may be negative, positive, or zero for an open, closed, or critical universe, respectively. The components of the energy-momentum tensor are to be in terms of local pressure and material density:

$$T_1^1 = T_2^2 = T_3^3 = -p \ , \ \text{and} \qquad T_4^4 = \rho c^2 \ . \tag{4.3}$$

Upon applying these expressions to Einstein's field equations [8, Eq.(98.6)], we obtain:

$$-\frac{8\pi G}{c^2}p = \frac{kc^2}{a^2} + 2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 - \Lambda c^2 , \tag{4.4}$$

$$\frac{8\pi G}{3}\rho = \frac{kc^2}{a^2} + \left(\frac{\dot{a}}{a}\right)^2 - \frac{\Lambda c^2}{3} \quad . \tag{4.5}$$

Manipulation of these gives the Friedmann- Lemaître equations;

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \quad , \tag{4.6}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad , \tag{4.7}$$

$$\dot{\rho} = -3\left(\frac{\dot{a}}{a}\right)\left(\rho + \frac{p}{c^2}\right) \quad , \tag{4.8}$$

where H is the Hubble-Lemaître parameter. These three expressions happen to be the same as they would have been for Eq.(4.1), but Eq.(4.2) ensures consistency with Newtonian gravitation.

To realise these expressions in physical terms, we will now let a*(t)* in Eq.(4.2) take units of length, and leave *r* dimensionless. In addition, a nominal mass (M_U = $(4/3)\pi\rho a^3$ = $1.085\text{x}10^{52}$ kg) for the whole universe and the observed values in Eq.(4.10) will be used. Then Figure 1 depicts the expansion outer radius, velocity and acceleration as a function of time; ('a' is explicitly taken to represent the radius of the material universe, and Λ is governed by M_U as in Eqs.(3.3) and (4.16)). Clearly, 'a' appears feasible but *super-luminal* expansion velocity can exist in this model universe, albeit Einstein's equations are valid up to the velocity of light; see Davis & Lineweaver [9].

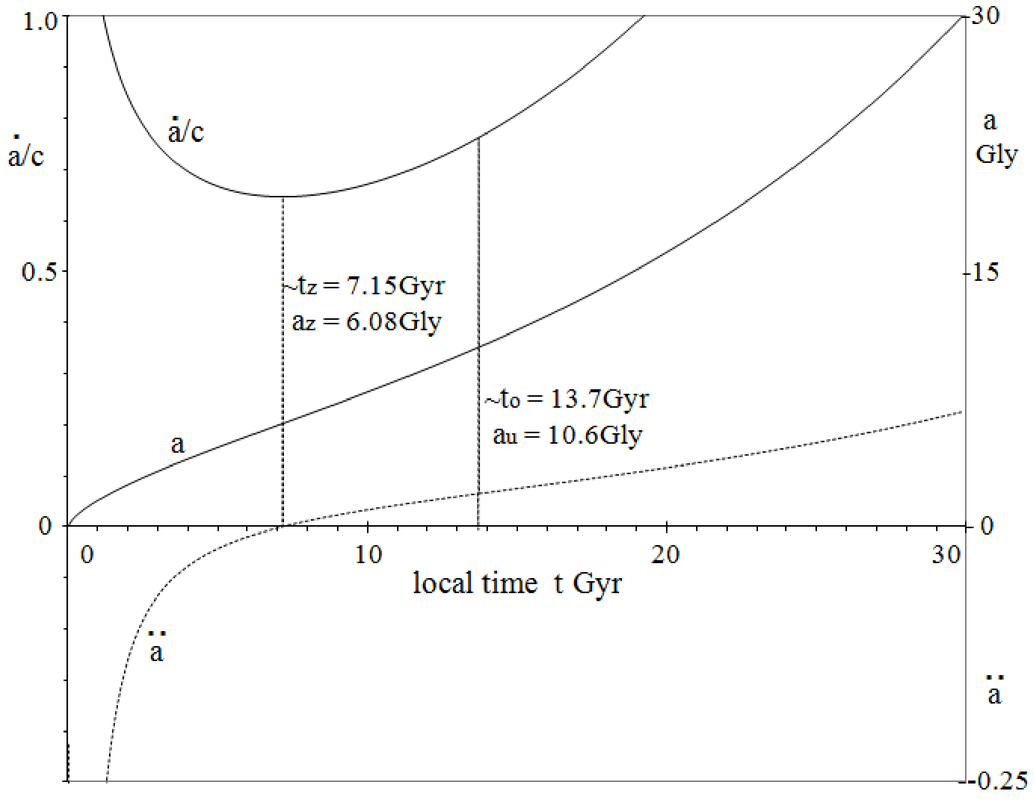

Figure 1. Friedmann- Lemaître model: variation of expansion velocity relative to the velocity of light (ȧ/c), radius (a, Gly), and acceleration (ä) with universal time (t, Gyr). Universal mass has been set at (M_U = $(4/3)\pi\rho a^3$ = $1.085\text{x}10^{52}$ kg), with the change from deceleration to accelerated expansion occurring at radius (a_z = 6.08Gly) corresponding to epoch (t_z = 7.15Gyr) from the big-bang. The present age of the universe is (t_0 = 13.7Gyr) and its radius is (a_u = 10.6Gly).

