A closed 3-form in spinorial geometry

A.Balan

April 1, 2019

Abstract

We define here a closed 3-form for any spinorial manifold.

1 The spinorial manifolds

For a riemannian manifold (M, g), we can define the spinorial fiber bundle if the second class of Stiefel-Whitney of the manifold M vanishes. We have a Clifford multiplication over this fiber bundle, it means that we can multiply a vector and a spinor to get a new spinor. The Levi-Civita connection can be defined over the spinor bundle, with compatibility with the usual one. So, the curvature is a 2-form with values in the endomorphisms

$$R(X, Y) = \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{(X,Y)}$$

2 The 3-form of the manifold

We can define a 3-form v with the curvature and the Clifford multiplication:

$$v(X, Y, Z)(\psi) = X.R(Y, Z)\psi + Y.R(Z, X)\psi + Z.R(X, Y)\psi$$

It is a 3-form because of antisymmetry of the curvature:

$$R(X, Y) = -R(Y, X)$$

This 3-form v takes its values in the endomorphisms of the spinor fiber bundle and is closed.

3 Characteristic classes

By mean of v, we can define characteristic classes:

$$a_k = tr(v^k)$$

They are topological invariants of the manifold M.

References

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