Unification of Electromagnetism, Weak Gravitation, and Classical and Quantum Mechanics by Diamond Equations

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Abstract: **It was found that electromagnetic and weak gravitational fields can be unified by diamond equations, which is one of extended Maxwell's equations. Classical mechanics is derived from the diamond equation for weak gravitational field by assuming that gravitational wave is not generated under the condition of stable motion. Quantum mechanics is derived from the same equation by assuming that the square of total energy consists of square of total momentum, square of the mass, and imaginary part of the energy creation-annihilation rate.**

I. Introduction

Maxwell's equations have the serious problem to prohibit carrier generation-recombination in semiconductors. We found that the carrier generation-recombination needs the charge creation-annihilation scalar field, [1-7] which is almost equivalent to Nakanishi-Lautrup field[8, 9] of quantum electrodynamics. The extended Maxwell's equations can be simply written by 4 × 4 complex differential operator matrix as a square root of d' Alembertian $\Box \equiv \partial_0{}^2 - \nabla^2$, which we call diamond operator $\Diamond$ by analogy of nabla ∇ as one of the square roots of Laplacian Δ.[10] It is found that the extended Maxwell's equations written by the diamond operator, which should be called diamond equations, also describe weak gravitational field. Furthermore, classical and quantum mechanics can be derived from the diamond equations for weak gravitational field by assuming that the gravitational wave is not generated under the condition of weak gravitational field and stable motion.

II. Extended Maxwell's equations

Maxwell's equations are given by

$$\mathbf{J} = \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t}, \tag{1}$$

$$\rho = \nabla \cdot \mathbf{D}, \tag{2}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{4}$$

where **E**, **B**, **D**, and **H** are electric, magnetic, displacement, and magnetizing fields, respectively, and **J** and ρ are current and charge densities, respectively.
(1) and (2) give

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0. \tag{5}$$

On the other hand, electron current density $\mathbf{J}_n$, hole current density $\mathbf{J}_p$, electron charge density ρ_n, and hole charge density ρ_p in semiconductors satisfy the following equation[11]

$$-\nabla \cdot \mathbf{J}_n - \frac{\partial \rho_n}{\partial t} = \nabla \cdot \mathbf{J}_p + \frac{\partial \rho_p}{\partial t} = GR, \tag{6}$$

where GR is carrier generation-recombination rate. Since electromagnetic fields induced by electrons and holes should individually satisfy (1) and (2), (5) and (6) contradict each other in the case of $GR \neq 0$. In other words, Maxwell's equations prohibit carrier generation-recombination in semiconductors. In order to solve the problem, we introduced charge creation-annihilation scalar field N, which is almost equivalent to Nakanishi-Lautrup field in quantum electrodynamics. Then, (1) and (2) are rewritten as

$$\mathbf{J} = \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} + \frac{1}{\mu}\nabla N, \tag{7}$$

$$\rho = \nabla \cdot \mathbf{D} - \varepsilon \frac{\partial N}{\partial t}, \tag{8}$$

where ε and μ are permittivity and permeability in the material, respectively. Then, carrier generation-recombination rate, in other words, charge creation-annihilation rate GR is given by

$$GR = \nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = -\frac{1}{\mu}\Box N, \tag{9}$$

where $\Box$ denotes d' Alembertian defined by

$$\Box \equiv \frac{1}{{c_m}^2}\frac{\partial^2}{\partial t^2} - \nabla^2 . \qquad (10)$$

Here, c_m is the speed of light in the material given by

$$c_m = \frac{1}{\sqrt{\varepsilon\mu}} . \qquad (11)$$

Then ∂_0 is defined as

$$\partial_0 \equiv \frac{1}{c_m}\frac{\partial}{\partial t} . \qquad (12)$$

III. Diamond operator and equations

We define the diamond operator $\diamondsuit$ as

$$\diamondsuit \equiv \begin{pmatrix} -\partial_0 & -i\partial_3 & i\partial_2 & -i\partial_1 \\ i\partial_3 & -\partial_0 & -i\partial_1 & -i\partial_2 \\ -i\partial_2 & i\partial_1 & -\partial_0 & -i\partial_3 \\ i\partial_1 & i\partial_2 & i\partial_3 & -\partial_0 \end{pmatrix} * , \qquad (13)$$