In a review article by Coles [10], it is shown how Eq.(4.6) can be conveniently expressed as:

$$1 \approx \Omega_m + \Omega_k + \Omega_\Lambda \quad , \tag{4.9}$$

where these components can take the (WMAP + BAO + SN Mean) observed values as representative, from Komatsu et al [11]:

$$\Omega_m = \left(\frac{8\pi G\rho_0}{3H_0^{\,2}}\right) \approx 0.274\,, \quad \Omega_k = \left(\frac{-kc^2}{a^2 H_0^{\,2}}\right) \approx 0\,, \quad \Omega_\Lambda = \left(\frac{\Lambda c^2}{3H_0^{\,2}}\right) \approx 0.726\ . \tag{4.10}$$

Consequently, we can evaluate Λ and ρ_0 using the Hubble-Lemaître parameter value (H_0 ~ 70.5kms^{-1}Mpc^{-1}):

$$\Lambda \approx 3(0.726)H_0^{\,2}/c^2 = 1.265\times 10^{-52}\,m^{-2} \quad , \tag{4.11}$$

$$\rho_0 \approx \frac{3}{8\pi G}(0.274)H_0^{\,2} = 2.56\times 10^{-27}\,kgm^{-3} \quad . \tag{4.12}$$

Given these values, Eq.(4.6) may be solved to get the expansion age of the universe, t_0. For negligible pressure and a universal mass ($M_U = (4/3)\pi\rho\, a^3$), we have:

$$H = \left(\frac{\dot{a}}{a}\right) \approx \left[\frac{2GM_U}{a^3} + \frac{\Lambda c^2}{3}\right]^{1/2} \quad , \tag{4.13}$$

then upon integration,

$$t_0 = \left(\frac{1}{3H_0\Omega_\Lambda^{\,1/2}}\right) \ln\left[\frac{(1+\Omega_\Lambda^{\,1/2})}{(1-\Omega_\Lambda^{\,1/2})}\right] \approx 13.7Gyr \ . \tag{4.14}$$

It is also possible to calculate the time when universal deceleration changed smoothly to acceleration. The general time /radius relationship is given by:

$$t = \left(\frac{2}{3H_0\Omega_\Lambda^{\,1/2}}\right) \ln\left[\left\{1 + \frac{\Lambda c^2/3}{2GM_U}(a^3)\right\}^{1/2} + \left\{\frac{\Lambda c^2/3}{2GM_U}(a^3)\right\}^{1/2}\right] , \tag{4.15}$$

and from Eq.(4.7), when ($\ddot{a} = 0$) at radius ($a = a_z$), and ($p \approx 0$), we have:

$$\frac{4\pi G}{3}\rho_z \approx \frac{GM_U}{a_z^{\,3}} \approx \frac{\Lambda c^2}{3} \quad . \tag{4.16}$$

Therefore by substitution, the zero-field time is governed by the Hubble-Lemaître parameter and cosmological constant:

$$t_z \approx (0.439)\left(\frac{\Lambda c^2}{3}\right)^{-1/2} = 7.15\,\text{Gyr}. \tag{4.17}$$

In Eq.(4.16), the value of M_U depends on radius (a_z) which will be set equal to the proposed value, 6.082Gly in Section 6.2. Thus the universal mass is (M_U = $1.085\text{x}10^{52}$ kg), then given the present density from Eq.(4.12), the current universal outer radius must be (a_u = 10.60Gly).

It is possible to calculate the observed redshift of any supernova which occurred at the time of zero-field, (t_z = 7.15Gyr). From Eq.(4.10) we have:

$$\frac{\Omega_\Lambda}{\Omega_m} = \frac{0.726}{0.274} = \frac{(\Lambda c^2/3)}{(8\pi G\rho_0/3)} = \frac{(\Lambda c^2/3)}{(2GM_U/a_u{}^3)} \quad , \tag{4.18}$$

which with the introduction of Eq.(4.16) yields a redshift:

$$z_z = \frac{a_u}{a_z} - 1 = 0.74 \quad . \tag{4.19}$$

This redshift is independent of our (a_z) value chosen from Eq.(6.9) because M_U and (a_u) compensate for variation of (a_z).

In conclusion, the standard big-bang model of the early universe has always had non-Einsteinian super-luminal expansion of the spacetime manifold, but now the expansion has to be super-luminal at large radii. The ethereal nature of spacetime originating at the big-bang singularity with inflation phase is inexplicable given that there was no empty space around the primeval singularity before the big-bang, yet infinite space and enough material were instantaneously created to produce the observed flat universe. There has been a problem with energy conservation, and inherent flatness- and horizon-problems; now continuous creation of dark energy throughout infinite space is required. All these serious problems need to be precluded by deriving an accurate model.

