Nullspace and Mitigation in seismic imaging and seismic inversion

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Abstract. Seismic inversion is the inverse problem in seismic data processing. Seismic inversion is normally used to recover the medium properties or reflectivity series given the input seismic traces. Nullspace is common in linearized inversion where many vectors could be mapped into the one vector. It is an indication that many models could fit the same data, regardless of the inversion method. Inversion could give odd geometric shapes that are not physical or geological. We will discuss mitigation of nullspace to yield results that are meaningful for interpretation of geologic layers for oil and gas exploration.

Introduction. Inversion could be classified as direct and indirect inversion. There are many concerns about direct and indirect inversion.

Direct inversion requires that the input data to be clean to satisfy the algorithm. This is called "preconditioning" of input data. The goal is to remove "noise" before direct inversion. In general, preconditioning is not well-defined and is a vague process dependent on the user. There is no guarantee that preconditioning will satisfy the direct inversion algorithm. Preconditioning is somewhat ad hoc.

Indirect inversion is dependent on the search algorithm to recover the medium. The most common approach is some form of gradient method which fits Lp optimization. Nonlinearity is the common objection to gradient method. There are other search methods like MCMC (Markov chain Monte Carlo), simulated annealing, genetic algorithm, machine learning, etc. With modern computing, there is proliferation of search algorithms which might not yield physically meaningful model or image.

Direct and indirect inversions have their own drawbacks and artifacts for real data. There is no guarantee that any search method yields physically/geologically meaningful result. We will focus on null space of the inversion and mitigation of the problem.

Focus

We will keep our discussion to inversion of anisotropic compressional velocity and no other parameters like density, shear velocity, anisotropy. To simplify our discussion further, we will use travel time inversion (tomography) of primaries only.

We will derive the approximate null space to understand the non-uniqueness of tomography. Then scanning of parameters is used to illustrate the importance of interpretation to pick the correct inversion parameters.

Nullspace

We have previously used linear moveout inversion (LMI) for tomography of first arrivals (refractions/diving waves) to perform tomography (see Lau/Gonzalez/Osypov). In this paper, we will use hyperbolic moveout inversion (HMI) also called stacking velocity inversion to illustrate null space which creates artifacts. We will also use Toldi's approach to inversion which is closer to analytic approach and easier to demonstrate. Artifacts are difficult to control, and the inversion result might not "look geologic" to the interpreter.

Parseval's Theorem:

(Roughly speaking) RMS of amplitude in time domain is equal to RMS of amplitude in frequency domain where RMS is root mean square.

The interpretation of the theorem is that if there is misfit error in time, there is misfit error in all frequencies (and wavenumbers). So the error in frequencies/wavenumbers appears to be sinusoidal geometrically in time/space. The artifacts from null space (small error in time) often have the geometric appearance of superposition of sinusoids. Typical geologic units do not look sinusoidal and the inversion results look artificial. We cannot escape Parseval's theorem but we will use empirical methods to mitigate the problem.

