The Dark side of Gravity (living review)

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Dark Gravity (DG) is a background dependent bimetric and semi-classical extension of General Relativity with an anti-gravitational sector. The foundations of the theory are reviewed. The main theoretical achievement of DG is the avoidance of any singularities (both black hole horizon and cosmic initial singularity) and an ideal framework to understand the cancellation of vacuum energy contributions to gravity and solve the old cosmological constant problem. The main testable predictions of DG against GR are on large scales as it provides an acceleration mechanism alternative to the cosmological constant. The detailed confrontation of the theory to SN-Cepheids, CMB and BAO data is presented. The Pioneer effect, MOND phenomenology and Dark Matter are also investigated in the context of this new framework.

Keywords: Anti-gravity, Janus Field, Negative energies, Time Reversal, Field Discontinuities

1. Introduction

In the seventies, theories with a flat non dynamical background metric and/or implying many kinds of preferred frame effects became momentarily fashionable and Clifford Will has reviewed some of them (Rosen theory, Rastall theory, BSLL theory ...) in his book [35]. Because those attempts were generically roughly conflicting with accurate tests of various versions of the equivalence principle, the flat non dynamical background metric was progressively given up. The Dark Gravity (DG) theory we support here is a remarkable exception as it can easily reproduce most predictions of GR up to Post Newtonian order (as we shall remind in the two following sections) and for this reason deserves much attention since it might call into question the assumption behind most modern theoretical avenues: background independence.

DG follows from a crucial observation: in the presence of a flat non dynamical background $\eta_{\mu\nu}$, it turns out that the usual gravitational field $g_{\mu\nu}$ has a twin, the "inverse" metric $\tilde{g}_{\mu\nu}$. The two being linked by :

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\left[g^{-1}\right]^{\rho\sigma} = \left[\eta^{\mu\rho}\eta^{\nu\sigma}g_{\rho\sigma}\right]^{-1} \tag{1}$$

are just the two faces of a single field (no new degrees of freedom) that we called a Janus field [3][4][7][14][15]. See also [5][8][9][6] [30][31][32][33][34][28] for alternative approaches to anti-gravity with two metric fields. In the following, fields are labelled with (resp without) a tilde if they are exclusively built from $\tilde{g}_{\mu\nu}$ (resp $g_{\mu\nu}$) and/or it's inverse and/or matter and radiation fields minimally coupled to $\tilde{g}_{\mu\nu}$ (resp $g_{\mu\nu}$). The exceptions are $\eta_{\mu\nu}$ and it's inverse $\eta^{\mu\nu}$.

The action treating our two faces of the Janus field on the same footing is achieved by simply adding to the usual GR and SM (standard model) action, the similar action with $\tilde{g}_{\mu\nu}$ in place of $g_{\mu\nu}$ everywhere.

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L}) \tag{2}$$

where R and $\tilde{R}$ are the familiar Ricci scalars respectively built from $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ as usual and L and $\tilde{L}$ the Lagrangians for respectively SM F type fields minimally coupled to $g_{\mu\nu}$ and $\tilde{F}$ fields minimally coupling to $\tilde{g}_{\mu\nu}$. This theory symmetrizing the roles of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ is Dark Gravity (DG) and the field equation satisfied by the Janus field derived from the minimization of the action is:

$$\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}G^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{G}^{\rho\mu} = -8\pi G(\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}T^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{T}^{\rho\mu}) \tag{3}$$

with $T^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ the energy momentum tensors for F and $\tilde{F}$ fields respectively and $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ the Einstein tensors (e.g. $G^{\mu\nu} = R^{\mu\nu} - 1/2g^{\mu\nu}R$). Of course from the Action extremization with respect to $g_{\mu\nu}$ (see the detailed computation in the Annex), we first obtained an equation for the dynamical field $g_{\mu\nu}$ in presence of the non dynamical $\eta_{\mu\nu}$. Then $\tilde{g}_{\mu\nu}$ has been reintroduced using (1) and the equation was reformatted in such a way as to maintain as explicit as possible the symmetrical roles played by the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ of the Janus field. The contracted form of the DG equation simply is :

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{4}$$

It is well known that GR is the unique theory of a massless spin 2 field. However DG is not the theory of one field but of two fields: $g_{\mu\nu}$ and $\eta_{\mu\nu}$. Then it is also well known that there is no viable (ghost free) theory of two interacting massless spin 2 fields. However, even though $\eta_{\mu\nu}$ is a genuine order two tensor field transforming as it should under general coordinate transformations[a], $\eta_{\mu\nu}$ actually propagates no degrees of freedom : it is really non dynamical, not in the sense that there is no kinetic (Einstein-Hilbert) term for it in the action, but in the sense that all it's degrees of freedom were frozen a priori before entering the action and need not extremize the action : we have the pre-action requirement that Riemm($\eta_{\mu\nu}$)=0 like in the BSLL, Rastall and Rosen theories [35]. So DG is also not the theory of two interacting spin 2 fields.

We will later carry out the complete analysis of the stability of the theory however we already found that, at least about a Minkowskian background common to

[a] in contrast to a background Minkowski metric $\hat{\eta}_{\mu\nu}$ such as when we write $g_{\mu\nu} = \hat{\eta}_{\mu\nu} + h_{\mu\nu}$, which by definition is invariant since only the transformation of $h_{\mu\nu}$ is supposed to reflect the effect of a general coordinate transformation applied to $g_{\mu\nu}$

the two faces of the Janus field, the worst kind of classical instabilities might be avoided (reduced to a well acceptable level) because:

- Fields minimally coupled to the two different sides of the Janus field never meet each other from the point of view of the other interactions (EM, weak, strong) so stability issues could only arise in the purely gravitational sector.
- The run away issue [10] [11] is avoided between two masses propagating on $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively, because those just repel each other, anti-gravitationally as in all other versions of DG theories [9][6] rather than one chasing the other ad infinitum.
- The energy of DG gravitational waves almost vanishes about a common Minkowski background (we remind in a forthcoming section that DG has an almost vanishing energy momentum pseudo tensor $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ in this case) avoiding or extremely reducing for instance the instability of positive energy matter fields through the emission of negative energy gravitational waves.

In particular the first two points are very attractive so we were not surprised discovering that recently the ideas of ghost free dRGT bimetric massive gravity [36] have led to a PN phenomenology identical to our though through an extremely heavy, unnatural and Ad Hoc collection of mass terms fine tuned just to avoid the so called BD ghost[b]. Anyway, all such kind of bimetric constructions seriously question the usual interpretation of the gravitational field as being the metric describing the geometry of space-time itself. There is indeed no reason why any of the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, which describe a different geometry should be preferred to represent the metric of space-time. At the contrary our non dynamical flat $\eta_{\mu\nu}$ is now the perfect candidate for this role.

We think the theoretical motivations for studying as far as possible a theory such as DG are very strong and three-fold : challenge the idea of background independence, bridge the gap between the discrete and the continuous and challenge the standard understanding of time reversal.

- Challenge the idea of background independence because DG is the straightforward generalization of GR in presence of a background non dynamical metric so either there is no such background and GR is most likely the fundamental theory of gravity or there is one and DG is the most obvious candidate for it.
- Bridge the gap between the discrete and the continuous because we here have both the usual continuous symmetries of GR but also a permutation

[b]Indeed the first order differential equation in [32] is exactly the same as our: see e.g eq (3.12) supplemented by (4.10) and for comparison our section devoted to the linearized DG equations. This is because the particular coupling through the mass term between the two dynamical metrics in dRGT eventually constrains them to satisfy a relation Eq (2.4) which for $\alpha = \beta$ [32] becomes very similar to our Eq (1) to first order in the perturbations which then turn out to be opposite (to first order) as Eq (4.10) makes it clear.

symmetry which is a discrete symmetry between the two faces of the Janus field.

- Challenge the standard understanding of time reversal because as we shall see the two faces of the Janus field are related by a global time reversal symmetry.

The two last points require more clarification and the reader may find enlightening sections in our previous publications (though most of their content is now outdated) however we may summarize the situation as follows:

Basically modern physics incorporates two kinds of laws: continuous and local laws based on continuous symmetries, most of them inherited from classical physics, and discrete and non local rules of the quanta which remain largely as enigmatic today as these were for their first discoverers one century ago. Though there are many ongoing attempts to "unify" the fundamental interactions or to "unify" gravity and quantum mechanics, the unification of the local-continuous with the non-local-discrete laws would be far more fundamental as it would surely come out with a genuine understanding of QM roots. However such unification would certainly require the identification of fundamental discrete symmetry principles underlying the discontinuous physics of the quanta just as continuous and local laws are related to continuous symmetries. The intuition at the origin of DG is that the Lorentz group which both naturally involves discrete P (parity) and T (time reversal) symmetries as well as continuous space-time symmetries might be a natural starting point because the structure of this group itself is already a kind of unification between discrete and continuous symmetries. However neither P nor the Anti-Unitary T in the context of QFT seem to imply a new set of dynamical discrete laws. Moreover our investigation in [7] (see also [14] section 3) revealed that following the alternative non- standard option of the Unitary T operator to understand time reversal led to a dead-end at least in flat spacetime: indeed there is an obvious unitary-T symmetric of the usual positive energy field of QFT and this is now a negative energy field creating and annihilating negative energy quanta however this field requires a negative kinetic term in the Lagrangian and accordingly a negative Hamiltonian: the problem then is that the Unitary time reversal alone is not able to link the positive Hamiltonian for the familiar positive energy field to the new negative Hamiltonian for the negative energy field.

However we concluded that it might eventually be possible to understand and rehabilitate negative energies and relate them to normal positive energies through time reversal but only in the context of an extension of GR in which the metric itself would transform non trivially under time reversal. This time reversal not anymore understood as a local symmetry but as a global symmetry implying a privileged time and a privileged origin of time, would jump from one metric to it's T-conjugate. Only such time reversal $x^\mu \Rightarrow {x^\mu}_T$ would retain it's discrete nature inherited from the local Lorentz group but now promoted to a global symmetry because at the contrary to a diffeomorphism, a mere reparametrisation which has no actual physical content as it does not affect the set of inertial coordinates i.e. $\zeta^\alpha(x^\mu) \Rightarrow \zeta^\alpha_T(x^\mu_T) = \zeta^\alpha(x^\mu)$ but rather like an internal symmetry it would really discretely transform one set of inertial coordinates $\zeta^\alpha(x^\mu)$ into another non equivalent one $\zeta^\alpha(x^\mu_T) = \zeta^\alpha_T(x^\mu) \neq \zeta^\alpha_T(x^\mu_T)$ (see [4] section 5), i.e. it would transform a metric into a really distinct one describing a different geometry. The DG solutions that we shall remind in the first sections in the homogeneous-isotropic case impressively confirm that our sought privileged time x^0 is a cosmological conformal time reversing according the global symmetry $x^0 \Rightarrow x^0_T = -x^0$ about a privileged origin of time $x^0 = 0$ and that the two faces of the Janus field are just this time reversal conjugate metrics we have been looking for: in particular the conjugate conformal scale factors are indeed found to satisfy $\tilde{a}(t) = 1/a(t) = a(-t)$ (also see [14] section 6.2). The interpretation of this new global time reversal is also very different from the interpretation of the familiar local time reversal: the later exchanges initial and final states as does the anti-unitary operator of QFT so it means going backward in time whereas our new global time reversal amounts to jump from t to -t and not to go backward in time.

The solutions in the isotropic case then also confirm the reversal of the gravific energy as seen from the conjugate metric i.e any F field is seen as a positive energy field by other F fields (as it produces an attractive potential well in $g_{\mu\nu}$) but as a negative energy field (as it produces a repelling potential hill in $\tilde{g}_{\mu\nu}$) from the point of view of $\tilde{F}$ fields and vice versa. In a sense DG had to reinvent an absolute zero and negative values for the time and mass-energies which only became possible thanks to the pivot metric $\eta_{\mu\nu}$. Eventually we are aware that we are not yet ready to derive the Planck-Einstein relations from this new framework but in the following we will have to keep in mind what was our initial motivation: understand the origin of the discrete rules of QM from discrete symmetries to not prohibit oneself the explicit introduction of discrete rules and processes any time the development of the theory seems to require them.

The article is organized as follows: in section 2 we remind and complement the results of previous articles as for the homogeneous-isotropic solution and present the full complete test of DG cosmology against the main data: SN, BAO, CMB. In section 3 we comment the local static isotropic asymptotically Minkowskian solutions of the DG equation. In section 4 we discuss the linearized theory about this common Minkowskian background for $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ and the prediction of the theory as for the emission of gravitational waves. In sections 5 and 6, we give up the hypothesis that the two conjugate metrics are asymptotically the same to derive the isotropic static solution again in this more general case and discuss our pseudo Black Hole and new predictions for gravitational waves. In section 7, we investigate the physics of matter exchanges between the two sides of our universe. These matter exchanges were found necessary to avoid static solutions in section 2. In section 8 we start to seriously consider the case of actual static background solutions in some delimited spatial domains and pursue this exploration in section 8 and 9 having in mind a possible explanation of the Pioneer anomaly and renewed understanding of expansion effects. Various other possible predictions are described in section 10. Section 11 explores a plausible MOND like phenomenology of DG. Section 12 discusses all kind of stability issues to conclude that the theory is safe once understood as a semi-classical theory of gravity. Section 13 outlines the DG linear theory of cosmological perturbations and then analyses a new plausible Dark Matter candidate and mechanisms mimicking the Dark Matter phenomenology within our framework. Before the conclusion, section 14 last remarks and outlooks among other topics rapidly cover in turn the old cosmological problem, the scale invariance of the primordial power spectrum, the potential issue of closed timelike curves (CTCs) and emphasizes the need for a theory of gravity such as DG which very principles being based on discrete as well as continuous symmetries, for the first time open a natural bridge to quantum mechanics and hopefully offer a royal road toward a genuine unification.

2. The homogeneous and isotropic case

2.1. *Unphysical background solutions*

We found that an homogeneous and isotropic solution is necessarily spatially flat because the two sides of the Janus field about our flat Minkowski background are required to be both homogeneous and isotropic whereas if one of the two metrics is homogeneous and isotropic with non vanishing spatial curvature $k \neq 0$ then the conjugate one is not an homogeneous and isotropic metric.

The conjugate homogeneous and isotropic spatially flat metrics are then assumed to take the form $g_{\mu\nu} = a(t)\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = a^{-1}(t)\eta_{\mu\nu}$. In the coordinate system in which the non dynamical background Minkowski metric $\eta_{\mu\nu}$ reads diag(-1,1,1,1), our metrics then have the conformal form. In the following the time variable t is therefore the conformal time and the Hubble parameters H and $\tilde{H}$ are understood to be conformal Hubble parameters. Then the two Friedman type equations the conformal scale factor should satisfy are:

$$a^2(2\dot{H} + H^2) - \tilde{a}^2(2\dot{\tilde{H}} + \tilde{H}^2) = -6K(a^4 p - \tilde{a}^4\tilde{p}) \tag{5}$$

$$a^2H^2 - \tilde{a}^2\tilde{H}^2 = 2K(a^4\rho - \tilde{a}^4\tilde{\rho}) \tag{6}$$

with $K = \frac{4\pi G}{3}$, but an equivalent couple of equations is:

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{7}$$

$$\dot{a}^2 - \dot{\tilde{a}}^2 = 2K(a^4\rho - \tilde{a}^4\tilde{\rho}) \tag{8}$$

The time derivative of the second equation leads to:

$$a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4\frac{\dot{\rho}}{H} - \tilde{a}^4\frac{\dot{\tilde{\rho}}}{H} + 4\rho a^4 + 4\tilde{\rho}\tilde{a}^4) \tag{9}$$

with $H = \frac{\dot{a}}{a} = -\frac{\dot{\tilde{a}}}{\tilde{a}}$. The energy conservation equations on both sides being:

$$\frac{\dot{\rho}}{H} = -3(\rho + p) \tag{10}$$

$$\frac{\dot{\tilde{\rho}}}{\tilde{H}} = -\frac{\dot{\tilde{\rho}}}{H} = -3(\tilde{\rho} + \tilde{p}) \tag{11}$$

we can replace the corresponding terms in (9),

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{12}$$

$$a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4(\rho - 3p) + \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{13}$$

then adding and subtracting the two equations we get the new equivalent couple of differential equations:

$$a\ddot{a} = Ka^4(\rho - 3p) \tag{14}$$

$$\tilde{a}\ddot{\tilde{a}} = K\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) \tag{15}$$

which makes clear that the two equations are not compatible with $\tilde{a} = 1/a$ and any usual equation of state except for empty and static universes. For instance in the $a(t) = e^{h(t)}$, $\tilde{a}(t) = e^{-h(t)}$ domain of small h(t), to first order in h, (14)(15) reduce to:

$$\ddot{h} = K(\rho_0 - 3p_0) \geq 0 \tag{16}$$

$$\ddot{h} = -K(\tilde{\rho}_0 - 3\tilde{p}_0) \leq 0 \tag{17}$$

The reason for that incompatibility is that we have no equivalent of the Bianchi identities to make the DG equations functionally dependent as in GR. It is therefore not surprising to get two independent equations for the scale factor (constraining it to remain static and the universe empty) when the matter and radiation fields equations of motion are satisfied on each metric. By the way we can notice that in general we have four additional independent equations relative to GR but also four additional independent degrees of freedom. Indeed, though DG equations are of course generally covariant, the gauge invariance of GR is lost[c]: our equations are not invariant under the transformations of $g_{\mu\nu}$ alone but under the combined transformations of $g_{\mu\nu}$ and $\eta_{\mu\nu}$. Therefore we expect that for instance the two scalar and two vector degrees of freedom under rotations about a gravitational wave direction of motion, that are pure Gauge within GR, become physical in DG.

2.2. *Matter-radiation exchange*

As they stand the DG equivalent (7) of GR Friedman equations are not viable. However following an original idea by Prigogin (see for instance [49] and multi-references

[c]Another example of theory with non dynamical degrees of freedom is for instance unimodular gravity [43][44]

therein) let's allow the gravitationally induced adiabatic creation or annihilation of particles on either side. Our conservation equations then get modified[d]:

$$\dot{\rho} = (\Gamma - 3H)(\rho + p) \tag{18}$$

$$\dot{\tilde{\rho}} = (\tilde{\Gamma} - 3\tilde{H})(\tilde{\rho} + \tilde{p}) \tag{19}$$

The next assumption is to relate the creation rates through $\tilde{\Gamma} = -\Gamma$ (just as $\tilde{H} = -H$) in such a way that there is no actual creation or annihilation of particles but merely a transfer from one metric to the conjugate so that the baryonic number conservation is for instance globally insured but not necessarily the absolute value of the particles energy as these are transferred from one metric to the other.

In [49] the creation is also done in such a way that the energy-momentum tensor is covariantly conserved on the right side of the Einstein equation as required by the Bianchi identities: the energy is therefore somehow transferred from gravity to the created particles and the final purpose is to mimic a cosmological constant. This obviously requires that the energy momentum tensor at the source of Einstein equation be modified to include not only ρ and p but also a creation pressure to be covariantly conserved. In our case the Bianchi identities are only approximately verified on the left hand side which implies that the right hand side can involve the energy-momentum conservation violating tensor (very weak violation when the ratio of the scale factors is very large) involving just ρ and p alone.

The adiabaticity condition is another essential assumption in [49] to insure that the created matter should not disturb the mean thermodynamical properties of the cosmic fluid given that a high creation rate is needed to produce the observed acceleration of the universe. At the contrary, in our case it is only used in a first attempt, as an arbitrary working assumption here allowing us to make direct use of the above relations from [49]. Indeed our creation and annihilation rates will turn out to be so small (at any time except near the origin of time) that an influx of particles with energies quite different from the mean energy of particles in our universe should not be problematic on observational ground.

Now replacing again in the differential equations and again adding and subtracting them we alternatively get:

$$a\ddot{a} = K(a^4(\rho - 3p) + \frac{1}{2}(C_r + \tilde{C_r})) \tag{20}$$

$$\tilde{a}\ddot{\tilde{a}} = K(\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) + \frac{1}{2}(C_r + \tilde{C_r})) \tag{21}$$

including the creation/annihilation terms $C_r = a^4 \frac{\Gamma}{H}(\rho + p)$, $\tilde{C_r} = \tilde{a}^4 \frac{\Gamma}{H}(\tilde{\rho} + \tilde{p})$.

When our side density source terms dominate ($a^4 d >> \tilde{a}^4 \tilde{d}$) where d (resp $\tilde{d}$) is any linear combination of densities ρ and p (resp $\tilde{\rho}$ and $\tilde{p}$) alone, we just need

[d]The equations are as valid in conformal time as in standard time. The conformal time Γ and H here are related to the standard time t' for our side metric Γ' and H' according $\Gamma = a\Gamma'$ and $H = aH'$. The standard time being t" for the conjugate metric we also have $\tilde{\Gamma} = \tilde{a}\tilde{\Gamma}''$ and $\tilde{H} = \tilde{a}\tilde{H}''$

$\frac{\Gamma}{H} << 1$ to recover from the first of these equations, the same evolution law of the scale factors we had before. The good new is that now the second equation can be compatible with this solution provided the C_r term is dominant in the second equation : $\frac{\Gamma}{H} >> \frac{\tilde{a}^4\tilde{d}}{a^4 d}$. Then for instance in matter dominated eras on both sides, the equations simplify a bit:

$$a\ddot{a} \approx Ka^4\rho \tag{22}$$

$$\tilde{a}\ddot{\tilde{a}} \approx K\frac{a^4\rho}{2}\frac{\Gamma}{H} \tag{23}$$

from which we get the required evolution of Γ:

$$\Gamma \approx 2H\frac{\tilde{a}\ddot{\tilde{a}}}{a\ddot{a}} = \frac{2H}{a^4}(\frac{1-\frac{\dot{H}}{H^2}}{1+\frac{\dot{H}}{H^2}}) \tag{24}$$

For a power law $a(t) \propto t^\alpha$ of the conformal scale factor,

$$\Gamma \approx \frac{2\alpha}{a^{4+1/\alpha}}(\frac{\alpha+1}{\alpha-1}) \tag{25}$$

is positive (transfer of particles from the conjugate to our side) for $\alpha > 1$ or $-1 < \alpha < 0$ and negative (transfer of particles from our to the conjugate side) otherwise. α positive (resp negative) translates to a decelerating (resp accelerating) universe in standard time t'. Hence in a cold matter dominated era, $\alpha = 2$ (the solutions are presented in greater detail in the next subsection) implies that particles are transferred from the conjugate to our side.

When the conjugate scale factor dominates, roles are exchanged so:

$$\tilde{a}\ddot{\tilde{a}} \approx K\tilde{a}^4\tilde{\rho} \tag{26}$$

$$a\ddot{a} \approx K\frac{\tilde{a}^4\tilde{\rho}}{2}\frac{\Gamma}{H} \tag{27}$$

then,

$$\Gamma \approx 2H\frac{a\ddot{a}}{\tilde{a}\ddot{\tilde{a}}} = \frac{2H}{\tilde{a}^4}(\frac{1+\frac{\dot{H}}{H^2}}{1-\frac{\dot{H}}{H^2}}) \tag{28}$$

For a power law $a(t) \propto t^\alpha$ of the conformal scale factor, the sign of

$$\Gamma \approx \frac{2\alpha}{a^{-4+1/\alpha}}(\frac{\alpha-1}{\alpha+1}) \tag{29}$$

behaves as before and now taking $\alpha = -2$ for an accelerating universe (see next subsection), particles are still transferred from the conjugate to our side.

We see that DG equations can be solved for physically acceptable solutions, i.e. a non static scale factor evolution : for that we need to introduce the transfer of particles between the two conjugate metrics. This conclusion is actually valid at all times as we could check by numerically integrating our differential equations. We did this assuming for instance $\tilde{p} = p = 0$ (this is just an example, the exercise would work as well for any equations of state) and then $\tilde{\rho} = \rho^{-1}$. Those equations of state of course can't be valid at anytime but the important point is that the same equations of state can be valid on both sides near the origin of time when we have the equality of conjugate densities there. The purpose of this example is actually just to understand the effect of Γ near the origin of time. The system of (necessarily) first order equations integrated thanks to Geogebra NresolEquadiff is:

$$\dot{a} = b \tag{30}$$

$$\dot{b} = \frac{a}{a^2 + \frac{1}{a^2}}\left(\frac{2b^2}{a^4} + K(a^4\rho - \frac{1}{a^4\rho})\right) \tag{31}$$

$$\dot{\rho} = \rho\frac{b}{a}(\Gamma/H - 3) \tag{32}$$

with $\Gamma/H = \frac{\frac{\dot{b}}{a}(a^2 - \frac{1}{a^2}) + 2\frac{b^2}{a^4}}{K(a^4\rho + \frac{1}{a^4\rho})} - 1$. The two first order equations of this system are equivalent to the second order equation $a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = K(a^4\rho - \tilde{a}^4\tilde{\rho})$ while $\frac{\Gamma}{H}$ is deduced from the other second order equation $a\ddot{a} + \tilde{a}\ddot{\tilde{a}} = K(a^4\rho + \tilde{a}^4\tilde{\rho})(1 + \frac{\Gamma}{H})$ still neglecting pressure terms. The resulting functions $a(t)$ and $\rho(t)$ of Figure 1 show that the density increases very sharply near t=0 because of the incoming matter from the dark side while the scale factor is almost constant. The density reaches a maximum for $\Gamma/H = 3$ then decreases as a^{-3} as expected for pressureless matter when matter exchange becomes negligible. This occurs as soon as our side scale factor has started to dominate over $\tilde{a} = 1/a$, and then this scale factor evolves as t^2 corresponding to $t'^{2/3}$ in standard comoving time coordinate.

It is important to understand that the only way to "reconcile" our two cosmological equations was to introduce an additional degree of freedom, which here is our scalar function Γ which must remain offshell (should not extremize the action) otherwise we would also have an additional equation for it, hence still more equations than degrees of freedom. In a sense it appears that the non dynamical $\eta_{\mu\nu}$ in the background requires the introduction of another non dynamical scalar.