5 External coordinate observer cosmology: the ECO-model

The above standard cosmology model has not included the possibility that our local observer's time might be dilated by the universal expansion velocity. Such time dilation could make sub-luminal velocities appear super-luminal. We shall now consider the universal expansion from the point of view of an external coordinate observer located at rest outside of the material universe, in field-free Minkowski spacetime. In order to satisfy Einstein’s most basic relativity principles and eliminate the problems, this model will be controlled by the velocity of light while excluding the cosmological principle.

The big-bang phenomenon is regarded here as an explosion of a primeval particle into pre-existing empty space, at some arbitrary origin of coordinates. Before exploding at the velocity of light, this particle of finite mass and complex internal structure was in stable equilibrium. Thus the horizon-problem has been removed by prescribing a granular primeval particle in equilibrium, which disintegrated to produce a viscous fireball. The current material universe now occupies a spherical volume which is still expanding into free space on the coordinate-frame time scale and location referred to the big-bang event. This obviously differs from the interior local universal time scale used in the previous section. Our own position within this material volume is unspecified and not clearly within sight of the material surface. Other regions of space beyond ours may be empty or occupied by separate material structures at various distances in a multiverse scheme. Such a simple model appears compatible with the real world, being supported by a variety of observations and describable by Einstein’s general relativity theory.

5.1 The metric

The metric for the ECO-model is to be:

$$ds^2 = -\frac{a^2(t)}{(1+kr^2/4)^2}\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right\} + \left(1-\frac{v^2}{c^2}\right)c^2 dt^2 \ . \tag{5.1}$$

As in Section 4, a*(t)* is initially a scale factor, but it will now take real units of radial length from *r* and represent the maximum radius of the expanding material universe:

$$a(t) \to R(t) \ , \text{and} \ \ \dot{a}(t) \to \dot{R}(t) = v \ , \tag{5.2}$$

for radius $(R_\alpha \le R(t) < \infty)$, and $(c \ge v \ge 0)$. The primeval particle radius R_α will be defined in Section 6. Coordinate-frame time *t* is that measured by an external observer situated at rest in field-free space far outside of the expanding universal material. Local time τ for a co-moving observer is therefore dilated, due to the velocity of expansion, as [$d\tau = dt(1-v^2/c^2)^{1/2}$]. Upon introducing metric Eq.(5.1) into Einstein's field equations [8, Eq.(98.6)], we get Friedmann-Lemaître-ECO equations for an external observer:

$$\left(\frac{\dot{R}}{R}\right)^2 / \left(1 - v^2/c^2\right) = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3} \quad , \tag{5.3}$$

$$\left(\frac{\ddot{R}}{R}\right) / \left(1 - v^2/c^2\right)^2 = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad , \tag{5.4}$$

$$\dot{\rho} / \left(1 - v^2/c^2\right)^{1/2} = -3\left(\frac{\dot{R}}{R}\right)\left(\rho + \frac{p}{c^2}\right) / \left(1 - v^2/c^2\right)^{1/2} . \tag{5.5}$$

And we shall specify a conserved universal mass for the expanding sphere of maximum radius R:

$$M_U = \frac{4}{3}\pi\left(\rho + \frac{3p}{c^2}\right)R^3 \quad , \tag{5.6}$$

where ρ is the average matter density in the matter dominated universe, and the pressure is relatively small ($3p \ll \rho c^2$). The expansion velocity and deceleration are controlled by the velocity of light, see Figure 2, where the general coordinate time versus radius relationship has been calculated numerically (for negligible *p* and *k*):

$$t = \int \frac{dR}{v} = \int_{R_\alpha}^{R} \left[1 + \left\{ \frac{2GM_U}{c^2 R} + \frac{\Lambda R^2}{3} \right\}^{-1} \right]^{1/2} \frac{dR}{c} \quad . \tag{5.7}$$

If $d\tau$ is substituted into Eqs.(5.3)-(5.5) in place of *dt*, then they look like Eqs.(4.6)-(4.8), and it follows that ρ, *k* and Λ must take the same local values as previously.

