Aristotle's law of contradiction and Einstein's special theory of relativity

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ABASTRACT

Objective: The aim of this study is to re-evaluate the relationship between Aristotle's law of contradiction and Einstein's special theory of relativity.

Methods: In order to clarify the relationship between Aristotle's law of contradiction and Einstein's special theory of relativity, several different approaches were chosen and appropriate theorems were developed.

Results.

It was possible to provide the proof that Aristotle's law of contradiction is observer dependent too but does not contradict Einstein's special theory of relativity. Furthermore, a derivation of Aristotle's law of contradiction from the identity law (principium identitatis) was provided.

Conclusions

Aristotle's law of contradiction and Einstein's special theory of relativity are compatible with each other.

Keywords: principium identitatis, principium contradictionis, causality, Einstein's special theory of relativity

Introduction

Faced with the serious difficulty of the task to answer the question can and how can a scientist live on forever and be immortal it is clear enough that the question of the immortality of a scientist has to do among other things with his scientific work too. So nearly all of us now and again are confronted every day with a difficult challenge to recognize what does truly defines a historical scientific work and can and how can the same be established? Producing a chain of non-ending none-sense has proved historically remarkably as not long-lived and appears not to be the way to eternal scientific live. By time, the historical development of science assures the survival of the fittest (Spencer, 1864) scientific concepts independently whether an individual scientist may refuse to accept that. Surely, all scientist dies, but only few of these scientists might continue to exist or at least will be remembered for ever. In fact, the majority of authors and academic writers working in different fields of science have reason to be deeply indebted to Aristotle (Aristotle, 1908), Leibniz (Leibniz, 1765), Einstein (Einstein, 1916) and other forerunners of science as such which many times were divided in several positions but still were united in their striving to find a generally acceptable common ground or a principle of scientific inquiry, reasoning and communication and of our scientific knowledge. In the scientific world, the path to truth is sometimes rocky, and errors occur frequently. Because of this, no doubt that it takes a lot of hard work to be able to detect and to avoid especially logical fallacies in science and it might turn out that the knowledge of fallacies needed to arm us against fundamental missteps one might take with arguments published one day in the distant future can be viewed as a fundamental criterion of good scientific skill and reasoning. In the narrow sense, the present opportunity is appropriate enough to address the assembly of scientists working in many different fields but united in their everyday struggle to clear up the misunderstandings which have arisen, to avoid apparent conceptual difficulties in the future and above all might help us to find a common foundation for our scientific knowledge. Before entering in more detail into the problems to be discussed, it is necessary to recall only briefly how often the development of science has taught us that any description of our daily experience or the progress in science as such is based on assumptions which are not transparent enough, hided beyond a lot of highly abstract mathematical stuff or initially completely unnoticed. Such a methodological attitude

thereby appears to contribute to the hyper-inflation of mysticism and mystic position in science and is incompatible with the true spirit of science. Sometimes poets are able to widen our unnecessary restricted view on things and processes. Ultimately, for this reason, we are invited to reflect about the words of the great German Poet Johann Wolfgang von Goethe: "*Ich bin der Geist der stets verneint! Und das mit Recht; denn alles was entsteht Ist werth daß es zu Grunde geht;*" (Goethe, 1808, p. 86). In broken English: "I am the Spirit that denies! And rightly too; for all that doth begin should rightly to destruction run;". Nonetheless, as it is, it is and the greater the scientific none-sense, the more glory in overcoming the same. However, even those who are already wise enough and no longer may love to consider any new wisdom are challenged to accept that *within any danger itself the rescue can be found*. In order to overcome todays obvious difficulties erected due the lack of use of appropriate logically and mathematically consistent methods and the mismatch of strict and non-strict inequalities (Harriot, 1631; Tanner, 1962), **the principle of causality** is one way to turn around and to get out of this Copenhagen interpretation of quantum mechanics initiated scientific dead-end street and non-locality disappointing research results kept alive historical scientific disaster. A number of objections have been raised and expressed from various sides especially against the principle of causality and causation as such as the most important and common foundation of our science. At the end, especially according to Copenhagen's quantum mechanics and Heisenberg, respect the following: "*Weil alle Experimente den Gesetzen der Quantenmechanik und damit der Gleichung (1) unterworfen sind, so wird durch die Quantenmechanik die Ungültigkeit des Kausalgesetztes definitiv festgestellt.*" (Heisenberg, 1927). For lack of a better translation and with the authority of the most errenous and dogmatic logical fallacy of science, Heisenberg demands us to accept without any sense or a without a clear proof the following: "Because all experiments are subject to the laws of quantum mechanics and hence to equation (1), quantum mechanics definitively establishes the invalidity of the principle of causality." Even Bohr himself points to the necessity of "*abandoning the causal description in atomic physics*" (Bohr, 1937) and of the principle of causality as such too. Heisenberg's uncertainty is meanwhile refuted (Barukčić, 2011; Barukčić, 2014; Barukčić, 2016b) for several times but is still not exterminated out of physics and out of science too.

2. Material and methods

The study of properties of the numbers (Number theory) can be clarified and optimized and is one way to rebuild the whole mathematics without prerequisite if (physical) experiments can be used to investigate and proof mathematical objects et cetera. In last consequence, *defining numbers in terms of natural, physical constants* will provide us with a deeper knowledge of objective reality far beyond any rules of number theory.

2.1. Definitions

DEFINITION 2.1.1. (NUMBER +0).

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number (Bombelli, 1579). The number +0 is defined as the expression

$$+0 \quad \equiv \quad (c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \quad \equiv \quad +1-1 \quad = \quad +i^2 - i^2 \quad (1)$$

while "=" denotes the equals sign or equality sign (Recorde, 1557; Rolle, 1690) used to indicate equality and "-" (Widmann, 1489; Pacioli, 1494; Recorde, 1557) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" (Widmann, 1489; Pacioli, 1494; Recorde, 1557) denotes the plus signs used to represent the operations of addition and the notions of positive as well.

DEFINITION 2.1.2. (NUMBER +1).

Let c denote the speed of light in vacuum, let ε_0 denote the electric constant and let μ_0 the magnetic constant, let i denote an imaginary number (Bombelli, 1579). The number +1 is defined as the expression

$$+1 \quad \equiv \quad (c^2 \times \varepsilon_0 \times \mu_0) \quad \equiv \quad -i^2 \quad (2)$$

DEFINITION 2.1.3. (ARISTOTLE'S LAW OF CONRADICTION).

Aristotle's law of contradiction (Aristotle, 1908) is defined as

$$+1 \quad \equiv \quad +0 \tag{3}$$

or according to Boolean algebra (Boole, 1854) as

$$1 \quad \times 0 \quad \equiv \quad 0 \quad \times 0 \quad = 0 \tag{4}$$

2.2. Methods

2.2.1. Thought Experiments

There are many different things one can say about the relation between premises and conclusions even if it is beyond the scope of this article to provide a brief formal characterization. In many respects even if leaving out a large number of philosophical debates and also leaving out almost some technical details the contemporary accounts of logical consequence are the heart of the interior logic of valid (quantum mechanical) arguments too. We should note that the most widespread and strongest narrower criterion for a good (quantum mechanical) argument is given if a conclusion drawn follows from its premises without any contradiction independently whether based on *a proof-centered approach* or *the absence of counterexample* et cetera. If the premises of a (quantum mechanical) argument are true, then the conclusion follows as a matter of fact in the absence of any technical errors deductively from the premises with the consequence that the conclusion drawn is also true (Tarski, 1937). Thus far we might be able to present some theoretical (thought experiment) or experimental circumstances in which the premises are true but the conclusion drawn is false because such circumstances does not support the validity, the soundness and completeness of a (quantum mechanical) argument. Thought experiments (Sorensen, 1999) valid devices of the scientific (Cargile, Horowitz, & Massey, 1994) investigation both in natural sciences and in philosophy to confront theorems or theories with circumstances which effectively can provide evidence in favor of or against a theorem, a theory et cetera.

2.2.2. Counterexamples

In general, the method of a counterexample (Romano and Siegel, 1986) is a simple but valid proof technique which philosophers and mathematicians use extensively to disproof some certain philosophical, mathematical (Stoyanov, 2013), physical and other arguments and was effectively used for the historically first refutation of Heisenberg's uncertainty principle (Barukčić, 2011). A scientific position, a theory or a theorem is generally valid or valid only under certain conditions. Still, if the conditions under which such a scientific position, a theory or a theorem are given, it should not be possible to show that the scientific position, the theory or the theorem does not apply in a certain single example. By using counterexamples under the conditions of a theory or of a theorem, researchers may avoid the scientific community from going down blind logical alleys and prevent us from losing time, money and effort by showing a scientific position, a theory or a theorem as wrong and as not (generally) valid.