where * denotes the complex conjugate operator which satisfies

$$*A = A^* * , \qquad (14)$$

and

$$** = 1 , \qquad (15)$$

for A as a complex scalar, vector, or matrix, and A^* as the complex conjugate of A. The diamond operator satisfies

$$\diamondsuit^2 = \Box . \qquad (16)$$

For electromagnetic and gravitational forces, the four current C and the four field F satisfy

$$gC = \diamondsuit F * . \qquad (17)$$

In (17), g is a coupling constant and

$$C \equiv \begin{pmatrix} \mathbf{C} \\ iC_0 \end{pmatrix} , \qquad (18)$$

$$F \equiv \begin{pmatrix} \mathbf{D} + i\mathbf{R} \\ -iS \end{pmatrix} , \qquad (19)$$

where **D**, **R**, and S are divergent, rotational, and scalar fields, respectively. (17) – (19) give

$$g\mathbf{C} = \nabla \times \mathbf{R} - \partial_0 \mathbf{D} + \nabla S , \qquad (20)$$

$$gC_0 = \nabla \cdot \mathbf{D} - \partial_0 S , \qquad (21)$$

$$\nabla \times \mathbf{D} + \partial_0 \mathbf{R} = 0 , \qquad (22)$$

$$\nabla \cdot \mathbf{R} = 0 . \qquad (23)$$

The four field $\overline{F}$ with gauge parameter ξ and four potential A satisfy

$$\overline{F} = \diamondsuit A * , \qquad (24)$$

where

$$\overline{F} \equiv \begin{pmatrix} \mathbf{D} + i\mathbf{R} \\ -i\xi S \end{pmatrix} , \qquad (25)$$

$$A \equiv \begin{pmatrix} \mathbf{A} \\ iA_0 \end{pmatrix} . \qquad (26)$$

(24) – (26) give

$$\mathbf{D} = -\partial_0 \mathbf{A} - \nabla A_0 , \qquad (27)$$

$$\mathbf{R} = \nabla \times \mathbf{A} , \qquad (28)$$

$$\xi S = -\nabla \cdot \mathbf{A} - \partial_0 A_0 . \qquad (29)$$

We call (17) and (24) diamond equations.

The extended Maxwell's equations are obtained by substituting four current J for C and permeability μ for g as

$$gC = \mu J = \mu \begin{pmatrix} \mathbf{J} \\ ic\rho \end{pmatrix} . \qquad (30)$$

Then four field F is substituted by electric and magnetic fields **E** and **B**, and charge creation-annihilation field N as

$$F = \begin{pmatrix} \mathbf{E}/c + i\mathbf{B} \\ -iN \end{pmatrix} . \qquad (31)$$

Therefore, the extended Maxwell's equations are given by

$$\mathbf{J} = \frac{1}{\mu}\nabla \times \mathbf{B} - \varepsilon\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\mu}\nabla N , \qquad (32)$$

$$\rho = \varepsilon\nabla \cdot \mathbf{E} - \varepsilon\frac{\partial N}{\partial t} , \qquad (33)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 , \qquad (34)$$

$$\nabla \cdot \mathbf{B} = 0 . \qquad (35)$$

Since displacement field **D** and magnetizing field **H** are equal to ε **E** and **B** / μ, respectively, in most cases, (32) – (35) are equivalent to (7), (8), (3), and (4).

IV. Linear gravitational field

Einstein's gravitational equation is given by

$$G_{\mu\nu} = \kappa T_{\mu\nu} \qquad (36)$$

where $G_{\mu\nu}$ is Einstein tensor and κ is Einstein's gravitational constant. $T_{\mu\nu}$ is momentum density tensor written by

$$T_{\mu\nu} = -\rho v_\mu v_\nu , \qquad (37)$$

where v_μ and v_ν are μ and ν component of the velocity. When the momentum density is enough small, metric tensor $g_{\mu\nu}$ is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} , \qquad (38)$$

where $\eta_{\mu\nu}$ and $h_{\mu\nu}$ are tensors which satisfy

$$\eta_{\mu\nu} \equiv \begin{cases} 1 & (\mu=\nu=1,2,3) \\ 0 & (\mu \neq \nu) \\ -1 & (\mu=\nu=0) \end{cases}, \quad (39)$$

$$\left|h_{\mu\nu}\right| \ll 1 . \quad (40)$$

Here we define $\bar{h}_{\mu\nu}$ as

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^{\lambda}{}_{\lambda} . \quad (41)$$