The Hubble-Lemaître parameter should now be defined as:

$$H_\tau = \frac{1}{R}\frac{dR}{d\tau} \quad , \tag{5.8}$$

so the left side of Eq.(5.3) may be written as H_τ^2 . Then Ω_Λ and Ω_m will take the same numerical values as previously, simply by changing H_0 to $H_{\tau 0}$ in Eqs.(4.10)-(4.12). Local time τ measured by a co-moving observer is analogous to Eq.(4.15) as:

$$\tau = \left(\frac{2}{3H_{\tau 0}\Omega_\Lambda^{1/2}}\right)\ln\left[\left\{1+\frac{\Omega_\Lambda}{\Omega_m}\left(\frac{R}{R_0}\right)^3\right\}^{1/2} + \left\{\frac{\Omega_\Lambda}{\Omega_m}\left(\frac{R}{R_0}\right)^3\right\}^{1/2}\right], \qquad (5.9a)$$

and the corresponding local age of the universe is now analogous to Eq.(4.14):

$$\tau_0 = \left(\frac{1}{3H_{\tau 0}\Omega_\Lambda^{1/2}}\right)\ln\left[\frac{(1+\Omega_\Lambda^{1/2})}{(1-\Omega_\Lambda^{1/2})}\right] = 13.7\text{Gyr} \quad . \qquad (5.9b)$$

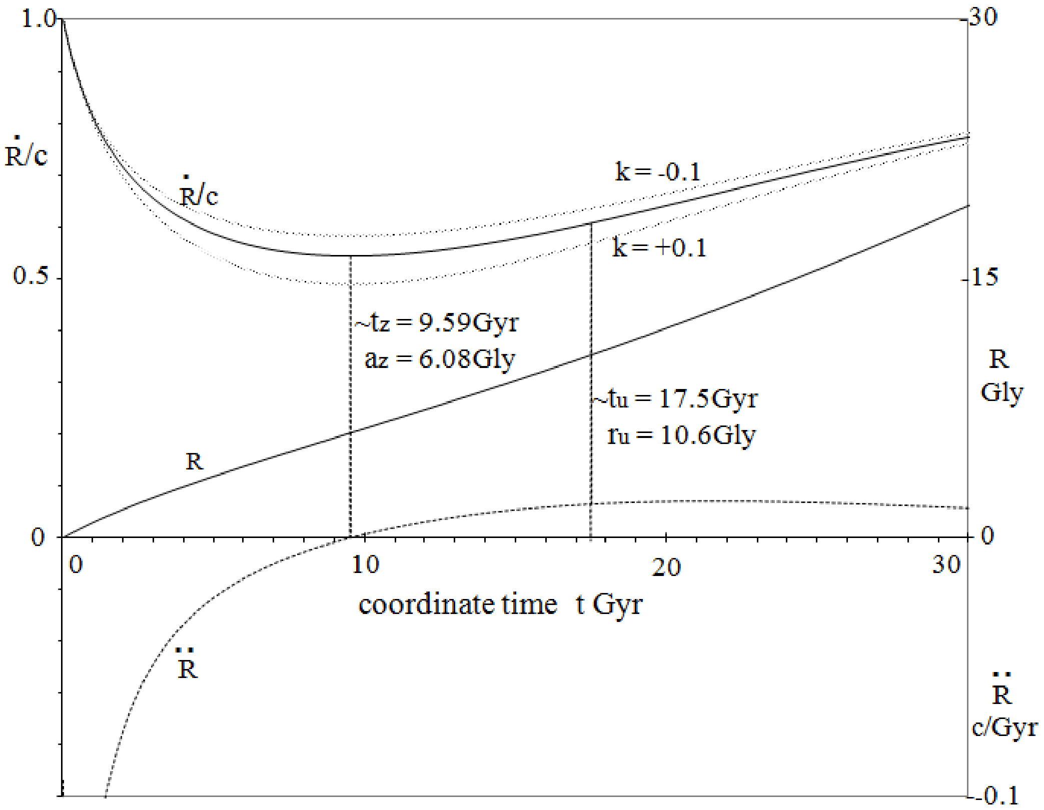

Figure 2. ECO-model: the variation of expansion velocity relative to the velocity of light $(\dot{R}/c)$, radius (R, Gly), and acceleration $(\ddot{R})$ with *coordinate-frame time (t, Gyr).* Universal mass is (M_U = 1.085x10^{52} kg), with the change from deceleration to accelerated expansion occurring at radius 6.08Gly corresponding to 9.59Gyr from the big-bang. The present coordinate age of the universe is 17.5Gyr, and its radius is 10.6Gly. For comparison, the effect of a finite *k* value (±0.1) is also shown.

It is time τ which has governed all atomic processes including star and galaxy evolution rate. Consequently, the graphs and velocities shown in Figure 1 represent transformations of the real fundamental values in Figure 2, due to time-dilation. That is, the 13.7Gyr quantity of evolution, which we observers have experienced according to our clocks, has really taken 17.5Gyr to perform. Evolution *rate* began low at $t \sim 0$ and grew to a maximum *rate* at $t \sim 9.59$Gyr, then declined thereafter. Co-moving local observers cannot be conscious of rate variation but can understand why the Standard Model has so many problems of interpretation.