2.3. Axioms

A merely historical look at development of human knowledge (Einstein, 1919) teaches us that big advances in science may originate by observing natural and experimentally generated individual facts and grouping and selecting the same together until a lawful connection may become clearly apparent. By time the complex of facts may become extremely large and may lead some scientist to the postulation of some hypothetical basic laws of nature that go beyond the observed. From such basic laws of nature (a system of axioms) it is possible to derive conclusions in a purely logically deductive manner which can be compared with (thought) experiments. *Deduction* as almost diametrically opposed to *induction* has contributed to the greatest advances in natural science too. In opposite to Einstein, Hume's (Hume, 1739) own erroneous and restricted understanding and analysis of the notions of cause and effect lead him to call into question the justification of any reasoning based on inductive inference. Under some conditions, the development and application of a scientific theory is determined by some basic law (axioms) and conclusions drawn from the same. In view of the fact that it is difficult to prove the truth of a theory forever and ignoring details, (incompatible) theories can very well

be found to be incorrect. Theorems or laws deduced from a theory can be tested for accuracy and comprehensiveness by comparing them to theoretical or observational data. **One single accurate (thought) experiment or observation is enough to disproof a theory or a theorem**.

2.2.1. Axiom I (Lex identitatis. Principium Identitatis. Identity Law)

$$+1 \quad = \quad +1 \qquad\qquad (5)$$

The number +1 is just identical with itself, it is +1=+1, or negatively: +1 cannot at the same time be +1 and not +1, another number (i.e. +0) different from +1. In other words +1 is equal to itself, it is completely identical with itself, no local hidden variable, no incompatible properties, just the pure itself. Something like difference or nonidentity in the features of the number +1 cannot be found. Thus far, any change or alteration as such of the number +1 in a very general way might raise subtle problems. Whatever we make by similar reasoning of these arguments, is it extremely implausible to claim that axiom I: +1=+1 denies any hidden variables or causal interpretation of quantum mechanics as discovered by Louis de Broglie (1892–1987) in 1927 (Conseil de Physique, 1928) as pilot-wave theory and as rediscovered by David Bohm (1917–1992) in 1952 as hidden variable theory (Bohm, 1952a; Bohm, 1952b) because axiom I is grounded on the non-existence of a local hidden variables? Especially, according to John von Neumann, Einstein's dream of a deterministic quantum theory is mathematically impossible (Neumann, 1932). Setting aside questions about a or the cause of a change or changes as such, many of the above problems come together in the consistency of change so pervasive in our lives. Historically, the law of identity is deeply connected with our understanding of the foundation of science. In most of what follows, the practice of sharply distinguishing theories and views about the principium identitatis is beyond the scope of this article. Traditionally the identity law or principium identitatis has been given several different and usually imprecise definitions. Principium identitatis became subject to clarification and even mathematical analysis by several authors, among them Plato (428/427 or 424/423 – 348/347 BC) and Aristoteles (384 - 322 BC) too. No matter what we do, no matter what we think, principium

identitatis has, roughly speaking, different features. Nevertheless, Leibniz's approach to the identity law appears to be crucial to our understanding of principium identitatis, and, more particularly, to our understanding of the 'Laws of Thought' in general. It is not hard to uncover the reasons why. The view of the law of identity as put forward by Gottfried Wilhelm Leibniz (1646-1716) is at the end: A=A. According to Leibniz: "*Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra A est A, B est B*" (Leibniz, 1765). For present purposes the important point to recognize is that various authors worked on the identity law too. We may usefully state Russell's position with respect to the identity law as mentioned in his book "The problems of philosophy" (Russell, 1912). In particular, according to Russell, "*... principles have been singled out by tradition under the name of 'Laws of Thought.' They are as follows:*

(1) The law of identity: 'Whatever is, is.'

(2) The law of contradiction: 'Nothing can both be and not be.'

(3) The law of excluded middle: 'Everything must either be or not be.'

*These three laws are samples of self-evident logical principles, but are not really more fundamental or more self-evident than various other similar principles: for instance, the one we considered just now, which states that what follows from a true premiss is true. The name 'laws of thought' is also misleading, for **what is important is** not the fact that we think in accordance with these laws, but **the fact that things behave in accordance with them**;*"

Russell's critique, that we tend too much to focus only on the formal aspects of the 'Laws of Thoughts' with the consequence that "… *we thing in accordance with these laws*" (Russell, 1912) is justified. Judged solely in terms of this aspect, it is of course necessary to think in accordance with the 'Laws of Thoughts'. But this is not the only aspect of the 'Laws of Thoughts'. The other and may be much more important aspect of these 'Laws of Thoughts' is the fact that quantum mechanical objects or that "**things behave in accordance with them**" (Russell, 1912). More commonly, so goes the story and this may be regarded as part of the basis of the popular wisdom, principium identitatis, principium contradictionis and principium exclusi tertii are the simplest, the most basic and the most general laws of objective reality or of nature too. In other words, once one appreciates to describe processes or circumstances et

cetera in terms of local hidden variables, principium identitatis is of use. In fact, it is far from partly mistaken and/or misleading and obviously self-evident that a quantum mechanical observable X_t which is identical only with itself ($X_t = X_t$) excludes local hidden variables. The reason is simple it is $X_t = X_t$ and not $X_t = {}_0x_t +$ (local hidden variable), while ${}_0x_t$ can denote something like an eigenvalue.

3. Results

Theorem 3.1. (*The identity law (principium identitatis I)*)

Einstein's special theory of relativity is based on the principium identitatis or

$$+1 \quad = \quad +1 \tag{6}$$

Proof.

According to Einstein's special theory of relativity, c_0, the speed of the light in vacuum as measured by a co-moving observer is equivalent to the speed of the light in vacuum as measured by the stationary observer c_R. It is

$$c_0 \quad = \quad c_R \tag{7}$$

Dividing by the speed of the light in vacuum, we obtain

$$+1 \quad = \quad +1 \tag{8}$$

Quod erat demonstrandum.

Remark 1.

Einstein himself demanded that it is possible that the constancy of the speed of the light itself is something relative and not something absolute otherwise we would have an absolute frame of reference. Einstein linked the constancy of the speed of the light in vacuum to a constant gravitational potential but not to a constant gravitational field. "*Dagegen bin ich der Ansicht, daß das Prinzip der Konstanz der Lichtgeschwindigkeit sich nur insoweit aufrecht erhalten läßt, als man sich auf raum-zeitliche Gebiete von* **konstantem Gravitationspotential** *beschränkt. Hier liegt nach meiner Meinung die Grenze der Gültigkeit… des Prinzips der Konstanz der Lichtgeschwindigkeit und damit unserer heutigen Relativitätstheorie*" (Einstein, 1912)

Theorem 3.2. (*The identity law (principium identitatis II)*)

Einstein's special theory of relativity is based on the principium identitatis or

$$+1 \quad = \quad +1 \tag{9}$$

Proof.

According to Einstein's special theory of relativity, it is

$$m_0 \quad = \quad \sqrt[2]{1 - \frac{v^2}{c^2}} \quad \times \quad m_R \tag{10}$$

were m_0 denotes the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, v is the relative velocity between the co-moving and the stationary observer, c is the speed of the light in vacuum. Multiplying by c, we obtain

$$\underbrace{m_0 \times c^2}_{E_0} \quad = \quad \sqrt[2]{1 - \frac{v^2}{c^2}} \quad \times \quad \underbrace{m_R \times c^2}_{E_0} \tag{11}$$

were E_0 denotes the rest-energy (Einstein, 1935a) as measured by i. e. by a co-moving observer. Thus far, in general, it is

$$E_0 \quad = \quad E_0 \tag{12}$$

Dividing by E_0, we obtain

$$+1 \quad = \quad +1 \tag{13}$$

Quod erat demonstrandum.

Remark 2.

Einstein's special theory of relativity supports and demands the validity of the axiom +1=+1, which can be tested by accelerator experiments too.