Notice that neglecting the effects of expansion and pressure, our exchange process implies that $\frac{\dot{\rho}}{\rho} = -\frac{\dot{\tilde{\rho}}}{\tilde{\rho}}$, rather than $\dot{\rho} = -\dot{\tilde{\rho}}$. Though the latter would be much easier to interpret in terms of particles exchange (with energy reversal from a given metric point of view) it would soon lead to unacceptable negative $\tilde{\rho}$. For $\frac{\dot{\rho}}{\rho} = -\frac{\dot{\tilde{\rho}}}{\tilde{\rho}}$ on the other hand there is no trivial instantaneous particle exchange interpretation though integrating on a full half cosmological cycle the total energy and particles may have indeed been exchanged (with energy reversal) between the two metrics. So either there exists a kind of buffer container between the two conjugate metrics (in η ?) or the energy exchange should not be actually interpreted in terms of a genuine particle exchange. This discussion is continued in a forthcoming section titled "Matter-radiation exchange or equivalent alternative mechanism".

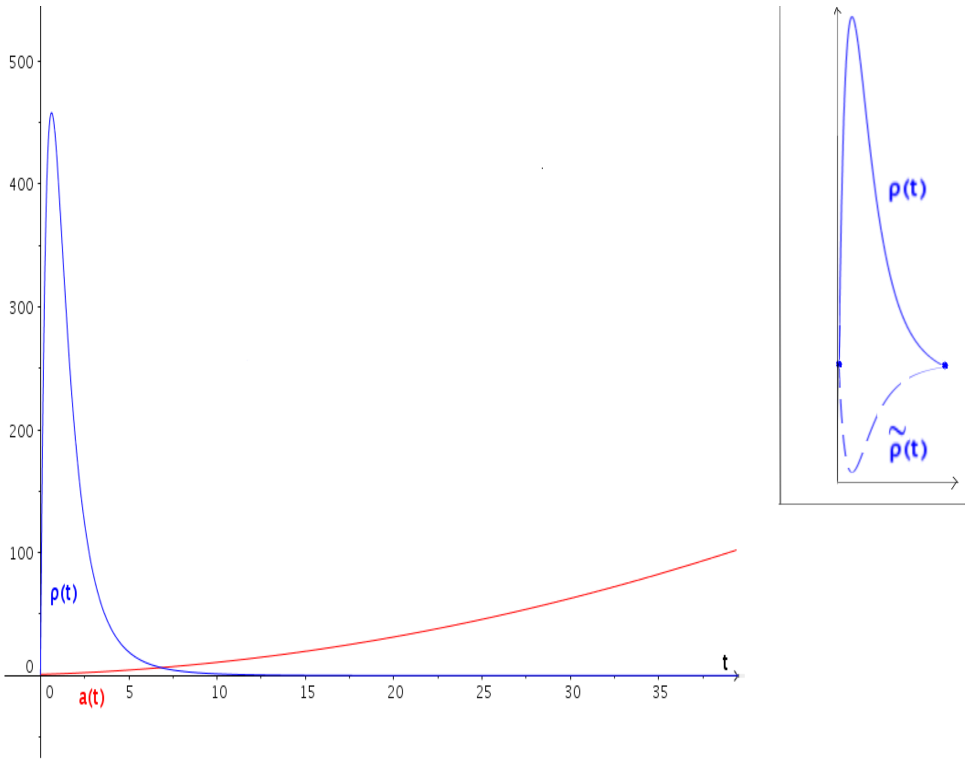

Fig. 1. a(t) and $\rho(t)$ when including the effect of the transfer rate Γ to restore the consistency of Friedmann and conservation equations.

2.3. *Cosmology*

We are then ready to investigate our cosmological solutions with the insurance that our introduced matter exchange mechanism makes these actual physical solutions. This subsection reviews and provides a more in depth analysis of results already obtained in [14][15].

2.3.1. *Reproducing GR cosmology*

The expansion of our side implies that the dark side of the universe is in contraction. Provided dark side terms and the exchange terms can be neglected which is certainly an excellent approximation far from t=0, our cosmological equations reduce to equations known to be also valid within GR. For this reason it is straightforward for DG to reproduce the same scale factor expansion evolution as obtained within the standard LCDM Model at least up to the redshift of the LCDM Lambda dominated era when something new must have started to drive the evolution in case we want to avoid a cosmological constant term. The evolution of our side scale factor before the transition to the accelerated regime is depicted in blue on the top of

Figure 2 as a function of the conformal time t and the corresponding evolution laws as a function of standard time t' are also given in the radiative and cold era. Notice however the new behaviour about t=0 meaning that the Big-Bang singularity is avoided.

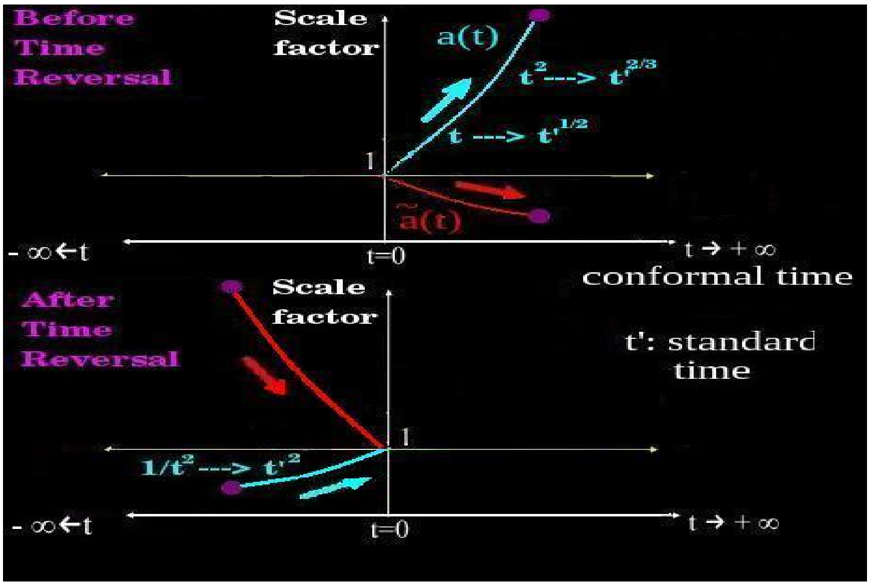

Fig. 2. Evolution laws and time reversal of the conjugate universes, our side in blue

2.3.2. *Continuous evolution and discontinuous permutation*

A discontinuous transition is a natural possibility within a theory involving truly dynamical discrete symmetries as is our permutation symmetry in DG. The basic idea is that some of our beloved differential equations might only be valid piecewise, only valid in the bulk of space-time domains at the frontier of which new discrete rules apply implying genuine field discontinuities. Here this will be the case for the scale factor. Of course a discontinuous process can't be consistent with the continuous process predicted by a differential equation but here the two kind of processes have their own domain of validity (the bulk vs the frontier) which avoids any conflicting predictions. However we would prefer the discontinuous process not to occur arbitrarily but to be governed by the same discrete symmetries readily readable from the equations of motion.

We postulated that a transition occurred billion years ago as a genuine permutation of the conjugate scale factors, understood to be a discrete transition in time modifying all terms explicitly depending on $a(t)$ but not the densities and pressures themselves in our cosmological equations: in other words, the equations of free fall apply at any time except the time of the discrete transition.

Let's be more specific. The equations of free fall for the perfect fluids on both sides of course apply as usual before and after the transition and for instance on our side in the cold era dominated by non relativistic matter with negligible pressure, we have $\frac{d}{dt}(\rho a^3) = 0$. Such conservation equation is valid just because it follows from our action for the matter fields on our side. But here we not only have the usual invariance of our action under continuous space-time symmetries from which we can derive the corresponding field conservation equations closely related to the continuous field equations of motion valid in the bulk of a space-time domain. We also have the invariance of the action under a permutation which is a discrete symmetry. To continuous symmetries can be associated continuous evolution, interactions and conservation equations of the fields thanks to variational methods. Such methods are of course not available to derive discontinuous processes from discrete symmetries so we postulate and take it for granted that our new permutation symmetry also allows a new kind of process to take place : the actual permutation of the conjugate a and $\tilde{a}$ while density and pressure terms remain unchanged. Because such process is not at all related to the continuous symmetries that generate the continuous field equation there is indeed no reason why the discrete version $(\rho a^3)_{before} = (\rho a^3)_{after}$ of a conservation equation such as $\frac{d}{dt}(\rho a^3) = 0$ should be satisfied by this particular process. The symmetry principles and their domain of validity are the more fundamental so we should not be disturbed by a process which violates the conservation of energy since this process is discontinuous, only valid at the frontier of a space-time domain and related to a new discrete symmetry for which we have no equivalent of the Noether theorem. Here the valid rule when the permutation of the scale factors occurs is rather $\rho_{before} = \rho_{after}$ and the same for the pressure densities.

This permutation (at the purple point depicted on figure 2) could produce the subsequent recent acceleration of the universe. This was already understood in previous articles [14] and [15] assuming our side source terms such as $a^4(\rho - 3p)$ have been dominant and therefore have driven the evolution up to the transition to acceleration. Specifically, just before the transition we have for instance: $a^4(\rho - 3p) >> \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ just because $a(t) >> \tilde{a}(t)$ and $\rho - 3p \approx \tilde{\rho} - 3\tilde{p}$ resulting in the usual (as in GR) expansion laws whereas just after the transition, $a^4(\rho - 3p) << \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ because now $a(t) << \tilde{a}(t)$ and $\rho - 3p \approx \tilde{\rho} - 3\tilde{p}$ resulting in the dark side source term now driving the evolution, producing a constant acceleration of our side scale factor in standard time coordinate t' following the transition redshift : $a(t') \propto t'^2$. In fact the reason why the densities do not change at the transition is that actually this transition is understood to be triggered by the crossing of conjugate densities ($\rho = \tilde{\rho}$ and $p = \tilde{p}$). Indeed, in general our cosmological equations are actually invariant under the combined permutations of densities and scale factors rather than permutation of scale factors alone so we might have expected from this symmetry that the allowed discontinuous process should exchange scale factors as well as densities simultaneously. However when the densities are equal our equations become invariant under the exchange of scale factors alone so the discontinuous process does not need to actually exchange the densities at this time but only the scale factors. Moreover we then have the bonus that the equality of densities is a perfect triggering condition for the transition to occur and we already knew from the previous section analyses that the crossing of densities is anyway expected.

2.3.3. *Global time reversal and permutation symmetry*

The evolution of the scale factor is largely determined by initial conditions at t=0. The parameters are the initial densities ρ_o, p_o, $\tilde{\rho}_o$, $\tilde{p}_o$ and initial expanding rate H_o (not to be confused with the usual present standard time t' Hubble rate H'_0). Considering a scenario with equal initial densities on both sides one needs a non vanishing H_o to get non static solutions which then turn out to satisfy the fundamental relation:

$$\tilde{a}(t) = \frac{1}{a(t)} = a(-t) \tag{33}$$

For this reason, already in our previous publications we could interpret our permutation symmetry as a global time reversal symmetry about privileged origin of conformal time t=0. But from such initial conditions (equal initial densities) it would erroneously appear that the densities (decreasing on our expanding side while increasing on the contracting dark side) will never have the opportunity to cross again. This is not exact however as soon as we acknowledge the crucial role of the significant continuous matter-radiation exchange near the origin of time. Indeed, thanks to matter-radiation exchange we can now have equal conjugate densities at the origin of time that will again be equal in the future according our previous subsection results and as can be readily seen from Figure 1.

Without such exchange mechanism, we know that our differential equations have no solutions except the trivial static ones but just out of curiosity we may consider the fictitious theory of conjugate "scalar-eta" fields $\phi\eta_{\mu\nu}$ and $\phi^{-1}\eta_{\mu\nu}$. This scalar field $\phi(t) = a^2(t)$ in the homogeneous case now only needs to satisfy the single differential equation 7. The value of considering such fictitious scalar theory is that it does not require us to postulate any exchange mechanism to get realistic solutions for the scale factor. For such theory, to get benefit from our scale factors permutation postulated process (A) we would need to break the initial equality between densities in such a way that the densities could again cross each other at a time different from t=0. Then however, we would realize that for $a(t) = e^{h(t)}$, h(t) is not anymore an odd function meaning that the condition Eq 33 for interpreting the permutation symmetry as a global time reversal would be broken. The only thing we would need to restore Eq 33 is to postulate another discrete process (B), again a density exchange process occurring at t=0 but now a discrete one. This is illustrated in fig 3 where h(t) is plotted with (plain line) and without (dotted line) assuming such exchange. The value of this fictitious scalar theory example is to make us realize that fortunately, thanks to the continuous matter exchange mechanism of our actual theory we get Eq 33 for free i.e. without any need to postulate an additional discrete process such as (B) at t=0.

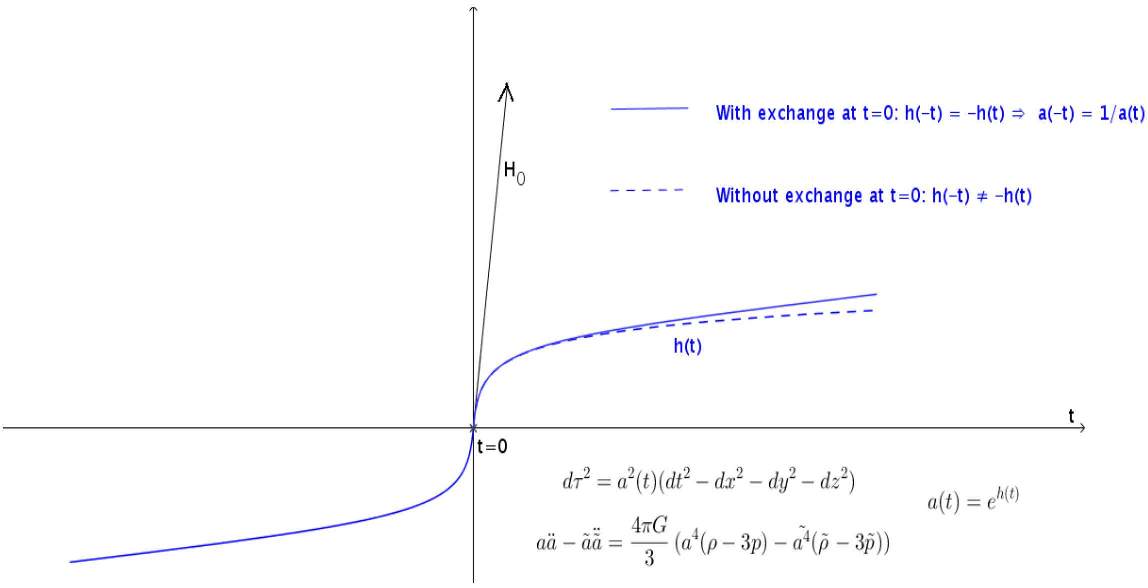

Fig. 3. h(t) with or without discrete exchange of densities at t=0 in a scalar-eta fictitious theory

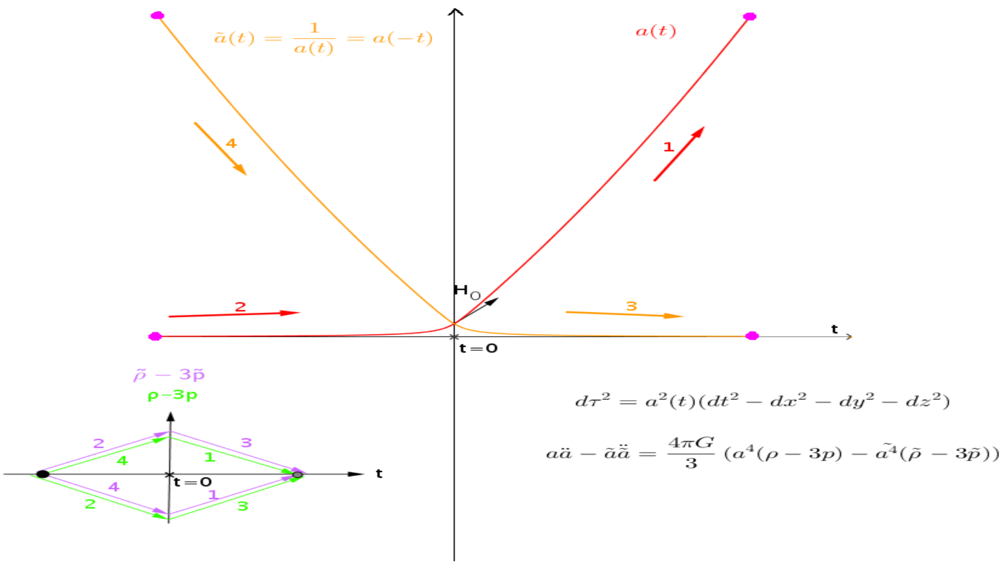

Fig. 4. Scale factors and densities evolution for a fictitious "scalar-eta" theory

Anyway, whether continuous or discontinuous, densities exchange processes result in the inversion of densities evolution laws i.e from decreasing to increasing or vice versa, so that the evolution of both densities and scale factors are cyclic as illustrated in fig 4. This also insures the stability of our homogeneous solutions in the sense that these remain bounded and confirms that we completely avoid any singularity issue.

By the way having equal initial densities is also ideal to have equal amounts of matter and anti-matter at the origin of time, but then, following the separation of the two sides, a small excess of matter on our side corresponding to the same exact small excess of anti-matter on the conjugate side. The small excess on our side would then presumably be the origin of the baryonic asymmetry of our universe after almost complete matter anti-matter annihilation.

Once our permutation symmetry is successfully reinterpreted as being associated with a time reversal symmetry, for the scale factors to exchange their respective values at the equality of densities, we just need to jump from t to -t as illustrated in fig 2 and 4. A mere permutation symmetry would also exchange the scale factors time derivatives producing an inversion of the arrow of time and therefore Hubble rates i.e. a transition from expansion to contraction on our side. So our time reversal symmetry is actually only a permutation of the scale factors while the Hubble rates and densities remain the same (symmetry also satisfied by our differential equations) resulting in our side still being expanding as promised following the transition redshift.

2.3.4. *Discontinuities and consistency checks*

To gain insight into the meaning of field discontinuities, let us first investigate the possibilities offered to us within GR. Assume space-time can be divided into two domains D_- and D_+ separated by a constant conformal time hypersurface t=T. In the domain $D_- =]-\infty, T_-[$ the laws of GR apply just as they also apply in $D_+ =]T_+, +\infty[$. The question is whether we may consider a non trivial i.e. non continuous relation linking the D_+ matter-radiation and gravitational fields and derivatives in the T_+ limit to D_- matter-radiation and gravitational fields and derivatives in the T_- limit. Of course the problem is severely constrained by equations of motion that must be satisfied in both domains however the solutions are not only determined by the equations but also by integration constants that we may chose differently in the two domains by imposing different asymptotic conditions at infinity, and these in turn could imply discontinuities i.e. non matching D_+ and D_- solutions in the limit t=T.

The homogeneous case with negligible pressures is the simplest one to start with. In D_+ and D_- the set of independent equations are the first Friedmann equation and the conservation equation of matter fields.

$$H_+^2 = \frac{8\pi G}{3}\rho_+ a_+^2 \qquad \frac{\dot{\rho}_+}{\rho_+} = -3H_+ \tag{34}$$

$$H_-^2 = \frac{8\pi G}{3}\rho_- a_-^2 \qquad \frac{\dot{\rho}_-}{\rho_-} = -3H_- \tag{35}$$

in which H_+ and H_- are still conformal Hubble parameters. In a conservative approach (also motivated by the kind of discontinuity we are interested in within DG), we are wondering whether a discontinuity of the scale factor implying $a_+(T_+) = Ca_-(T_-)$ and implying a mere renormalization by a constant C of the total gravitational field from one domain to the other, while matter and radiation densities (and their derivatives) would be continuous is a possibility within GR. This of course also implies the continuity of the Hubble parameters : $H_+(T_+) = H_-(T_-)$.

The conservation equations do not forbid $\rho_+(T_+) = \frac{c_+}{a_+^3(T_+)} = \frac{c_-}{a_-^3(T_-)} = \rho_-(T_-)$ as the discontinuity of the scale factor ($a_+(T+) \neq a_-(T_-)$) can be compensated by different integration constants $c_+ \neq c_-$ to maintain the continuity of the density $\rho_+(T_+) = \rho_-(T_-)$. However then the Friedmann equations in the two domains obviously can't be consistent ! Such kind of discontinuity is therefore forbidden within GR but what about DG ? Again we know that thanks to various integration constants a discontinuity (by a renormalization constant) of the scale factor leaving the densities and Hubble rates continuous just as in the above GR case is not an issue as far as the matter and radiation conservation equations are concerned. Now the corresponding first Friedmann-DG equations are:

$$a_+^2 H_+^2 - \tilde{a}_+^2 \tilde{H}_+^2 = \frac{8\pi G}{3}(\rho_+ a_+^4 - \tilde{\rho}_+ \tilde{a}_+^4) \tag{36}$$

$$a_-^2 H_-^2 - \tilde{a}_-^2 \tilde{H}_-^2 = \frac{8\pi G}{3}(\rho_- a_-^4 - \tilde{\rho}_- \tilde{a}_-^4) \tag{37}$$

and again the equations can't be consistent for an arbitrary renormalization coefficient C in $a_+(T_+) = Ca_-(T_-) \Rightarrow \tilde{a}_+(T_+) = C^{-1}\tilde{a}_-(T_-)$. There is however the remarkable exception corresponding to the permutation case $a_+(T_+) = \tilde{a}_-(T_-), \tilde{a}_+(T_+) = a_-(T_-) \Rightarrow C = \frac{a_+(T_+)}{\tilde{a}_+(T_+)} = \frac{\tilde{a}_-(T_-)}{a_-(T_-)}$. This is exactly the kind of discontinuity in time we have postulated within DG and we now see how a new kind of process, a discontinuous one, is made possible by our permutation symmetry while no such thing was even thinkable within GR!

Admittedly, in the GR case, the real reason behind the block was to force the continuity of densities and Hubble rates which was a quite arbitrary demand. In our theory, in [6] , H^2 and $\tilde{H}^2$ are always equal by definition while ρ and $\tilde{\rho}$ are equal at the crossing time T which makes the equation invariant under the exchange of the scale factors values at T as long as ρ, $\tilde{\rho}$, H^2 and $\tilde{H}^2$ remain unchanged. Inspection of Eq [6] alone therefore strongly suggests that the non arbitrary requirement is indeed the continuity of densities and squared Hubble rates rather than Hubble rates implying that the Hubble rate may either be continuous or flip sign at the transition and we shall keep open minded to this last possibility in the following sections.

Then however, the other Friedmann-DG equation [5] implies that the approximate equations of motion before and after the transition are $\dot{H} \approx -H^2/2 - 3Ka^2 p$ and $\dot{\tilde{H}} \approx -\tilde{H}^2/2 - 3K\tilde{a}^2\tilde{p}$ respectively. Indeed, since the densities are continuous at the transition, so must be the pressures: $p_+ = p_-$, $\tilde{p}_+ = \tilde{p}_-$ and though pressures might not cross each other ($p_- \neq \tilde{p}_-$) at the same exact time the densities cross each other ($\rho_- = \tilde{\rho}_-$), we expect not so different pressures at this time insuring that the dominant source terms are still those multiplied by the greatest scale factor both before and after the transition which makes our approximations valid.

Then, since at any time $H = -\tilde{H} \Rightarrow \dot{H} = -\dot{\tilde{H}}$, $\tilde{H}_+^2 = \tilde{H}_-^2 = H_-^2$ we have:

$\dot{H}_+ = -\dot{\tilde{H}}_+ \approx \tilde{H}_+^2/2 + 3K\tilde{a}_+^2\tilde{p}_+ = H_-^2/2 + 3K\tilde{a}_+^2\tilde{p}_+ \approx -\dot{H}_- + 3K(\tilde{a}_+^2\tilde{p}_+ - a_-^2 p_-)$.
But $\tilde{a}_+ = a_-$, $\tilde{p}_+ = \tilde{p}_-$ so eventually:

$$\dot{H}_+ \approx -\dot{H}_- + 3Ka_-^2(\tilde{p}_- - p_-) \tag{38}$$

This means that the time derivatives of the Hubble rates flip sign in very good approximation in a cold matter dominated universe and are therefore discontinuous at the transition. We cannot however exclude a very small contribution of pressures to this discontinuity, in case $p_- \neq \tilde{p}_-$. We see that there is no obvious physical motivation for requiring that the pressures should cross each other at T_- since a discontinuity of $\dot{H}$ is anyway unavoidable in contrast to the continuity of H^2. May be it's not really annoying to have a discrete symmetry only meaningful in the first Friedmann-DG equation because just as in GR, it is well known that this equation involving only first derivatives of the metric is rather a constraint that must be satisfied at any time than an evolution equation involving second derivatives of the metric as the second Friedmann-DG equation. Even in GR those equations don't have the same status (see ([2]) p163) and since our discontinuity only defines the new initial conditions for the subsequent evolution after the transition, it's natural that it is rather constrained by the first Friedmann-DG equation. However in the following we still want to require $p_- = \tilde{p}_-$ because then $\dot{H}_+ = -\dot{H}_- = \dot{\tilde{H}}_-$ is exact meaning that not only the H^2 but also the $\dot{H}$ are exchanged between the two sides at the transition so the complete geometrical terms of our equations as well (but not the H: we are still in an expanding universe)! Interestingly the two equations $p(T_-, V_-) = \tilde{p}(T_-, V_-)$, $\rho(T_-, V_-) = \tilde{\rho}(T_-, V_-)$ can have a solution if we have two or more free parameters : not only the time of the transition T_- but also extra-parameters defining the volume V_- of a spatial sub-domain of the universe in which the transition takes place.

From a phenomenological point of view the continuity of mean densities and pressures but also their perturbations insures that the discontinuous process itself has no observable effect at the time it occurs except two phenomena. First, following the transition the universe will start to accelerate: again the Hubble rate is continuous but not it's time derivative. Yet frequencies of clocks and light, energy levels of matter and radiation are cosmologically continuous from T_- to T_+: no unusual contribution to the redshifts. Second, the gravity from sources on our side (F fields perturbations) is expected to be almost switched off at the transition but we shall see later how this problem can be solved.