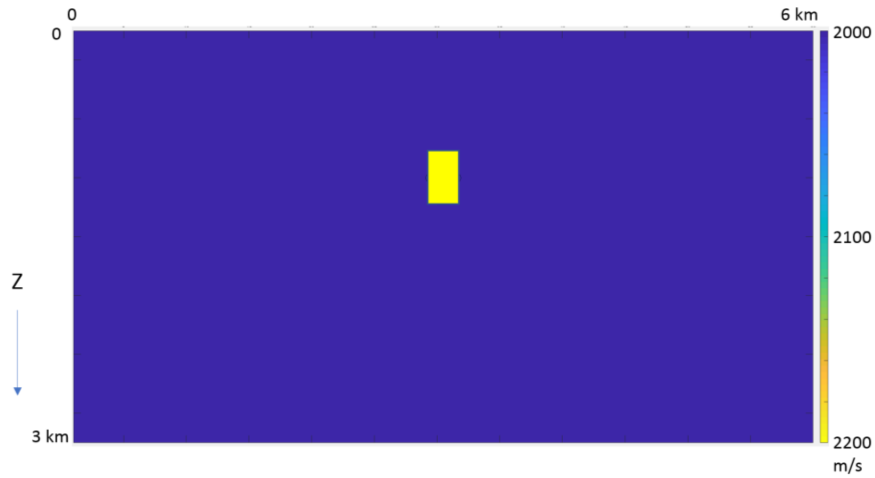
Mitigation

Mitigation is a collection of tools. We will illustrate various methods to mitigate nullspace problem. Nullspace problem exhibits itself as "artifacts" which do not look geologic.

Method 1: Constrained inversion

Our constrained inversion example is highly underdetermined. We do this on purpose to emphasize the difficulty of controlling numerical artifacts during inversion. In practice, seismic data is always incomplete and do not provide enough observations to completely constrain a solution. This is true regardless of what inversion method is used, from simple Dix's rms-to-interval velocity conversion, to more sophisticated anisotropic full waveform inversion. All algorithms face similar restrictions. Our simple example also illustrates the power of adding geometric constraints to the solution.

Interval Velocity. Model.



Anomaly: 200 m x 200 m

Fig. 1. Interval velocity model. Background has 2,000 m/s constant velocity. Anomaly is 10% higher velocity.

Figure 1 shows a simple Earth model. The model has a constant velocity background of 2,000 m/s and a velocity anomaly with 10% higher velocity. The model has a single flat reflector at 3,000 m depth. Seismic CMP gathers are simulated with 3,000 m offset at 12.5 m interval. Figure 2 shows three sample CMPS.

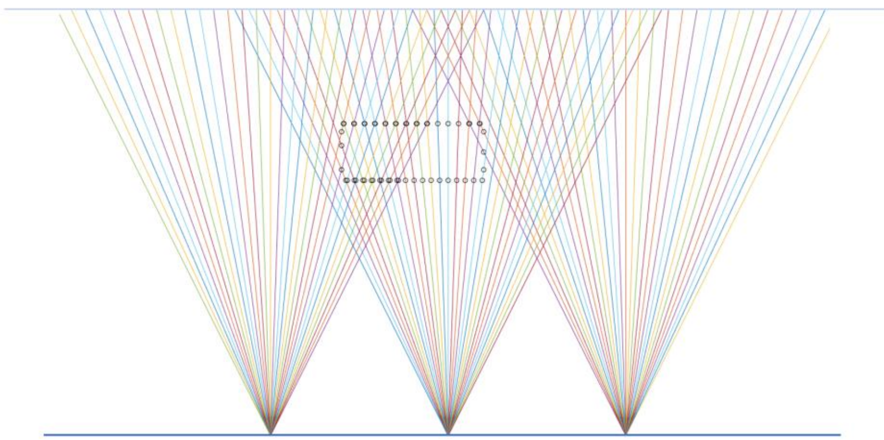


Fig. 2. Sample shots. Cable length is 3,000 m. Single reflector at 3,000 m. Figure displays the rays intersecting the boundaries of the interval velocity rectangular anomaly.

Stacking velocities are picked continuously along the profile. These stacking velocities, shown in Figure 3, are input to stacking velocity inversion, as described by Toldi (1989). The presence of a lateral velocity anomaly, interacting with the shooting geometry, produces a well-known response. These observations cannot be used in Dix's inversion to convert RMS to interval velocities. The lateral velocity variation invalidates the laterally-homogeneous velocity assumption of Dix's inversion.