Theorem 3.3. (*Local hidden variable I*)

According to Einstein's mass–energy equivalence (Einstein, 1935a) we are invited until a better explanation is published to consider the following: "Gibt ein Körper die Energie L in From von Strahlung ab, so verkleinert sich seine Masse um L/V² " (Einstein, 1905). Under conditions of Einstein's special theory of relativity, let $E_0 = m_0 \times c \times c$ denote the energy "at rest" of an entity as measured by Bob (B), an observer at rest in the moving system, moving with constant velocity v relatively to the stationary system were Alice (A) is located. Let m_0 denote the "rest mass", c is the speed of light. Let $E_R = m_R \times c \times c$ denote the total relativistic energy (Lewis and Tolman, 1909; Tolman, 1912) the same entity as measured by in the stationary system by Alice (A) at the same (period of) time where m_R denote the "rest mass". Furthermore, let $\underline{E}_0 = E_R - E_0$ denote the local hidden variable.

Proof.

Taken axiom 1 to be true, it is

$$+1 \quad = \quad +1 \tag{14}$$

The same axiom 1 may serve us as a starting point or as a premise for our further reasoning and arguments. Multiplying equation by total relativistic energy $+E_R$, we obtain

$$+E_R \quad = \quad +E_R \tag{15}$$

Adding zero to this equation, the situation doesn't change. It is

$$+E_R \quad = \quad +E_R + 0 \tag{16}$$

Since $+E_0 - E_0 = 0$, the equation simplifies as

$$+E_R \quad = \quad +E_R + E_0 - E_0 \tag{17}$$

or as

$$+E_R \quad = \quad +E_0 \quad +E_R \quad -E_0 \tag{18}$$

According to our definition $\underline{E}_0 = E_R - E_0$ it is

$$+E_R \quad = \quad +E_0 \quad +\underline{E}_0 \tag{19}$$

Quod erat demonstrandum.

Remark 3.

As soon as the relative velocity v between a co-moving observer B and a stationary observer A is **$v > 0$**, both observers will not agree on the energy content of a (quantum mechanical) object. Under these circumstances, every time when B measures the total energy of a system from his own, co-moving standpoint, B will obtain E_0 while A, the stationary observer, will obtain E_R. and both energies are not equal to each other, it is $E_R \geq E_0$. It is the same energy which is measured only from two different standpoints. Even if a co-moving observer B knows about the existence of $\underline{E_0} = E_R - E_0$ when performing some measurements on E_0, the co-moving observer B is not able to measure $\underline{E_0}$. For the co-moving observer B, $\underline{E_0} = E_R - E_0$ is the local hidden variable, otherwise the differences cannot be explained in a logically consistent manner. Thus, even if a co-moving observer B cannot measure both E_0 and E_R simultaneously and precisely, this does not justify a conceptual understanding of the special theory of relativity as dominated by uncertainty or similar mysterious stuff. Especially, as long as centered on observation and measurement, *the variance* of a random variable $\boldsymbol{\sigma(E_0)^2 = E(E_R - E(E_0))^2}$, where $E(E_0)$ denotes the expectational value of "rest energy" is a measure of the degree of existence of a *local hidden variable* too. The greater the variance of a random variable, the more a local hidden variable is effective.

Theorem 3.4. (*Local hidden variable II*)

Let the distribution of a quantum mechanical observable X, a physical quantity which can be measured, contains all of the probabilistic information about X. Corresponding to each quantum mechanical observable X is an operator, which can be designated by the same letter and which can be represented by Hermitian operators in a complex linear vector space. In agreement with classical ideas of reality let the quantum-mechanical observable $_RX_t$ as viewed from the standpoint of an *stationary observer A* be determined by a countable set of finite outcomes or eigenvalues, i. e. $_1X_t, _2X_t, _3X_t, _4X_t, _5X_t, _6X_t$ at the Bernoulli trial t or quantum state t occurring with probabilities $p(_RX_t = _1X_t)$, $p(_RX_t = _2X_t)$, $p(_RX_t = _3X_t)$, $p(_RX_t = _4X_t)$, $p(_RX_t = _5X_t)$, $p(_RX_t = _6X_t)$ respectively. In other words, the observable $_RX_t$ which is corresponding to some physical dynamical variable to be measured is itself in a state of (quantum) superposition before any

measurement. To each eigenvalue $_1X_t$, $_2X_t$, $_3X_t$, $_4X_t$, $_5X_t$, $_6X_t$ is assigned *an own co-moving observer* B, to $_1X_t$ we assign B_1, to $_2X_t$ we assign B_2, to $_3X_t$ we assign B_3, to $_4X_t$ we assign B_4, to $_5X_t$ we assign B_5, to $_6X_t$ we assign B_6. Every measurement of a (quantum mechanical) observable $_RX_t$ can yield only one of the known eigenvalues $_1X_t$, $_2X_t$, $_3X_t$, $_4X_t$, $_5X_t$, $_6X_t$.

Claim.

The expectation value $E(_1\underline{X}_t)$ of a local hidden variable/s follows as

$$E\left({}_1\underline{x}_t \right) \equiv E\left({}_RX_t \right) \times \left(1 - \left(\frac{E\left({}_1x_t \right)}{E\left({}_RX_t \right)} \right) \right) \tag{20}$$

Proof.

Taken axiom 1 to be true, it is

$$+1 = +1 \tag{21}$$

Multiplying equation before by an eigenvalue $(_1X_t)$, we obtain $1\times(_1X_t) = 1\times(_1X_t)$ or

$$\left({}_1x_t \right) = \left({}_1x_t \right) \tag{22}$$

Adding the rest of all possible eigenvalues of the quantum mechanical observable above, it is

$$\left({}_1x_t \right) + \left({}_2x_t \right) + \left({}_3x_t \right) + \left({}_4x_t \right) + \left({}_5x_t \right) + \left({}_6x_t \right)$$
$$=$$
$$\left({}_1x_t \right) + \left({}_2x_t \right) + \left({}_3x_t \right) + \left({}_4x_t \right) + \left({}_5x_t \right) + \left({}_6x_t \right) \tag{23}$$

Taking the expectation value, we obtain

$$E\left(\left({}_1x_t \right) + \left({}_2x_t \right) + \left({}_3x_t \right) + \left({}_4x_t \right) + \left({}_5x_t \right) + \left({}_6x_t \right) \right)$$
$$=$$
$$E\left({}_1x_t \right) + E\left({}_2x_t \right) + E\left({}_3x_t \right) + E\left({}_4x_t \right) + E\left({}_5x_t \right) + E\left({}_6x_t \right) \tag{24}$$

A quantum mechanical observable $_RX_t$ is determined by its own possible outcomes or eigenvalues or in a state of superposition, it is $_RX_t = {}_1X_t + {}_2X_t + {}_3X_t + {}_4X_t + {}_5X_t + {}_6X_t$. Substituting, we obtain

$$E\left({}_RX_t \right) \equiv E\left({}_1x_t \right) + E\left({}_2x_t \right) + E\left({}_3x_t \right) + E\left({}_4x_t \right) + E\left({}_5x_t \right) + E\left({}_6x_t \right) \tag{25}$$

or in other words it is

$$E\left({}_RX_t \right) \equiv \left({}_1x_t \times p\left({}_RX_t = {}_1x_t \right) \right) + \left({}_2x_t \times p\left({}_RX_t = {}_2x_t \right) \right) + \left({}_3x_t \times p\left({}_RX_t = {}_3x_t \right) \right) + \left({}_4x_t \times p\left({}_RX_t = {}_4x_t \right) \right) + \left({}_5x_t \times p\left({}_RX_t = {}_5x_t \right) \right) + \left({}_6x_t \times p\left({}_RX_t = {}_6x_t \right) \right) \tag{26}$$

In general, *the expectation value of a quantum mechanical observable* is equivalent with itself or it is

$$E\left(\,_{R}X_t\,\right) \;\equiv\; E\left(\,_{R}X_t\,\right) \tag{27}$$

Adding zero to this equation, the situation doesn't change at all. It is

$$E\left(\,_{R}X_t\,\right) \;\equiv\; E\left(\,_{R}X_t\,\right) \;+\; 0 \tag{28}$$

The same quantum mechanical observable is determined by different eigenvalues as measured i.e. by a co-moving observer. Especially, if a certain outcome or an eigenvalue is considered, it has to be that **+E($_1$x$_t$) - E($_1$x$_t$)= 0**. Substituting this relationship into the equation before, we obtain

$$E\left(\,_{R}X_t\,\right) \;\equiv\; E\left(\,_{R}X_t\,\right) \;+\; E\left(\,_{1}x_t\,\right) - E\left(\,_{1}x_t\,\right) \tag{29}$$

or

$$E\left(\,_{R}X_t\,\right) \;\equiv\; E\left(\,_{1}x_t\,\right) \;+\; E\left(\,_{R}X_t\,\right) - E\left(\,_{1}x_t\,\right) \tag{30}$$