2.3.5. *A testable cosmological scenario*

The transition being triggered by equal densities and pressures on both sides of the Janus field, the dark side is also dust dominated at the transition and we also have the continuity of the Hubble rate[14]. This leads to a constantly accelerated universe $a(t') \propto t'^2$ in standard coordinate following the transition redshift.

Constraining the age of the universe to be the same as within LCDM the transition redshift can be estimated (see [15] equation 6) and confronted to the measured value $z_{tr} = 0.67 \pm 0.1$. The prediction is $z_{tr} = 0.78$ in very good agreement with the measured transition redshift.

The conjugate side being in contraction, should reach the radiative regime in the future, then our cosmological equation will simplify in a different way[e] :

$$\tilde{a}^2 \frac{\ddot{\tilde{a}}}{\tilde{a}} \approx \frac{4\pi G}{3} \tilde{a}^4 (\tilde{\rho} - 3\tilde{p}) = K\tilde{a}^2 \tag{39}$$

The solution $\tilde{a}(t) = C.sh(\sqrt{K}(t-t_0)) \approx C\sqrt{K}(t-t_0)$ for $1/C << \sqrt{K}(t-t_0) << 1$ so $a(t) \propto 1/(t-t_0)$ which translates into an exponentially accelerated expansion regime $e^{t'}$ in standard time coordinate.

We believe that our transition to a constantly accelerated universe is the most satisfactory alternative to the cosmological constant as it follows from first principles of the theory and eventually should fit the data without any arbitrary parameter, everything being only determined by the actual matter and luminous contents of the two conjugate universes, such content so far not being directly accessible for the dark side. More specifically, the parameter which replaces the cosmological constant in our framework is merely the redshift of densities equality i.e. the transition redshift z_{tr}. But in contrast to a cosmological constant which just corresponds to one theoretical possibility out of a myriad of other terms that one could add either on the left or the right of the Einstein equation hence implying a high degree of arbitrariness, everything in our framework follows from a different conceptual choice from the beginning: the existence of a non dynamical background.

2.3.6. *Confrontation with the SN, Cepheids, BAO and CMB data*

In this section we now denote t the standard time rather than conformal time and present the detailed confrontation of our best motivated transition scenario, the transition to a t^2 acceleration regime, to the most accurate current cosmological

[e] That a quantity such as $\tilde{\rho} - 3\tilde{p}$ is expected to follow a $1/\tilde{a}^2$ evolution in the limit where all species are ultra-relativistic can be deduced from Eq (21)-(25) of [40] and the matter and radiation energy conservation equation rewritten as $\tilde{\rho} - 3\tilde{p} = 4\tilde{\rho} + \tilde{a}\frac{d\tilde{\rho}}{d\tilde{a}}$ in a radiation dominated dark side of the universe when $\tilde{\rho}$ and $\tilde{p} \approx 1/\tilde{a}^4(t)$.

data: the cosmological microwave background spectrum, the Hubble diagram of Cepheid calibrated supernovae and baryonic acoustic oscillations. We already noticed a long time ago the remarkable (and not expected within LCDM) agreement between the supernovae Hubble diagram up to z=0.6 and a constantly accelerated universe [53] .ie. with $a(t) \propto t^2$ meaning a deceleration parameter q=-0.5. This is also confirmed by fig 2 from [54] with 740 confirmed SN IA of the JLA sample, some models fit functions (fig 2 bottom) even apparently indicating that our universe q(z) is asymptotically q=-0.5 at low redshift.

Just to confirm this tendency we use the same sample to fit α of a power law t^α evolution of the scale factor for redshifts restrained to the $[0, z_{max}]$ interval and get:

$\alpha = 1.85 \pm 0.15$ for z_{max}=0.6 (one standard deviation from 2.)

$\alpha = 1.78 \pm 0.11$ for z_{max}=0.8; (two standard deviations from 2.)

As expected, beyond redshift 0.8 the power low is deviating from 2 by more than two sigmas : we may be reaching the decelerating $t^{2/3}$ regime.

The next step is therefore to fit the transition redshift between a fixed $t^{2/3}$ and subsequent t^2 evolution laws, and we get: $z_{tr} = 0.67 + 0.24 - 0.12$ with a $\chi^2 = 740.8$ slightly larger than that of the LCDM fit (739.4) but we notice by the way that allowing for two different normalization parameters on both sides of z_{tr} to account for possible imperfections of the inter-calibration of different instruments, thus an additional free parameter, the fit χ^2 is improved to 734.1 while z_{tr} is unchanged and the two normalization parameters are compatible (within 1 σ).

The next step is to use our Geogebra graphical tool to play with cursors and hopefully determine a z_{tr} value lying in the allowed interval according our previous SN fits, a H_0 close to the directly obtained value by Riess et all [55] (local distance ladder method through Cepheids and SN) and simultaneously allowing a good agreement to both the CMB data (angular position of first acoustic peak $\theta*$ at decoupling and comoving sound horizon r_{drag}) [56] and BAO data (H(z), $D_M(z)$)[57]. We first of course need to correct the BAO data, obtained assuming the r_{drag} of a fiducial LCDM cosmology, to adapt them to our r_{drag}. Ω_{rad} is fixed as usual from the present day photon and neutrino densities. What's new is that Ω_M is then not anymore a free parameter. Indeed, we may define $\Omega_M(z_{tr}) = \frac{8\pi G \rho_M(z_{tr})}{3H_{tr}^2} = 1 - \Omega_r(z_{tr}) \approx 1$ since, beyond the transition redshift, we are indistinguishable from a mere CDM flat cosmology without any dark energy nor cosmological constant. We can then extrapolate this to the usual present $\Omega_M = \frac{8\pi G \rho_M(0)}{3H_0^2}$ given that $\rho_M(z_{tr}) = \rho_M(0)(1 + z_{tr})^3$ and $H_{tr} = H_0(1 + z_{tr})^{1/2}$ for a constantly accelerated regime between z=0 and z=z_{tr}. Then, $\Omega_M = (1 + z_{tr})^{-2}$.

Our attempts resulted in one of the best fits for $z_{tr} = 0.83$ (see Figure 6) for which we nevertheless cannot avoid a potential tension at the two sigma level for the lowest z D_M point (our prediction in the $D_M(z)$ plot is the red band) but we notice that this kind of tension appears almost unavoidable for any model that would fit the high H_0 value from Riess. The most likely origin of this tension is that linear regime perturbations from the contracting dark side start to grow differently than

[!p]

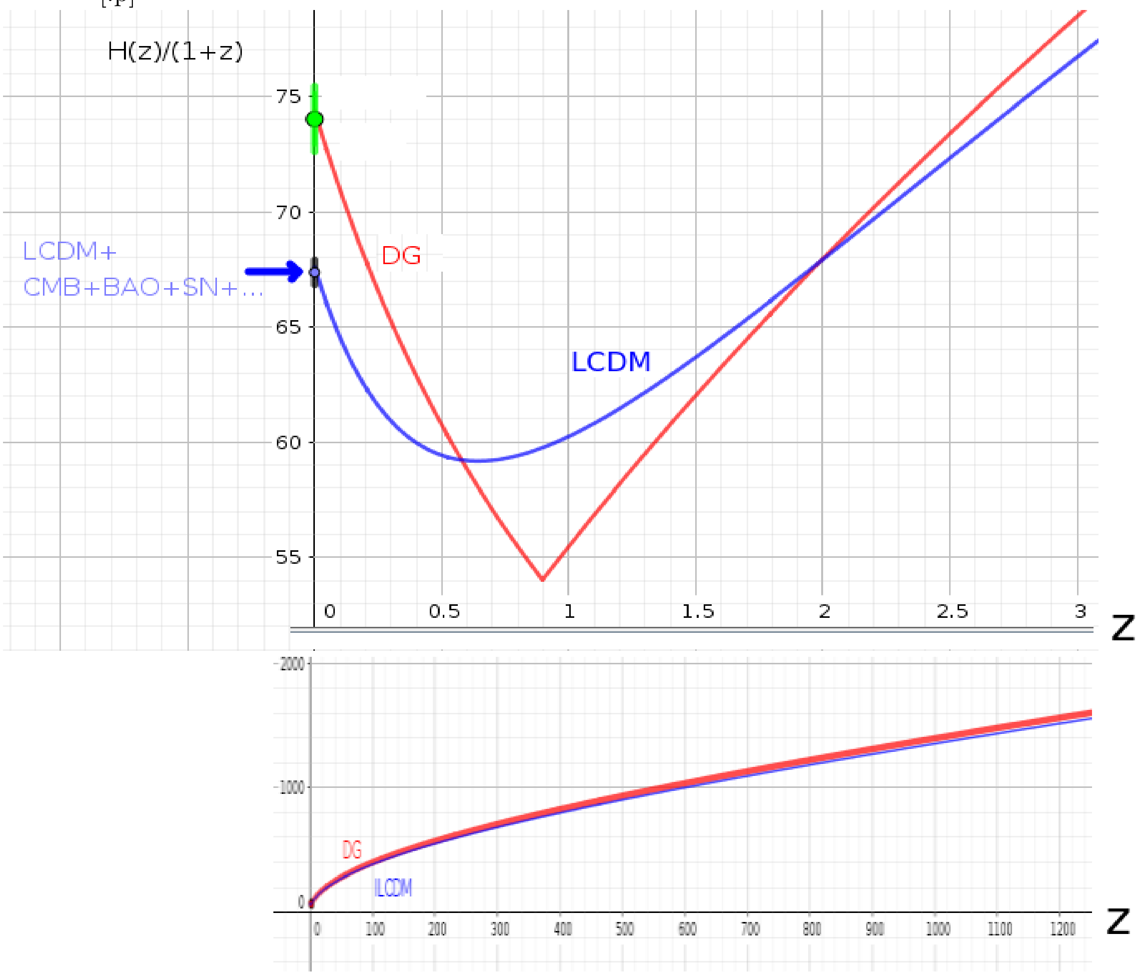

Fig. 5. A transition scenario vs the LCDM best fit

within LCDM after the transition redshift and as their gravity dominates over our side dark matter gravity as we shall see, those may deform the BAO peak in an unexpected way for those who analyze the data with LCDM as fiducial model to estimate various systematics.

The small tension in H(z=0.7) corresponding to the full shape analysis of the BAO data remains acceptable but becomes more serious with the value obtained through reconstruction techniques [57] [61] [62], not only correcting various non-linear effects and reducing the errors but also assuming a growth rate of linear perturbations and correcting for redshift space distorsions (RSD) in a way which is valid for LCDM but certainly not for Dark Gravity.

Actually all current BAO analysis would need to be re-investigated within our

[!p]

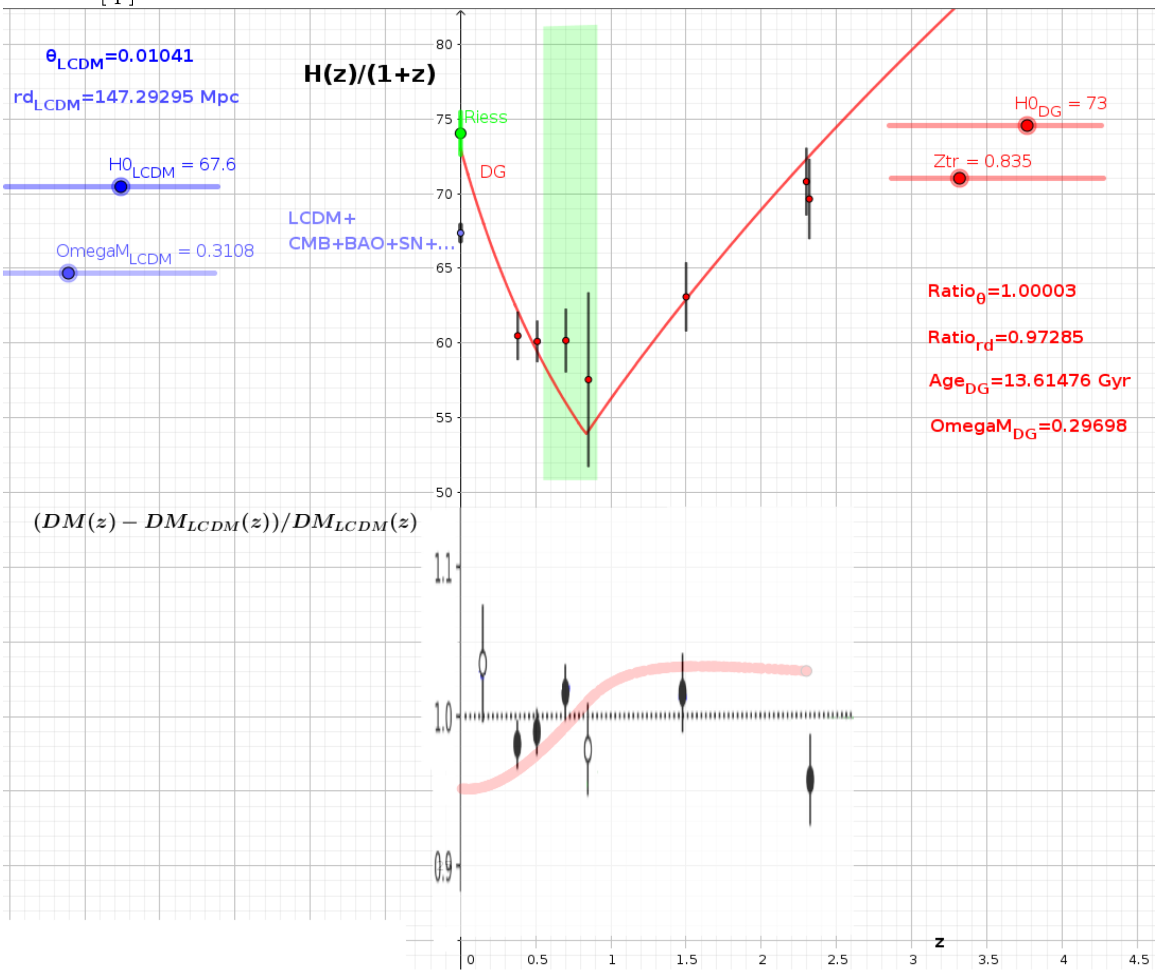

Fig. 6. A transition scenario confronted to CMB and BAO data, the red band is our prediction for $D_M(z)$ (bottom). The red data points in the H(z)/(1+z) plot are corrected for r_d DG cosmology and not expected to fit LCDM anymore. The green band is the allowed interval for the transition redshift (within 1 standard deviation) according our SN Hubble diagram fit

framework. New BAO points at higher redshifts will prove crucial to eventually validate or rule-out our predictions, given that on the other hand, before the transition redshift, we don't expect different linear or non linear effects than within LCDM.

The confrontation with Big Bang nucleosynthesis data is also granted to be successful given how close to the LCDM one is our H(z) at high redshift (Figure 5). Our r_d is only less than three percent lower than the LCDM one and the age of the Universe is still reasonable given the oldest stars ages (13.6 billion years) : this is because the much higher than LCDM H(z) that we have at low redshifts is compensated by a much lower H(z) than LCDM between 0.6 and 1.6 (Figure 5) :

needless to say that this good property is not shared by most models trying to solve the H_0 tension with new physics at low redshifts.

3. Isotropic solution about Minkowski

We are now interested in the isotropic solution in vacuum (equivalent of the GR Schwarzschild solution) of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$.

$$A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} \tag{40}$$

$$-B = -\frac{1}{A} = -e^{\frac{-2MG}{r}} \approx -1 + 2\frac{MG}{r} - 2\frac{M^2G^2}{r^2} + \frac{4}{3}\frac{M^3G^3}{r^3} \tag{41}$$

perfectly suited to represent the field generated outside an isotropic source mass M. This is different from the GR one, though in good agreement up to Post-Newtonian order. The detailed comparison will be carried out in section 6. It is straightforward to check that this Schwarzschild new solution involves no horizon. The solution also confirms that a positive mass M in the conjugate metric is seen as a negative mass -M from its gravitational effect felt on our side.

4. Local gravity : linear equations about Minkowski

The linearized equations about a common Minkowskian background look the same as in GR, the main differences being the additional dark side source term $\tilde{T}_{\mu\nu}$ and an additional factor 2 on the linear lhs:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu} + t_{\mu\nu} - \tilde{t}_{\mu\nu}) \tag{42}$$

however to second order in the perturbation $h_{\mu\nu}$ (plane wave expanded as usual) and given that $\tilde{h}_{\mu\nu} = -h_{\mu\nu} + h_{\mu\rho}h_{\nu\sigma}\eta^{\rho\sigma} + O(3)$ we found that the only non cancelling contributions to $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ on the rhs, vanish upon averaging over a region of space and time much larger than the wavelength and period (this is the way the energy and momentum of any wave are usually evaluated according [2] page 259). This $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ is standing as usual for the energy-momentum of the gravitational field itself because the Linearized Bianchi identities are still obeyed on the left hand side and it therefore follows the local conservation law:

$$\frac{\partial}{\partial x^\mu}(T^{\mu\nu} - \tilde{T}^{\mu\nu} + t^{\mu\nu} - \tilde{t}^{\mu\nu}) = 0 \tag{43}$$

We can try to go beyond the second order noticing that the DG equation (3) has the form $X^{\mu\nu} - \tilde{X}^{\nu\mu} = -8\pi G(Y^{\mu\nu} - \tilde{Y}^{\nu\mu})$ and can be split in a $\mu \leftrightarrow \nu$ symmetric, $X^{\mu\nu}_s - \tilde{X}^{\mu\nu}_s = -8\pi G(Y^{\mu\nu}_s - \tilde{Y}^{\mu\nu}_s)$, and a $\mu \leftrightarrow \nu$ anti-symmetric $X^{\mu\nu}_a + \tilde{X}^{\mu\nu}_a = -8\pi G(Y^{\mu\nu}_a + \tilde{Y}^{\mu\nu}_a)$, in which the s (resp a) indices refer to the symmetric (resp anti-symmetric) parts of the tensors. Though the antisymmetric equation could in principle source gravitational waves, its production rate is expected to be extremely reduced vs GR because the dominant source term is at most of order hT rather than T in the Y term.

The value of the $\mu \leftrightarrow \nu$ symmetric equation is the manifest anti-symmetry of its lhs under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. Replacing $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$, this translates into the odd property of the lhs to all orders in $\bar{h}_{\mu\nu}$. Then we are free to use the plane wave expansion of this new $\bar{h}_{\mu\nu}$ (not to be confused with $h_{\mu\nu}$ nor $\tilde{h}_{\mu\nu}$) instead of $h_{\mu\nu}$ and because each term of the perturbative series has an odd number of such $\bar{h}$ factors, such term will always exhibit a remaining $e^{i\mathbf{kx}}$ factor which average over regions much larger than wavelength and period vanishes (in contrast to [1] page 259 where the computation is carried on for quadratic terms for which we are left with some x^μ independent, hence non vanishing, cross-terms).

Our new interpretation is that any radiated wave of this kind (sourced from the symmetric rather than the anti-symmetric part of the equation) will both carry away a positive energy in $t^{\mu\nu}$ as well as the same amount of energy with negative sign in $-\tilde{t}^{\mu\nu}$ about Minkowski resulting in a total vanishing radiated energy. Thus the DG theory, so far appears to be dramatically conflicting with both the indirect and direct observations of gravitational waves.

Actually, we shall show in the next two sections that, since the asymptotic behaviours of the two sides of the Janus field are not necessarily the same, we could both expect from the theory an isotropic solution approaching the GR Schwarzschild one with it's black hole horizon and the same gravitational wave solutions, including the production rate, as in GR but also, whenever some particular yet to be defined conditions are reached, the above DG solutions, with a vanishingly small production rate of gravitational waves and the B=1/A exponential DG Schwarzschild solution without horizon. Both will be limiting cases of a more general solution.

5. Differing asymptotic values

5.1. *The C effect*

Due to expansion on our side and contraction on the dark side the common Minkowskian asymptotic value of our previous section is actually not a natural assumption. At the contrary a field assumed to be asymptotically $C^2\eta_{\mu\nu}$ with C constant (here we neglect the evolution of the background as usual in the very non linear regime) has its conjugate asymptotically $\eta_{\mu\nu}/C^2$ so their asymptotic values should differ by many orders of magnitude. Given that $g_{\mu\nu}^{C^2\eta} = C^2 g_{\mu\nu}^{\eta}$ and $\tilde{g}_{\mu\nu}^{\eta/C^2} = \frac{1}{C^2}\tilde{g}_{\mu\nu}^{\eta}$, where the $< g^\eta, \tilde{g}^\eta >$ Janus field is asymptotically η, it is straightforward to rewrite the local DG Janus Field equation now satisfied by this asymptotically Minkowskian Janus field after those replacements. Hereafter, we omit all labels specifying the asymptotic behaviour for better readability and only write the time-time equation satisfied by the asymptotically $\eta_{\mu\nu}$ Janus field.

$$C^2\sqrt{g}\frac{G_{tt}}{g_{tt}} - \frac{1}{C^2}\sqrt{\tilde{g}}\frac{\tilde{G}_{tt}}{\tilde{g}_{tt}} = -8\pi G(C^4\sqrt{g}\delta\rho - \frac{1}{C^4}\sqrt{\tilde{g}}\tilde{\delta\rho}) \tag{44}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\delta\rho$ is the energy density fluctuation for matter and radiation. The tilde terms again refer to the same tensors except that they are built from the corresponding tilde (dark side) fields.

Then for $C >> 1$ we are back to $G_{tt} = -8\pi G C^2 g_{tt}\delta\rho$, a GR like equation for local gravity from sources on our side because all terms depending on the conjugate field become negligible on the left hand side of the equation while the local gravity from sources on the dark side is attenuated by the huge $1/C^8$ factor (in the weak field approximation, $G_{tt} = 8\pi G\frac{\delta\tilde{\rho}}{C^6}$). From $g^{\eta}_{\mu\nu}$ we then can get back $g^{C^2\eta}_{\mu\nu}$ and of course absorb the C constant by the adoption of a new coordinate system and redefinition of G, so for $C >> 1$ we tend to GR : we expect almost the same gravitational waves emission rate and almost the same weak gravitational field. However on the dark side everything will feel the effect of the anti-gravitational field from bodies on our side amplified by the same huge factor relative to the gravity produced by bodies on their own side.

The roles are exchanged in case $C << 1$. Then the GR equation $\tilde{G}_{tt} = -\frac{8\pi}{C^2}G\tilde{g}_{tt}\delta\tilde{\rho}$ is valid on the dark side while the anti-gravity we should feel from the dark side is enhanced by the huge $1/C^8$ factor relative to our own gravity (given in the weak field approximation by solving $\tilde{G}_{tt} = 8\pi G C^6\delta\rho$ for $\tilde{g}_{\mu\nu}$ from which we derive immediately our side $g_{\mu\nu}$ of the Janus field).

Only in case C=1 do we recover our local exponential Dark Gravity, with no significant GW radiations and also a strength of gravity ($G_{tt} = -4\pi G\delta\rho$) reduced by a factor $2C^2$ relative to the above GR gravity ($G_{tt} = -8\pi G C^2\delta\rho$).

It's important to stress that the phenomenology following from different asymptotic behaviours of the two faces of the Janus field here has no peer within GR in which a mere coordinate transformation is always enough to put the gravitational field in an asymptotically Minkowskian form in which a redefinition of the gravitational constant G gives back the usual gravitational potentials. This would still be possible in DG for one face of the Janus field but not for both at the same time. The new physics emerges from their relative asymptotic behaviour which can't be absorbed by any choice of coordinate system.

Eventually, depending on the local C value in a given space-time domain, a departure from GR predictions could be expected or not both for the gravitational waves radiated power and the local static gravitational field e.g. depending on a context able or not to trigger a reset to C=1, we could get either the DG exponential elements or the GR Schwarzschild solution for the static isotropic gravity; and get either no gravitational waves at all or the same radiated power as in General Relativity[f].

[f]For $C >> 1$ we also approximately recover the gauge invariance of GR, meaning that the scalar and vector degrees of freedom tend to decouple, leaving the pure tensor modes as in GR

5.2. *Frontier effects*

We are here interested in specifying the kind of effects related to the occurrence of C and 1/C asymptotic gravity spatial domains and more specifically at the frontier between two such domains. We anticipate that we shall soon be led to admit that such configuration actually occurs.

Let's assume a 1/C asymptotic domain neighbouring a C asymptotic domain and a weak field so that we can for instance approximate the g_{00} metric element by an exponential function. Let's assume we have point masses M_1 on our side and M_2 on the dark side, both being in the C domain (of our side metric). Then according the previous section results, we have :

$$g_{00} \approx C^2 e^{-G(C^2 M_1/r_1 - C^{-6} M_2/r_2)} \tag{45}$$

anywhere in the C domain at distance r_1 from M_1 and r_2 from M_2.

Switching from a formula like (44) valid for density fluctuations to a formula valid for point mass sources as we just did requires justification. We may notice that, when C is not anymore a constant but a genuine scale factor, in order to recover the Mac Vittie metric behaviour for g_{00}, which is considered to be the best effort metric in GR when the source is a point mass in a perfect fluid with homogeneous density, a useful trick is to replace $\delta\rho$ by M/C^3 instead of just replacing $\delta\rho$ by M as we did. Then of course one should replace $\delta\tilde{\rho}$ by $\tilde{M}/\tilde{C}^3$ and the g_{00} metric element formula would rather be:

$$g_{00} \approx C^2 e^{-G(C^{-1} M_1/r_1 - C^{-3} M_2/r_2)} \tag{46}$$

Since the dominance relationships among the two terms are not modified in this new formula with respect to the former, our qualitative results will not be modified in the sense that the negligible terms will be the same so in the following we stick to the first formula. Indeed a bit of caution is not superfluous as the $ra(t)$ dependency of the Mac Vittie metric is related to the questionable requirement that there should be no radial flow, no energy accretion toward the mass in such solution: this is in line with the perfect fluid hypothesis hence a vanishing Einstein tensor element G_{tr} and this in turn requires a non homogeneous pressure to resist the accretion. In fact this question brings us back to an open and difficult problem in GR : how to correctly describe the metric of an isotropic mass in an homogeneous expanding background which we do not claim to solve here. Moreover , we actually never have an isotropic mass in an homogeneous fluid in realistic situations such as for a star: in the solar system for instance even the baryonic density alone in the sun neighbourhood is orders of magnitude greater than the critical density and decreases as $1/r^2$.