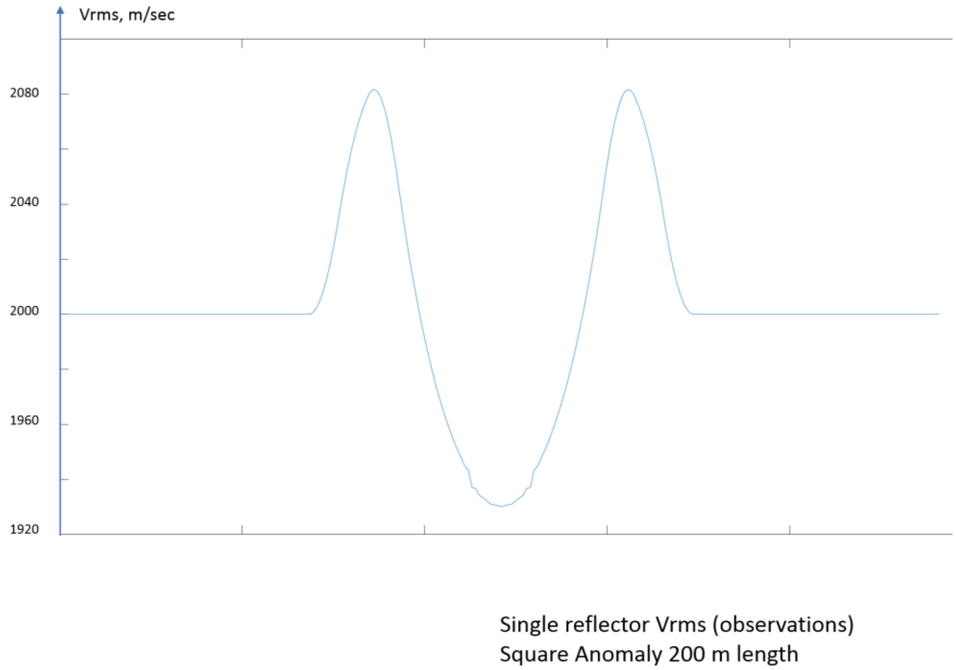


Fig. 3. Measured stacking velocity from hyperbolic velocity analysis. Input to stacking velocity inversion.

Because of its simplicity, stacking velocity inversion is particularly useful to illustrate problems with direct inversion. In this inversion an operator is constructed linking stacking velocities (data space) to interval velocities (model space) in the depth domain, as shown in Figure 4.