Figure 2 shows how minimum expansion velocity, and zero acceleration in Eq.(5.4), occurred when

$$\frac{\Lambda c^2}{3} = \frac{GM_U}{a_z{}^3} \quad , \tag{5.10}$$

where $a_z = 6.082$Gly, and $M_U = 1.085\text{x}10^{52}$kg, as in Sections (4) and (6.2). The effect on the velocity of finite k values (–0.1, +0.1), is also demonstrated such that little difference occurs near the origin because the primeval particle's internal circulation was at the velocity of light before the expansion began. The term kc^2 in Eq.(5.3) is observed to be comparatively small, and if negative it could be attributed to an extra impulse of KE from the reactive fireball, or if positive to drag by the cohesive constituents of the particle which may have led to great strings of galaxies. If k really is zero, the total energy of the expanding matter is M_Uc^2, after any radiation has subsided. This total energy is divided between the rest mass and kinetic energy because gravity is an inductive force field, see [7]. Consequently, kinetic energy was steadily converted into rest mass as far as radius a_z but thereafter the repulsive Λ-term has reversed the trend and induced conversion of mass to kinetic energy.

The primeval particle described in Section (6.1) had all its material in viscous thermodynamic equilibrium while circulating coherently at velocity *c*, before exploding and converting to mass plus radiation which would have been mostly lost into space ahead of the expanding mass. No inflationary phase is necessary because the expansion is moderated by the velocity of light, allowing time for equalisation of the radiation temperature.

In Eq.(5.3) the expansion velocity does not overtly depend on pressure *p*, but by substituting ρ from Eq.(5.6) we can see how pressure shares some of the potential energy:

$$\left(\dot{R}\right)^2/\left(1-v^2/c^2\right)+\frac{8\pi G}{c^2}pR^2=\frac{2GM_U}{R}-kc^2+\frac{\Lambda c^2}{3}R^2\ . \qquad (5.11a)$$

In Eq.(5.4), *p* contributes to the gravitational force because pressure is stored energy. Pressure terms in these equations apply to gravitational processes in a fluid before separation into contactless particles. According to Figure 2, the expansion velocity $(\dot{R}/c)$ at the beginning decreases almost linearly with radius R. Therefore, substitute ($v = c - \delta v$) into Eq.(5.11a) with ($3p << \rho c^2$, k and $\Lambda \sim 0$), then separate the parts to get:

$$\frac{\delta v}{c}\approx\frac{c^2}{4GM_U}\cdot R\quad , \qquad (5.11b)$$

and the pressure decreases with R^2 :

$$\frac{p}{c^2}\approx\left(\frac{c^2}{8\pi G}\right)\frac{1}{R^2}\ . \qquad (5.11c)$$

Thus for small R, the pressure term in Eq.(5.11a) is reduced to a constant kinetic energy factor (c^2), small relative to the preceding term:

$$\left(\dot{R}\right)^2/\left(1-v^2/c^2\right)+c^2\approx\frac{2GM_U}{R}-kc^2+\frac{\Lambda c^2}{3}R^2. \qquad (5.11d)$$

5.2 Cosmological redshift

The standard model calculation of redshift shows that measured light wavelengths are increased in proportion to the scale factor a(t), see Tolman [8, p389] and Narlikar [12, p113]. We need to see what an external coordinate observer in the ECO-model would calculate for the redshift.

According to the line element Eq.(5.1) with Eq.(5.2), for a null geodesic we get:

$$cdt\left(1-v^2/c^2\right)^{1/2}=\frac{R(t)\,dr}{(1+kr^2/4)}\quad . \qquad (5.12)$$

This leads to the redshift equation:

$$\frac{c\Delta t_0(1-v_0{}^2/c^2)^{1/2}}{c\Delta t_1(1-v_1{}^2/c^2)^{1/2}}=\frac{R(t_0)}{R(t_1)}=1+z\ , \tag{5.13}$$

where (t_1) is the coordinate time of photon emission, and (t_0) the coordinate time of detection. The first quotient in Eq.(5.13) is also equal to local redshift $(\Delta\tau_0/\Delta\tau_1)$ and the second term is the ratio of scale factors seen by a coordinate or local observer. Thus the theoretical coordinate redshift defined as $(\Delta t_0/\Delta t_1)$ involves (v_0, v_1) and differs from the local redshift. However, we observe local redshift $(\Delta\tau_0/\Delta\tau_1)$ as for the standard model; for example, a supernova which occurred at the time of zero-field has redshift given by the ratio of ($R(t_0)$ = 10.6Gly), and ($R(t_1)$ = 6.08Gly), as in Eq.(4.19). Regarding photon energy, the decrease during its travel from source at $R(t_1)$ to detection at $R(t_0)$ may be due to interaction with the universal copious graviton field; but this interaction cannot cause much photon scattering because distant galaxies are still discernible.