Anyway what matters for us is that the g_{00} metric element can be extended anywhere in a neighbouring 1/C domain by

$$g_{00} \approx C^{-2} e^{-G(C^2 M_1/r_1 - C^{-6} M_2/r_2)} \tag{47}$$

In other words the metric is simply renormalized by a constant factor at the frontier between two domains. Now let's assume we have two point masses, M_3 on our side and M_4 on the dark side, both being in the 1/C domain (of our side metric). Then we get:

$$g_{00} \approx C^{-2} e^{-G(C^{-6} M_3/r_3 - C^2 M_4/r_4)} \tag{48}$$

anywhere in this 1/C domain at distance r_3 from M_3 and r_4 from M_4. Again this can be extended anywhere in the neighbouring C domain by

$$g_{00} \approx C^{2} e^{-G(C^{-6} M_3/r_3 - C^2 M_4/r_4)} \tag{49}$$

At last if we both have the previous two couples of masses we can merely combine the above results in the C domain to get:

$$g_{00} \approx C^2 e^{-G(C^2(M_1/r_1 - M_4/r_4) + C^{-6}(M_3/r_3 - M_2/r_2))} \approx C^2 e^{-G(C^2(M_1/r_1 - M_4/r_4))} \tag{50}$$

and in the 1/C domain to get:

$$g_{00} \approx C^{-2} e^{-G(C^2(M_1/r_1 - M_4/r_4) + C^{-6}(M_3/r_3 - M_2/r_2))} \approx C^{-2} e^{-G(C^2(M_1/r_1 - M_4/r_4))} \tag{51}$$

the last approximations being for $C >> 1$. We realize that in both domains the strengths of gravity and anti-gravity respectively from M_1 and M_4 are the same! The above combination reflects our intuition that the frontier surface behaves as a secondary source (Huygens principle) when it propagates (renormalizing it in passing) the field from one domain to the neighbouring one so that eventually in a given domain the fields from masses in any domains, non linearly mix just as in GR.

Now that we have clarified how the metric transforms at domain frontiers it just remains to clarify how the matter and radiation fields behave there. Just as the discontinuity in time of the scale factor triggering the acceleration of the universe had no effect on densities, the discontinuity in space from C^2 to C^{-2} implied by the different normalization between the two domains (itself implied by the scale factors permutation) is again required not to affect the energy levels of particles crossing the frontier and their associated densities.

6. Back to Black-Holes and gravitational waves

6.1. *Back to Black-Holes*

Let's consider the collapse of a massive star which according to GR should lead to the formation of a Black Hole. As the radius of the star approaches the Schwarzschild radius the metric becomes singular there so the process lasts an infinite time according to the exterior observer. If the local fields both outside and inside the star have huge asymptotic C values, we already demonstrated that the gravitational equations tend to GR. However this can't be the case when we approach the Schwarzschild radius because C is finite and the metric elements can grow in such a way that we could not anymore neglect the dark side geometrical term. Therefore presumably the horizon singularity is avoided as well for C $\neq$ 1. To check this we need the exact differential equations satisfied in vacuum by C-asymptotic isotropic static metrics of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$. With $A = C^2 e^a$ and $B = C^2 e^b$, we get the differential equations satisfied by a(r) and b(r):

$$a'' + 2a' + \frac{a'^2}{p} = 0 \tag{52}$$

$$b' = -a' \frac{1 + a'r/p}{1 + 2a'r/p} \tag{53}$$

where $p = 4\frac{e^{a+b}C^4+1}{e^{a+b}C^4-1}$. GR is recovered for C infinite thus p=4. Then the integration is straightforward leading as expected to

$$A = (1 + U)^{p=4}; \tag{54}$$

$$B = (\frac{1-U}{1+U})^{(p=4)/2} \tag{55}$$

where $U = GM/2r$ and the infinite C can be absorbed by opting to a suitable coordinate system : then there is no dark side. DG C=1 corresponds to b=-a, p infinite and the integration, as expected, gives $A = e^U$, $B = e^{-U}$.

The integration is far less trivial for intermediary Cs because then p is not anymore a constant, however in the weak field approximation, treating p as the constant $4\frac{C^4+1}{C^4-1}$ the PPN development of the above solutions brings to light a possible departure from GR at the PostPostNewtonian level since:

$$A_{GR} \approx 1 + 4U + 6U^2 \tag{56}$$

$$B_{GR} \approx 1 - 4U + 8U^2 - 12U^3 \tag{57}$$

$$A_{p\neq4} \approx 1 + pU + \frac{p(p-1)}{2}U^2 \tag{58}$$

$$B_{p\neq4} \approx 1 - pU + \frac{p^2}{2}U^2 - p\frac{2+p^2}{6}U^3 \tag{59}$$

This makes clear that for $p \neq 4$ redefining the coupling constant to match GR at the Newtonian level, which amounts to replace U by 4U/p in the above expressions, a discrepancy would remain at the PPN level relative to GR predictions.

$$A_{p\neq4} \approx 1 + 4U + 8(\frac{p-1}{p})U^2 \tag{60}$$

$$B_{p\neq4} \approx 1 - 4U + 8U^2 - \frac{32}{3}(\frac{2+p^2}{p^2})U^3 \tag{61}$$

For $4 \leq p = 4\frac{1+1/C^4}{1-1/C^4} \leq \infty$ the departure from GR is the greatest for p infinite (C=1) :

$$A_{DG} \approx 1 + 4U + 8U^2 \tag{62}$$

$$B_{DG} \approx 1 - 4U + 8U^2 - \frac{32}{3}U^3 \tag{63}$$

but should hopefully soon become testable with the data from neutron stars or black holes mergers if C is not too big.

In the strong field regime we need to rely on numerical approximation methods to understand what's going on near the Schwarzschild radius. The numerical integration in Geogebra (using NRésolEquaDiff) was carried on and the resulting b(r) are shown in Figure 7 for various C values. It is found that as C increases b(r) will closely follow the GR solution near the Schwarzschild radius over an increasing range of b(r) which can be many orders of magnitude and perfectly mimic the GR black hole horizon, however at some point the solution deviates from GR and crosses the Schwarzschild radius without singularity. Therefore, as far as the numerical integration is reliable our theory appears to avoid horizon singularities (true Black Holes) for any finite C and not only C=1. This means that the collapsed star will

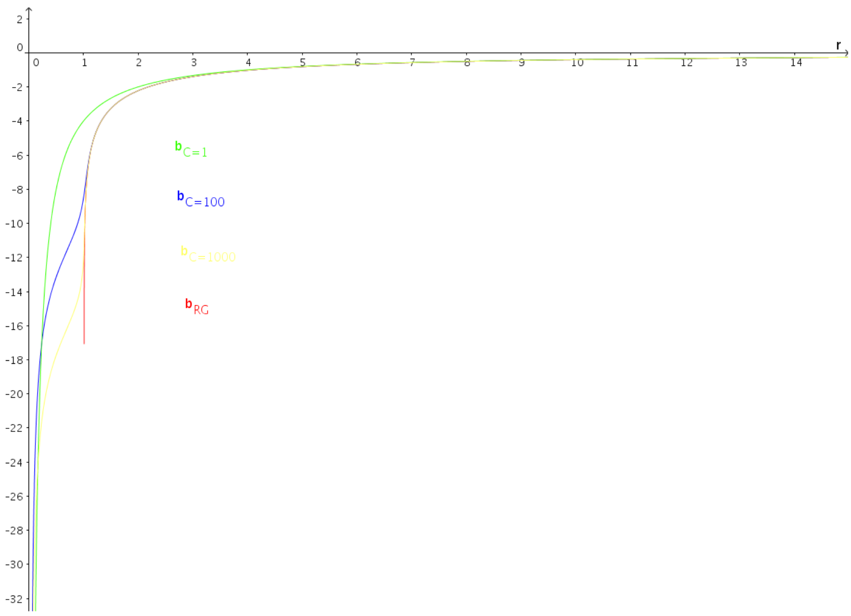

Fig. 7. b(r) near the Schwarzschild radius (r=1) for various C values

only behave as a Black Hole for a finite time after which the external observer will be able to learn something about what's going on beyond the pseudo Horizon. Indeed, the resulting object having no true horizon is in principle still able to radiate extremely red-shifted and delayed light or gravitational waves emitted from inside the object.

The classical picture of a collapse toward a central singularity could therefore also be probed which is interesting because we can imagine various different original mechanisms to avoid the central singularity. The most obvious one would be a massive transfer of the star matter to the dark side near the horizon where the g_{00} metric elements are expected to cross each other. This process would be extremely fast from the point of view of an observer accompanying the collapse whereas it would take billion years for the far away observer facing an apparently stable black hole.

Another more sophisticated mechanism within our framework could stop the collapse: when the metric reaches some threshold, the inner region (the volume defined by the star itself) global and local fields could respectively be reset to Minkowski and C=1. This discrete transition would produce a huge discontinuity at a spherical surface with radius very close to the Schwarzschild Radius (because this is where the postulated metric threshold is expected to be reached). This surface would behave like the hard shell of a gravastar [45] and likely produce the same kind of phenomenological signatures such as echoes following BH mergers which might already have been detected [23].

Then at the center of such object, the two faces of the Janus field should get very close to each other just because C=1 and because this is where the own star potential vanishes. The crossing of the metrics is the required condition to allow the transfer of matter and radiation between the star and the conjugate side there. The lost of a significant part of its initial mass along with the strength of gravity being reduced by a factor 2C for DG relative to GR should eventually stop the collapse as it would allow new stability conditions to be reached.

To still behave as a very gravific object while it has lost most of it's matter and gravitational strength, the discontinuity itself must be gravific and behave as an equivalent gravific mass as the original one[g]. This is expected as the discontinuity is at a domain boundary and just needs to "store" the original value of the metric and it's derivative at the surface at the time it became this domain boundary. Then the external Schwarzschild type solution in vacuum is obtained merely thanks to these boundary conditions.

Shocks and matter anti-matter annihilation at the discontinuity (an excess of gamma radiation from our Milky Way giant black hole has indeed been reported [22]) which we remember is also a bridge toward the dark side and it's presumably anti-matter dominated fluid, could also produce further GWs radiation which would be much less natural from a regular GR Black Hole [23].

Eventually in the vicinity of stars as well as in "Black Holes" we can't exclude a transfer of matter and radiation through the discontinuity at crossing metrics that would proceed in the opposite way feeding them and increasing their total energy : a possible new mechanism to explain the unexpectedly high gravific masses of recently discovered BH mergers but also an attractive simple scenario to explain the six SN like enigmatic explosions of the single massive star iPTF14hls if they resulted from

[g]or an even greater gravific mass which then might lead to pseudo BHs much more massive than we believed them to be.

a succession of injections of antimatter from the dark side[41]. Such discontinuities in the vicinity of stars could also block matter accumulating in massive and opaque spherical shells around stars : a possible scenario to explain the reduced light signal from the recently discovered neutron stars merger.

Of course a Kerr type solution also remains to be established in our framework which is postponed for some future paper. But it is already clear that both conjugate metrics as well as the Minkowski metric in between them must be expressed in ellipsoidal coordinates (remind that our theory is generally covariant) hence in the form given by [46] Eq 21 for the Minkowski metric and Eq 22 or similar for the ensatz in input to our differential equations.

6.2. *Back to Gravitational Waves*

On 17 August 2017, LIGO/Virgo collaboration detected a pulse of gravitational waves,[72] named GW170817, associated with the merger of two neutron stars in an elliptical galaxy 40Mpc from the earth. GW170817 also seemed related to a short (≈ 2 second long) gamma-ray burst, GRB 170817A, first detected 1.7 seconds after the GW merger signal, and a visible light observational event first observed 11 hours afterwards, SSS17a.

The association of GW170817 with GRB 170817A in both space and time is strong evidence that neutron star mergers do create short gamma-ray bursts and that light propagated in this case at the same speed as the gravitational waves within 10^{-15} times the speed of light: 10^{-8} probability to obtain this by chance [73].

If confirmed (no other such coincidence occurred since then, three years later, despite a significant upgrade of the detectors and the detection of many other neutron star merger candidates) the consequence for DG is that light and GW can propagate on the same geodesics over distances as long as 40Mpc. This is expected before the transition redshift because at this epoch our side scale factor dominates by at least $a^2 \propto 10^{20}$ the dark side one so the dark side geometrical terms are suppressed relative to our side terms by $det(g) \propto a^8$ hence at least 80 orders of magnitude. In that case GWs and our side light propagate on almost the same geodesics.

However, following the transition redshift, GW are now supposed to propagate essentially along the geodesics of the dark side metric because now the relative strength of our and the dark side geometrical terms is inverted while, in principle, the light that we can see still propagates on our side. Of course the background being in conformal form on the two sides does not produce any difference for massless waves however the fluctuations i.e. the potentials encountered by light and GW during their propagation are supposed to be opposite: GW see potential hills when light sees potential wells and vice-versa and this alone is expected to produce delays much larger than observed between light and GW, given the typical potentials on the largest scales at the level of 10^{-5}. The effect of our galaxy alone outside a radius of 100kpc would be greater than observed by 10 orders of magnitude.

Then there are only two possible ways to save the theory: either the light received with GRB 170817A, against all odds, mainly propagated on the dark side metric as the GW of GW170817 (first option) or the GW propagated on our side metric as the light of GRB 170817A (second option) just as would have been the case before the transition redshift.

- The first case would imply that a binary neutron star merger into a black hole is able to emit light on the dark side which is not so surprising our pseudo black holes being the perfect places (near the pseudo Horizon or the BH center) for transfers between the two metrics. The fact that this light could be detected on earth, hence on our side, is however much more surprising: if true it would imply that most structures from the dark side are actually visible and detectable and we would expect to be able to see many dark side structures, for instance those situated near the center of our side large scale voids which are expected to be mainly filled by dark side matter. This is difficult to imagine except if for yet unknown reasons, matter on the dark side is essentially in the form of dark matter. This last possibility is however plausible given that in DG, our side and the dark side don't have symmetric roles : the symmetry of the equations is broken by the initial conditions: our side is expanding while the dark side is in contraction (may be eternally) and the cosmological transfer of matter is always from the dark side to our side on the mean...so we have no strong reason to believe that the ratio of normal to dark matter should be the same on the dark side as it is on our side while it remains likely that when radiation on the dark side meets a field discontinuity on it's trajectory, it's transfer to our side will be much favoured relative to the reversed process. So apart from the exceptional case (extreme pressures and gravitational fields) of a neutron star collapse to a Black Hole that would produce the transfer of matter and radiation to the dark side, the normal behaviour of matter or radiation meeting a discontinuity would be a transfer to our side. Now since such discontinuities are expected to be localized in the vicinity of the most condensed forms of matter (planets and stars) the light from GRB 170817A which has propagated on the dark side, presumably was transferred to our side just before reaching us in which case we expect no significant time delay and are motivated to seek for a discontinuity near and around the solar system. It remains that we have no reason to forbid part of the emitted photons to travel also on our side and those may arrive several years later relative to the GWs and photons that propagated on the dark side: this could explain the recently reported observation that, very unexpectedly, the X rays signal from GW170817, now several years later shows an excess increasing with time which is difficult to explain within the current paradigm[74].
- We may not be in position to completely exclude however the second option meaning that not the whole universe transited at the transition redshift

but only a sub-part of it and that regions in which the scale factor was not renormalized allowing light and GW to propagate at the same speed on our side, can extend over as much as 40 Mpcs. The option of a partial transition over a spatial sub-domain is actually unavoidable as it is also actually required to solve another issue that we already identified : if our side had transited over the whole universe, all stars and planets would have lost their gravitational strength and exploded at the transition redshift. It's rather the possibility that such sub-domains could extend over beyond 40 Mpcs distances which is both difficult to explain in our framework and disappointing because then the inside dynamics of smaller structures such as galaxies could not be helped by the dark side. At such smaller scales instead all our predictions would not depart from the LCDM predictions. So the only remaining difference with LCDM for the growth of structures would be in larger structures like voids that presumably define those regions that transited (renormalized their scale factor) while regions in which our side matter dominates, galaxy clusters along filament, did not transit and we would have to assume that this is where the GW and GRB from 170817 propagated.

Therefore, the remaining question for the following sections is whether such sub-domains really need to extend over as much as 40 Mpc (option two) or alternatively (option one) whether we can rely on plenty of small sub-domains about galaxies. In the much more interesting first option (sill trusting the GW-GRB coincidence of august 17 2017) in which almost all the universe transited except small domains about galaxies or even individual stars, the dark side could hopefully help us understand the rotation of galaxies and the MOND empirical law...

In the following we do not decide between the two options to avoid missing any interesting new phenomenology but let's keep in mind that GW and GRB 170817 has far reaching implications for DG and wait and see if this can be confirmed by other similar events.

7. Matter-radiation exchange or equivalent alternative mechanisms?

7.1. *Particle exchange*

The rate of matter exchange would, as we have seen, be driven globally by the expansion rate but we would like to better understand and describe how this could work locally.

In principle it is possible to describe the exchange by a specific new term in our action coupling our to the dark sector as in [33] or [48] : for instance the occurrence of Γ in the matter equations of motion but not in the gravity equations of motion is not a serious issue as one could just add an action piece built from the matter fields and Γ in η instead of g and $\tilde{g}$ just to avoid the occurrence of Γ in our DG Janus field equations : actually just an additional simple piece of action built as $\int d^4x\sqrt{\eta}\Gamma F\tilde{F}$ (here for scalar fields F and $\tilde{F}$ for instance) could describe the exchange of scalar particles between the two metrics and of course such term would have to reverse the energy (from a given metric point of view) when an annihilated F field particle is recreated as an $\tilde{F}$ field particle. But the real issue is that the energy of each such transferred particle actually would have to be re-scaled to properly describe a process involving $\frac{\dot{\rho}}{\rho} = -\frac{\dot{\tilde{\rho}}}{\tilde{\rho}}$, rather than $\dot{\rho} = -\dot{\tilde{\rho}}$. It then becomes rather unnatural and unclear how to describe such particle exchange process (with a quantum probability and energy re-scaling related to Γ) by an action. An alternative to the description by an action is to consider that our actions are only valid in the bulk of space-time domains in which the process described by Γ does not take place. The exchange would rather exclusively occur at frontiers where we find the discontinuities and, there, could as well be described by the new transfer quantum rules not requiring any action. Only when integrating over all space (including bulk and frontiers) as we do to get a cosmological equation would the new term involving Γ appear in the matter equations of motion exclusively. Restricting the exchange process at surfaces is a nice idea as $\dot{\rho} \propto \rho$, $\dot{\tilde{\rho}} \propto \tilde{\rho}$ could simply result from the probability of particles to meet the surface which is proportional to the density of particles. The idea works more naturally to explain an annihilation of particles rather than a creation related to an exchange between conjugate metrics and we would anyway need η to play the role of a buffer container between the conjugate metrics and separately describe the exchange between each metric and the container to get $\frac{\dot{\rho}}{\rho} = -\frac{\dot{\tilde{\rho}}}{\tilde{\rho}}$, rather than $\dot{\rho} = -\dot{\tilde{\rho}}$ while still conserving the energy and number of particles in such two separate exchanges. Actions can describe this but in a more complicated way than we might have wished.

May be no actual exchange nor buffer is needed if the effect of meeting the surfaces is just to accelerate or decelerate the particles in each metric to produce the required increase or decrease of the energy density.

A variant of that idea could apply to a cosmological domain with internal frontiers such as shown in Figure 8.

A centripetal drift of such frontier toward over-dense regions on our side and therefore under-dense corresponding regions on the dark side should increase the mean density of the cosmological domain on our side while decreasing the mean density on the dark side cosmological domain, thereby mimicking a transfer of energy and matter. Again $\dot{\rho} \propto \rho$, $\dot{\tilde{\rho}} \propto \tilde{\rho}$ is expected while $\Gamma = -\tilde{\Gamma}$ would result from opposite density gradients however it's still difficult to imagine how such process alone could eventually turn a cold universe into a close to Big Bang hot universe except if unexpectedly huge amounts of energy are concentrated in and could be released from the small domains with red borders.

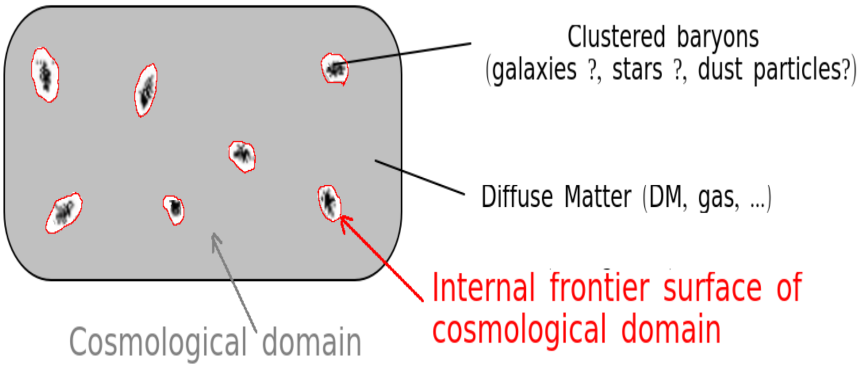

Fig. 8. Cosmological domain with drifting internal surface

7.2. *No particle exchange*

An alternative option is to completely give up the interpretation in terms of particle exchange (between domains or conjugate metrics), if we notice that we could easily introduce a new non dynamical Janus scalar field ϕ,$\tilde{\phi}$ (hence $\tilde{\phi} = \frac{1}{\phi}$) defined by $\Gamma = -3\frac{\dot{\phi}}{\phi}$. Then since $\frac{\dot{\phi}}{\phi}$ produces the same kind of effect as $\frac{\dot{a}}{a} = H$ in our matter-radiation equations it is natural to postulate that matter-radiation fields do not only minimally couple as usual to $g_{\mu\nu}, \tilde{g}_{\mu\nu}$ but also and in the same way to $\phi^2\eta_{\mu\nu}, \tilde{\phi}^2\eta_{\mu\nu}$ and then we would easily get an excellent approximation of our postulated matter radiation equations with the bonus that this can still be described by adding the actions of matter-radiation fields in the eta-scalar non dynamical Janus field. So our total action would just be:

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L_g + \sqrt{\tilde{g}}\tilde{L}_{\tilde{g}}) + \int d^4x(\phi^4\sqrt{\eta}L_{\phi\eta} + \tilde{\phi}^4\sqrt{\eta}\tilde{L}_{\tilde{\phi}\eta}) \tag{64}$$

in which subscripts such as g or $\phi\eta$ just specify to which metric the matter and radiation fields are minimally coupled in the Lagrangian. Of course the DG field equations are left unchanged. Only the matter and radiation field equations are now:

$$a^4(\dot{\rho} + 3H(\rho + p)) + \phi^4(\dot{\rho} - \Gamma(\rho + p)) = 0 \tag{65}$$

$$\tilde{a}^4(\dot{\tilde{\rho}} + 3\tilde{H}(\tilde{\rho} + \tilde{p})) + \tilde{\phi}^4(\dot{\rho} - \tilde{\Gamma}(\tilde{\rho} + \tilde{p})) = 0 \tag{66}$$

or, introducing $r = \frac{\phi}{a} = \frac{1}{\tilde{r}}$,

$$\dot{\rho} = (\frac{\Gamma}{1 + \tilde{r}^4} - \frac{3H}{1 + r^4})(\rho + p) \tag{67}$$

$$\dot{\tilde{\rho}} = (\frac{\tilde{\Gamma}}{1 + r^4} - \frac{3\tilde{H}}{1 + \tilde{r}^4})(\tilde{\rho} + \tilde{p}) \tag{68}$$

Then the numerical integration of these equations gives similar solutions as before for the scale factors and ρ but $\tilde{\rho}$ is found to decrease for ever because Γ remains of the same order as H and the conjugate densities will nether cross each other again. If we require $\frac{\Gamma}{1+\tilde{r}^4} = -\frac{\tilde{\Gamma}}{1+r^4}$ instead of $\Gamma = -\tilde{\Gamma}$, densities will cross again but we do not recover the good phenomenology producing the acceleration of the universe. So the idea of describing matter-radiation exchange from an additional piece of action in η leads to a dead end.

A variation of the gravitational constants ($G(t)$ and $\tilde{G}(t)$ now different for our and the dark sector, varying but still non dynamical) if we absorb them in the definition of densities (hence $\rho G \to \rho$, $pG \to p$, $\tilde{\rho}\tilde{G} \to \tilde{\rho}$, $\tilde{p}\tilde{G} \to \tilde{p}$) will obviously imply an additional contribution in the matter-radiation equations of motion now reading:

$$\frac{\dot{\rho}}{\rho + p} = \Gamma \frac{\rho}{\rho + p} - 3H \tag{69}$$

$$\frac{\dot{\tilde{\rho}}}{\tilde{\rho} + \tilde{p}} = \tilde{\Gamma} \frac{\tilde{\rho}}{\tilde{\rho} + \tilde{p}} - 3\tilde{H} \tag{70}$$

with $\Gamma = \frac{\dot{G}}{G}$ and $\tilde{\Gamma} = \frac{\dot{\tilde{G}}}{\tilde{G}}$, and with these equations along with $\tilde{\Gamma} = -\Gamma$ and our still unmodified DG equations, we are led to almost the same phenomenology as the one following from the postulated matter radiation exchange : the slight difference is only in the influence of the matter and radiation equations of state. Actually any combined variation of other fundamental constants producing a variation of densities and pressures would do the job, for instance $\Gamma = \frac{\dot{h}}{h} = -\frac{\dot{\tilde{h}}}{\tilde{h}} = -\tilde{\Gamma}$ changes the energies of free massless or massive particles at the same rate (any rest energy m_0 can presumably be written as h ν_0) along with the atomic energy levels provided the fine structure constant α remains a constant (hence a variation of the electric charge is also implied) .