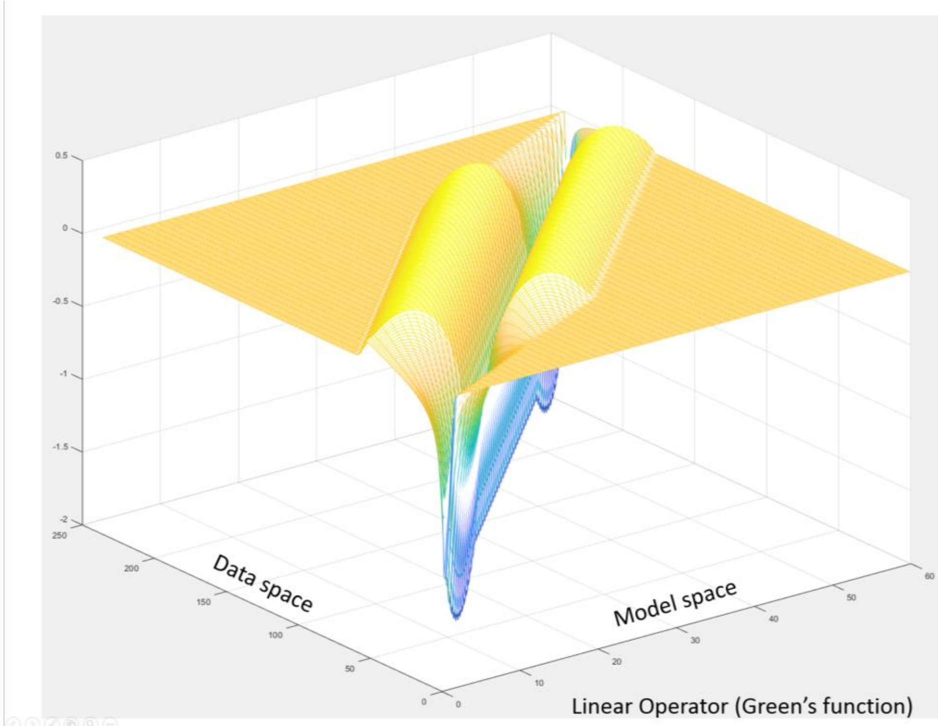


Fig. 4. Stacking velocity inversion operator. Model space is the interval velocity anomaly field. Data space is the stacking velocity anomaly field.

Direct inversion of our data is shown in Figure 5. The model space consists of a 100 m x 200 m dx-dz grid. In the inversion we assumed no knowledge of the lateral velocity anomaly, and used a constant initial velocity field. As expected, with only observations from a single reflector and no other constraints, the solution is dominated by numerical artifacts. Even though there is a hint of the lateral velocity anomaly in the solution field, its impact has been diminished by the least-squares averaging property, spreading the anomaly across the entire solution grid. This solution satisfies the observations almost perfectly, not surprising given the high degree of freedom and the non-uniqueness of the solution. However, the velocity field is not useful for depth-migration or interpretation.

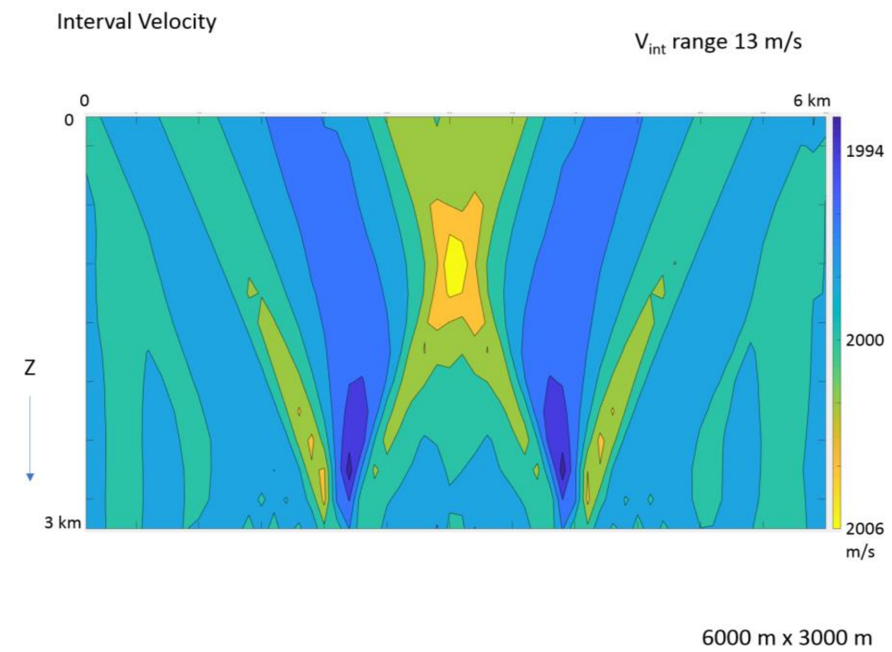


Fig. 5. Interval velocity field from unconstrained inversion. The strong numerical artifacts are associated with the ray coverage.

There is a significant number of papers that describe the non-uniqueness of the inversion problem, the impact of the null-space, and the numerical artifacts in the solution. Most papers try to control the situation by adding some form of numerical constraint and smoothing operators. These strategies work more or less depending on how much seismic data constraints the solution. With synthetic data it is common to simulate seismic data with low frequencies, starting the inversion with a velocity model that has the correct background at these low frequencies, and iterating from low to higher frequencies to increase resolution. This is highly effective, but with real seismic data we do not have an exact low-frequency background to start the iterations, and the data never has the important very-low frequencies needed for velocity inversion.