Thus far, every time when a certain outcome $_1$x$_t$ occurred or when the eigenvalue $_1$x$_t$ is **measured**, the rest of all possible eigenvalues of a quantum mechanical observable is equally **not measured**, which can be considered as a local hidden variable/s. We **define** *the expectation value of the local hidden variable/s* in the following as E($_1\underline{x}_t$) = E($_R$X$_t$) – E($_1$x$_t$) and do obtain

$$E\left(\,_{R}X_t\,\right) \;\equiv\; E\left(\,_{1}x_t\,\right) \;+\; E\left(\,_{1}\underline{x}_t\,\right) \tag{31}$$

or

$$\frac{E\left(\,_{R}X_t\,\right)}{E\left(\,_{R}X_t\,\right)} \;\equiv\; \frac{E\left(\,_{1}x_t\,\right)}{E\left(\,_{R}X_t\,\right)} \;+\; \frac{E\left(\,_{1}\underline{x}_t\,\right)}{E\left(\,_{R}X_t\,\right)} \tag{32}$$

or

$$1 \;\equiv\; \frac{E\left(\,_{1}x_t\,\right)}{E\left(\,_{R}X_t\,\right)} \;+\; \frac{E\left(\,_{1}\underline{x}_t\,\right)}{E\left(\,_{R}X_t\,\right)} \tag{33}$$

or

$$\frac{E\left(\,_{1}\underline{x}_t\,\right)}{E\left(\,_{R}X_t\,\right)} \;\equiv\; 1 \;-\; \frac{E\left(\,_{1}x_t\,\right)}{E\left(\,_{R}X_t\,\right)} \tag{34}$$

Multiplying the equation by E($_R$X$_t$), a quantum theory and a theory of special relativity consistent expectation value of the local hidden variable/s follows as

$$E\left(_1\underline{x}_t \right) \;\equiv\; E\left(_RX_t \right) \;\times\; \left(1 - \left(\frac{E\left(_1\underline{x}_t \right)}{E\left(_RX_t \right)} \right) \right) \tag{35}$$

Quod erat demonstrandum.

Remark 4.

The fundamental philosophical concept of **negation** (Newstadt, 2015) which found its own melting point in Hegel's dialectic is more than just a formal logical process which converts only false to true and true to false, negation is equally an engine of changes of objective reality. A generally accepted link between this fundamental philosophical concept and an adequate counterpart in mathematics, mathematical statistics or physics et cetera has still not been established. Especially the relationship between determination and negation has been discussed in science since ancient (Horn, 2001) times. **Benedict de Spinoza** (1632 – 1677), one of the philosophical founding fathers of *the Age of Enlightenment*, addressed these notions in his lost letter of June 2, 1674 to his friend Jarig Jelles (Förster & Melamed, 2012) by the discovery of his fundamental insight that "***determinatio negatio est***" (Spinoza,1802, p. 634). Spinoza's slogan was extended by Hegel to "*Omnis determinatio est negatio*" (Hegel, 1812). From the equation above follows too, that

$$E\left(_1x_t \right) \;\equiv\; E\left(_RX_t \right) \;\times\; \left(1 - \left(\frac{E\left(_1\underline{x}_t \right)}{E\left(_RX_t \right)} \right) \right) \tag{36}$$

The relativity theory consistent correction factor (rtccf) defined as

$$rtccf \equiv \left(1 - \left(\frac{E\left(_1\underline{x}_t \right)}{E\left(_RX_t \right)} \right) \right) \tag{37}$$

can be treated something as *the general for of negation* which under certain physical circumstances reduces to *the relativistic correction or the Lorenz factor squared or (1-(v²/c²))*. From this definition follows that

$$E\left(_RX_t \right) \;\equiv\; E\left(_1x_t \right) \;\times\; \frac{1}{\left(1 - \left(\frac{E\left(_1\underline{x}_t \right)}{E\left(_RX_t \right)} \right) \right)} \;\equiv\; E\left(_1x_t \right) \;\times\; \frac{1}{rtccf} \tag{38}$$

with the consequence that

$$rtccf \quad \equiv \quad \frac{E\left(\,_1x_t\,\right)}{E\left(\,_RX_t\,\right)} \tag{39}$$

Multiplying equation above derived as

$$E\left(\,_1x_t\,\right) \quad \equiv \quad E\left(\,_RX_t\,\right) \quad \times \quad \left(1 - \left(\frac{E\left(\,_1x_t\,\right)}{E\left(\,_RX_t\,\right)}\right)\right) \tag{40}$$

by an eigenfunction $\psi(_1x_t)$, it is

$$E\left(\,_1x_t\,\right) \times \psi\left(\,_1x_t\,\right) \quad \equiv \quad E\left(\,_RX_t\,\right) \times \left(1 - \left(\frac{E\left(\,_1x_t\,\right)}{E\left(\,_RX_t\,\right)}\right)\right) \quad \times \quad \psi\left(\,_1x_t\,\right) \tag{41}$$

Each eigenvalue is associated with an eigenfunction $\psi(_1x_t)$ which provides an adequate starting-point even **under conditions where $E(_1x_t) = (_1x_t)$**. Under these circumstances, the equation before changes too

$$\left(\,_1x_t\,\right) \times \psi\left(\,_1x_t\,\right) \quad \equiv \quad E\left(\,_RX_t\,\right) \times \left(1 - \left(\frac{E\left(\,_1x_t\,\right)}{E\left(\,_RX_t\,\right)}\right)\right) \quad \times \quad \psi\left(\,_1x_t\,\right) \tag{42}$$

There are experimental conditions (i. e. eigenfunction is a function for that operator) where the general operator Q_{op} can be defined as

$$\left(\,Q_{op}\right) \quad \equiv \quad E\left(\,_RX_t\,\right) \times \left(1 - \left(\frac{E\left(\,_1x_t\,\right)}{E\left(\,_RX_t\,\right)}\right)\right) \tag{43}$$

with the consequence that *a relativity theory consistent Eigenvalue equation* follows as

$$\left(\,_1x_t\,\right) \times \psi\left(\,_1x_t\,\right) \quad \equiv \quad \left(\,Q_{OP}\right) \quad \times \quad \psi\left(\,_1x_t\,\right) \tag{44}$$

In general, the time independent Schrödinger Equation is something like an example of an Eigenvalue equation. The eigenvalue concept of quantum theory must not be limited only to quantum theory. **The eigenvalue concept can be extended to macro-physics and other sciences too**. Especially, under conditions where the relationship

$$\left(1 - \left(\frac{E\left(\,_1x_t\,\right)}{E\left(\,_RX_t\,\right)}\right)\right) \times \psi\left(\,_1x_t\,\right) \quad = \quad \Psi\left(\,_RX_t\,\right) \tag{45}$$

is valid, **the relativity theory consistent general form of the Schrödinger's equation** (Schrödinger, 1926) follows as

$$E\left({}_1x_t \right) \times \psi\left({}_1x_t \right) \equiv E\left({}_RX_t \right) \times \Psi\left({}_RX_t \right) \tag{46}$$

while $\Psi({}_RX_t)$ denotes the wavefunction. This **relativity theory consistent general form of the Schrödinger's equation** is of use outside from quantum physics, especially in macro-physics but in other sciences too (human medicine, macro-economy, testing of drugs, stock exchange market et cetera). However it may be, a relativity theory consistent quantum theory appears not to possible without something like "local hidden variables". Under the assumption that *Pythagorean theorem* is valid for quantum theory too, we change the equation above derived as

$$\frac{E\left({}_1\underline{x}_t \right)}{E\left({}_RX_t \right)} \equiv 1 - \frac{E\left({}_1x_t \right)}{E\left({}_RX_t \right)} \tag{47}$$

too

$$\frac{E\left({}_1\underline{x}_t \right) \times E\left({}_RX_t \right)}{E\left({}_RX_t \right) \times E\left({}_RX_t \right)} \equiv 1 - \frac{E\left({}_1x_t \right) \times E\left({}_RX_t \right)}{E\left({}_RX_t \right) \times E\left({}_RX_t \right)} \tag{48}$$

and define $b^2 = E({}_1\underline{x}_t) \times E({}_RX_t)$ and $a^2 = E({}_1x_t) \times E({}_RX_t)$ and $C^2 = E({}_RX_t) \times E({}_RX_t)$ while the **normalized** (Barukčić, 2013; Barukčić, 2016d; Barukčić, 2017) *Pythagorean theorem* is known to be derived as $(a^2/c^2) + (b^2/c^2) = (C^2/C^2) = +1$. Under circumstances where

$$\frac{E\left({}_1\underline{x}_t \right) \times E\left({}_RX_t \right)}{E\left({}_RX_t \right) \times E\left({}_RX_t \right)} \equiv 1 - \frac{E\left({}_1x_t \right) \times E\left({}_RX_t \right)}{E\left({}_RX_t \right) \times E\left({}_RX_t \right)} = \frac{v^2}{c^2} \tag{49}$$

it is equally

$$\frac{E\left({}_1\underline{x}_t \right)}{E\left({}_RX_t \right)} \equiv \frac{v^2}{c^2} \tag{50}$$

and at the end

$$E\left({}_1\underline{x}_t \right) \equiv \frac{v^2}{c^2} \times E\left({}_RX_t \right) \tag{51}$$

In other words, according to the *Pythagorean theorem* and the *special theory of relativity*, there are certainly circumstances (relative velocity v > 0) where local hidden variables are not completely absent from the entire objective reality. To put it very briefly, objective reality cannot function without local hidden variables. Only under conditions, were the relative velocity *v* between a stationary observer A (Alice) and a co-moving observer B (Bob) is equal to *v=0,* a local hidden variable appears not to be effective but this does not mean not-existent.