In Lorentz gauge condition of $\bar{h}^{\mu\alpha}{}_{,\alpha}=0$, we obtain

$$-\Box\bar{h}_{\mu\nu} = 2\kappa T_{\mu\nu} . \quad (42)$$

The above equation is regarded as the wave equation for linear gravitational field.[12] In order to obtain Lorentz vector, we assume small volume Ω. Then the gravitational vector potential A_g and gravitational current C_g are given by

$$A_{g\mu} \equiv \frac{1}{2\kappa c}\bar{h}_{\mu 0}\Omega , \quad (43)$$

$$C_{g\mu} \equiv -\frac{1}{c}T_{\mu 0}\Omega = \rho v_{\mu}\Omega . \quad (44)$$

Therefore

$$C_{g\mu} = \Box A_{g\mu} . \quad (45)$$

Then the gravitational fields $F_g = (\mathbf{D}_g + i\mathbf{R}_g, -iS_g)^t$ and $\bar{F}_g = (\mathbf{D}_g + i\mathbf{R}_g, -i\xi S_g)^t$ satisfy

$$C_g = \Diamond F_g * , \quad (46)$$

$$\bar{F}_g = \Diamond A_g * , \quad (47)$$

where $\mathbf{D}_g$, $\mathbf{R}_g$, and S_g are the divergent, rotational, and scalar fields of the linear gravitational field. Since the four current vector C_g is equivalent to the four momentum vector $P \equiv (\mathbf{P}, iP_0)^t$, where $\mathbf{P}$ and cP_0 are 3D momentum and energy,

$$\mathbf{P} = \nabla\times\mathbf{R}_g - \partial_0\mathbf{D}_g + \nabla S_g , \quad (48)$$

$$P_0 = \nabla\cdot\mathbf{D}_g - \partial_0 S_g . \quad (49)$$

V. Classical and quantum mechanics

If we assume existence of the four potential $V \equiv (\mathbf{V}, i\psi/c)^t$, the total four momentum π is given by

$$\pi \equiv \begin{pmatrix} \boldsymbol{\pi} \\ i\dfrac{E}{c} \end{pmatrix} = \begin{pmatrix} \mathbf{P}+\mathbf{V} \\ i\left(P_0 + \dfrac{\psi}{c}\right) \end{pmatrix} . \quad (50)$$

3D total momentum $\boldsymbol{\pi}$ and total energy E satisfy

$$\boldsymbol{\pi} = \nabla\times\mathbf{R}_{total} - \partial_0\mathbf{D}_{total} + \nabla S_{total} , \quad (51)$$

$$\frac{E}{c} = \nabla\cdot\mathbf{D}_{total} - \partial_0 S_{total} , \quad (52)$$

where $\mathbf{D}_{total}$, $\mathbf{R}_{total}$, and S_{total} are the total divergent, rotational, and scalar fields considering the four potential, respectively. If the four potential is appropriate and the motion is stable, the wave sources of the total divergent and rotational fields should be zero as

$$\Box\mathbf{D}_{total} = \Box\mathbf{R}_{total} = 0 . \quad (53)$$

Therefore

$$c\partial_0\boldsymbol{\pi} + \nabla E = 0 , \quad (54)$$

$$\nabla\times\boldsymbol{\pi} = 0 . \quad (55)$$

By using special relativity, we obtain

$$\left(E-\psi\right)^2 = \left(\boldsymbol{\pi}-\mathbf{V}\right)^2 c^2 + m^2c^4 . \quad (56)$$

In the case of classical mechanics of $|P| \ll mc$,

$$E \approx \frac{\left(\boldsymbol{\pi}-\mathbf{V}\right)^2}{2m} + \psi + mc^2 . \quad (57)$$