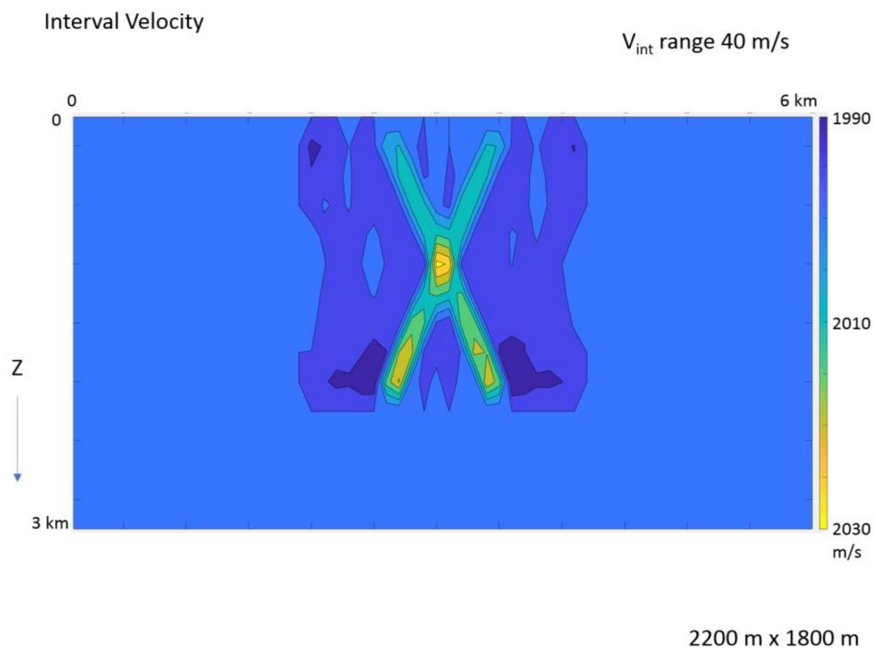


Fig. 6. Constrained inversion. Velocity anomalies restricted to a box 2,200 m x 1,800 m. Strong numerical artifacts are still apparent, but inversion is starting to resolve the location of the main anomaly.

We propose to use geometric constraints in the inversion. There is little discussion of these type of constraints in the literature, perhaps because they are not readily automatic as numerical constraints are. Geometric constraints require interpretation of the seismic image to guide the inversion. However, these constraints do not have to be hard, even soft geometric constraints are effective in mitigating the strong artifacts of an un-constrained inversion. Figures 6 and 7 illustrate this property. In Figure 6 the solution has been constrained to a 2,200 m x 1,800 m. Even though the constrained is relatively soft, most of the numerical artifacts are attenuated and resolution is increasing, giving guidance to the location of the anomaly. In Figure 7 the box has been reduced to 600 m x 600 m. All these velocity solutions satisfy the observations accurately.