In this context, let us consider as an example a six-sided *magic* quantum mechanical particle/entity called "*Zei Wei*". Let this deterministically defined six-sided magic quantum mechanical object represent a quantum mechanical system with observables represented by Hermitian operators in a complex linear vector space. Corresponding to each physically observable quantity X of this magic quantum mechanical cube there exists a quantum mechanical operator, which might be designated by the same letter. Let every measurement of an observable of this magic quantum mechanical die yield one of the eigenvalues 1, 2 ,3, 4, 5, 6 of the corresponding operator. In general, observables with commuting operators be measured simultaneously. The state of this magic quantum mechanical cube is supposed to be completely characterized by its own wave function Ψ. The sample space S of such a six-sided magic quantum mechanical die can be regarded as containing the six numbers 1, 2, 3, 4, 5, 6, each representing a possible side of the die after the roll, i. e. after the measurement. The six sides of each die which are numbered 1–6 are equally the eigenvalues. To each eigenvalue $_1x_t$, $_2x_t$, $_3x_t$, $_4x_t$, $_5x_t$, $_6x_t$ is assigned *an own co-moving observer* B, to $_1x_t$ we assign B_1, to $_2x_t$ we assign B_2, to $_3x_t$ we assign B_3, to $_4x_t$ we assign B_4, to $_5x_t$ we assign B_5, to $_6x_t$ we assign B_6. Symbolically, we write the sample space S as

$$S(\text{six sided die}) \;=\; \left\{\; _1x_t = 1,\; _2x_t = 2,\; _3x_t = 3,\; _4x_t = 4,\; _5x_t = 5,\; _6x_t = 6 \right\} \qquad (52)$$

The outcomes of an experiment or eigenvalues can be treated as elements of a set and events as subsets of a set. *The set theory should be able provide a natural context for the development of relativity consistent quantum theory*. The celebrated, fair and well-balanced die is rolled. In this context the different possible outcomes or eigenvalues are regarded as equally likely while it is impossible to obtain zero or a negative number. Hence, the event or eigenvalue that zero or a negative number will be obtained is defined by a subset of S which itself contains no outcomes.

A fair 6-sided die is rolled in a large number of trials. The expected value of the number rolled is 3.5 even though it is not possible to roll a 3.5 on a 6-sided die.

The general situation:

Outcome/Eigenvalue	$_1x_t$	$_2x_t$	$_3x_t$	$_4x_t$	$_5x_t$	$_6x_t$
Probability	$p(_1X_t = _1x_t)$	$p(_2X_t = _2x_t)$	$p(_3X_t = _3x_t)$	$p(_4X_t = _4x_t)$	$p(_5X_t = _5x_t)$	$p(_6X_t = _6x_t)$
Expected value	$\left(_1x_t \times p(_1X_t = _1x_t)\right)$	$\left(_2x_t \times p(_2X_t = _2x_t)\right)$	$\left(_3x_t \times p(_3X_t = _3x_t)\right)$	$\left(_4x_t \times p(_4X_t = _4x_t)\right)$	$\left(_5x_t \times p(_5X_t = _5x_t)\right)$	$\left(_6x_t \times p(_6X_t = _6x_t)\right)$

$$(53)$$

A concrete six-sided die.

$$
\begin{array}{c|cccccc}
Outcome/Eigenvalue & {}_1x_t = 1 & {}_2x_t = 2 & {}_3x_t = 3 & {}_4x_t = 4 & {}_5x_t = 5 & {}_6x_t = 6 \\
\hline
Probability & p({}_1X_t = 1) = 1/6 & p({}_2X_t = 2) = 1/6 & p({}_3X_t = 3) = 1/6 & p({}_4X_t = 4) = 1/6 & p({}_5X_t = 5) = 1/6 & p({}_6X_t = 6) = 1/6 \\
Expected\ value & (1 \times (1/6)) & (2 \times (1/6)) & (3 \times (1/6)) & (4 \times (1/6)) & (5 \times (1/6)) & (6 \times (1/6)) \\
Expected\ value = & 3,5 & & & & &
\end{array}
\tag{54}
$$

After measuring on "Zei Wei", the system will be in an eigenstate of "Zei Wei". "Zei Wei" is a combination of multiple different and definite eigenstates, but after the measurement we have with certainty a concrete eigenstate and "Zei Wei" is no longer in a state of superposition. The *act of measurement* may be a crucial aspect in the framework of today's quantum theory, but if one certain of multiple and definite eigenstate of a quantum mechanical entity is measured, what happened with the rest of all the other eigenstates. Are the same no longer existent? "Zei Wei" is defined as having six-sides, if only one aspect or one side of "Zei Wei" is measured this does not imply that the other sides are destroyed or no longer existent, the other sides are only not measured but at the same still existent. "Zei Wei" with all its properties is existing independently and outside of human mind and consciousness, independently of any act of measurement. It is necessary to distinguish between the act of measurement and existence of a quantum mechanical entity as such. *Without* existence of a quantum mechanical entity *no* measurement of the same. The existence of our moon, an accumulation of quantum mechanical objects, is a necessary condition thus that some measurements on the moon can be performed. But such measurements neither destroy the moon nor does the same create the moon. Our moon exit's independently of any measurement.

Theorem 3.5. (*The principle of contradiction (principium contradictionis) I*)

Aristotle's law of contradiction can be derived from principium identitatis as

$$
(+1) \times (+0) \quad = \quad (+0)
\tag{55}
$$

Proof.

Taken axiom 1 *to be true*, then it is *true = true* or

$$
+1 \quad = \quad +1
\tag{56}
$$

Multiplying equation by +0, we obtain

$$(+1) \times (+0) \quad = \quad (+1) \times (+0) \tag{57}$$

According to today's laws of algebra and mathematics, it is $1 \times 0 = 0$ and *Aristotle's law of contradiction according to Boolean algebra* (Boole, 1854) *follows* as

$$(+1) \times (+0) \quad = \quad (+0) \tag{58}$$

Quod erat demonstrandum.

Remark 5.

It is possible to derive Aristotle's law of contradiction from the identity law.

Theorem 3.6. (*The principle of contradiction (principium contradictionis) II*)

Let $_RX_t$ denote a binomial random variable which can take only the values either +1 (i. e. TRUE) or +0 (i.e . FALSE) at a certain Bernoulli trial (or period of time) t. Under conditions where $_RX_t = +1$, the law of contradiction (according to George Boole) follows as

$$\left({}_RX_t \right) \times \left(+1 - {}_RX_t \right) \quad = \quad 0 \tag{59}$$

Proof.