5.3 Luminosity distance

The luminosity distance d_L derived by Carroll, Press & Turner [13] for the standard model may be adapted for the present model, wherein $H_{\tau 0} \equiv H_0$. When $\Omega_k = 0$, and $\Omega_M + \Omega_\Lambda = 1$, and Z = 1+z, we can get from [13, Eqs.25+23]:

$$d_L=\frac{cZ}{H_{\tau 0}}\int_1^Z\frac{dZ}{\left(Z^3\Omega_M+\Omega_\Lambda\right)^{1/2}}\quad . \tag{5.14}$$

Then for d_L in megaparsecs, the predicted distance modulus is:

$$\mu_p=5\log d_L+25\quad . \tag{5.16}$$

Given $H_{\tau 0}$ = 70.5km^{-1}Mpc^{-1} , these equations will produce the same fit to the data gathered by Riess et al. [14, Figure 7].

5.4 Flatness problem

This problem has effectively been removed for the new model because the primeval particle is well specified in mass and internal mechanism. Let the density parameter be given as usual by $(\Omega_M = \rho/\rho_c)$, where critical density ρ_c exists for $(k = \Lambda = 0)$. Then from Eq.(5.3), we can derive:

$$(\Omega_M + \Omega_\Lambda - 1) = k\left(c^2/v^2 - 1\right) \quad . \tag{5.17}$$

According to Figure 2, (c/v) is always between 1.0 and 2.0 and therefore $(\Omega_M - 1)$ approaches zero in a steady manner as $(R \to 0$ and $\Omega_\Lambda \to 0)$. There is no problem with this because mass is conserved in Eq.(5.6), and kc^2 is a constant amount of energy which becomes relatively unimportant as $(R \to R_\alpha)$. The gradient of Eq.(5.17) remains finite:

$$\frac{\delta(\Omega_M - 1)}{\delta(v/c)} = -2k\left(\frac{c}{v}\right)^3 \underset{(v \to c)}{\to} -2k \quad . \tag{5.18}$$

This contrasts with the standard model, wherein (c/v) approaches zero asymptotically as $(R \to 0)$, thereby requiring great accuracy of $(\Omega_M \to 1)$ for an un-specified singularity.

6. Properties of the primeval particle and gravitons

6.1 Primeval particle

The size of R_α in Section 5.1 can be specified if the primeval particle was of mass ($M_{\alpha u} \approx 7.748 \times 10^{52}$ kg), such that a gravitational strength factor may be expressed as:

$$\frac{GM_{\alpha u} m_\ell}{\hbar c} = \left(\frac{e^2}{\hbar c}\right) \times \left(\frac{e^2}{Gm^2}\right) = \frac{1}{137}\left(\frac{E}{G}\right) \quad , \tag{6.1}$$

where ($m_\ell = m_p/9$) is the proton-pearl mass [15], $\hbar$ is Planck's constant/2π, $(e^2/\hbar c \approx 1/137)$ is the fine structure constant or electromagnetic strength factor, ($e/m = E^{1/2}$) is the electronic charge/mass ratio, and ($E/G = 4.1659 \times 10^{42}$). The primeval mass relative to a proton-pearl mass is then:

$$\frac{M_{\alpha u}}{m_\ell} = \left(\frac{m}{m_\ell}\right)^2 \left(\frac{E}{G}\right)^2 = 4.169 \times 10^{80} \quad . \tag{6.2}$$

This primeval mass $M_{\alpha u}$ exploded to yield free radiation plus the residual universal mass, ($M_U = 1.085 \times 10^{52}$kg) used in Sections 4, 5, such that ($M_U / m_\ell = 5.838 \times 10^{79}$).

A pearl classical *electromagnetic* radius would be given by

$$r_{o\ell} = e^2 / m_\ell c^2 = 1.3812 \times 10^{-17}\,\mathrm{m} \quad , \tag{6.3a}$$

whereas a *gravitational* radius for mass $M_{\alpha u}$ may be defined as

$$R_{\alpha u} = GM_{\alpha u} / c^2 = 6.082\,\mathrm{Gly} \quad . \tag{6.3b}$$

These characteristic parameters are connected by

$$R_{\alpha u} = r_{o\ell} \times (E/G) \quad , \tag{6.4}$$

therefore we will postulate that the primeval particle of mass $M_{\alpha u}$ was like a *supermassive-pearl* of radius ($R_\alpha = r_{o\ell}$), although its charge and structure were not those of the proton-pearl. Such a particle with its surrounding gravitational field *required* coordinate space to contain it and whatever else existed, such as its twin anti-matter particle or other primeval particles in a multiverse. This is different from Lemaître's hypothesis of the 'primeval atom', which proposed that space and time only came into being following disintegration; see Godart & Heller [16].

According to our proton model [15], the supermassive-pearl might have consisted of helical loops of matter comprised of many smaller spinning elemental seeds, all tied together by a strong viscous gluon field. During disintegration, the seeds started decaying into radiation plus lesser particles but the pressure generated between seeds by deflagration caused segregation and prevented total conversion during the fireball expansion and cooling stage. Therefore, separate matter volumes remaining from decaying seeds survived the fireball. The viscous gluon material first helped equalise overall density, but then to initiate great strings and super-clusters of galaxies with large-scale velocity flow, plus low density voids. Consequently, observed great structures did not have to form entirely from accreted *homogeneous* matter. Early structure formation was thereby amplified above the standard model; see Perivolaropoulas [17]. Indeed, such great structures did not occur in the Millennium Simulation produced by Springel et al. [24].