We see that there are several more or less natural mechanisms that could exactly or similarly result in our postulated matter and radiation equations of motion and it is difficult at the moment to decide which one was chosen by mother nature.

7.3. *Asymptotically static domains*

Anyway, if for any reason, those transfer mechanisms were to be interrupted, the scale factor evolution would be frozen. This leads us to seriously consider the possibility that regions of our universe might indeed be completely frozen in a perfectly static background, all the more since, as we shall soon see, this is amazingly required by the most obvious interpretation of the Pioneer effect.

Following this idea, we may then have two kind of spatial domains. The evolving one thanks to matter transfers and the frozen ones. The frozen ones would be finite spatial domains (the islands) in which the homogeneous assumption for a background anyway does not make sense for any scales inside such domains. On the other hand for the cosmologically evolving domain around the islands (the ocean) including a priori unbounded scales, a cosmological metric would still make sense.

In the islands, the metrics are therefore asymptotically Minkowskian but rather in standard cosmological time coordinate (hence the expansion effects are switched off in such domains while their clock rates are still not drifting with respect to clocks in the evolving domain). This is possible if high density regions, for instance about stars, cut-out of the rest of the expanding universe, implying a discontinuity at their frontier surface defining a new volume which is not anymore submitted to the expanding:

$$d\tau^2 = a^2(t)(dt^2 - d\sigma^2) = dt'^2 - a'^2(t')d\sigma^2 \tag{71}$$

cosmological metric ($d\sigma^2 = dx^2 + dy^2 + dz^2$), but to the new Minkowski metric.

$$d\tau^2 = a^2(t)dt^2 - C_{frozen}^2 d\sigma^2 = dt'^2 - C_{frozen}^2 d\sigma^2 \tag{72}$$

where C_{frozen} stands for the reached value of the scale factor at the time it froze. Again, this is very natural given that for a finite bounded domain, the very notion of a dynamical homogeneous background is ill-defined so that it is instead natural to treat it as asymptotically Minkowskian but also because for a small domain even in GR the background expansion effects are negligible.

What is then crucial for us is that the domain of validity of the evolving background solutions according (71) has frontiers in such a way that all the local physics responsible for matter transfers may be taking place at those frontiers rather than in the bulk of the domain and may not require any additional action terms. We are of course strongly suspecting that the particle transfers could be taking place at our BH pseudo-horizons since this is where at least the g_{00} elements of the conjugate metrics cross each other so this could be as well the frontier between an outside domain with evolving scale factor and the inside one with frozen scale factor.

However there is an even more fascinating alternative which would not require any actual transfer at all between our and the dark side. Indeed anything carrying energy-momentum crossing the frontiers of the evolving background domain on our side (resp on the conjugate side) could then contribute to the effective Γ (resp $\tilde{\Gamma}$). And even more the frontiers could be dynamical, moving just in such a way as to contribute to these effective creation-annihilation operators as needed to insure the compatibility of our two cosmological differential equations.

The new question that arises then is what determines the density threshold for producing a frozen area and what determines the exact frontier of such domain. The answer might be related to quantum mechanics if the only contributors to the evolving domain are those particle wave functions that are dispersed rather than in their collapsed state. Indeed any object less than 1 micron (except may be a PBH) in the very rarefied intergalactic medium has a decoherence time more than 1 second (and more than 10 days for 0.1 micron particles) so that it's mass energy (we are following a realistic interpretation of QM) is most often diluted in a large volume insuring it should not represent a large fluctuation from the mean universe density which order of magnitude is atoms per cube meter. So most of the diffuse matter-energy in the form of gaz and dark matter should actually be in this un-collapsed state and would not produce frozen regions at the contrary to the collapsed forms of matter. At last any variation of the fundamental collapse triggering parameter will result in an increase or decrease of the fraction of energy matter in the evolving domain rather than in the static domains and then result in a contribution to the now effective Γ and $\tilde{\Gamma}$. Eventually we are led to the fascinating idea that the physics of the QM wave function collapse is what could ultimately make possible the evolution of the scale factor in the Dark Gravity theory.

The existence of static domains could also be the solution to another problem that we did not already mention. At the transition from deceleration to acceleration regime of the universe, the scale factors have exchanged their roles in such a way that the mean density of the dark side now leads the game because it is enhanced by a huge factor in equation 110. But, according what we explained earlier this also implies that any mass on our side should also have it's local gravitational field damped by a huge factor as it is now in the 1/C domain and corresponds to the M_3 kind of mass in equations (50) and (51). Certainly our earth, sun, and all stars of our galaxy do not belong to this type of mass as their gravity was never switched off and must still be of the M_1 kind of masses still in the C domain. So the question is : which ones are the actual energy-masses that must have flipped to the 1/C domain at the transition redshift resulting in switching off almost all the density of our side of the universe in the cosmological equation (110). The most natural answer to this question is that the transition from C to 1/C occurred everywhere except the static domains. It only concerns the far more homogeneous contributions of what we call dark matter whatever it is but also probably essentially most of, if not all of the diffuse intergalactic gas in the universe : the two contributions adding up to more than 99 % of the mass of the universe! As a result, from the transition redshift to now the gravific masses at work which effects we can probe in the universe are the fluctuations on the dark side (of type M_4) (we shall see in a next section that a void in that distribution can perfectly mimic a halo of dark matter on our side), but also the condensed forms of matter on our side (of type M1) : stars, planets...

Eventually static domains are able to solve several issues at the same time:

- They provide frontiers allowing to understand matter radiation exchange not only between our and the dark sector but also between the finite bounded static domains and the rest of the universe with dynamically evolving homogeneous background thanks to these matter-radiation exchanges.
- The static domains can remain C-domains on our side rather than 1/C domains insuring that their masses are still gravific. Even though those domains were not renormalized from C to 1/C at transition redshift, their clocks need to remain synchronized with the evolving domains background clocks driven by a scale factor in the accelerated expansion regime. This is actually needed for our reference clocks which happen to be in the static

domains to allow us to see the universe expansion accelerated by comparing the frequencies of cosmological photons to these reference clocks frequencies. In the next sections we shall deal with this issue and explain how all clocks can remain synchronized.

- We might not only need the equality of densities but also the equality of pressures from both sides of the Janus field to trigger the transition to acceleration. It is unlikely that those two conditions can be met simultaneously and exactly in the whole universe even though we expect the pressures to be similar when the densities are equal. However CMB photons are a dominant part of the distribution of pressure which is expected to have a significantly different distribution than that of cold matter. It is therefore likely that when we reach pressures equality, there exists a cosmological domain frontier also allowing the equality of densities within such domain. Since such condition seems able to determine the location of domain frontiers, it is competing with the other mechanism described above: the QM wave function collapse triggering.
  Anyway, only the highly clustered forms of matter e.g. stars, planets, micro PBHs and may be up to even dust particles of a sufficient size should be able to generate their own static domain of the scale factor evolution in their vicinity in which these can remain in the frozen regime described by (72).

We shall later explore all the consequences and new related predictions among which the Pioneer effect as a natural outcome.

8. The physics of static domains

Because we want to understand the Pioneer anomaly, and for several other reasons discussed earlier we are led to seriously consider that the static domains introduced in a previous section are real. These obviously require new synchronization mechanisms between clocks from the static and evolving background domains which we shall detail now. In subsequent sections we shall focus on some of the very rich phenomenological related outcomes.

8.1. *Actions and space-time domains*

We earlier explained why, anywhere we can't rely anymore on the matter exchange mechanism, the background of a fully dynamical gravitational field can't evolve anymore. In such kind of space-time domain D_{int} cut out from the expanding rest of the universe D_{ext} we still have as usual the Einstein Hilbert (EH) action for the asymptotically Minkowskian Janus Field $g^{\eta}_{\mu\nu}$ added to SM actions for F and $\tilde{F}$ type fields respectively minimally coupled to $g^{\eta}_{\mu\nu}$ and $\tilde{g}^{\eta}_{\mu\nu}$ (the superscript here does not mean that the two sides of the Janus field are asymptotically identical but merely both asymptotically flat and static). However we may add to such action, an independent Einstein Hilbert action for a pure scalar-η homogeneous and isotropic Janus field which we write $a_{int}^2\eta$. The purpose of this action is just to extend to D_{int} the effect of the background which dynamics was determined by extremizing the D_{ext} action and solving the implied equations for the FRW ensatz to get the external scale factor evolution $a_{ext}(t)$. In other words in the D_{int} action for the scalar-η field the scalar field is not dynamical but it's evolution is driven by the external background field. Indeed to insure the synchronization of interior and exterior clocks we postulate that the Hubble rates H_{int} and H_{ext} are still equal implying that $a_{int} = C^2 a_{ext}$ just because only the exterior scale factor was renormalized by $1/C^2$ at the transition redshift. Then the total action in D_{int} is [h]:

$$\int_{D_{int}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a_{int}^2\eta} + \tag{73}$$

$$\int_{D_{int}} d^4x(\sqrt{g}(R + L) + \sqrt{\tilde{g}}(\tilde{R} + \tilde{L}))_{g^\eta} \tag{74}$$

The advantage of adding a separate action for an independent non dynamical $\eta - scalar$ field in D_{int} is not clear at this level because there is no shared field between the two kinds of actions. The point is that g^η is not only determined by its equations of motion. It could be asymptotically identical to any Minkowskian metric, for instance any of the form :

$$d\tau^2 = f^2(t)dt^2 - C^2 d\sigma^2 \tag{75}$$

in which the f(t) function is of course pure Gauge inside D_{int} however it is needed to determine how clocks within D_{int} may actually drift in time with respect to clocks in D_{ext}. Since f(t) is free as of now our purpose is indeed to introduce an additional driving mechanism relating f(t) to $a_{int} = C^2 a_{ext}$. We could just postulate these are equal again to prevent the local clocks in D_{int} to drift with respect to D_{ext} clocks, however we are interested in a more involved mechanism actually allowing such drifts to occur at least momentarily as this is needed to produce Pioneer like effects. Our total action will be helpful just to later introduce such mechanism and establish a somewhat less trivial connection between f(t) and $a_{int}(t)$ in D_{int}.

Instead of the always Minkowski metric of (75), in an earlier version of this work, we have been considering a metric of the kind

$$d\tau^2 = f^2(t)(dt^2 - d\sigma^2) \tag{76}$$

which is acceptable as long as f(t) would be a constant piecewise function of time. f(t) would be periodically discontinuously updated to $a(t)$ in such a way that it

[h]There is may be one alternative possible way to obtain a background metric in D_{int} in a fully dynamical way by adding source terms which densities would be averages over $D_{int} + D_{ext}$. Then the implied equations of motion for a dust universe, $\rho_{[D_{int}+D_{ext}]}/a^3_{[D_{int}+D_{ext}]}$=Const could still be compatible with $\rho_{D_{ext}}/a^3_{D_{ext}}$=Const, the scale factors $a_{[D_{int}+D_{ext}]}$ and $a_{[D_{ext}]}$ evolution being slightly different.

would closely follow the evolution of $a(t)$ through a series of fast discrete transitions on a regular basis. The idea is natural because f(t) is constrained to remain a mere integration constant C by the equations of motion in D_{int} whereas it is also a boundary condition imposed at the boundary of D_{int} requiring it to not remain constant but to actually evolve in time, for instance to follow the scale factor $a(t)$ from D_{ext} so there are conflicting constraints on f(t). However the conflict can be solved if C can take different constant values in successive time slots, provided the actions and differential equations being only valid piece-wise i.e. only within those time slots. Only at the frontier between two such time slots or space-time domains do we need to apply new additional discrete rules to update the new C to the current value of the scale factor and accordingly to propagate the effect to all other physical quantities in D_{int}. The idea is fascinating because it just appears to be a genuine physically motivated quantization postulate that should shed light on the origin of quantum mechanics itself (remember that was one of our initial strongest motivations) [i] The quantization postulate however should be implemented carefully to insure that the effect of the step by step evolving f(t) in a D_{int} domain as for instance in our solar system will not be very different from those expected from GR. Indeed a naive implementation could lead to strongly excluded expansion effects of orbital planetary periods relative to atomic periods: the gravitational constant G would seem to vary at a rate similar to H_0 which is not the case[j].

In the following we shall stick to the always continuous evolution option of (75) rather than (76) but the results we shall obtain are also valid and straightforward to obtain in the other case. There is however an important difference, in one approach the metric is purely Minkowski in the solar system while in the other approach we would presumably (the full quantization program must be completed to get firm predictions) closely follow the predictions and expectations from GR with expansion effects only significant on scales beyond those of galaxy clusters and almost completely negligible but not strictly vanishing in the solar system.

[i] There is a striking analogy with what Quantum Field Theory actually describes : the succession of continuous local and discontinuous non local processes respectively described by the propagation of free fields according classical wave equations and the annihilation/creation of these fields wherever interactions take place, i.e. respectively propagators and vertices in the Feynman language. So our postulate is not at all a conceptually revolutionary one and we even feel tempted to name our discrete transition of C, a quantization rule even though it is quite an unusual one as it applies to a zero frequency component in contrast to what we learned from the Planck-Einstein relations predicting vanishing quanta in the zero frequency limit.

[j] according to [29] "If G were to vary on a nuclear timescale (billions of years), then the rates of nuclear burning of hydrogen into helium on the main-sequence would also vary. This in turn would affect the current sun central abundances of hydrogen and helium. Because helio-seismology enables us to probe the structure of the solar interior, we can use the observed p-mode oscillation frequencies to constrain the rate of G variation." Again the relative variation of G at a rate similar to H_0 is completely excluded the precision being two orders of magnitude smaller.

8.2. *Field discontinuities*

If the mechanism which translates the $a_{int}(t)$ evolution into f(t) evolution is momentarily switched off, we expect a field discontinuity for the g_{00} metric element at the frontier between a momentarily stationary scale factor domain D_{int} and evolving outside D_{ext} domain.

Let's stress that those new kind of discontinuities are not related at all to our permutation symmetry and the related discrete cosmological transition process that could trigger the acceleration of the universe. Now the usual conservation equations for matter or radiation apply when crossing such frontiers though in presence of genuine potential discontinuities. Indeed it's possible to describe the propagation of the wave function of any particle crossing this new kind of discontinuous gravitational potential frontier just as the Schrodinger equation can be solved exactly in presence of a squared potential well : we just need to require the continuity of the matter and radiation fields and continuity of their derivatives at such gravitational discontinuity. Since the differential equations are valid everywhere except at the discontinuity itself where they are just complemented by the former matching rules we obviously avoid the nuisance of any infinite potential gradients and eventually only potential differences between both sides of such discontinuity will physically matter. For instance we can now have have $(\rho a^3)_{before-crossing} = (\rho a^3)_{after-crossing}$ in contrast to what we had following the permutation transition ($\rho_{before-crossing} = \rho_{after-crossing}$).

8.3. *Space-time domains and the Pioneer effect*

The following question therefore arises: suppose we have two identical clocks exchanging electromagnetic signals between one domain submitted to the expanding $a_{int}(t)$ and another without such effect. The reader is invited to visit the detailed analysis in our previous publication [15] starting at page 71. We shall only remind here the main results. Electromagnetic periods and wavelengths are not impacted in any way during the propagation of electromagnetic waves even when crossing the inter-domain frontier. Through the exchange of electromagnetic signals, the period of the clock decreasing as $a(t)$ can then directly be tracked and compared to the static clock period and should be seen accelerated with respect to it at a rate equal to the Hubble rate H_0. Such clock acceleration effect indeed suddenly appeared in the radio-wave signal received from the Pioneer space-crafts but with the wrong magnitude by a factor two: $\frac{\dot{f}_P}{f_E} \approx 2H_0$ where f_P and f_E stand for Pioneer and earth clocks frequencies respectively. This is the so called Pioneer anomaly [12][13]. The interpretation of the sudden onset of the Pioneer anomaly just after Saturn encounter would be straightforward if this is where the spacecraft crossed the frontier between the two regions. The region not submitted to $a_{int}(t)$ (at least temporarily) would therefore be the inner part of the solar system where we find our earth clocks and where indeed various precision tests have shown that expansion or contraction effects on orbital periods are excluded during the last decades. Only the origin of the factor 2 discrepancy between theory and observation remains to be elucidated in the following sections as well as a PLL issue we need to clarify first.

8.4. *Back to PLL issues*

As we started to explain in our previous article [15] in principle a Pioneer spacecraft should behave as a mere mirror for radio waves even though it includes a frequency multiplier. This is because its re-emitted radio wave is phase locked to the received wave so one should not be sensitive to the own free speed of the Pioneer clock.

Our interpretation of the Pioneer effect thus requires that there was a failure of on board PLLs (Phase Lock Loop) to specifically "follow" a Pioneer like drift in time or even a failure that forced the analysis of the data in open loop mode. As for the first hypothesis, we already pointed out that nobody knows how the scale factor actually varies on short time scales: in [15] we already imagined that it might only vary on very rare and short time slots but with a much bigger instantaneous Hubble factor than the average Hubble rate. This behaviour would produce high frequency components in the spectrum which might have not passed a low pass filter in the on board PLL system, resulting in the on board clocks not being able to follow those sudden drifts. The on board clocks would only efficiently follow the slow frequency variations allowing Doppler tracking of the spacecrafts. Only when the integrated total drift of the phase due to the cumulative effect of many successive clock fast accelerations would reach a too high level for the system, this system would "notice" that something went wrong, perhaps resulting in instabilities and loss of lock at regular intervals [15]. This view would be even better supported if our clocks and rods are submitted to the scale factor evolution not continuously but rather through the succession of discontinuous steps we considered earlier. The failure of the PLL system is then even better understood for discontinuous variations of the Pioneer clock frequency with respect to the earth clock frequency. As a result, the frequency of the re-emitted wave is impacted by the Pioneer clock successive drifts and the earth system could detect this as a Pioneer anomaly.

8.5. *Cyclic expanding and static regimes*

We are now ready to address the factor two discrepancy between our prediction and the observed Pioneer clock acceleration rate. We know from cosmology that, still in the same coordinate system, earth clocks must have been accelerating at a rate H_0 with respect to still standing electromagnetic periods of photons reaching us after travelling across cosmological distances (thus mainly in D_{ext}): this is nothing but the description of the so called cosmological redshift in conformal time rather than usual standard time coordinate.

On the other hand the Pioneer effect itself requires that not all regions have their clocks submitted to the same scale factor at the same time but some regions instead have their clocks drifting at rate $2H_0$ with respect to those from other regions.

This seems to imply that through cosmological times, not only earth clocks but also all other clocks in the universe, may have spent exactly half of the time in the $2H_0$ regime and half of the time in the static regime, in a cyclic way. It would follow that the instantaneous expansion rate $2H_0$ as deduced from the Pioneer effect is twice bigger than the average expansion rate (the average of $2H_0$ and zero respectively in the expanding and static halves of the cycle) as measured through a cumulative redshift over billions of years.

In our previous article we presented a very different more complicated and less natural explanation on how we could get the needed factor two which we do not support anymore. This article also discussed the expected field discontinuities at the frontier between regions with different expansion regimes, and likely related effects which we still support. Those discontinuities do not necessarily imply huge potential barriers even though the scale factors have varied by many orders of magnitude between the Big Bang and now. At the contrary they could be so small to have remained unnoticed as far as our cycle is short enough to prevent some regions to accumulate a too much drift relative to others. We are now at last ready, having introduced the main ideas, to detail the mechanism relating $f(t)$ to $a_{int}(t)$ in a D_{int} domain.

9. Driving mechanism for frozen domains and frontier dynamics

9.1. *A sophisticated periodic mechanism*

- First postulate : A D_{int} domain has a new own non dynamical Minkowski metric in addition to the non dynamical Minkowski metric still there in both D_{int} and D_{ext}. This new metric is just (72):

$$d\tau^2 = a_{int}^2(t)dt^2 - C_{frozen}^2 d\sigma^2 = dt'^2 - C_{frozen}^2 d\sigma^2 \tag{77}$$

while the old non dynamical Minkowski metric is still :

$$d\tau^2 = dt^2 - d\sigma^2 \tag{78}$$

Obviously the dynamics of the background in D_{ext} (the scale factor $a_{ext}(t)$) is what determines the new non dynamical metric.

- Second postulate: The dynamical metric in D_{int} is asymptotically successively:

$$d\tau^2 = D_{frozen}^2 dt^2 - C_{frozen}^2 d\sigma^2 \tag{79}$$

which is completely frozen and:

$$d\tau^2 = \frac{a_{int}^4(t)}{D_{frozen}^2} dt^2 - C_{frozen}^2 d\sigma^2 \tag{80}$$

in which clocks are found drifting at the double rate $2H_0$. D_{frozen} in (80) stands for the last frozen value of $a_{int}(t)$ at the time the metric switched from (79) to (80). Of course D_{frozen} has a new value at each cycle.

Therefore, in D_{int} we have an alternate cyclic succession of what would seam to be the two sides of a new emergent Janus field about (77) except

that at any time only one physically shows up and only as an asymptotic value of the D_{int} dynamical field.
This field is of course always asymptotically Minkowskian at the contrary to the background of the Janus field in D_{ext} just because this is required by the complete field equations in D_{int} as we learned earlier. However as we also noticed earlier the asymptotic behaviour is not determined by those equations and as promised our postulates provide the needed constraints according to which $a_{ext}(t)$ from D_{ext} drives this asymptotic bahaviour.
The cyclic succession of (79) and (80) makes the D_{int} dynamical field asymptotically evolve as (77) on cosmological times but this is a mean.
Of course the fact that metrics (79) and (80) look like the two sides of a new D_{int} Janus field about (77) is not an accident. Presumably the existence of (80) is just the consequence of the existence of the other side (79) and (77) in between. In other words we have a kind of baby universe in D_{int} which background is not (may be not yet) able to evolve by itself but which evolution is completely dictated by D_{ext} according our postulates. Presumably the baby universe will eventually acquire it's full autonomy when the two sides really become the two sides of a genuine new dynamical Janus field starting it's own evolution according it's own action and derived field equations.

- Third postulate : In general the dynamical field is not necessarily asymptotically (79) or (80) in the whole domain D_{int}. Rather half of the time D_{int} is in the static regime and the other half of the time the domain progressively passes in the double rate regime: when this occurs there is a domain frontier that scans the whole D_{int}: upstream (not yet reached area of) this propagating frontier we are still in the static regime while downstream all clocks have been synchronized and are in the double rate regime. At the end of the scan the whole D_{int} is frozen again in the static regime for the next half cycle.
To describe this the action in D_{int} is the one we have already written in (73) and (74) which we can rewrite now only retaining the double rate regime area in D_{int} and the geometrical terms (the matter actions and static regime area play no role in the following so we drop them out hereafter just for the sake of conciseness)[k]:

$$\int_{D_{int:2H_0}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a_{int}^2\eta} + \tag{81}$$

$$\int_{D_{int:2H_0}} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g^\eta} \tag{82}$$

[k]In the step by step Minkowskian alternative we would not need to introduce a new non dynamical Minkowski metric as is (72) since $a_{int}^2\eta$ that we already have is just what we need in that case.

Our third postulate is to require this action to be extremum i.e. stationary under any infinitesimal displacement of the hypersurface defined by the frontier of this action validity domain $D_{int:2H_0}$.

Our purpose is to understand the physics that governs the location of the frontier surface of $D_{int:2H_0}$ at any time. Of course determining it will at the same time determine the frontier of the complementary $D_{int:static}$ area. If such surface is moving it will of course scan a space-time volume as time is running out. Having extended the extremum action principle thanks to the third postulate allows to determine this hypersurface.

Indeed the arbitrarily displaced hypersurface might only differ from the original one near some arbitrary point, so that requiring the action variation to vanish actually implies that the total integrand should vanish at this point and therefore anywhere on the hypersurface. Eventually, anywhere and at any time at the domain boundary we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} + (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g^\eta} = 0 \tag{83}$$

This equation is merely a constraint relating the Janus field gravity (terms 3 and 4) to the non dynamical metric (terms 1 and 2) at the hyper surface. Here and from now on we shall omit the "int" subscript for the scale factor unless otherwise specified. Now provided one scale factor dominates the other side one we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \approx \pm_{a<<\tilde{a}}^{a>>\tilde{a}}(\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \tag{84}$$

and then we can make use of the contracted equation 4 to replace:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{g=a^2\eta} \approx \pm_{a<<\tilde{a}}^{a>>\tilde{a}} 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T})_{g=a^2\eta} \tag{85}$$

in equation(83) and we can do the same for the g^η part provided $C(t) = a^2(t)/D_{frozen}$ and D_{frozen} dominate their inverse (the common order of magnitude of $C(t)$ and D_{frozen} is simply named C hereafter). Then equation (83) becomes:

$$\pm_{a<<\tilde{a}}^{a>>\tilde{a}}(a^4 < \rho - 3p >_{ext} - \tilde{a}^4 < \tilde{\rho} - 3\tilde{p} >_{ext}) \tag{86}$$

$$\pm_{C<<\tilde{C}}^{C>>\tilde{C}}(C(t)\ D^3_{frozen}F(r)(\rho - 3p) - \tilde{C}(t)\tilde{D}^3_{frozen}\tilde{F}(r)(\tilde{\rho} - 3\tilde{p})) = 0 \tag{87}$$

The $F(r) = e^{2\Phi(r)}$ and $\tilde{F}(r) = e^{-2\Phi(r)}$ here account for the effect of a local assumed static isotropic gravitational potential $\Phi(r)$. The $<>_{ext}$ denote averages over D_{ext}. First and third terms involve a factor which currently has approximately the same magnitude as $a(t)$ in our cold side of the universe (even though third term is actually momentarily evolving at twice the rate of a hence rather as a^2) while second and fourth terms involve a factor which currently has approximately the same magnitude as $\tilde{a}(t)$ (even though fourth term is actually momentarily evolving at twice the rate of $\tilde{a}(t)$ hence rather as $\tilde{a}^2(t)$) if the dark side is also in a cold matter dominated era.

