Super accelerated motion in rindler spacetime

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ABSTRACT

In the general relativity theory, we discover formulas that the super accelerated matter moves with the acceleration $a_0{}'$ about Rindler space-time. We can represent the super accelerated motion about coordinates x, ct, ξ^0.

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1. Introduction

In the general relativity theory, we discover formulas that the super accelerated matter moves with the acceleration $a_0{}'$ about Rindler space-time.

At first, Rindler coordinate is

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0\xi^0}{c}) \tag{1}$$

$$x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \tag{2}$$

$$y = \xi^2, z = \xi^3 \tag{3}$$

In Eq(1),

$$\xi^1 + \frac{c^2}{a_0} = \frac{ct}{\sinh(\frac{a_0\xi^0}{c})} \tag{4}$$

If we insert Eq(4) in Eq(2),

$$x = ct\coth(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \tag{5}$$

If we insert Eq(5) in Eq(2),

$$ct\coth(\frac{a_0\xi^0}{c}) = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0\xi^0}{c}) \tag{6}$$

Hence, the result is

$$\xi^1 = \frac{ct}{\sinh(\frac{a_0\xi^0}{c})} - \frac{c^2}{a_0} \tag{7}$$

2. The super accelerated motion about an uniformly accelerated frame

Rindler spacetime is

$$d\tau^2 = (1 + \frac{a_0\xi^1}{c^2})^2(d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] \tag{8}$$

Hence, if the super accelerated matter moves with the acceleration $a_0{}'$ about an uniformly accelerated frame,

$$a_0' = \frac{d}{d\xi^0}\left[\frac{(\frac{d\xi^1}{d\xi^0})}{\sqrt{(1+\frac{a_0\xi^1}{c^2})^2 - (\frac{d\xi^1}{d\xi^0})^2 / c^2}}\right] \quad (9)$$

If we compute,

$$(a_0'\xi^0)^2 = \frac{(\frac{d\xi^1}{d\xi^0})^2}{(1+\frac{a_0\xi^1}{c^2})^2 - (\frac{d\xi^1}{d\xi^0})^2 / c^2} \quad (10)$$

Then,

$$(\frac{d\xi^1}{d\xi^0})^2 = (a_0'\xi^0)^2[(1+\frac{a_0\xi^1}{c^2})^2 - (\frac{d\xi^1}{d\xi^0})^2 / c^2] \quad (11)$$

If we compute about $\frac{d\xi^1}{d\xi^0}$,

$$\frac{d\xi^1}{d\xi^0} = \frac{a_0'\xi^0(1+\frac{a_0\xi^1}{c^2})}{\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}}} \quad (12)$$

In Eq(12), if we multiply $\frac{d\xi^0}{(1+\frac{a_0\xi^1}{c^2})}$,

$$\frac{d\xi^1}{1+\frac{a_0\xi^1}{c^2}} = \frac{a_0'\xi^0 d\xi^0}{\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}}} \quad (13)$$

If we integrate Eq(13),

$$\frac{c^2}{a_0}\ln\left|1+\frac{a_0\xi^1}{c^2}\right| = \frac{c^2}{a_0'}\left(\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}} - 1\right) \quad (14)$$

Hence, if we compute about the coordinate ξ^1, we can represent the super accelerated motion by Rindler

coordinates ξ^1, ξ^0.

$$\xi^1 = \frac{c^2}{a_0}[\exp\frac{a_0}{a_0'}(\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}}-1)-1] \qquad (15)$$

If we insert Eq(7) in Eq(15),

$$\xi^1 = \frac{c^2}{a_0}[\exp\frac{a_0}{a_0'}(\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}}-1)-1] = \frac{ct}{\sinh(\frac{a_0\xi^0}{c})} - \frac{c^2}{a_0} \qquad (16)$$

Hence, we can represent the super accelerated motion about coordinates ct, ξ^0.

$$ct = \frac{c^2}{a_0}\exp\frac{a_0}{a_0'}(\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}}-1)\sinh(\frac{a_0\xi^0}{c}) \qquad (17)$$

If we insert Eq(17) in Eq(5), we can represent the super accelerated motion about coordinates x, ξ^0.

$$x = ct\coth(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0}$$

$$= \frac{c^2}{a_0}\exp\frac{a_0}{a_0'}(\sqrt{1+\frac{(a_0'\xi^0)^2}{c^2}}-1)\cosh(\frac{a_0\xi^0}{c}) - \frac{c^2}{a_0} \qquad (18)$$

3. Conclusion

In the general relativity theory, we discover formulas that the super accelerated matter moves with the acceleration in an uniformly accelerated frame.

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