Low-order multipole maps derived by Bielewicz et al. [18], Eriksen et al. [19], and Tegmark et al. [20], may be interpreted in terms of the hot-spots due directly to the surviving matter volumes. Some evidence of vorticity and toroidal field might eventually be detected in the cosmic microwave background anisotropy maps from WMAP; see Jaffe et al. [21] and de Oliveira-Costa et al. [22]. Fine granularity in the form of minor seeds and gluons would help account for some correlation between the CMB anisotropy and galaxy clusters; see Cole et al. [23]. The observed small degree of anisotropy is all that remains of granular structure, so thermalisation of the cosmic microwave background radiation involved multiple scattering of the radiation by photons, matter, and dark matter. In the next section we will show how 86% of the primeval particle mass must have been completely lost from the fireball as radiation into surrounding free space.

The above segregation of seed structure could account for dark matter which survived the fireball without conversion into normal matter; but even now, dark matter falling into stars might convert to normal matter, or simply convert to kinetic energy then radiation.

6.2 *Evaluation of r_m , r_a and Λ*

For simple interpretation of Eqs.(2.4) to (2.12), the cosmological constant Λ should depend on the central mass M through r_0 as in Eq.(2.8); but r_m and r_a should be properties of the fundamental particles constituting M. An estimate of graviton maximum radius r_m may be derived from proton theory [15] to satisfy astronomical observations.

First of all, *for an electron*, the electric field strength relative to the gravitational field is given by:

$$(e^2/Gm^2) = (E/G) \approx 4.1659x10^{42} . \qquad (6.6)$$

An application of this ratio is possible if the electromagnetic field from an electron also ends at radius r_m , rather then extending to infinity. Then the electric field energy saved beyond r_m is given by:

$$\int_{r_m}^{\infty} \frac{1}{8\pi}\left(\frac{e^2}{r^4}\right) 4\pi r^2 dr = \frac{1}{2}\frac{e^2}{r_m} \quad . \qquad (6.7a)$$

This saving could conveniently provide the total *gravitational* field energy for the electron, which is emitted from an effective internal source radius r_s . Namely from Eq.(2.10a) we integrate field energy density T_4^4 and propose:

$$\int_{r_s}^{r_m} \frac{1}{8\pi}\left(\frac{Gm^2}{r^4}\right)4\pi r^2 dr \approx \frac{1}{2}\frac{Gm^2}{r_s} \quad . \tag{6.7b}$$

Consequently, by equating Eqs.(6.7a) and (6.7b), the ratio in Eq.(6.6) may be expressed as (r_m/r_s = E/G) for the *electron*.

Now if most of the universal mass comprises proton-pearls in matter and dark matter, we will relate r_s and r_m to *pearl* dimensions. Thus our proton model [15] consists of 9 'pearls' of mass ($m_\ell = m_p/9$); and a pearl *electromagnetic* radius has been given in Eq.(6.3a). Then a pearl graviton is proposed to have a wavelength $(\lambda_{G\ell} = 2\pi r_{o\ell})$, which will be taken as r_s the characteristic source dimension. The graviton's maximum extent is therefore to be equal to (E/G) wavelengths:

$$r_m = \lambda_{G\ell}(E/G) = 2\pi\, r_{o\ell} \times (E/G) = 38.21\text{Gly} \; . \tag{6.8}$$

Further, the radius at zero acceleration will be proposed arbitrarily as:

$$r_a = r_m / 2\pi = r_{o\ell} \times (E/G) = 6.082\text{Gly} \; , \tag{6.9}$$

which is also the theoretical gravitational radius $R_{\alpha u}$ of the primeval particle Eq.(6.3b). This radius has also been employed previously as (a_z) in Eqs.(4.16) and (5.10) such that the current universal mass ($M_U = 1.085\text{x}10^{52}$kg) follows from the measured value for Λ in Eq.(4.11). If the *original* primeval mass was ($M_{\alpha u} \sim 7.748\text{x}10^{52}$kg) as in Eq.(6.1), then 86% of the mass must have been lost from the fireball material as radiation. This leaves a 14% portion of surviving mass, which is not a specified value for the future of the universe. For example, the expansion velocity at the lowest point a_z is not sensitive to variation of ($M_U / M_{\alpha u}$), thus from Eq.(5.3) with Eq.(5.10) we can derive:

$$\frac{\dot{R}_z}{c} = \left(\frac{3M_U}{M_{\alpha u} + 3M_U}\right)^{1/2} \; . \tag{6.10}$$