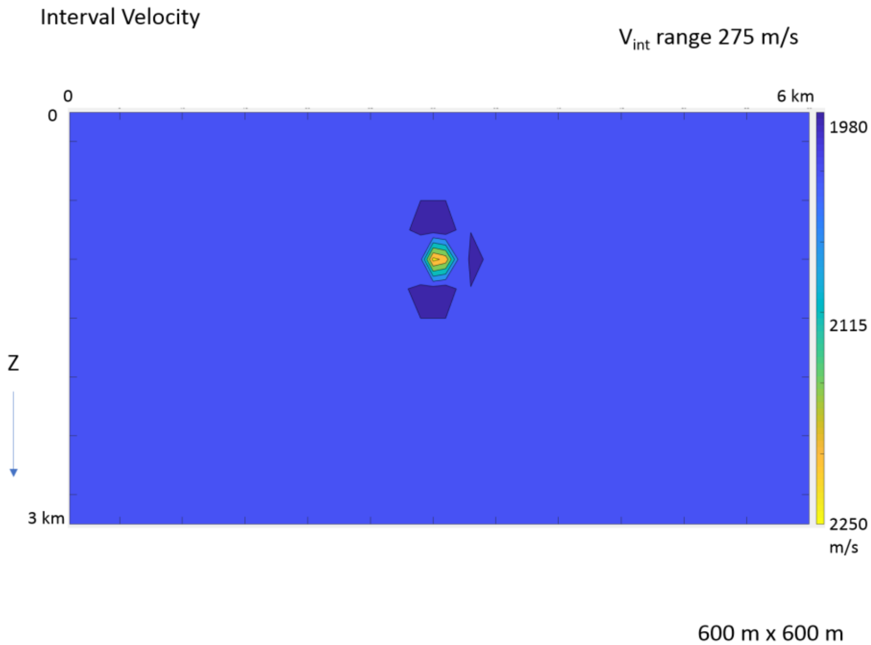


Fig. 7. Constrained inversion. Velocity anomalies restricted to a box 600 m x 600 m. Constraining approximately the location of the anomaly reduces significantly numerical artifacts and starts to resolve better the magnitude of the anomaly.

For illustration, Figure 8 shows an inversion assuming we know where the anomaly is. This is not realistic with real seismic data, but a comparison with Figure 7 demonstrates that even a box three times bigger produces a satisfactory solution that is suitable for depth migration.

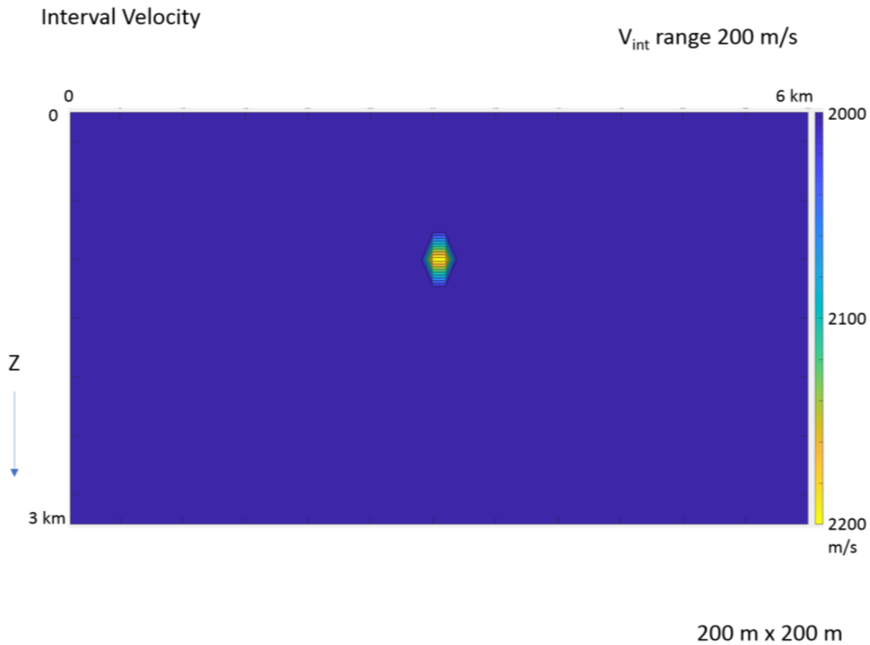


Fig. 8. Constrained inversion. Assuming we know the location of the anomaly but not the magnitude of the velocity, inversion can resolve accurately the magnitude of the anomaly with few numerical artifacts.

As a reference we also include in Figure 9 the velocity inversion with no constraints, but with Tikhonov regularization. The regularization has smoothed the solution, but our example is highly unconstrained and regularization cannot guide the solution to a more desirable solution. Different types of regularization will give variations of this result, but will not overcome the problem of our highly unconstrained inversion with few observations.