The relative magnitudes of the local densities can be very different from the relative magnitudes of the averages <> given the extremely non linear structures in the current universe. Is this enough to make the relative magnitudes of terms 1 and 2 in the opposite way to the relative magnitudes of terms 3 and 4 ? Unlikely at first sight given the huge expected current ratio $a(t)/\tilde{a(t)} \approx C(t)/\tilde{C(t)} \approx z_{crossing}^2 >> 10^{18}$, if $z_{crossing}$ is the redshift of the conjugate scale factors equality probably much greater than the BBN redshift. Then as term 3 >> term 4, just as term 1 >> term 2 the equation today (with negligible pressures) simplifies to :

$$a^4 < \rho >_{ext} + C(t) D^3_{frozen} F(r) \rho = 0 \tag{88}$$

Such equation is satisfactory because the two terms don't evolve in the same way as a function of time: the first and second terms imply clocks drifting at rate H_0 and $2H_0$ respectively. So this can lead us to a trajectory r(t) for our hypersurface. Therefore, for instance in the external gravity of a massive spherical body, planet or star on our side, which radial a-dimensional potential is $\Phi(r) = -GM/rc^2$ and a quite uniform $\rho(r)$ so we may neglect it's radial dependency (for instance in the empty space surrounding a star), and using the fact that $C(t)$ momentarily evolves as $a^2(t)$ we are led to:

$$a(t) \propto e^{\frac{2MG}{rc^2}} \tag{89}$$

This equation gives us nothing but the "trajectory" r(t) of the hypersurface we were looking for. Here obtained in the conformal time t coordinate system, it is also valid in standard time t' coordinate since the standard scale factor and the "conformal scale factor" are related by $a(t) = a'(t')$. It is valid to PN order being understood that the exponential metric is here used for simplicity as a weak field PN approximation of a GR Schwarzschild solution rather than really the DG exponential Schwarzschild solution. This equation I=J implies $\dot{I}/I = \dot{J}/J$ so that:

$$H_0 = -2\frac{d\Phi}{dr}\frac{dr}{dt} \tag{90}$$

From this we learn that the frontier between the two domains is drifting at speed:

$$\frac{dr}{dt} = -\frac{1}{2}\frac{H_0}{[\frac{d\Phi(r)}{dr}]} \tag{91}$$

and therefore could involve a characteristic period, the time needed for the scale factor to scan $e^{\frac{2MG}{r}}$ from the asymptotic value to the deepest level of the potential at which point a new scan cycle is started. Thus we are able to understand both the Pioneer effect when we compare clocks in $D_{int:2H_0}$ and in $D_{int:static}$ but also the average H_0 expansion rate of the universe. Video of an animation is available at [17].

We may estimate an order of magnitude of the characteristic period of this cyclic drift assuming that the cycle is over when the frontier reaches the deepest potential levels. For collapsed stars such as white dwarfs or neutron stars this would give a far too long cycle exceeding billions of years because their surface potential is so deep and even much worse for black holes. But the majority of stars have very similar surface potentials even though there is a large variability in their masses and sizes. So we may take the value of our sun a-dimensional surface potential which is about 2.10^{-6} as indicative of a mean and common value. To that number we should add the potential in the gravitational field of the Milky Way and the potential to which the local cluster of galaxies is subjected. Knowing the velocities: 220 km/s of the sun about the center of the galaxy and 600km/s of the local cluster vs the CMB, the virial approximation formula $\frac{v^2}{c^2} \approx GM/rc^2$ may lead us to a crude estimation of each contribution and a total potential near 6.10^{-6}. Then the order of magnitude of the cycle period would be in between 10^4 and 10^5 years.

9.2. *Alternative: a trivial but exceptional mechanism*

Of course the Pioneer effect could be a rare and exceptional event and in this case we could account for it in the most trivial way, just arguing that exceptionally and for yet not clear reasons in some static bounded domains clock frequencies may momentarily evolve (lock their Hubble rate) according the contracting side laws instead of other clocks evolving according the laws of the expanding side, and of course in this case it is trivial to get a $2H_0$ drifting rate between such two kinds of clocks.

10. Other predictions related to frozen metrics

The metrics of (79) and (80) lead to likely testable new phenomenological outcomes. If, as we already pointed out, those are alternating at a high frequency cycle, the g_{00} element mean evolution is almost the same as within GR with short-lived transient deviations that should remain small.

A remarkable exception could occur in the vicinity of compact star surfaces (white dwarfs, neutron stars or our pseudo Black Holes) because it takes much longer time for the scale factor to scan such star strong gravitational potentials up to the star surface. So for instance the asymptotically evolving according (80) and stationary according (79) regions on either sides of the drifting frontier can accumulate an extremely large relative drift of their g_{00} metric elements relative to each other over such a long time, but also a very large drift with respect to the g_{00} metric elements of the D_{ext} region evolving as $a^2(t)$.

This would not only result in much larger discontinuous barriers, able to block or instantaneously accelerate matter, but also large accumulating gravitational red-shifts of regions submitted to (79) relative to the external universe. Eventually any kind of radiation emitted from within such region is going to be red-shifted as usual along it's cosmological path to the observer implying an "emission" redshift z_e. However the total redshift should also receive an additional very significant contribution due to the source itself being already shifted if it remained frozen for billion years relative to earth clocks before the emission (we are still reasoning in the conformal time coordinate system) and this should extend the total to the freeze redshift z_{fr}. Now the luminosity distance to BH mergers should be given by $d_L = (1+z_{fr})a_0 r_1(z_e)$. Using the usual $d_L(z)$ formula ignoring that there are actually two different redshifts entering it, the deduced z from the luminosity distance is then in between z_{fr} and z_e and seriously systematically underestimates the physically relevant z_{fr} resulting in overestimating the mergers chirp mass: this is analogous to the argument in [51] except that we don't need lensing and magnification for that in our case. So similarly the true BH masses may remain in the 10 - 12 solar masses range.

We also have a discontinuity for g_{ii} metric elements[1] because of frozen C_{frozen} and this could be responsible for a different kind of effects: Shapiro delay or deflection of photons crossing the discontinuous potential. Because D_{int} evolves as (77) on the mean, there is a potentially cumulative hence large effect on cosmological times. On the other hand if the metric in D_{ext} is just as within GR the result of a non linear non trivial superposition of background and local gravity, the effects of the expansion are expected to be highly suppressed if we are not very far away from the sun which is also almost equivalent to a frozen scale factor. So the effect when crossing the discontinuous frontier might remain small though this remains to be investigated in more details!

In particular, it will prove interesting to check whether the implied distortions could actually explain the CMB low multipole anomalies [60][59], for instance the low quadrupole power and correlations with the ecliptic and galactic planes, and more specifically the order of magnitude of g_{ii} discontinuities related to the presence of the sun (but not anymore necessarily constrained to be at the level of the sun a-dimensional surface potential which is 10^{-6}) needed to get such effect from light rays being deviated according to the Descartes refraction law with effective gravitational indices given by differing g_{ii} on both sides of the frontier. This also obviously requires the frontier surface to not look isotropic from the Planck experiment view point which indeed is not centered at the sun.

Near a BH such discontinuities could be much larger not only implying refraction but also a significant reflection if the effective gravitational optical indices actually differ by a large amount. The question remains opened whether this could help produce echoes of a gravitational wave signal.

11. The MOND phenomenology

As already pointed out DG crucially differs from GR in the way global expansion and local gravity work together. Any anomaly in the local physics of the solar system or galaxy seemingly pointing to effects related to the Hubble rate is completely puzzling in the context of GR while it may be naturally explained within DG. Not

[1]avoided however in the alternative step by step evolution scenario

only the Pioneer effect but also MOND phenomenology seem related to the H_0 value.

We derived in a former section the speed $\frac{dr}{dt} = -\frac{1}{2}\frac{H_0}{d\Phi(r)/dr}$ at which a frontier sitting at an isopotential between internal and external regions should radially propagate in the potential well of a given body. From this formula the speed of light $\frac{dr}{dt} = c$ is reached anywhere the acceleration of gravity equals $cH_0/2$. This appears to be the order of magnitude of the MOND acceleration and the corresponding radius even closer to the MOND radius beyond which gravity starts to be anomalous in galaxies [20][28]. Also remember that we assumed a radially uniform fluctuation to derive the speed formula for our hypersurface which amounts to consider that $d\Phi(r)/dr$ is its leading contribution so such estimation can only be very approximate. We are therefore tempted to suspect that something must be happening near the MOND radius due to frontier discontinuities propagating (and dragging matter) at a speed approaching the speed of light. Our best guess is that this is the radius beyond which our adiabatic particle exchanges allow a completely dynamical metric to take over.

Another kind of argument could explain a MOND like frontier even though in a less predictive way as for its exact location. The mean universe density $\bar{\rho}$ should now be dominated by the conjugate one $\bar{\tilde{\rho}}$ by a $1.7^6 \approx 25$ factor if the equality of global densities was reached at the transition redshift $z \approx 0.7$. Yet we know for sure that planets and stars are still gravific meaning that the asymptotic values C^2 and $\frac{1}{C^2}$ of the conjugate metrics did not exchange their roles at the place of such condensed bodies. In other words the existence of static bounded domains anyway implies frontiers delimiting regions in which the cosmological permutation between $a(t)$ and $\tilde{a}(t)$ already occurred and others where it did not. It is not even clear at this stage whether such frontiers are propagating and in the affirmative what determines the location of such frontiers. But anyway such frontier must exist and could be located at the MOND radius in galaxies. Then as we explained in a previous section it should result in the gravitational field from the dark side in the region beyond such radius to be enhanced by a huge factor C^8 relative to the gravity due to our side matter in this region. Eventually this leads to a new picture in which only our side matter can be considered to be significantly gravific below the transition radius while only the dark side matter is significantly gravific beyond this radius. Then because a galaxy on our side implies a slightly depleted region on the dark side by it's anti-gravitational effects, even such a slightly underdense fluctuation on the dark side would result in an anti-anti-gravitational effect on our side. This effect exclusively originating from beyond the transition radius would be difficult to discriminate from the effect of a Dark Matter hallow as an underdense fluctuation in a distribution of negative mass is perfectly equivalent to an overdensity of normal positive mass matter. Also the most spectacular features of Dark Matter and MOND Phenomenology in galaxies such as galaxies that seem to be dominated at more than 99 percent by Dark Matter [21] or unexpectedly high acceleration effects in the flyby of galaxies [24] are more naturally interpreted in a framework where the gravitational effects from the hidden side are dominant beyond the MOND radius.

12. Stability issues about distinct backgrounds: C $\neq$ 1

12.1. *Stability issues in the purely gravitational sector*

Our action for gravity being built out of two Einstein Hilbert terms, each single one is obviously free of Ostrogradsky ghost. This also means that all degrees of freedom have the same sign of their kinetic term in each action.

There might still remain issues in the purely gravitational sector when we add the two actions and express everything in terms of a single dynamical field $g_{\mu\nu}$: everything is all right as we could demonstrate for C=1, but otherwise what we need to insure stability is that in the field equation resulting from the total action, all degrees of freedom will have their kinetic term tilting to the same sign. Again adopting $\bar{h}_{\mu\nu}$ from $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ and $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$ as the dynamical field puts forward that we have exactly the same quadratic (dominant) terms in $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ except that for $C > 1$ (resp $C < 1$) all terms in $t_{\mu\nu}$ are enhanced (resp attenuated) by a C-dependent factor while all terms in $\tilde{t}_{\mu\nu}$ are attenuated (resp enhanced) by a 1/C dependent factor, so that we will find in $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ all such quadratic terms tilting to the same sign, ensuring that the theory is still free of ghost in the purely gravitational sector.

Of course there remains an instability menace whenever $C \neq 1$ in the interactions between matters and gravity which we shall inspect now.

12.2. *Stability issues in the interactions between matter and gravity: the classical case*

Generic instability issues arise again when C is not anymore strictly equal to one. This is because the positive and negative energy gravitational terms $t^{\mu\nu}$ and $\tilde{t}^{\mu\nu}$ do not anymore cancel each other as in the DG C=1 solution. Gravitational waves are emitted either of positive or negative (depending on C being less or greater than 1) energy whereas on the source side of the equation we have both positive and negative energy source terms. Whenever two interacting fields (here the gravitational field and some of the matter and radiation fields) carry energies with opposite sign, instabilities would seem unavoidable (see [26] section IV and V for a basic description of the problem and [27] for a more technical approach) and the problem is even worsen by the massless property of the gravitational field.

Yet, the most obvious kind of instability, the runaway of a couple of matter particles with opposite sign of the energy, is trivially avoided in DG theories [5][8][9][6] [30][31][32][33][34][28] in which such particles propagate on the two different sides of the Janus field and just gravitationally repel each other.

It is also straightforward to extend the theory of small gravitational fluctuations to DG in the Newtonian approximation for C=1 (neglecting expansion): the equations governing the decay or grow of compressional fluctuations are :

$$\ddot{\delta\rho} = v_s^2 \Delta\delta\rho + 4\pi G < \rho > (\delta\rho - \delta\tilde{\rho}) \tag{92}$$

$$\ddot{\delta\tilde{\rho}} = \tilde{v}_s^2 \Delta\delta\tilde{\rho} + 4\pi G < \tilde{\rho} > (\delta\tilde{\rho} - \delta\rho) \tag{93}$$

which in case the speeds of sound v_s and $\tilde{v}_s$ would be the same on both sides allows to subtract and add the two equations with appropriate weights resulting in two new equations governing the evolution of modes $\delta\rho^- = \delta\rho - \delta\tilde{\rho}$ and $\delta\rho^+ = \delta\rho + \frac{<\rho>}{<\tilde{\rho}>}\delta\tilde{\rho}$.

$$\Box_s \delta\rho^- = 4\pi G(< \rho > + < \tilde{\rho} >)\delta\rho^- \tag{94}$$

$$\Box_s \delta\rho^+ = 0 \tag{95}$$

Where $\Box_s$ is a fake Dalembertian in which the speed of sound replaces the speed of light. Because $\delta\rho^+$ does not grow we know that $\delta\rho \approx -\frac{<\rho>}{<\tilde{\rho}>}\tilde{\delta\rho}$ and the two can grow according the growing mode of $\delta\rho^-$. The complete study, involving attenuation of gravity between the two sides due to differing scale factors and the effect of expansion will be the subject of the next section. It is already clear that in the linear domain anti-gravity by itself does not lead to a more pathological growth of fluctuations than in standard only attractive gravity: eventually we would expect the growth of a gravitational condensate on one side to proceed along with the corresponding growth of a void in the conjugate side and vice versa[m]. In other words our "instabilities" in the linear domain are nothing but the usual instabilities of gravity which fortunately arise since we need them to account for the growth of matter structures in the universe. These instabilities could be classified as tachyonic (the harmless and necessary ones for the formation of structures), non gradient (fortunately because those instabilities are catastrophic even at the classical level), and ghost (energy unbounded from below which is only catastrophic for a quantum theory) in the terminology of [37] reviewing various kind of NEC violations in scalar tensor theories which confirms that these are acceptable for a classical theory.

From this it appears that DG is not less viable than GR in the linear domain as a classical theory and that the real concern with all DG models proposed to this date will actually arise for the quantized DG theories for which ghost instabilities are of course prohibitive, and may be in the strong field regime for the classical theories. Only then the real energy exchange between the gravitational field itself (it's kinetic energy quadratic terms) and other fields kinetic energies should start to become significant relative to the Newtonian like energy exchange between kinetic energy of the fields and their gravitational potential energy that drives the evolution

[m]The situation is less dramatic than Ref [26] section IV might have led us to think mainly because our leading order terms are linear in a gravitational field perturbation h whereas the leading order coupling term is quadratic in the lagrangian (22) of [26] leading to equations of motion of the form $\ddot{\Psi} \propto \Psi^3$.

of the compressional modes according Eq [92] and [93]. In the strong field regime the problem is thus related to the radiation of gravitational waves when they are carrying non zero energy (for C≠1) while they can couple to matter sources with both positive and negative energies[n].

However, we expect that high density regions produced by compact objects on our side are always in the $C > 1$ domain (remind that the scale factors hence C permutation is triggered at the crossing of densities i.e. wherever the conjugate side density starts to dominate our side density) so that the interaction between this matter and the positive energy gravitational field (due to $C > 1$) is not a ghost interaction. For the same reason, high density regions produced by compact objects on the dark side are expected to remain in the $C < 1$ domain so that the interaction between the dark side negative energy (from our point of view) matter and the negative energy gravitational field (due to $C < 1$) is again not a ghost interaction. Eventually the only remaining ghost interactions with the gravitational field could be those from density fluctuations too small to locally flip the sign of C-1 in the safe direction, but these fluctuations do not produce strong gravity and therefore are not problematic, all the more since their gravity is expected to be suppressed by a huge C^8 factor.

12.3. *Stability issues in the interactions between matter and gravity: the quantum case*

12.3.1. *Problem statement*

The next step is therefore to try to understand how we might solve stability issues in the quantum case. In the quantized theory the problematic couplings would produce divergent decay rates by opening an infinite space-phase for for instance the radiation of an arbitrary number of negative energy gravitons by normal matter (positive energy) particles. To avoid such instabilities may be the most natural way would be to build the quantum Janus field operator also as a double-faced object, coupling it's positive energy face to usual positive energy particles and it's negative one (from our side point of view) to the negative energy particles (from our side point of view) of the dark side thereby avoiding any kind of instabilities. However the picture described by our classical Janus field equation which in principle really allows the direct exchange of energy between GW (with a definite sign of the energy depending on C > 1 or $C < 1$) and matter fields with different signs of the energy

[n]This remains true even when great care is being taken to avoid the so-called BD ghost in the massive gravity approach particularly when the perturbations of the two metrics about a common background have different magnitudes i.e. when one parameter of the couple α, β dominates the other in [32]. By the way there is a much worse problem in models having two independent differential equations instead of one to describe the dynamics of two fields assumed independent, i.e. not related from the beginning by a relation such as Eq (1). Then the energy losses through the generation of gravitational waves predicted by each equation are different so that such models are inconsistent [5][8][9][6] [30] as shown in [16].

does not actually fit into such quantization idea. The most straightforward way to avoid such fatal quantum instabilities is to consider that the gravity of DG is not a quantum but remains a classical field. Semi-classical gravity indeed treats matter fields as being quantum and the gravitational field as being classical, which is not problematic as far as we just want to describe quantum fields propagating and interacting with each others in the gravity of a curved space-time (within GR) considered as a spectator background. To describe the other way of the bidirectional dialog between matter and gravity i.e how matter fields source gravity, semi-classical gravity promotes the expectation value of the energy momentum tensor of quantum fields as the source of the Einstein equation and this is considered problematic by many theorists. In the last section we shall ask ourselves whether the usual prescription for semi-classical GR e.g. exploiting the quantum fields energy momentum expectation value is the right way to go or if there are more natural alternatives in our case to make quantum and classical fields live together and describe their interactions.

13. Evolution of fluctuations

13.1. *Evolution for negligible dark side gravity*

Except for matter-radiation transfers which are only non-negligible near t=0, our DG equations are negligibly deviating from GR equations before the transition redshift. Dark Matter is required just as in the standard model to have almost the cosmological critical density implied by k=0 the measured value of the Hubble expansion rate and the low density of radiation at late times (but still before the transition redshift). Presumably, this Dark Matter did the same good job as within LCDM to help the formation of potentials already in the radiative era and then thanks to these potentials the growth of baryonic fluctuations falling into these potentials. We then have potentially all the successes of CDM phenomenology before the transition redshift with the bonus that we have a new natural candidate for Dark Matter and shall present it in an upcoming section. We also naturally expect almost the same sound horizon at decoupling even though a true singularity is avoided at t=0.

Also remember that the dark side reaches the same density of pressureless matter as on our side at the transition redshift. So even though the dark side growing of fluctuations could of course have been boosted by it's contracting scale factor especially on the largest scales the mean dark side density can be extrapolated to extremely small values at high redshift with $\tilde{\rho} \approx z^{-6}\rho \ = 10^{-18}\rho$ at $z \approx 1000$. Then it is quite obvious that the growth of our side fluctuations starting from $\frac{\delta\rho}{\rho} \approx 10^{-5}$ of the CMB, could not be helped at high z.

As in LCDM, for the evolution of fluctuations the background evolution only becomes important in the matter dominated era arising as usual as an additional friction term $H\dot{\delta\rho}$ where H is the Hubble rate. So we can readily rewrite Eq (92) and (93) taking into account all non negligible effects depending on the scale factor but neglecting sound speeds on both sides assumed to be dominated by non relativistic matter: (see for instance equation (5.1.8) of [42], also written in term of the conformal scale factor for comparison)

$$\ddot{\delta} + H\dot{\delta} = 4\pi G(a^2 < \rho > \delta - \tilde{a}^2 < \tilde{\rho} > \tilde{\delta}) \quad (96)$$

$$\ddot{\tilde{\delta}} + \tilde{H}\dot{\tilde{\delta}} = 4\pi G(\tilde{a}^2 < \tilde{\rho} > \tilde{\delta} - a^2 < \rho > \delta) \quad (97)$$

We here have introduced the relative density fluctuations e.g. $\delta = \frac{\delta\rho}{<\rho>}$. We can justify these equations they have to satisfy in the following way. Those relative densities as usual are sourced by the potential which is just opposite on the dark side relative to our side so δ and $\tilde{\delta}$ are of the same order of magnitudes. Therefore the absolute density fluctuations satisfy $\delta\rho \gtrsim \delta\tilde{\rho}$ before the transition that is as long as $< \rho >$ is greater than $< \tilde{\rho} >$ and the dominance is reversed after the transition. Then inspection of a formula like (44) shows that we can always completely neglect the subdominant terms damped by huge ratios of the scale factors both on the left and the right hand side in order to obtain the potential before or after the transition. As a result, equations (96) and (97) are always valid with an excellent level of approximation.

Those equations confirm that though the dark side gravitational influence on our side can be neglected from the early universe up to the transition redshift (because then $a >> \tilde{a}$), the converse is not true: the dark side is negligibly submitted to it's own gravity but feels the anti-gravitational forces from our side matter structures so:

$$\ddot{\delta} + H\dot{\delta} \approx 4\pi G a^2 < \rho > \delta \quad (98)$$

$$\ddot{\tilde{\delta}} + \tilde{H}\dot{\tilde{\delta}} \approx -4\pi G a^2 < \rho > \delta \quad (99)$$

A common practice is to reformulate those differential equations with derivatives with respect to the scale factor instead of time:

$$\frac{d^2\delta}{da^2} + \frac{3}{2a}\frac{d\delta}{da} \approx \frac{3}{2}\frac{\delta}{a^2} \quad (100)$$

$$\frac{d^2\tilde{\delta}}{d\tilde{a}^2} + \frac{3}{2\tilde{a}}\frac{d\tilde{\delta}}{d\tilde{a}} \approx -\frac{3}{2}\frac{\delta}{\tilde{a}^2} \quad (101)$$

The equation is the usual one for the evolution of our side fluctuations with well known growing solution modes $\delta \propto a$. Those are also driving the evolution of the dark side side $\tilde{\delta}$ in the second equation as $\tilde{\delta} = -3\delta \propto a$. Eventually the dark side matter merely develops over-densities $\tilde{\delta}^+$ with a maximum density contrast three times bigger than our side void under-densities but is negligibly non linearly clustering under it's negligible self-gravity all over this period so that $\delta^- \geq -1 \Rightarrow \tilde{\delta}^+ \leq 3$.

At the same time it is developing voids everywhere we have our side over-densities, for instance around our galaxies but then of course $\tilde{\delta}^- \geq -1$ and the growth factor of those voids must asymptotically tend to 0 significantly faster than on our side given that the dark side linear fluctuations are in advance by a factor 3, so many small scale dark side voids must already have reached a density contrast close to -1 at which point they have no more ability to significantly grow anymore.

13.2. *Evolution with dark side gravity*

It remains to investigate the influence of fluctuations from the dark side after the transition redshift. For that we need to rely on the extremely efficient effect of the scale factors permutation to understand the gravitational effect of dark side fluctuations (voids) starting to play a significant role and produce the MOND empirical laws in galaxies. But in accordance with what we also explained earlier we have two kinds of regions for fluctuations : those static regions around our side concentrations of baryonic matter in which the gravity from our side $\delta\rho_{static}$ remains hugely enhanced over the gravity from the dark side $\delta\tilde{\rho}_{static}$ because the scale factor was not renormalized there, and the rest of the universe in which at the contrary, it is the gravity from the dark side $\delta\tilde{\rho}_{evol}$ that hugely dominates that from $\delta\rho_{evol}$. Close to the transition redshift, we would therefore expect similar strengths for $\delta\rho_{static}$ and $\delta\tilde{\rho}_{evol}$ gravity, however since the static domains are likely to house highly non linear fluctuations we can't include them in our linear equations so keeping the linear dominant terms only, we have:

$$\frac{d^2\delta}{da^2} + \frac{3}{2a}\frac{d\delta}{da} \approx -\frac{3}{2}\frac{\tilde{\delta}}{a^2} \tag{102}$$

$$\frac{d^2\tilde{\delta}}{d\tilde{a}^2} + \frac{3}{2\tilde{a}}\frac{d\tilde{\delta}}{d\tilde{a}} \approx \frac{3}{2}\frac{\tilde{\delta}}{\tilde{a}^2} \tag{103}$$

Therefore the dark side linear fluctuations are now submitted essentially to their own gravity and in a contracting background are expected to grow very fast. The equation is indeed the usual one with solution modes $\tilde{\delta} \propto \tilde{a}$ and $\tilde{\delta} \propto \tilde{a}^{-3/2}$. The latter $\tilde{\delta} \propto a^{3/2}$ are now the growing ones driving the evolution of our side δ in the first equation also as $\delta = -\frac{\tilde{\delta}}{2} \propto a^{3/2}$ when the steady state regime is reached. This should be compared to $\delta \propto a$ before the transition redshift just as within LCDM during the matter dominated era whereas the LCDM growth factor (this is defined to be f in $\delta \propto a^f$), as Λ becomes dominant, is expected to decrease progressively from f=1 to $f \approx 0.5$ now.