By defining $\mathbf{v}$ as a 3D velocity vector, ∇E is calculated as

$$\begin{aligned} \left(\nabla E\right)_i &= \sum_{j=1}^{3}\frac{P_j}{m}\left(\frac{\partial\pi_j}{\partial x_i} - \frac{\partial V_j}{\partial x_i}\right) + \frac{\partial\psi}{\partial x_i} \\ &= \sum_{j=1}^{3} v_j\left(\frac{\partial\pi_j}{\partial x_i} - \frac{\partial V_j}{\partial x_i}\right) + \frac{\partial\psi}{\partial x_i} . \end{aligned} \quad (58)$$

Therefore, (54) and (58) give

$$\begin{aligned} \frac{dP_i}{dt} &= \left(\frac{d\pi_i}{dt} - \frac{dV_i}{dt}\right) \\ &= \sum_{j=1}^{3}\frac{dx_j}{dt}\frac{\partial\pi_i}{\partial x_j} + \frac{\partial\pi_i}{\partial t} - \left(\sum_{j=1}^{3}\frac{dx_j}{dt}\frac{\partial V_i}{\partial x_j} + \frac{\partial V_i}{\partial t}\right) \\ &= \sum_{j=1}^{3} v_j\frac{\partial\pi_i}{\partial x_j} - \sum_{j=1}^{3} v_j\frac{\partial\pi_j}{\partial x_i} + \sum_{j=1}^{3} v_j\frac{\partial V_j}{\partial x_i} - \sum_{j=1}^{3} v_j\frac{\partial V_i}{\partial x_j} - \frac{\partial V_i}{\partial t} - \frac{\partial\psi}{\partial x_i} \\ &= -\left\{\mathbf{v}\times\left(\nabla\times\boldsymbol{\pi}\right)\right\}_i + \left\{\mathbf{v}\times\left(\nabla\times\mathbf{V}\right)\right\}_i - \frac{\partial V_i}{\partial t} - \frac{\partial\psi}{\partial x_i} . \end{aligned} \quad (59)$$

Since the first term of (59) is zero by using (55), we obtain

$$\frac{d\mathbf{P}}{dt} = \mathbf{v}\times\left(\nabla\times\mathbf{V}\right) - \frac{\partial\mathbf{V}}{\partial t} - \nabla\psi . \quad (60)$$

The above equation shows Newton's second law of motion. In electromagnetic field case, the right side of (60) is equivalent to the sum of Lorentz and Coulomb forces.

By using (54), (55), and a appropriate scalar field S_c, $\boldsymbol{\pi}$ and E are written as

$$\boldsymbol{\pi} = \nabla S_c , \quad (61)$$

$$E = -\frac{\partial S_c}{\partial t} . \quad (62)$$

(61) and (62) are equivalent to Hamilton-Jacobi equations, where S_c is Hamilton's principle function. Here we call S_c energy

creation-annihilation field, because energy creation-annihilation rate σ is defined by

$$\sigma \equiv c^2\nabla\cdot\boldsymbol{\pi}+\frac{\partial E}{\partial t}=-c^2\Box S_c\,. \quad (63)$$

Next we consider about quantum mechanics. When we define the wave function ϕ as

$$\phi \equiv \exp\left(\frac{iS_c}{\hbar}\right), \quad (64)$$

we obtain

$$\Box\phi=\left(\frac{-E^2+\pi^2c^2}{c^2\hbar^2}+\frac{i}{\hbar}\Box S_c\right)\phi\,. \quad (65)$$

In the case of $\Box S_c = 0$, we obtain Klein-Gordon equation of

$$\Box\phi+\frac{m^2c^2}{\hbar^2}\phi=0\,. \quad (66)$$

If we assume existence of the potential U, the above equation is rewritten as[13]

$$\Box\phi+\frac{m^2c^2}{\hbar^2}\phi=-\frac{U}{\hbar^2c^2}\phi\,. \quad (67)$$

Since $\Box S_c \neq 0$ in the above case, we obtain

$$E^2=\pi^2c^2+m^2c^4+U-i\hbar\sigma\,. \quad (68)$$

The above equation suggests the principle of quantum mechanics, that it is equivalent to classical mechanics when the absolute value of $\hbar\sigma$ is much smaller than π^2c^2, otherwise the imaginary part of energy creation-annihilation field creates or annihilates quantized interactive energy depending on the potential U.