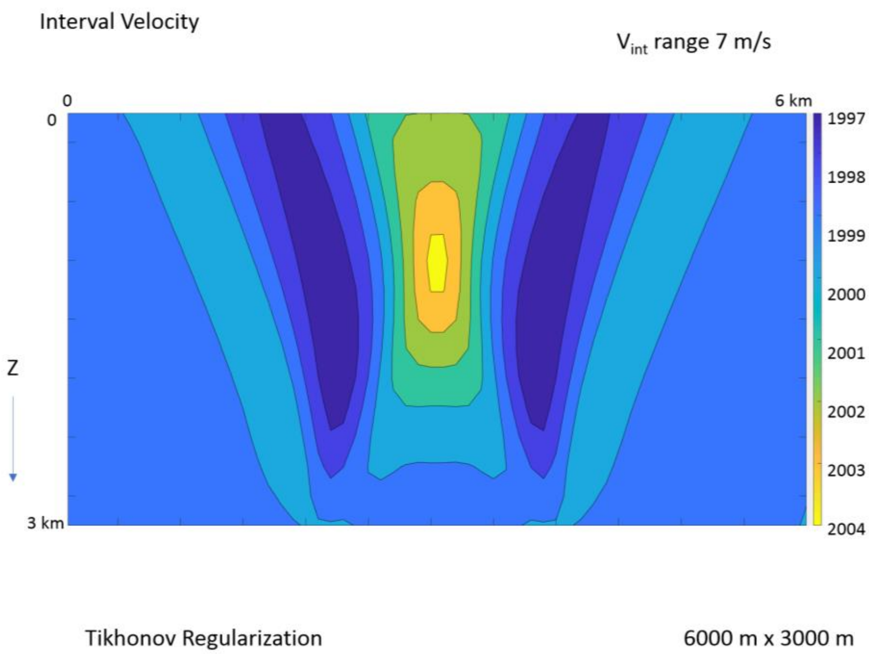


Fig. 9. Unconstrained inversion with Tikhonov regularization. Without additional geometric constraints, and for this particular example, regularization helps smooth the solution, however, the resulting velocity field is still dominated by artifacts and is not accurate for depth migration or time-to-depth conversion.

Method 2: Transformation

FK migration uses the Fourier transform which converts a second order pde (space and time) into a second order algebraic equation (frequency and wavenumber). The original motivation is to speed up migration with constant velocity using fast Fourier transform. Simple transform like FK has less migration artifacts. Interpolation can be used to approximate heterogeneous velocity model.

Another transformation method is tau-p transform which converts a quadratic algebraic equation like NMO (normal moveout) from time-offset domain to tau-p domain. Multiples exhibit different periodicities for different offsets in time domain. Tau-p domain makes the periodicities more regular. So we can perform deconvolution in tau-p domain and transform it back to time domain. The shortcoming of tau-p is 1-D deconvolution. What is 3-D deconvolution (dip-dependent deconvolution)?

Method 3: Data modeling data

In general, real data is complex and no single equation could accurately model it. We have to model the “noise” which is not included in our primary equation. Using input data to model other types of data like multiple was viewed as “signal processing” method. One such approach was used in Gasparotto/Lau to predict surface-related multiples. Then mute the predicted multiples in tau-p domain. Muting is effective in difficult real data like diffraction multiples. There have been many papers which have emphasized accuracy for multiple prediction. Despite best efforts, they do not address artifacts in real data problems. The artifacts of the residual multiples appear to have geometric “smiles” or “swings” after migration, e.g., in subsalt imaging.

Method 4: editing and smoothing
We could manually edit and smooth the inversion model result even though it seems subjective.

Method 5: Operator and data decomposition
Operator and data could be decomposed into eigenvectors. We could select only the larger eigenvalues to invert and ignore the small values.