It is interesting to calculate a real value of the repulsive field. The universal *cosmic* repulsion field term at radius a_z is from Eq.(5.4):

$$F_\Lambda = \left(\Lambda c^2/3\right)a_z = GM_U / a_z^2 \approx 2.173 \times 10^{-10}\,\text{ms}^{-2} \, . \tag{6.11a}$$

For comparison, the gravitational field in a spherical galaxy, of included mass $10^{11} M_\Theta$ within a 10kpc radius, is of the same order:

$$F = \frac{G \times 10^{11} M_\Theta}{r^2} \approx 1.4x10^{-10}\, ms^{-2}. \qquad (6.11b)$$

Overall, according to these different solutions, the cosmological *constant* is proportional to the source mass which generates it, as given by:

$$\Lambda = 3GM / c^2 r_a{}^3 = 3r_o / r_a{}^3 \quad , \qquad (6.12)$$

where r_a depends upon the type of source particle (eg. proton-pearl, electron). The change in graviton behaviour from attraction to repulsion may be understood as a change of helicity within its structure due to longitudinal stress, see Figure 3.

Figure 3. Pictorial representation of a graviton's reversal of helicity/attraction due to internal stress; cf. tendril of passiflora.

Gravitons emitted by *electrons* are proposed to have a wavelength $(\lambda_{Ge} = 2\pi\, r_{oe})$, where $(r_{oe} = e^2 / mc^2)$ is the classical electromagnetic radius of an electron. This will be taken as effective source radius for the gravitational field, so the maximum extent is equal to (E/G) wavelengths:

$$r_{me} = \lambda_{Ge} (E / G) = 7796 Gly \; . \qquad (6.13)$$

The corresponding radius at zero acceleration will be:

$$a_{ze} = r_{me} / 2\pi = 1241 Gly \; . \qquad (6.14)$$

Clearly these dimensions might allow gravitational interaction between members of a multiverse; although the main component of the gravitational field from our primeval particle $M_{\alpha u}$ of radius ($R_\alpha = r_{o\ell}$) was probably limited in range by Eq.(6.8).

7. Cosmic flow and variation of the fine structure constant

7.1 Cosmic flow

Cosmic flow, dark flow, has recently been observed and looks real, see [25] [26] [27]. Its magnitude implies that it must be due to a distant mass attracting large clusters of galaxies in a particular direction. This mass appears to be beyond our own universal material and is not visible to us.

Our model only describes a single primeval supermassive-pearl having complex circulating structure, which would account for early seeding and large-scale structure in this universe. On the other hand, there could have existed a neighbouring pearl or anti-pearl to produce an extra (anti-)universe not yet visible to us. The gravitational field of our supermassive-pearl existed before it disintegrated into the Big Bang, so any other universes will also have attracted gravitationally to affect our expansion throughout. Today this process might reveal itself as flow of galaxy clusters against the general cosmic expansion. Given the proposed complex primeval particle structure [15], the cosmic flow may be branched into several directions, appearing as rings or arcs in the cosmic microwave background [28].

7.2 Variation of alpha (α)

The apparent variation of the fine structure constant reported by Webb et al [29] might be attributed to the various absorption lines originating from inhomogeneous clouds with different turbulence and radial velocities. In addition, the spectrograph is sensitive to variable illumination of the slit by the QSO scintillating image position, plus variable vignetting, see Suzuki et al [30]. This problem could be attenuated by using a light-pipe image diffuser to mix the QSO light, and reference source, into a uniform source on the slit, see Wayte et al [31] [32]. Intuitively, the tiny apparent variation in α over 5Gyr is indicative of zero actual variation. One specific derivation of natural α makes it a constant [33], which can only be increased in high pressure environments like $e+e^-$ collisions, [34].

8. Conclusion

Repulsive gravity at large distances has been accommodated within the universal solution of Einstein's equations by introducing a cosmological constant which represents inherent graviton-graviton repulsion at large radii, rather than dark energy. A cosmological model for the external coordinate observer was then developed to replace Friedmann-Lemaître cosmology with its theoretical problems. It was logically necessary to limit the graviton field extent from matter to a definite maximum radius. This radius was related to proton structure and led to an estimated onset of universal repulsion at 7.15Gyr (local time) after the big-bang, or 9.59Gyr in the coordinate-frame, when the universe radius was 6.08Gly. The present age of the universe is 13.7Gyr (local time), corresponding to 17.5Gyr coordinate time. Evolution of stars and galaxies is governed by the local time rate so cosmological redshift and luminosity distance take the values found for the standard model. The horizon-problem has been removed by prescribing a granular primeval particle in equilibrium, which disintegrated at the velocity of light to produce a viscous fireball. The singularity problem is thereby redundant, and the flatness problem is no more. Observations of the Hercules–Corona Borealis Great GRB Wall and Huge-LQG plus other massive structures helped justify abandoning the cosmological principle.

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