Taken axiom 1 *to be true*, it is

$$+1 \quad = \quad +1 \tag{60}$$

Multiplying equation by $_RX_t$, we obtain

$$_RX_t \quad = \quad _RX_t \tag{61}$$

The identity of something with itself ($_RX_t = {}_RX_t$) to many seems in itself an utterly unproblematic notion even if it is equally at the center of different debates. To say that something (i.e. $_RX_t$) is identical with itself, is to say that the same something has only a relation to itself but equally not to another, not to a third, not to a local hidden variable. Whatever position one may take in the controversy concerning *the unrestricted and general validity of the law of identity*, for present purposes the important point to recognize is, as just done, that, however identity might be characterized, the equivalence relation which everything has to itself might not assure that circularity is avoided to a necessary extent. Nevertheless, there is no very

straightforward argument for such a conclusion. As noted, various interrelated problems may be at the center of discussion of the law of identity and circularity itself appears to be crucial to our misunderstanding of identity, but, more particularly, the circularity is entirely on the surface and sometimes the result of our unacknowledged mental fear of accepting the world the way the same is. If there is only $_RX_t$ and if there is not another, then there is only $_RX_t$ and there is not another. In this case, $_RX_t$ cannot have any relation to another because there is not another. The other side of the identity with itself is indeed that there is no identity to another. The view of identity just put forward (henceforth "the stationary view") characterizes the same from the standpoint of a stationary observer. Accordingly, it is better to become more concrete. The same observer, in our case a stationary observer A, is performing some measurements and has been able to record at the trial t that $_RX_t = +1$. We obtain

$$+1 \quad = \quad _RX_t \tag{62}$$

Simultaneously and it is not completely clear how, the same stationary observer A is claiming to have found _at the same trial_ t that $_RX_t = +0$. In general, it is claimed that

$$+1 \quad = \quad +0 \tag{63}$$

with the consequence that axiom 1 _cannot be taken to be true_. Our starting point was that axiom I is true, now we must accept that the same axiom is equally not true and both has to be taken as correct. Under such circumstances it is difficult to recognize anything if there is no correction factor (i.e. in classical logic negation: $1 = \neg 0$) which assures that it is equally $+1=+1$. The equation

$$+1 \quad = \quad +0 \tag{64}$$

is therefore the simplest mathematical formulation of _the principle of contradiction_. Multiplying equation by $+0$, we obtain

$$(+1) \times (+0) \quad = \quad (+0) \times (+0) \tag{65}$$

or

$$(+1) \times (+1 - 1) \quad = \quad (+0) \times (+0) \tag{66}$$

Under conditions where $_RX_t = +1$, we obtain Boole's formulation of the law of contradiction as

$$\left(_RX_t \right) \times \left(+1 - \,_RX_t \right) \;=\; 0 \tag{67}$$

Quod erat demonstrandum.

Remark 6.

Historically, the first documented and self-consistent binary number system representing all numeric values while using typically 0 (zero) and 1 (one) was published by Leibniz (Leibniz, 1703) himself in 1703. In the following, George Boole (1815-1864), an English mathematician, was able to develop in a very short time an impressive algebra of logic (Boole, 1854) as an mathematical extension of the traditional (Aristotelian) logic. According to Boole, "… *the principle of contradiction … affirms that it is impossible for any being to possess a quality, and at the same time not to possess it … * " (Boole, 1854, p. 49). Accordingly, "*Hence x(1 - x) will represent the class whose members are at once 'men,' and 'not men,' and the equation (1) thus express the principle, that a class whose members are at the same time men and not men does not exist. In other words, that it is impossible for the same individual to be at the same time a man and not a man.*" (Boole, 1854, p. 49). Aristotle's earliest formal study of logic has had an unparalleled influence on science. While some authors (i. e. Kant) where of the opinion that Aristotle has discovered everything that is possible to know about logic other (Russell) pointed to many serious limitations of Aristotle's logic. It is worth to mention that Lukasiewicz's allegations that Aristotle's *the law of contradiction* has no logical worth (Lukasiewicz & Wedin, 1971) are unfounded (Seddon, 1981). In general, even if something like a many-valued or dialectical logic as a non-classical logic which does not restrict the number of truth values to only two, either true or false, usually denoted by "0" and "1", is necessary, this does not falsify Aristotle's logic completely. The relationship between Aristotle's logic and a consistent multi-valued logic is similar to the relationship between Newtonian mechanics and Einstein's special theory of relativity, the one passes over into the other and vice versa.

Theorem 3.7. (*Aristotle's principle of contradiction (principium contradictionis) III and Einstein's special theory of relativity*)

*Let $_RX_t$ denote the path of an object as existing or measured by stationary observer A at a certain (period of) time under conditions of special theory of relativity, while the relative velocity v between observers is v > 0. Let $_0X_t$ denote the same path of an object as existing or measured by a co-moving observer B at a **simultaneous** (period of) time under conditions of special theory of relativity.   The path is under the superposition of at least two different states, the path as such can be a straight vertical line denoted by +1 or something other i. e. somehow curved and denoted by +0.*

Claim.

According to special relativity, something, the path of a steel ball, is equally both, the path is curved and the path is not curved and both is given simultaneously, which is a contradiction.

Proof.

A steel ball is mounted on a cart which is moving horizontally with constant relative velocity with respect to *a stationary observer A* under perfect conditions of special relativity. On the cart, a *co-moving observer B* is located and at rest while equally moving with the same relative velocity *v* with respect to the stationary observer A. The *co-moving observer B* performs some measurements and finds correctly, that the path of a steel ball is not curved, the path of a steel ball is a straight line.

This experiment can be studied by the movie "Reference Frames" (Barukčić, 2010) available at YouTube ® (https://www.youtube.com/watch?v=YU-7vfXawuA). The situation as measured by a co-moving observer B starts especially at 4:31 minutes and ends at 5:32 minutes. As it can be seen, the path of a steel ball is a vertical straight line, the path is not curved.

The following picture (Figure 1) may visualize the experiment.
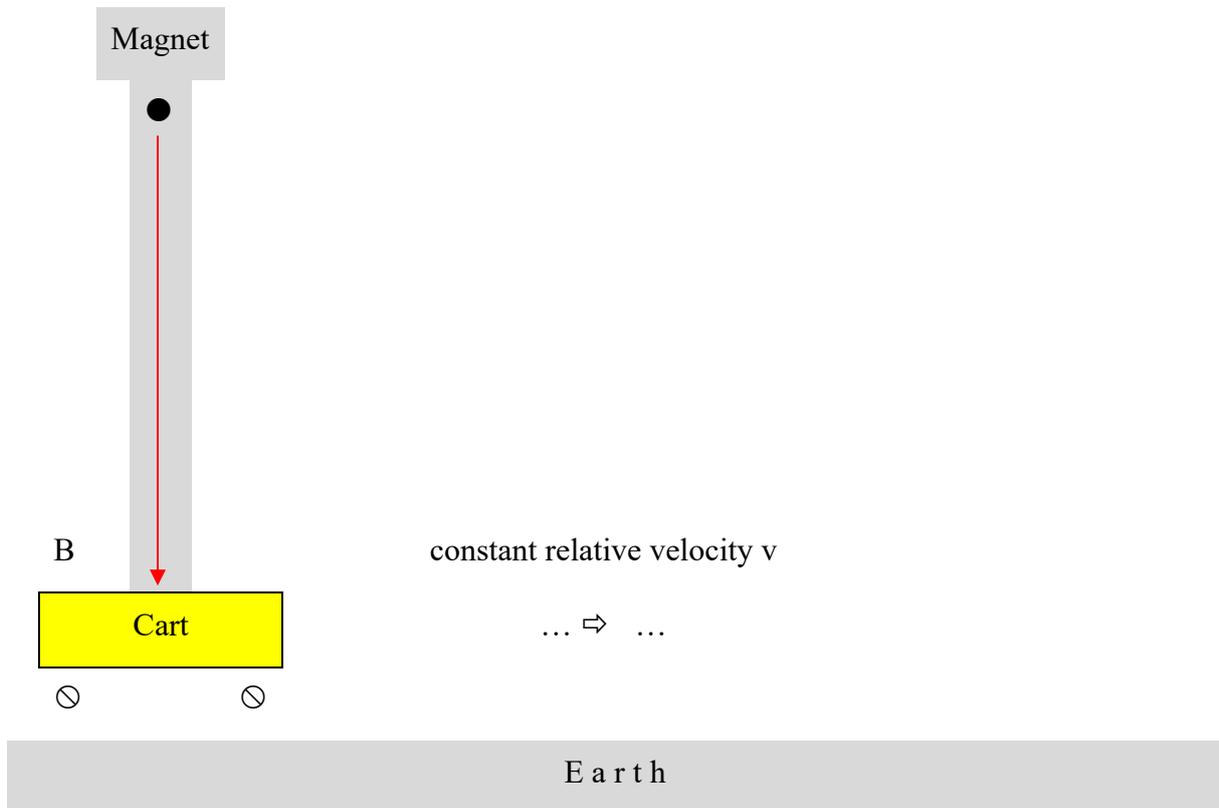
Figure 1. The contradiction as the foundation of objective reality (co-moving observer B).