Remarks. Non-uniqueness (existence of null space) is one theoretical reason why data alone cannot solve the problem. We need external help like qualitative method which uses interpretation, machine learning, geometric methods, data selection (e.g. mute), etc.

Qualitative method still uses mathematics and algorithms. Here are some examples. Interpretation uses mapping (picking algorithm) to guide imaging and inversion. Machine learning like neural network classifies lithology as 0 or 1 (yes or no for lithology type). Betti numbers indicate geometric/topological complexity (high complexity dictating operator reduction). Data selection like muting requires geologic decision to judge quality of the image after muting inside or outside traces.

The algorithms of qualitative method are sometimes called "soft" equations because they do not give “precise” answers. Quantitative method uses "hard" equations like wave equation or raytracing (e.g., inversion, migration, stack) to give precise answers. But precise answers might have more uncertainty and could exaggerate the impact of null space and artifacts.

Conclusion. We have illustrated the non-uniqueness of inversion using hyperbolic moveout inversion. There are ways to mitigate the null space so that the imaging or inversion results appear to have less artifacts. These are somewhat ad hoc methods. We might not like ad hoc methods since they seem to be “unscientific” trial and error. However, the ultimate optimization is our mind and how our mind interprets the output as geologically meaningful or not. Until machine learning could achieve results similar to human interpretation, we will live with ad hoc methods to alleviate the imperfection of our imaging algorithms.

Appendix:
There are some imaging problems that are subtle manifestations. They are observable obstructions to seismic inversion to recover the true model and true reflectivity. They do not have formal definition like nullspace. They remain as footprints after seismic processing and inversion have been done.

Apparent attenuation is probably the most difficult problem in seismic imaging. Vertical seismic profile (VSP) is seismic data collection by putting a geophone deep into a drilled well to record the sound wave from the surface. We have observed in some situations that VSP has good signal and yet the surface recorded seismic cannot image the deeper structure (e.g., through salt layer). Apparent attenuation could be not be fully accounted for by intrinsic attenuation of rocks, e.g., poor subsalt image with salt having less attenuation than normal sediment. We have not solved the problem of seismic imaging where VSP shows excellent signal and surface seismic cannot recover the image.

Coherent noise includes multiples, converted waves, diffractions and other waves. The modeling equations and input data do not predict the coherent noise precisely. Hence other methods like statistical subtraction are used for practical reasons to empirically remove the coherent noise.

Mistie is observed as mismatch between different cubes when we collect 3-dimensional data. This is most evident in azimuthal mistie with many source-to-receiver azimuth directions. The processed image cubes do not tie in depth and waveform even when we estimate and account for anisotropy (different wave speeds in different directions). The different azimuth cubes do not tie even in geologically simple structures.

References

Almasy, A., 2009, Inverse Problems in Classical and Quantum Physics, arXiv.

Diaconis, P., 2009, The Markov Chain Monte Carlo Revolution, Bull. Amer. *Math. Soc*. 46, 179-205

Gasparotto, F.A. and Lau, A., 2000, Multiple prediction for velocity analysis, SEG Annual Meeting

Lau, A., A. Gonzalez and K. Osypov , 1999, Unconstrained near-surface velocity inversion of linear moveout velocities and its impact on structural mapping, SEG Annual Meeting

Lau, A. and C. Yin, 2017, Quantitative and Qualitative Seismic Imaging and Seismic Inversion, arXiv:1706.05276v1

Ramirez, A.C., Sadikhov, E., Brunsvik, F.M.H., Sigernes, L.W. and Ramani, K., 2017, Internal multiples The details that matter! - Part I: Fast prediction and attenuation for quantitative interpretation, SEG Annual Meeting

Toldi, John L., 1989, Velocity analysis without picking, GEOPHYSICS, 54(2), 191-199

Volkwein, S., 2013, Proper orthogonal decomposition: theory and reduced-order modeling. Lecture notes. University of Konstanz, Dept. of Mathematics and Statistics