If π^2c^2 and the absolute value of $\hbar\sigma$ are much smaller than m^2c^4, we obtain

$$E=\sqrt{\pi^2c^2+m^2c^4+U-i\hbar\sigma} \approx mc^2+\frac{\pi^2+i\hbar\Box S_c}{2m}+\frac{U}{2mc^2}\,. \quad (69)$$

When we assume E and $\partial_0 S_c$ do not depend on time, and redefine the total energy excluding the rest energy $\hat{E} \equiv E - mc^2$ and the potential $V \equiv U/2mc^2$, we obtain

$$\hat{E}=\frac{\pi^2-i\hbar\nabla^2S_c}{2m}+V\,. \quad (70)$$

Since $\nabla^2\phi$ is given by

$$\nabla^2\phi=\left(-\frac{\pi^2}{\hbar^2}+\frac{i}{\hbar}\nabla^2S_c\right)\phi\,, \quad (71)$$

the time independent Schrödinger equation is obtained as

$$\hat{E}\phi=-\frac{\hbar^2}{2m}\nabla^2\phi+V\phi\,. \quad (72)$$

VI. Conclusion

We found that electromagnetic and weak gravitational fields can be unified by the diamond equations, which include a 4 × 4 complex differential operator matrix as a square root of d'Alembertian. The diamond equations for weak gravitational field derive classical and quantum mechanics, including Hamilton-Jacobi, Klein-Gordon, and time independent Schrödinger equations. It was found that imaginary part of the energy creation-annihilation field creates or annihilates quantized interactive energy depending on the potential.

References

1) H. Mutoh, "Physical Models and Algorithms for Device Simulation for High-Speed Image Sensors," *J. Institute of Image Information and Television Engineers*, **67**, no.3, pp. J89-J94, 2013. https://www.jstage.jst.go.jp/article/itej/67/3/67_J89/_pdf
2) H. Mutoh, "Device simulation with electromagnetic field propagation models for high-speed image sensors and FDA noise analysis," Proc. Int. Image Sensors Workshop, **7.01**, pp. 189-192, 2013.
3) H. Mutoh, "Modelinig and algorithms of device simulation for ultra-high speed devices," Proc. Int. Conf. Simulation Semiconductor Processes and Devices (SISPAD), pp. 225-228, 2014.
4) H. Mutoh, "Confinement of Charge Creation and Annihilation Centers by Nakanishi-Lautrup Field," viXra, (hep) 1403.0118, March 2014. http://vixra.org/pdf/1403.0118v1.pdf
5) H. Mutoh, "Extension of Maxwell's equations for charge creation and annihilation," viXra, (class) 1407.0051, July 2014. http://vixra.org/pdf/1407.0051v1.pdf
6) H. Mutoh, "Device simulations for ultrahigh-speed image sensors," *IEEE Trans. Electron Devices*, **63**, no. 1, pp. 49-56, 2016.
7) H. Mutoh, "Extension of Maxwell's Equations for Charge Creation-annihilation and Its Applications," Proceeding of 2018 Progress In Electromagnetics Research Symposium (PIERS), pp. 2537-2545, 2018.
8) N. Nakanishi, "Covariant quantization of the electromagnetic field in the Landau gauge," *Progr. Theor. Phys*., **35**, no. 6, pp. 1111-1116, 1966.
9) I. J. R. Aichison, *An informal introduction to gauge field theories*, Cambridge Univ. Press, 1982.
10) H. Mutoh: "Diamond operator as a square root of d' Alambertian for Bosons," viXra, (math-ph) 1708_0031, May 2018. http://vixra.org/pdf/1708.0031v4.pdf
11) S. M. Sze: *Physics of Semiconductor Devices*, Wiley, New York, 2nd ed., pp. 50-51, 1981.
12) C. W. Misner and K. S. Thorne and J. A. Wheeler, *Gravitation*, W. H. Freeman and Company, 1973.
13) I. J. R. Aichison and A. J. G. Hey, *Gauge theories in particle physics*, Adam Hilger Ltd., 1982.