The same experiment as before is recorded **simultaneously**, at the same period of time, according to special relativity by a *stationary observer* A. The situation as seen by the stationary observer A is *under actual experimental conditions* different from that of the *co-moving observer* B and something like the following (Figure 2).
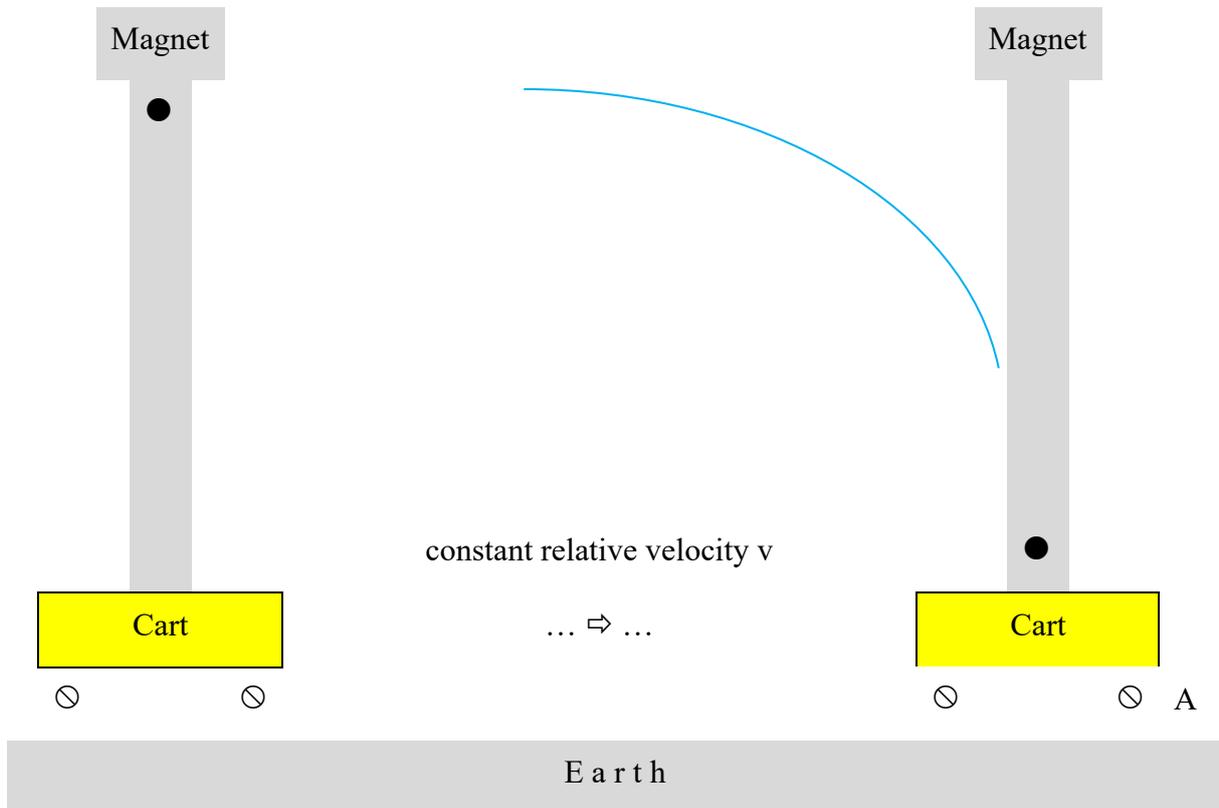
Figure 2. The contradiction as the foundation of objective reality (stationary observer A).

The experiment as performed simultaneously can be viewed by the movie "Reference Frames" (Barukčić, 2010) available at YouTube ® (https://www.youtube.com/watch?v=YU-7vfXawuA). The situation as measured by a *stationary observer* A starts especially at 3:46 minutes and ends at 4:31 minutes. As it can be seen, the path of a steel ball is not a vertical straight line, the path of a steel ball is simultaneously somehow curved.

Both experiments are conducted **simultaneously** or at the same time according to special theory of relativity. In fact, we must accept that both is true, *the path is not curved* (co-moving observer B) *and the path is curved* (stationary observer A) and both is true at the same time. This is a contradiction which may be viewed by the following illustration (Figure 3).
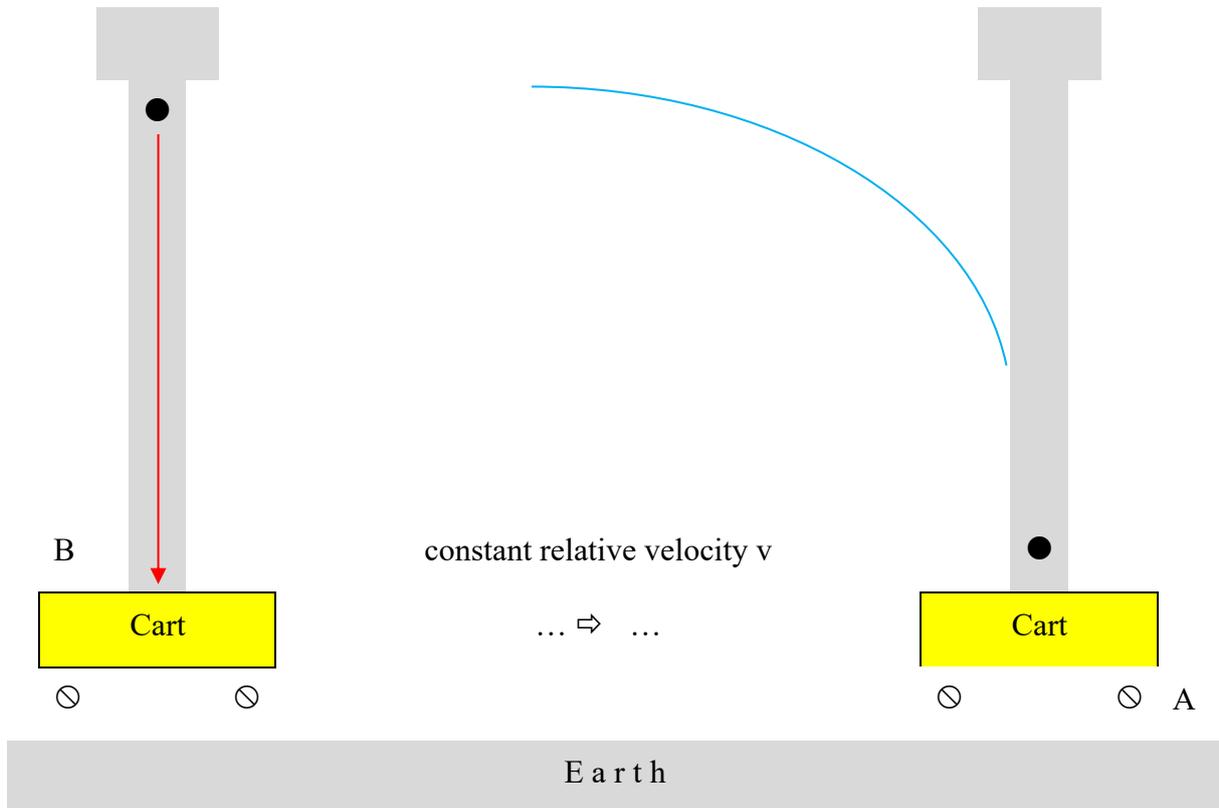
Figure 3. The contradiction as the foundation of objective reality.

Quod erat demonstrandum.

Remark 7.

Many times, *Aristotle's the law of contradiction* (hereafter sometimes simply LC) has been treated as an indemonstrable principle of Aristotelian philosophy, even by Aristotle himself, which in fact is not true. For Aristotle, LC is the most important and the first among all principles of science and has to be taken as the most primitive axiom rather than being derived from any other axiom. In contrast to Aristotle's position, the theorem above outlines the role of Pythagorean theorem and Einstein's special theory of relativity with respect to Aristotelian law

of contradiction and depicts the relation between objective reality and Aristotelian law of contradiction. According to Einstein's special theory of relativity and in contrast to Aristotle's law of contradiction, we must accept that both is true, *the path is not curved* (co-moving observer B) *and the path is curved* (stationary observer A) and both is true at the same time according to special relativity. **It should be noted that Aristotle's law of contradiction is demonstrable by a reproduceable physical experiment.** In general, it is asserted that there is nothing which is contradictory. But the experiment above demonstrates that contradiction is something objective and real, the contradiction exists independently and outside of human mind and consciousness. The contradiction "*is the root of all movement and vitality; it is only in so far as something has a contradiction within it that it moves, has an urge and activity*" (Hegel, 1812). Still, neither the objective existence of contradictions in nature nor the experiment before justifies a superficial conclusion that *either* Einstein's special relativity theory is correct *or* Aristotle's law of contradiction is correct but not both at the same time. Aristotle's main and most famous discussion of his three known versions of *principle of contradiction* can be found in *Metaphysics IV (Gamma) 3–6, especially 4*. In generals, the following version Aristotle's principle of contradiction is usually taken to be the main version of LC and it runs as follows:

"*Evidently ... the same attribute cannot <u>at the same time</u> belong and not belong to the same subject in the same respect; ... This, then, is the most certain of all principles...*"   (Aristotle, 1908, *IV 3 1005b 16–22*)

The experiment before has demonstrated that the same attribute (*straight line*) belongs (from the standpoint of co-moving observer B) and does not belong (*not a straight line* from the standpoint of a stationary observer A) to the same subject (*the path* of a steel ball) in the same respect and at the same (period) time according to special relativity. Aristotle has had the possibility even at his time to recognize this relationship. Besides of all, Aristotle's law of contradiction is correct but only relatively and from the standpoint of an observer and not absolutely. Every time, when a measurement is performed, a <u>single</u> especially co-moving observer will find always that the path of a ball is *either* a straight line *or* the path of a ball is not a straight line but not both.