This high growth factor of the dark side fluctuations should produce anomalies of our side voids growth factor exceeding LCDM expectations specially at low redshift as has indeed been reported for instance for cosmic voids below z=0.4[63]. Moreover when these fluctuations reach the non linear regime they are expected to cluster near the center of our voids producing an increasing repelling force on our side nearby matter. All those linear as well as non linear effects have replaced the own repulsive effect of our side voids that has been switched off following the transition redshift. Remember however that $\delta^- \geq -1$ so the growth factor of our voids should start to decrease in the future and asymptotically tend to zero.

On the other hand, the dark side under-densities must also have relayed as gravific actors our side, now switched off, over-densities for the same reasons. And then the resulting growth factor tending to 1.5 over super-clusters scales, and then largely exceeding the expectations from LCDM should remain much smaller on somewhat smaller scales corresponding to dark side voids that cannot grow anymore having almost reached the limit -1 for $\tilde{\delta}^-$.

Actually as a result of those $\tilde{\delta}^-$ capping under -1 already before the transition redshift, for many of those voids having reached the limit of the linear domain, $\tilde{\delta}^-$ must have been frozen at density contrasts well greater than $-3\delta^+$. In that case the transient regime for δ^+ following the transition redshift will merely be a convergence from $\delta^+ \geq \frac{1}{3}$ to $\delta^+ = -\frac{\tilde{\delta}^-}{2} \approx \frac{1}{2}$ implying that all δ^+ between 0.5 and 1 at the transition redshift have then started to decay toward 0.5.

All these considerations should therefore motivate a serious re-investigation within our framework of numerous recently reported anomalies of the growth rates [64] [65] and is also a plausible origin for the mild tensions between some of our predicted and observed BAO points at low redshift due to people influenced by GR and the standard model expectations not correctly understanding the shape of BAO peaks even in the linear regime.

On the smallest scales, fluctuations $\tilde{\delta}$ in the dark side distribution are also expected to produce gravific effects mimicking so well the gravity of DM halos that those are probably wrongly attributed to Dark Matter Halos within LCDM. Indeed, around our galaxies, the voids that formed before the transition redshift on the dark side have started to exert their confining force helping the rotation of galaxies after the transition redshift, as these exactly behave as dark matter halos but without cusps, from our side point of view. Again such effect must replace that of genuine dark matter that was gravitationnally active before but not anymore after the transition redshift.

13.3. *The confrontation to growth data: σ_8, $f\sigma_8$*

13.4. *BLK instabilities*

BKL (Belinksii, Khalatnikov, and Lifshitz)[69] instabilities are known to be generic in theories with a matter dominated contracting universe before a bounce that are interesting alternatives to inflation theories (see [66] and references therein). The catastrophic growth of anisotropic stress i.e. the universe starting to contract at different rates in the three spatial directions is a serious problem that would require an extreme suppression (hence fine tuning) of anisotropies already in the initial state of a contraction phase to avoid them growing up to an unacceptable level thereafter [70 chapter 2.2]. This becomes a concern for DG but only in the contraction phase following the transition redshift in which the dark side fluctuations are gravitationnally active. Fortunately this phase is preceded by our side dominated phase in which the evolution of the fluctuations on both sides is determined and driven by our side fluctuations in an expanding universe which is reducing such kind of anisotropies by exactly the same factor the next phase is expected to amplify them. This problem is not easy to solve in the context of bouncing universe theories as it turns out to be extremely unnatural to have a phase dominated by a field able to extremely homogeneize the anisotropies followed by a phase in which the matter field unavoidably becomes dominant (needed to get the correct spectrum of initial fluctuations after the bounce) and amplifies the anisotropies... extremely unnatural indeed except within a very special theory as is DG specifically thanks to it's discontinuous transition which precisely allows this in an amazing way!

13.5. *Cosmological Dark Matter reinterpretation*

We already pointed out that baryonic matter is, just as within GR, cosmologically not abundant enough to account for the Hubble rate before the transition redshit, so we still need a "Dark Matter" cosmological density $\bar{\rho}_{DM}$.

13.5.1. *Pseudo BH as DM candidates ?*

Primordial Black Holes (PBH) were recently considered a possible candidates for Dark Matter because these are collisionless, stable, and at least until recently, not yet completely ruled out by astrophysical and cosmological constraints. Just as GR Black Holes, our pseudo BH could exist in any size but in principle the smallest ones would not escape the main observational constraint on the fraction of PBH: the Hawking radiation flashes (these would on the other hand evade detection by their microlensing signature, too small to be detected for small objects) expected to be the same for our pseudo black holes as for true black holes. This is however questionable if our PBH can vanish i.e. completely disappear from our side as their matter content is transferred to the conjugate side before the emission of the Hawking flash which would make our PBH a better DM candidate than GR PBH. Even in that case however our PBH could presumably still not contribute a significant part of Dark Matter because if their masses spread over a range which upper bound is 10^{11}kg (at this mass their lifetime is comparable to the age of our universe), as they are transferred to the dark side in a lifetime smaller than the age of the universe we would see the matter density on our side significantly deviating from the $\rho \approx 1/a^3$ conservation law. On the other hand if their mass range extends beyond 10^{11} kg they will not escape the exclusions from lensing experiments. Thus our PBHs are not a good DM candidate.

13.5.2. *Heavy elements baryonic matter as DM candidate ?*

Figure 3 from [47] summarizing all existing constraints on the existence of Macros i.e. massive Dark matter objects possibly made of standard model particles assembled in a high density object (from beyond atomic to well beyond nuclear densities) leaves open the possibility that Dark Matter could be made of condensed matter with usual atomic densities and heavy elements such as iron if this was injected from the conjugate side Pseudo Black Holes in our radiative era. Then the distribution of this injected baryonic with high metallicity DM is expected to have been extremely inhomogeneous in our radiative era because highly concentrated on spots, much smaller than the Planck experiment resolution, making related small scale perturbation detection hardly possible. This concentration of DM in spots with very high metallicity is needed to make the idea viable as otherwise we would hardly understand why the universe is almost everywhere we look nowadays at a very low level of metallicity (compatible with the predictions of Big-Bang nucleosynthesis and stellar nucleosynthesis) both in the diffuse intergalactic gas as well as in stars. If this hypothesis is true the corresponding high metallicity and dark regions remain to be discovered. The high metallicity is also required to insure that these nuclei have a low charge over mass ratio making them much less dragged by the primordial acoustic fluctuations and then contributing to DM rather than normal baryonic matter from the analysis of the CMB spectrum.

The serious difficulty with this DM candidate making it unlikely is that the impulse response to an initial DM perturbation at a much higher redshift than the redshift of decoupling has been studied and should produce a spreading of this DM other scales extending to tens of Mpc. So this form of DM could not have remained localized until today (as we need to understand why it evades detection) and would have been vaporized by the high temperatures unless we assume that the injection occurred at a redshift not too much higher than 1000. But then the distribution of this DM would be quite different from the one predicted within LCDM as implied by it's initial spectrum of fluctuations but also the impulse response understood to evolve as depicted in fig 1 of [52] from which we also see that it would also have influenced very differently the mass profile of baryonic matter fluctuations already at decoupling. This argument therefore seems to rule out normal matter as DM.

13.5.3. *Micro lightning balls as DM candidates ?*

In previous papers we also described objects called micro lightning balls (mlb) that would also be collisionless in their collapsed state (they would "decouple" from the baryon photon fluid due to their small "cross-section") and deserve much attention since these as well might be perfect Dark Matter candidates. Some of those objects, as well as pseudo BH, might have been created as the result of density fluctuations producing a gravitational potential rising above a fundamental threshold triggering the discontinuous potential trapping and stabilizing the object. Some are likely to behave as miniature stars, presumably as dense and cold as black dwarfs and extremely difficult to detect either through their black body radiation of an extremely cold object, their negligible gravitational lensing given their surface gravity much smaller than that of a pseudo Black Hole of the same size and the absence of Hawking radiation even for the smallest of these objects. Of course a much more detailed characterization of long living micro lightning balls would be needed to make firm predictions as for both their spatial and mass distribution and the best way to detect them.

The intriguing possibility that our mlbs may constitute dark matter is also again supported by figure 3 from [47]. Presumably this high density form of matter could have been injected in our universe in it's radiation dominated era (hence with a negligible influence on the scale factor evolution at this epoch) from pseudo black holes and compact stars of the dark side which was very cold at this time. This era indeed corresponds to the beginning of a contraction phase of the dark side having followed a very long lasting expansion era having resulted in a dark side universe in which most of the matter had been swallowed by Pseudo Black Holes.

The mlbs only remain a plausible candidate provided their injection occurred at a sufficiently high redshift (see our discussion of normal matter as DM candidate in the previous subsection) but not too high to avoid the destruction of mlbs by a high energy particles bombardment. These also should have been injected according a nearly scale invariant spectrum (except on the very small scales marking the initial spots) determined by the distribution of pseudo BH on the Dark Side.

Interacting with matter, mlbs can decay and release their normal matter content with presumably high metallicity in their environment.

By the way, it is worth mentioning that discontinuities not only allow mlbs but might have helped the fast formation of stars in general and large mass ones in particular leading to many large mass pseudo BHs such as the ones recently discovered by Ligo or giant black holes at the centers of large galaxies. This is because the dragging effect of drifting discontinuities is presumably an effective mechanism to concentrate matter at all scales or to merge already formed pseudo BHs.

14. Last remarks and outlooks

14.1. *Frame dragging and gravitational waves anomalies?*

Earlier in this article we considered what we called a scalar-η field and investigated the consequences of having such a solution plus perturbation instead of the full metric with all it's degrees of freedom in the radiative era. We saw that such field would lead to anomalies such as the absence of gravitational waves but also frame dragging effects. We also noticed that a C=1 domain would also have almost vanishing gravitational waves solutions. At last, in some static domains cut out of the rest of the expanding universe, we might also have local rotating preferred frame attached to a rotating body with respect to the universe such that frame-dragging would also vanish in the vicinity of such body. So the DG theory asks us to seek many kinds of possible gravitational anomalies which are not absolutely excluded a priori in many different contexts and that could even be transient. In this spirit, we are tempted to interpret the zero frame dragging effect which was initially observed by Gravity Probe B on one of its four gyroscopes as evidence for DG. See our section 12 devoted to gravitomagnetism and preferred frame effects in [4] for further details.

14.2. *Status of the Janus field*

We already pointed out that none of the faces of our gravitational Janus field could be seriously considered as a candidate for the spacetime metric. Yet, though the gravitational field loses this very special status (be the spacetime metric) it had within GR, it acquires another one which again makes it an exceptional field : it is the unic field that makes the connection between the positive and negative energy worlds (this definition is relative: for any observer the negative field is the one that lives on the other side), the only one able to couple to both the dark side SM fields and our side SM fields. This special status alone implied that the gravitational interaction might need a special understanding and treatment avoiding it to be quantized as the other interactions. Avoiding ghost instabilities related to the infinite phase space opened by any interaction between quantum fields that do not carry energies with the same sign, is a requirement which also confirms that the gravitational Janus field in Eq (1) and (2) could not interact with matter as a quantum field. So the old question whether it is possible to build a theory with a classical gravitational field interacting with all other fields being quantum, was back to the front of the stage just because the usual answer "gravity must be quantized because everything else is quantum" fails for the Janus theory of the gravitational field.

14.3. *Gravity of quantum fluctuations*

Another point that deserves much attention is that within DG, wherever the two faces of the Janus field are equal, vacuum energy terms trivially cancel out as we already noticed in [15] so we might have good reasons to suspect that a mechanism is at work to insure that this cancellation is preserved even when the two faces depart from each other. First, cosmological constant terms are strictly constant within GR because of the Bianchi identities which is not necessarily the case in DG. Such terms might vary (because of varying cutoffs for instance) in order to preserve the cancellation between our side and the dark side vacuum energy terms. The context is anyway much more favourable than within GR where no such kind of cancellation could possibly occur.

Moreover the old cosmological constant problem is not necessarily a concern for a semi-classical theory of gravity as is DG. Indeed, the usual formulation of the problem is that we have no reason to doubt the existence of vacuum Feynman graphs since we see their effect for instance through the Casimir and Lambshift effects. However, it should be specified that the actual Feynman graphs probed this way have external legs of particles so that the extrapolation to gravity becomes straightforward: we just need to replace those external particles by gravitons to estimate how much gravity we can expect from such quantum fluctuations. The extrapolation is far less trivial when we don't have gravitons, as we should replace in this case the external particle legs by new kinds of legs actually representing an external classical field. The problem then is that the purely quantum part of the graph is really a vacuum graph: it has no external legs and we don't have any evidence that such graphs without any real particle actually exists: in particular it's not the kind of graph probed by the Lambshift and Casimir effects. Eventually the old cosmological constant problem might already be a strong clue that gravity is not quantum. As we already noticed the elusiveness of Dark Matter particles could be an additional clue that not everything is quantum.

14.4. *Scale invariance of the primordial power spectrum*

Because of the expansion laws of our universe in the radiative and matter dominated era we know that the cosmological perturbation scales presently within the Hubble radius crossed it in the past and that before that crossing time those fluctuations were beyond the Hubble radius with corresponding curvature perturbations being scale invariant. This approximate scale invariance (spectral index $n_s = 0.96 \pm 0.007$ hence close to 1) of the primordial dimensionless power spectrum of curvature perturbations is what we have learned from the detailed study of CMB and large scale structures.

By far the most popular theories naturally producing such kind of spectra are inflation theories in which the initial quantum vacuum fluctuations of a fundamental scalar field see their physical scales expand faster than a constant Hubble radius, thanks to an inflationary (exponential regime of the scale factor as a function of standard cosmological time) primordial phase resulting from the dynamics of the scalar field and then those fluctuations start to grow and tend to a scale invariant power spectrum (also resulting in a scale invariant spectrum of curvature perturbations) once they find themselves beyond the Hubble radius. Such inflationary scenarios not only can reproduce the correct power spectra of fluctuations but also solve the homogeneity and flatness problems of standard cosmology and actually were just designed to do so.

The question is whether we need to introduce a similar mechanism within the Dark Gravity theory or the theory as it stands is able to perform just as good or even better. First we already know that DG predicts a flat background and stable against anisotropies (see our section devoted to BKL instabilities). Large scale homogeneity on the other hand is presumably resulting from the matter-radiation exchange process which has transferred most of the content of a very dense dark side universe (at the end of its contraction phase) to our side near the origin of time. Such universe had plenty of time to thermally homogeneize over the largest scales we can probe today.

Because of this transfer the history of our universe is actually equivalent to that of a non singular bouncing universe theory in which an expanding phase followed a contraction phase. Such theories perform even better than inflation theories as they solve the same problems but without some of the well known issues introduced or not addressed by inflation (the trans-Planckian problem, the singularity problem, the initial amplitude or fine tuning problem, the unfalsifiability problem [67][68]).

In particular, these theories can just as well produce the correct scale invariant primordial power spectra as their perturbations beyond the Hubble radius just behave as in the inflationary scenario and as well originated from vacuum perturbations inside the Hubble radius : the comoving Hubble radius $\mathcal{H}^{-1}$ in which $\mathcal{H} = aH$ is also the "conformal time Hubble rate" decreases with time in a contracting universe so comoving fluctuation scales $\lambda = k^{-1}$ outside the Hubble radius at the bounce must have been inside it past the bounce).

The common problem to inflationary and bouncing cosmologies is that they both rely on initial quantum vacuum fluctuations at the origin of cosmological perturbations whereas we know that vacuum energy does not gravitate. This is again the old cosmological constant problem, now back to create distrust and suspicion as for the actual validity of all models relying on initial quantum vacuum fluctuations to explain the primordial scale invariant power spectrum.

We were earlier pleased to realize that if gravity is not quantum as is the case in DG may be it's not so surprising that pure vacuum graphs without any external incoming or outgoing real particle legs do not source the classical gravitational field. But this simple solution of the old cosmological constant problem within DG seems to dispel any hope of exploiting the initial quantum vacuum fluctuations as seeds for primordial scale invariant fluctuations, except if a more involved mechanism is may be at work: Indeed, it remains that within DG other fundamental fields could be classical in essence (Dark Matter itself ?) or even better that all known quantum fields could have a classical field counterpart only able to interact gravitationnally. Such classical fields would also have fluctuations that could source gravity but in a much safer way than quantum fluctuations as the energy of a classical field is limited by it's amplitude whereas a non limited number of quanta created and annihilated in vacuum make the energy of a quantum field divergent in vacuum. In other words the old cosmological constant problem is an issue completely related to the quantum nature of fundamental quantum fields. But if only the classical fluctuations are able to source gravity, then the only thing we would need to obtain the scale invariant spectra just as within inflationary or bouncing cosmologies is to justify that the normalization of those fundamentally classical initial fluctuations need to be the same as the normalization of states of the Bunch-Davies vacuum $\propto \frac{1}{\sqrt{k}}$ usually assumed as initial conditions in non singular bouncing as well as in inflationary cosmologies. Such normalization is easy to understand in the context of a quantum field theory over flat space-time as it is required to get an Hamiltonian which eigen-values are Planck-Einstein energies ($h\nu$) but is completely unjustifiable in the context of a classical field theory ... unless the field both exists in a classical and quantum version and the classical version vacuum which is the only one able to source gravity (as needed to avoid the old cosmological constant problem) retains the normalization of the quantum one. If this idea would turn out to be completely achievable we would have it both ways : the successes of the non singular bouncing universe theories without one of their most serious remaining drawbacks: the old cosmological constant problem.

A remaining very serious issue is that just as for bouncing universe theories the initial fluctuations before the bounce that need to be dominant are the fluctuations associated to this new kind of vacuum. Indeed, not only this "classical vacuum" would be impossible to define except by relating it to the normalization of a quantum field counterpart but more seriously the power of those "classical vacuum" fluctuations must dominate over all other fluctuations inherited from the past history of the contracting universe which is difficult to justify and actually not expected in DG.

Given such challenges, may be should we better rely on a far more trivial alternative way to get the scale invariant primordial curvature spectrum : postulate that the matter-radiation transfer is a process able to efficiently homogeneize not only baryons and radiation on both sides of the universe but also Dark Matter at all scales and able to directly produce scale invariant fluctuations. After all, if this process washes out all traces of the past history of the conjugate universes, we do expect the new initial states to be either perfectly homogeneous or at the very least inhomogeneous in a scale invariant way just because this process itself has no obvious privileged scale associated to it ... but this question for sure would deserve further investigation !

14.5. *Discrete symmetries, discontinuities and quantum mechanics*

Dark Gravity from the beginning is a theory involving both discrete (a permutation symmetry now understood as a global time reversal) and usual continuous symmetries unified in the same framework thanks to the crucial role payed by our background non dynamical metric.

It was then natural to wonder whether such discrete symmetry could have a genuine dynamical role to play, and we postulated a global time reversal process exchanging the two faces of our Janus field [7][14][15] and producing field discontinuities at the frontier of space-time domains. We now then have a unique and remarkable framework unifying not only the discrete and continuous symmetries but also the related continuous and discontinuous processes. Continuing to build on our successes we later considered various new possible discrete physical laws i.e. we may not only have discontinuous transitions in time when the conjugate scale factors exchange their roles but also other kind of discontinuities in space at the frontier between static and expanding spatial regions. We did not encounter any serious obstacle proceeding along this way and for instance we already drew the reader attention to the harmlessness of discontinuous potentials as for the resolution of wave function equations in the presence of discontinuities. Of course the exploration of this new physics of discontinuities in relation to discrete symmetries is probably still at a very early and fragile stage and requires an open minded effort because it obviously questions habits and concepts we used to highly value as physicists.

Discontinuous fields also put into question the validity of the Noether theorem implying the violation of local conservation laws wherever the new physics rules apply. However, we should remind ourselves that the most fundamental postulates of quantum physics remain today as enigmatic as they appeared to physicists one century ago: with the Planck-Einstein quantization rules, discontinuous processes came on to the scene of physics as well as the collapse of a wave function taken at face value obviously implies a violation of almost all local conservation laws.

Based on these facts, a new theoretical framework involving a new set of discrete and non local rules which, being implied by symmetry principles are not anymore arbitrary at the contrary to the as well discontinuous and non-local quantum mechanics postulates, might actually be a chance. A real chance indeed as they open for the first time a concrete way to hopefully derive the so arbitrary looking quantum rules from symmetry principles and may be eventually relate the value of the Planck constant to the electrical charge, in other words compute the fine structure constant. We are certain that only our ability to compute the fine structure constant would demonstrate that at last we understand where quantum physics comes from rather than being only able to use it's rules like a toolbox. With the classical discontinuous field of DG we are confident that we are much closer to establish a connection with the quantum fields than ever before. Again the unification of the continuous and the discontinuous seems to us a much more fundamental goal than simply trying to make gravity work with quantum rules if the later remain completely enigmatic.

In this perspective, it may be already meaningful to notice that our Pseudo Black Hole speculated discontinuity at the pseudo horizon, which would lie at the frontier between approximate GR and DG C=1 domains, behaves as a wave annihilator for incoming GW waves and a wave creator for outgoing waves. In the DG C=1 domain, the waves carry almost no energy while in the GR domain they carry energy and momentum as usual. This is a fascinating remark because this would make it the only known concrete mechanism for creating or annihilating waves à la QFT or even a significant step toward a real understanding of the wave function collapse i.e. in line with a realistic view of quantum mechanics. Such collapse is indeed known to be completely irreducible to classical wave physics because it is non local, and in fact just as non local as would be a transition from GR $C >> 1$ to DG, C=1 in the inside domain.

14.6. *The Janus field and the Quantum*

In the previous section we emphasized our theoretical motivation for bridging the gap between our classical discontinuous Janus field and true quantum fields. We also have now an additional strong couple of phenomenological motivations : dark matter could be the contribution of a classical field while the old cosmological constant problem might just disappear if gravity is classical.

We already mentioned semi-classical gravity as a candidate theory to describe the interactions between usual quantum fields and a classical gravitational field.

The idea is to have the expectation value of the energy-momentum tensor rather than the tensor itself sourcing the gravitational field equations. One often raised issue with semi-classical gravity is that it is incompatible with the Multi Worlds Interpretation (MWI) of QM since within the MWI the other terms of quantum superpositions which are still alive and represent as many parallel worlds would still be gravific as they contribute to the energy momentum tensor expectation value and should therefore produce large observational effects in our world. The MWI, considered as a natural outcome of decoherence is adopted by a large and growing fraction of physicists mainly because is considered the only alternative to avoid the physical wavefunction collapse. For this reason incompatibility with the MWI is often deemed prohibitive for a theory. Since we have nothing against a physically real wave function collapse (our theory even has opened new ways to hopefully understand it; discontinuity and non locality are closely related) we are not very sensitive to the incompatibility between semi-classical gravity and the MWI. The wave function collapse might eventually be triggered at the gravitational level: a simple achievement of something similar to the Penrose idea (gravitationally triggered collapse) seems within reach in our framework, thanks to a transition to C=1 which is tantamount to a gravitational wave collapse.

So we can still alternatively consider semi-classical gravity and the Schrodinger-Newton equation it implies [39] in the context of a true physical collapse interpretation of QM, all the more so as the usual arguments based on the measurement theory often believed to imply that gravity must be quantized have recently been re-investigated in [38] and the authors to conclude that "Despite the many physical arguments which speak in favor of a quantum theory of gravity, it appears that the justification for such a theory must be based on empirical tests and does not follow from logical arguments alone." This has even reactivated an ongoing research which has led to experiment proposals to test predictions of semiclassical gravity, for instance the possibility for different parts of the wave functions of a particle to interact with each other non linearly according classical gravity laws. However "together with the standard collapse postulate, fundamentally semi-classical gravity gives rise to superluminal signalling" [38] so the theoretical effort is toward suitable models of the wavefunction collapse that would avoid this superluminal signalling. From the point of view of the DG theory this effort is probably unnecessary because superluminal signalling would not lead to inconsistencies as long as there exists a unic privileged frame for any collapse and any instantaneous transmission exploiting it. We indeed have such a natural privileged frame since we have a global privileged time to reverse, so it is natural in our framework to postulate that this frame is the unic frame of instantaneity. Then the famous gedanken experiments claimed to unavoidably lead to CTCs (Closed Timelike Curves) do not work any more : the total round trip duration is usually found to be possibly negative only because these gedanken experiments exploit two or more different frames of instantaneous signaling. Let's be more specific : Does instantaneous hence faster than light signalling unavoidably lead to causality issues? : apparently not if there is a single unic privileged frame where all collapses are instantaneous. Then i (A) can send a message to my colleague (B) far away from me instantaneously and he can send it back to me also instantaneously still in this same privileged frame using QM collapses (whatever the relative motions and speeds of A and B and relative to the global privileged frame): the round trip duration is then zero in this frame so it is zero in any other frames according special relativity because the spatial coordinates of the two end events are the same: so there is no causality issue since there is actually no possible backward in time signalling with those instantaneous transmissions... in case there is some amount of time elapsed between B reception and re-emission, eventually A still receives it's message in it's future: no CTC here.

Anyway semi-classical gravity is known to entail many serious difficulties already in the context of GR among which bad non linearities, divergences and instabilities and the concern is as serious for DG [71]. A collapse of the quantum fields should also result in a discontinuous non-local behaviour of the energy-momentum tensor vacuum expectation value, hence of the corresponding gravitational field which is not acceptable because the Bianchi identities satisfied by the classical gravitational field (rigorously in GR, and in a very good approximation in DG) are local conservation equations. Eventually semiclassical gravity dashes our hopes for a simple solution of the old cosmological constant problem as the vacuum expectation value is then still an unavoidable contribution at the source of our Janus field equations.

