Algunas relaciones del tipo arcotangente

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Resumen

En esta nota se muestran algunas relaciones del tipo arcotangente.

Introducción

El número π se define por: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.141592...$, en esta nota mostramos algunas fórmulas del tipo arcotangente para π .

Fórmulas

1. Sean $a,b>0$ tales que

$$a^3+1=a+3ab^2 \quad , \quad 1+3a^2b=b+b^3 \tag{1}$$

Entonces se tiene

$$\pi = 4\tan^{-1}\left(\frac{b}{a}\right) - 4\tan^{-1}\left(\frac{b}{1-a}\right) + 4\tan^{-1}\left(\frac{b}{1+a}\right) \tag{2}$$

$$\pi = -4\tan^{-1}\left(\frac{a}{b}\right) + 4\tan^{-1}\left(\frac{1-a}{b}\right) + 4\tan^{-1}\left(\frac{b}{1+a}\right) \tag{3}$$

$$\pi = 4\tan^{-1}\left(\frac{a}{b}\right) + 4\tan^{-1}\left(\frac{2ab}{1-a^2+b^2}\right) \tag{4}$$

$$\pi = 4\tan^{-1}\left(\frac{b}{1+a}\right) + 4\tan^{-1}\left(\frac{b-2ab}{a-a^2+b^2}\right) \tag{5}$$

$$\pi = 4\tan^{-1}\left(\frac{1-a}{b}\right) + 4\tan^{-1}\left(\frac{b^2-a-a^2}{b+2ab}\right) \tag{6}$$

$$\pi = \frac{4}{3}\tan^{-1}\left(\frac{b}{1-a}\right) + \frac{4}{3}\tan^{-1}\left(\frac{b+2ab}{b^2-a-a^2}\right) \tag{7}$$

2. Sean $a, b > 0$ tales que

$$a^3 + 1 = a + 3ab^2 \quad , \quad 1 + b + b^3 = 3a^2 b \tag{8}$$

Entonces se tiene

$$\pi = 4\tan^{-1}\left(\frac{b}{a}\right) - 4\tan^{-1}\left(\frac{a-1}{b}\right) + 4\tan^{-1}\left(\frac{b}{1+a}\right) \tag{9}$$

$$\pi = \frac{4}{3}\tan^{-1}\left(\frac{b}{a}\right) + \frac{4}{3}\tan^{-1}\left(\frac{b}{a-1}\right) + \frac{4}{3}\tan^{-1}\left(\frac{b}{1+a}\right) \tag{10}$$

$$\pi = -4\tan^{-1}\left(\frac{b}{a}\right) + 4\tan^{-1}\left(\frac{2ab}{1-a^2+b^2}\right) \tag{11}$$

$$\pi = 4\tan^{-1}\left(\frac{b}{a}\right) + 4\tan^{-1}\left(\frac{1-a^2+b^2}{2ab}\right) \tag{12}$$

3. Sean $a, b > 0$ tales que

$$a^3 = 1 + a + 3ab^2 \quad , \quad 1 + b + b^3 = 3a^2 b \tag{13}$$

Entonces se tiene

$$\pi = 4\tan^{-1}\left(\frac{b}{a}\right) + 4\tan^{-1}\left(\frac{b}{a-1}\right) + 4\tan^{-1}\left(\frac{b}{1+a}\right) \tag{14}$$

$$\pi = 4\tan^{-1}\left(\frac{b}{a}\right) + 4\tan^{-1}\left(\frac{2ab}{a^2-1-b^2}\right) \tag{15}$$

$$\pi = 4\tan^{-1}\left(\frac{b}{1+a}\right) + 4\tan^{-1}\left(\frac{2ab-b}{a^2-a-b^2}\right) \tag{16}$$

$$\pi = 4\tan^{-1}\left(\frac{a^2 - a - b^2}{2ab - b}\right) - 4\tan^{-1}\left(\frac{b}{1+a}\right) \tag{17}$$

$$\pi = 4\tan^{-1}\left(\frac{b}{a-1}\right) + 4\tan^{-1}\left(\frac{b+2ab}{a+a^2-b^2}\right) \tag{18}$$

4. Sean $a, b, c, d, e, f > 0$ tales que

$$a^3 + 1 = a + 3ab^2 \quad , \quad 1 + 3a^2 b = b + b^3 \tag{19}$$

$$c^3 + 1 = c + 3cd^2 \quad , \quad 1 + d + d^3 = 3c^2 d \tag{20}$$

$$e^3 = 1 + e + 3ef^2 \quad , \quad 1 + f + f^3 = 3e^2 f \tag{21}$$

Entonces se tiene

$$\pi = 4\tan^{-1}\left(\frac{a}{b}\right) + 4\tan^{-1}\left(\frac{d}{c}\right) - 4\tan^{-1}\left(\frac{f}{e}\right) \tag{22}$$

$$\pi = \frac{4}{3}\tan^{-1}\left(\frac{b}{1-a}\right) + \frac{4}{3}\tan^{-1}\left(\frac{d}{c-1}\right) - \frac{4}{3}\tan^{-1}\left(\frac{f}{1+e}\right) \tag{23}$$

$$\pi = 4\tan^{-1}\left(\frac{1-a}{b}\right) + 4\tan^{-1}\left(\frac{c-1}{d}\right) + 4\tan^{-1}\left(\frac{f}{1+e}\right) \tag{24}$$

Referencias

1. Olver, F.W.J., Lozier, D.W., Boisvert, R.F., Clark, C.W. : NIST Handbook of Mathematical Functions. Cambridge University Press, 2010.