The two last arguments clearly rather support a description of the interaction between gravity and quantum fields that would be completely protected if it is somehow isolated from those fields quantum behaviour itself. Quantum Field Theory should allow that as its Feynman diagrams actually clearly describe the succession of classical processes (the propagators) and quantum processes (the vertices). So we apparently would just need to modify the classical propagation of the wave packets to introduce the classical energy exchange between these fields and gravity at this level, in a way that is completely isolated from the quantum transitions taking place at the vertices. This is also the solution that is clearly favoured by what we learned from DG: that its differential equations could be only piecewise valid, i.e. in some delimited space-time domains while the discontinuous transitions (just as the vertices) are better understood as isolated processes taking place at the frontier of such domains. This way is ensured the solution to both the old cosmological constant problem and continuous local equations that always need to be satisfied by the gravitational field.

The same reasoning applies also to the quantum measurement if it is due to a physical, then most probably discontinuous and non local collapse but an alternative no collapse MWI interpretation as well would still be tenable as then the quantum field superpositions would extend to classical superpositions of the gravitational field only one term of which being accessible to an observer resulting in the illusion of a collapse while there would be actually no collapse at all.

14.7. *Closed timelike curves*

At last, the issue of CTCs (closed timelike curves) is worth a few more words: in the context of GR it is known that a necessary condition to avoid CTCs is to ban negative energies at the source of Einstein equation (Hawking theorems). It is therefore interesting that in the limit of infinite C, in which DG tends to GR, negative energy terms also tend to decouple at the source. It is therefore left as an open mathematical problem whether for finite C values, the modification of the geometrical part of DG equations vs Einstein equations is just what we need to still avoid CTCs even in presence of negative energy source terms.

15. Conclusion

New developments of DG not only solve the tension between the oldest version of the theory and gravitational waves observations but also provide a renewed and reinforced understanding of the Pioneer effect as well as the recent cosmological acceleration. An amazing unification of MOND and Dark Matter phenomenology seems also at hand. The most important theoretical result remains the avoidance of both the Big-Bang singularity and Black Hole horizon.

Appendices

A. Field equations derivation

To get our field equation we demand that the action variation δS should vanish under any infinitesimal variation $\delta g_{\mu\nu}$. But the variation of $g_{\mu\nu}$ implies a variation of $\tilde{g}_{\mu\nu}$ resulting in the following variation of the total action integrand which must vanish:

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\delta g_{\mu\nu} + \sqrt{\tilde{g}}(\tilde{G}^{\mu\nu} + 8\pi G\tilde{T}^{\mu\nu})\delta\tilde{g}_{\mu\nu} = 0 \tag{104}$$

The variations are related by

$$\delta\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\delta g^{\rho\sigma} = -\eta_{\mu\rho}\eta_{\nu\sigma}g^{\rho\tau}g^{\sigma\kappa}\delta g_{\tau\kappa} \tag{105}$$

since the Minkowski metric not being dynamical, does not vary. Replacing in 104, we get :

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\delta g_{\mu\nu} - \sqrt{\tilde{g}}(\tilde{G}^{\mu\nu} + 8\pi G\tilde{T}^{\mu\nu})\eta_{\mu\rho}\eta_{\nu\sigma}g^{\rho\tau}g^{\sigma\kappa}\delta g_{\tau\kappa} = 0 \tag{106}$$

Or, after a convenient renaming of the indices $(\mu,\nu) \leftrightarrow (\tau,\kappa)$ in the second term:

$$\left[\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu}) - \sqrt{\tilde{g}}(\tilde{G}^{\tau\kappa} + 8\pi G\tilde{T}^{\tau\kappa})\eta_{\tau\rho}\eta_{\kappa\sigma}g^{\rho\mu}g^{\sigma\nu}\right]\delta g_{\mu\nu} = 0 \tag{107}$$

The resulting single equation of motion can be reshaped in a more elegant form multiplying it by $\eta^{\delta\lambda}g_{\delta\mu}$, and using $\eta_{\kappa\sigma}g^{\sigma\nu} = \eta^{\sigma\nu}\tilde{g}_{\sigma\kappa}$ (inverse metrics).

$$\sqrt{g}(G^{\mu\nu} + 8\pi G T^{\mu\nu})\eta^{\delta\lambda}g_{\delta\mu} - \sqrt{\tilde{g}}(\tilde{G}^{\lambda\kappa} + 8\pi G\tilde{T}^{\lambda\kappa})\eta^{\sigma\nu}\tilde{g}_{\sigma\kappa} = 0 \tag{108}$$

Of course this field equation is invariant under the permutation of F and $\tilde{F}$ fields (both metrics and matter-radiation fields) just as the action we started from. We can also contract the term in square brackets in (107) with $g_{\mu\nu}$ to get:

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{109}$$

B. An alternative to exchange mechanisms

Here we investigate whether alternative ideas could help us save a DG cosmology without relying on matter-radiation exchange.

B.1. *The fundamentally homogeneous η-scalar field*

In this section we introduce the concept of emerging dynamics, an alternative idea to get viable cosmological solutions in the hypothetical case matter-radiation exchange could not occur. This is an interesting option as it comes with its own new testable predictions though it appears to be only sustainable for the early universe as it does not allow the propagation of spin 2 gravitational waves.

To save cosmology we may introduce an η-scalar Janus field built from $\eta_{\mu\nu}$ and a scalar Φ such that $g_{\mu\nu} = \Phi\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \frac{1}{\Phi}\eta_{\mu\nu}$. Then our fundamental cosmological single equation obtained by requiring the action to be extremized under any variation of $\Phi(t) = a^2(t)$ is just the same as (7):

$$a\ddot{a} - \tilde{a}\ddot{\tilde{a}} = \frac{4\pi G}{3}(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{110}$$

where $\tilde{a}(t) = \frac{1}{a(t)}$. With this scalar cosmology we avoid all the normal degrees of freedom of a metric and corresponding two Friedmann type equations (7)(8) yet our single equation as soon as our side scale factor dominates the dark side one, can reproduce with an excellent level of approximation all predictions of GR cosmology as we shall check in the next subsection.

Now this field is also understood to be "genetically homogeneous" i.e. the spatially independent $\Phi(t)$ at any scale insuring that there are no scalar waves associated to this field. The fundamental homogeneity of the scalar field is interesting in an approach to rehabilitating field discontinuities: in a sense the field would sometimes need to vary discontinuously just because it cannot vary continuously in space. Of course in a given domain it is possible to require this fundamental homogeneity in a fully covariant way : the conjugate metrics should share the killing vector of a maximally symmetric sub 3d-space insuring that for each metric there is a coordinate system in which it can be written the way we did and it just remains to assume that in this coordinate system for one of the metrics, we also have $\eta_{\mu\nu} = diag(-1, +1, +1, +1)$ for it to be the common conformal coordinate system for both metrics. The difference with the GR treatment of a cosmological metric is that in the context of GR homogeneity and isotropy are isometries of the background but do not prevent the total metric to fluctuate in any way it wants.

Let's assume in a first stage that the sources are also homogeneous. For $a >> \tilde{a}$ we can again neglect $\tilde{a}$ terms in our equation to get an equation that is also valid within GR. For the scale factor in standard (comoving) time coordinate, this equation just becomes:

$$\dot{H} + 2H^2 = \frac{4\pi G}{3}(\rho - 3p) \tag{111}$$

We now want to understand the implications if any of only having this second order equation for a. Unsurprisingly this equation can be deduced by using the equation of motion (10) and taking the first derivative of GR first Friedmann equation, a first order equation for a which for k=0 reads:

$$H^2(t) = \frac{8\pi G\rho}{3} \tag{112}$$

So any solution of the GR first Friedmann equation is also solution of our scalar-DG cosmological equation which insures that we are indeed able to reproduce all GR cosmology as far as we are only interested in the background evolution so far. However, the converse is not true and the general solution of our equation could involve additional integration constants and terms relative to a GR solution. Indeed, since we only have a second order equation, in principle the initial conditions i.e. $a(t)$ and $\dot{a}(t)$ could be chosen at will at some particular time yielding $H^2(t)$ very different from $\frac{8\pi G\rho}{3}$ at this time. In the dust and radiation dominated eras it is straightforward to check that the following equations are respectively integrals of our second order equation (111):

$$H^2(t) = \frac{8\pi G\rho}{3} + \frac{K}{a^4} \qquad \textit{dust dominated} \tag{113}$$

$$H^2(t) = \frac{8\pi G\rho}{3}(1 - K'e^{-\frac{a^2}{2}}) + \frac{K}{a^4} \qquad \textit{radiation dominated} \tag{114}$$

Since we know that the solutions for K=K'=0 correctly fit the data before the acceleration of the universe, presumably the K' term is only significant near the Big Bang. The K term is however interesting as it could mimic a radiation component. The resulting expected shift in matter-radiation equality redshift is however severely constrained by Planck so this term must be very small and even much smaller than the contribution from the three neutrino species which effect on the CMB power spectrum observable are well measured. Therefore even in this alternative scalar cosmology we would be led to the usual deduction that the baryonic matter is cosmologically not abundant enough to account for the measured Hubble rate: in other words we again have a missing mass issue at the cosmological scale.

B.2. *Emerging dynamics*

B.2.1. *The basic idea*

Let's remind the first order cosmological perturbation GR equations for k=0 with the metric written in the Newtonian Gauge:

$$d\tau^2 = a^2(t)((1+2\Psi)dt^2 - (1-2\Psi)d\sigma^2) \tag{115}$$

The equations are : (4.4.169;4.4170;4.4.171 from [42]):

$$\nabla^2\Psi - 3H(\dot{\Psi} + H\Psi) = 4\pi G a^2 \delta\rho \tag{116}$$

$$\dot{\Psi} + H\Psi = -4\pi G a^2(\bar{\rho} + \bar{p})\mathbf{v} \tag{117}$$

$$\ddot{\Psi} + 3H\dot{\Psi} + (2\dot{H} + H^2)\Psi = 4\pi G a^2 \delta p \tag{118}$$

Of course even if we could perturb our scalar there would be no hope to get more than one field equation so we can't reproduce the phenomenology of these GR perturbative equations. On the other hand working with the full Janus field (with all degrees of freedom dynamical) we know that if we can't rely on matter-radiation exchange processes we could get similar equations neglecting the conjugate side terms but then with background solutions forced to remain static as we realized earlier.

The concept of emerging dynamics will provide us with an elegant solution at least plainly valid and satisfactory as far as the physics of the very small fluctuations, tested through CMB studies, is concerned. The idea which is quite natural in a background dependent framework, is that some of the degrees of freedom which were frozen in the primordial metrics have only later gained their independence and have been released as dynamical dof either because the fluctuations became stronger than some threshold value or due to the scale factors differing from their initial value (at crossing point) by more than yet another fundamental threshold. We can actually already identify three metrics that could fit in such theoretical construction: the completely non dynamical background $\eta_{\mu\nu}$, the scalar-η field which is a dynamically very limited metric having a single dof which is moreover constrained to be homogeneous, and the fully dynamical metric which degrees of freedom are all completely released in such a way that it's equations of motion constrain it to be asymptotically static. The idea of emerging dynamics is that there could exist yet another dynamically intermediate metric between the last two defined as:

$$\Phi(t)(\eta_{\mu\nu} + \Delta g_{\mu\nu}(\mathbf{r}, t)) \tag{119}$$

where $\Delta g_{\mu\nu}(\mathbf{r}, t)$ stands for an in-homogeneous perturbation to the background but not yet a dynamical one in the sense that we shall still only require the action to be extremized by any variation of $\Phi(t)$ alone. We therefore again have a single scalar equation to be satisfied:

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{120}$$

Now suppose we can write our metric in the Newtonian form (115) as in GR theory of cosmological fluctuations. From the single equation (120)taken at zeroth order we then get the scale factor evolution equation (110) while at first order the equation we get is all we need to describe the evolution of $\Psi(r, t)$. As we could check, neglecting dark side terms, this is unsurprisingly the same equation as the one obtained from combining the first and third equation of (118) to get $4\pi Ga^2(t)(\delta\rho - 3\delta p)$ at the source. Because this equation is also valid within GR we obviously recover the same predictions for the evolution of $\Psi(r, t)$ as in the Standard Model in the linear regime of small fluctuations as far as the dark side terms can be neglected. However we need to keep in mind that this theory as well as GR in it's contracted version doesn't have enough equations to include modes other than the compressional ones described by $\Psi(r, t)$. So the anisotropic dofs such as the radiative modes (gravitational waves) and rotational modes are not accounted for by such theory which therefore can only remain sustainable in the extremely weak field domain as long as B modes are not detected in the CMB.

We however need to justify the Newtonian metric form in DG. In GR it follows from neglecting anisotropic stress. In our case, even in the absence of anisotropic stress an equation is lacking which is eq 4.2.135 from [42]. For us a similar constraint originates from the way the dofs are frozen for our primordial metric to be in the most symmetrical form in our privileged coordinate system. Indeed, beyond the metric of the pure $scalar - \eta$ field, the next most symmetrical one we could consider is the metric in the isotropic form:[o]

$$d\tau^2 = a^2(t)(B(r,t)dt^2 + A(r,t)d\sigma^2) \quad (121)$$

All spatial coordinates are treated on the same footing in the expression of this metric and our additional extra constraint is the result of extending to space-time such kind of requirement on the form of the metric in our privileged coordinate system. This is achieved with the space-time exchange symmetry: a new kind of symmetry that was introduced and explained at length in section 6.2 of [4]. It implies that in our privileged coordinate system $B(r,t) = -A^{-1}(r,t)$[p]. Then our metric in the weak field approximation with $A(r,t) = 1 - 2\Psi(r,t)$ is just the same as the Newtonian Gauge metric.

B.2.2. *Advantages, drawbacks and restricted validity*

Eventually to understand the CMB fluctuations spectrum an economic option is merely a single scalar equation (120) describing both the background and compressional fluctuations dynamics for an order two tensor field satisfying the space-time exchange symmetry in the privileged frame. This theory could be valid in the sufficiently weak gravity domain. The a priori advantage is that being based on a fundamentally homogeneous single scalar field, a discontinuous transition to acceleration (time reversal) would have been a bit easier to understand for such field. One drawback is that it really requires the two kinds of discontinuous processes: not only (A) but also a (B) process exchanging densities in a discrete way at the origin of time which we already mentioned in our section devoted to cosmology. This process was described as the two metrics exchanging in a discrete way their matter and radiation contents when the scale factors crossed each others. An additional drawback of such scenario is that densities from both sides are not equal at $t = 0^+$ or $t = 0^-$, whereas continuous matter-radiation exchange allows equal densities to meet at t=0 which is hopefully better to help understand the matter-antimatter asymmetry.

Anyway, the observation of gravitational waves today means that if our new alternative based on the homogeneous scalar field is correct, the space-time exchange symmetry must have been broken at some point and previously frozen dofs must have emerged as truly dynamical field elements. Then, to account for gravitational

[o]We are here interested in how the form assumed by the metric in our privileged coordinate system treats the various coordinates on the same footing rather than by isometries strictly speaking.

Moreover, if isotropy ensures the existence of a coordinate system in which the metric can be written in that simple isotropic form, there is also within DG the implicit understanding that this is as well the coordinate system in which the DG pivot metric satisfies $\eta_{\mu\nu} = (-1, 1, 1, 1)$.

[p]Of course as it is written here, this is not a generally covariant constraint but we don't care as any non covariant equation can be considered to be the formed assumed by a generally covariant one in some particular coordinate system. Here we don't need to specify the generally covariant version of the equation as we shall remain in our privileged coordinate system.

waves we either need the DG extension allowing matter-radiation exchange between our and the dark sector or the physics of static domains that we have detailed earlier which could actually only become valid at late times. The two possess the radiative, compressional and rotational modes of GR.

References

1. Weinberg, S. 1972, Quantum Field Theory, Vol 1., Cambridge University Press
2. Weinberg, S. 1972, Gravitation and cosmology
3. Henry-Couannier, F. 2004, Int.J.Mod.Phys, vol. A20, no. NN, pp. 2341-2346, 2004.
4. Henry-Couannier, F. 2013, Global Journal of Science Frontier Research A. Vol 13, Issue 3, pp 1-53.
5. Petit, J. P. 1995, Astr. And Sp. Sc. Vol. 226, pp 273.
6. Hossenfelder, S. 2008, Phys. Rev. D 78, 044015.
7. Henry-Couannier, F. 2012, Dark Gravity, Global Journal of Science Frontier Research F. Vol 12, Issue 13, pp 39-58.
8. Petit, J.P. 1977, C. R. Acad. Sci. Paris t.285 pp. 1217-1221.
9. Petit, J.P. and D'Agostini, G. 2014, Astr. And Sp. Sc. 354: 611-615.
10. Bondi H., Rev. of Mod. Phys., Vol 29, N3 (1957)12.
11. Bonnor W.B., General Relativity and Gravitation Vol.21, N11 (1989)
12. Anderson, J D., Laing, P A., Lau, E L., Liu, A S., Nieto M M., Turyshev, S G. , 2002 Phys.Rev.D65:082004,2002
13. Anderson, J D., Feldman, M R., 2015 Int.J.Mod.Phys, vol. D24 1550066
14. Henry-couannier, F., 2015, J. Condensed Matter Nucl. Sci. 18 (2016) 1–23
15. Henry-couannier, F., 2016, J. Condensed Matter Nucl. Sci. 21 (2016) 59–80
16. http://www.darksideofgravity.com/Consistency.pdf
17. Animation at www.darksideofgravity.com/altern.mp4
18. A. G. Riess, L. M. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, D. O. Jones, J. M. Silverman, R. Chornock, P. Challis, W. Yuan, P. J. Brown, and R. J. Foley, APJ 826, 56 (2016), arXiv:1604.01424.
19. V. Bonvin, F. Courbin, S. H. Suyu, P. J. Marshall, C. E. Rusu, D. Sluse, M. Tewes, K. C. Wong, T. Collett, C. D. Fassnacht, T. Treu, M. W. Auger, S. Hilbert, L. V. E. Koopmans, G. Meylan, N. Rumbaugh, A. Sonnenfeld, C. Spiniello, 2016, arXiv:1607.01790
20. S. McGaugh, F. Lelli, J. Schombert, Physical Review Letters, Volume 117, Issue 20, arxiv:1609.05917
21. M. A. Beasley, A. J. Romanowsky, V. Pota, I. M. Navarro, D. M. Delgado, F. Neyer, A. L. Deich The Astrophysical Journal Letters, Volume 819, Issue 2, article id. L20, 7 pp. (2016) arxiv:1602.04002
22. Daylan, T., Finkbeiner, D. P., Hooper, D., Linden, T., Portillo, S. K. N., Rodd, N. L., Slatyer, T. R. 2016, Physics of the Dark Universe, 12, 1 astro-ph:1402.6703
23. Abedi, Jahed, Dykaar, Hannah and Afshordi, Niayesh, "Echoes from the Abyss: Evidence for Planckscale structure at black hole horizons" (2016). arXiv:1612.00266
24. Indranil Banik, Hongsheng Zhaoa, Anisotropic Distribution of High Velocity Galaxies in the Local Group astro-ph:1701.06559
25. R. Genzel, N.M. Förster Schreiber, H. Übler, P. Lang, T. Naab, R. Bender, L.J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, A. Beifiori, S. Belli, G. Brammer, A. Burkert, C.M. Carollo, J. Chan, R. Davies, M. Fossati, A. Galametz, S. Genel, O. Gerhard, D. Lutz, J.T. Mendel, I. Momcheva, E.J. Nelson, A. Renzini, R. Saglia, A. Sternberg, S. Tacchella, K. Tadaki, D. Wilman, Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago Accepted for publication in Nature, astro-ph:1703.04310
26. S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68 (2003) 023509 arxiv:0301273
27. R. J. Gleiser, G. Dotti, Class.Quant.Grav. 23 (2006) 5063-5078 arXiv:0604021
28. M. Milgrom, Monthly Notices Roy. Astronomical Soc. 405 (2) (2010) 1129–1139 arXiv:1001.4444v3

29. D.B. Guenther, L.M. Krauss, P. Demarque, Astrophys. J., 498, 871–876 (1998).
30. L. Bernard and L. Blanchet, Phys. Rev.D91, 103536 (2015), arXiv:1410.7708
31. L. Blanchet and L. Heisenberg, Phys. Rev.D91, 103518 (2015), arXiv:1504.00870
32. L. Blanchet and L. Heisenberg, JCAP 1512, 026 (2015) arXiv:1505.05146
33. L. Blanchet and L. Heisenberg, arXiv:1701.07747
34. L. Bernard, L. Blanchet and L. Heisenberg Contribution to the proceedings of the 50th Rencontres de Moriond, "Gravitation: 100 years after GR" arXiv:1507.02802
35. C.M. Will Theory and experiment in gravitational physics
36. C d Rham Living Rev. Relativity, 17, (2014), 7
37. V.A. Rubakov Physics-Uspekhi, Volume 57, Number 2 (2014) arXiv:1401.4024
38. M. Albers, C. Kiefer, M. Reginatto Phys.Rev.D78:064051,2008 arXiv:0802.1978
39. M. Bahrami, A. Großardt, S. Donadi, A. Bassi, New Journal of Physics, Volume 16, November 2014 arXiv:1407.4370
40. E. Komatsu et al Astrophys.J.Suppl.192:18,2011 arXiv:1001.4538
41. I. Arcavi et al, Nature 551, 210–213 09 November 2017 arxiv:1711.02671
42. Daniel Baumann Cosmology course, chapter 4 and 5 http://cosmology.amsterdam/education/cosmology/
43. R. Bufaloa, M. Oksanena, A. Tureanu Eur. Phys. J. C 75 (2015) 477
44. T. Josset, A. Perez, D. Sudarsky Phys. Rev. Lett. 118, 021102 (2017)
45. P. O. Mazur, E. Mottola Gravitational Condensate Stars: An Alternative to Black Holes (2001) arXiv:gr-qc/0109035
46. B. D. Nikolić, M. R. Pantić A Possible Intuitive Derivation of the Kerr Metric in Orthogonal Form Based On Ellipsoidal Metric Ansatz (2013) arXiv:1210.5922
47. D. M. Jacobs, G. D. Starkman , B. W. Lynn, Macro Dark Matter (2015) arXiv:1410.2236
48. T. Damour, I.I. Kogan Effective Lagrangians and universality classes of non linear bigravity (2002) arXiv:hep-th/0206042
49. S. Pan, J. de Haro, A. Paliathanasis, R. J. Slagter (2016) arXiv:1601.03955
50. M. Carrera, D. Giulini (2006) On the influence of the global cosmological expansion on the local dynamics in the Solar System arXiv:gr-qc/0602098
51. T. Broadhurst, J. M. Diego, G. F. Smoot (2019) Twin LIGO/Virgo Detections of a Viable Gravitationally-Lensed Black Hole Merger arXiv:1901.03190
52. D. J. Eisenstein, H-j Seo, M. White (2006), On the Robustness of the Acoustic Scale in the Low-Redshift Clustering of Matter arxiv:astro-ph/0604361
53. F. Henry-couannier, C. Tao, A. Tilquin, Negative Energies and a Constantly Accelerating Flat Universe arXiv:gr-qc/0507065
54. J. Magana, M. H. Amante, M. A. Garcia-Aspeitia, V. Motta The Cardassian expansion revisited: constraints from updated Hubble parameter measurements and Type Ia Supernovae data arXiv:1706.09848
55. A.G.Riess, A 2.4 percent Determination of the Local Value of the Hubble Constant,arXiv:1604.01424
56. Planck collaboration, Planck 2018 results. VI. Cosmological parameters arXiv:1807.06209
57. S.Alam et all, The Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cos- mological Implications... arXiv:2007.08991
58. https://en.wikipedia.org/wiki/Scalar field dark matter In particular arXiv:astro-ph/0006024 and arXiv:astro-ph/9711102
59. D.N. Spergel et all, Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Are There Cosmic Microwave Background Anomalies? arXiv:astro-ph/1001.4758

60. Hao. Liu, Ti-Pei Li, arxiv:astro-ph.CO/0907.2731 arxiv:astro-ph.CO/1003.1073 arxiv:astro-ph.CO/1203.5720
61. Improving Cosmological Distance Measurements by Reconstruction of the Baryon Acoustic Peak D. J. Eisenstein , H-j Seo, E. Sirko , D. Spergel arXiv: astro-ph / 0604362
62. N. Padmanabhan, M. White, J.D. Cohn Reconstructing Baryon Oscillations: A Lagrangian Theory Perspective arXiv:0812.2905
63. N. Hamaus, M-C. Cousinou, A. Pisani, M. Aubert, S. Escoffier, J. Weller Multipole analysis of redshift-space distortions around cosmic voids arXiv:1705.05328
64. L. Perenon , J. Bel , R. Maartens , A. de la Cruz-Dombriz, Optimising growth of structure constraints on modified gravity arXiv: 1901.11063
65. L. Kazantzidis, L. Perivolaropoulos Evolution of the $f\sigma_8$ tension with the Planck15/ΛCDM determination ... arxiv:1803.01337
66. R. Brandenberger, P. Peter, Bouncing Cosmologies: Progress and Problems. arxiv:1603.05834
67. R. Brandenberger, Cosmology of the Very Early Universe arxiv:1003.1745
68. Towards a Nonsingular Bouncing Cosmology Y-F Cai, D A. Easson, R Brandenberger arxiv:1206.2382
69. V. A. Belinskii, I. M. Khalatnikov, and E. M. Lifshitz, Oscillatory approach to a singular point in the relativistic cosmology", Advances in Physics, vol. 19, no. 80, pp. 525–573, 1970
70. A.M. Levy, Thesis : Generating curvature perturbations in a contracting universe, 2017
71. J. Mattingly, Is quantum Gravity Necessary? 5th International Conference on the History and Foundations of General Relativity, 1999
72. Abbott, B. P.; et al. , 2017. "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral". Physical Review Letters. 119 (16): 161101. arXiv:1710.05832
73. Abbott, B. P.; et al. , 2017. The Astrophysical Journal Letters . 848 (2): L13. arXiv : 1710.05834
74. E. Troja,et al. 2020. submitted to MNRAS arxiv : 2006.01150