It may not be completely clear how but it was impossible for Aristotle to deduce LC from anything else and one might follow Aristotle in his understanding of the peculiar status of LC to be the first and firmest principle of all principles which applies to everything that is and to be the common ground for all the special sciences even today. In response to Aristotle, one might wonder that it is was possible to take up the challenge and to derive Aristotle's principle of contradiction in an alternate way from another principle (*principium identitatis*) mathematically.

Theorem 3.8. (*Aristotle's principle of contradiction (principium contradictionis) IV*)

*A **stationary observer A** is located somewhere on our earth or somewhere else above our earth and is performing some measurements by his own eyes at a certain (period of) time with respect to our moon. At the same time or simultaneously according to special theory of relativity, a **co-moving observer B1** is at rest and located on the surface at the bright side or at the bright part of our moon. Another co-moving observer **B2** is located on the surface on the dark part (reduction or absence of light) or on **the far side of our moon**. Even if more or less the same side of our moon is always facing the Earth, about 18 percent of the far side of our moon are sometimes visible from Earth due to effect called libration. The observers are able to communicate with each other about the measurements obtained.*

Claim.

In contrast to a co-moving observer, there are circumstances, where a stationary observer A is able to observe simultaneously two each other excluding properties of a (quantum mechanical) object.

Proof.

The co-moving observer B1 is able to measure a lot of photon (visible light) while at the same time the co-moving observer B2 finds only view or none photons (visible light). In contrast to both co-moving observers, the stationary observer A is able to detect both each other excluding states simultaneously. Still, is our moon there, if nobody looks?

The following picture is able to illustrate the experimental setup in more detail.

Image Credit: © NASA, Heavenly Half Moon (Archive: NASA, International Space Station, Expedition 20)

https://creativecommons.org/licenses/by-nc/2.0/

In contrast to the co-moving observers B1 and B2, there are circumstances where the stationary observer A (depending upon his position) is able to observe both, each other excluding properties or sides of an object.

Quod erat demonstrandum.

Remark 8.

The foundations of logic, mathematics and physics are affected by observations which goes beyond or sometimes even against what is usually accepted or held to be as unproblematic. But the discovery of a number of these difficulties or the so-called paradoxes even if not all involve logical contradictions too. The experiment above is strikingly simple, but provides evidence that a stationary observer (*depending on his position*) can perform simultaneous measurement of incompatible quantities. Even if we accept the view that to date there is no wavefunction which is describing this scenario, the facts are there too. The moon is an accumulation of

quantum mechanical objects and it must be, according to Jordan, that "*Observations not only disturb what has to be measured, they produce it.*" (Jammer, 1974, p. 151). Jordan, rightly or wrongly, devoted to the philosophical doctrines of Subjective Idealists such as Plato, Berkeley, Hume, and Kant and other is offering us such a naive worldview with the consequence that the observer is producing the moon itself and its properties by an act of measurement. Put differently, especially natural sciences are not all the time and hermetically sealed off from the historical background and human society to a necessary extent with the consequence that ideologies are trying continuously to become part of the formation of sciences and scientific knowledge. In ideology or in religion it may be valid *whoever believes will be saved* or in German words: "*Wer .. glaubt ... wird selig*" (Luther, 1545) but not in science. Science taken more seriously than before must distinguish between believe and knowledge. At the end, must we believe or are we allowed to know that our moon is there where the same is even when nobody looks? Following our everyday experience and Einstein's position, our moon is there where the same is, even if nobody looks. "*We often discussed his notions on objective reality. I recall that during one walk **Einstein** suddenly stopped, turned to me and **asked whether I really believed that the moon exists only when I look at it**.*" (Pais, 1979, p. 907). Given the perspectives above, N. David Mermin, director of the Laboratory of Atomic and Solid State Physics at Cornell University, brings it to the point: "*... spooky actions at a distance ... that what bothered Einstein is not a debatable point but the observed behavior of the real world.*" (Mermin, 2008) Altogether, it is not easy to distinguish where ideology begins and where science ends. However, **simultaneous** measurements of incompatible quantities appear to be possible under certain circumstances, which may be difficult to accept. Thus far, Aristotle's attack on Heraclitus was not justified. According to Aristotle, "*... the Heraclitean doctrine ... is ... that ... things ... are in a state of flux.*" (Aristotle, 1908, *XII IV* 1078b, 14-19). Formally, it was possible to observe the phenomenon above at the times of Aristotle too.

Theorem 3.9. (*Bell's inequality is self-contradictory and mathematically incorrect*)

Bell's inequality/theorem is treated as **generally valid**. The principle of **causality** is **generally valid** too but not both at the same time. According to Bell himself, "... *causality (is, Barukčić) incompatible with the statistical predictions of quantum mechanics.*" (Bell, 1964) and his own inequality/theorem. In other words, Bell's inequality/theorem excludes causality and vice versa. If we follow the advocates of Bell's inequality/theorem, it is not possible to find one single counter-example where Bell's inequality/theorem does collapse, otherwise it is proofed that Bell's inequality/theorem is mathematically formally incorrect and completely worthless.

Claim.

Bell's inequality (i.e. theorem) is refuted because it is possible to derive a logical contradiction out of the same in the form

$$+0 \quad \geq \quad +1 \tag{68}$$

Proof.

According to Bell's inequality (Bell, 1964), we must accept that

$$1 \quad + \quad E(b,c) \quad \geq \quad |E(a,b) - E(a,c)| \tag{69}$$

where E(…) denotes the quantum mechanical expectation values. Bell's inequality is treated as generally valid and much more than this. Following Whitaker, "… *it was John Bell who investigated quantum theory in the greatest depth and established what the theory can tell us about the fundamental nature of the physical world. Moreover, by stimulating experimental tests of the deepest and most profound aspects of quantum theory, Bell's work led to the possibility of exploring seemingly philosophical questions, such as the nature of reality, directly through experiments.*" (Whitaker, 1998) There are arguably many advocates of Bell's inequality, but Stapp himself brings it to the point. "***Bell's theorem (2) is the most profound discovery of science. It shows that, if the statistical predictions of quantum theory are approximately correct, then, in certain cases, the principle of local causes must fail.*"** (Stapp, 1975) and follows word by word Bell's own dictum that "... *causality (is, Barukčić) incompatible with the statistical predictions of quantum mechanics.*" (Bell, 1964). This is not really something new since Heisenberg and other claimed already something similar. Bell's

inequality (or theorem, the reader may take it the way it is preferred) is derived under the assumption of the validity of *the law of independence*. According to Bell's own words, "*The vital assumption [2] is that the result **B** for particle 2 **does not depend on** the setting $\vec{a}$ of the magnet for particle 1, nor **A** on $\vec{b}$.*" (Bell, 1964). Under the conditions of independence, it is valid too that

$$E(a,b) \quad = \quad E(a) \quad \times \quad E(b) \tag{70}$$

or that

$$E(b,c) \quad = \quad E(b) \quad \times \quad E(c) \tag{71}$$

Thus far, it is

$$1 \quad + \quad \big(E(b) \times E(c)\big) \quad \geq \quad \big|\big(E(a) \times E(b)\big) - \big(E(a) \times E(c)\big)\big| \tag{72}$$

Bell's inequality is treated as generally valid implicates the consequence that it is not possible to present *one single counter-example* where the same breaks down. Thus far we analyze the general validity of Bell's inequality under conditions where **E(a) = E(c) and |-E(a)^2| > 1**. Our measurements obtained the result **E(a) = E(c).** Substituting into equation before, we obtain

$$1 \quad + \quad \big(E(b) \times E(a)\big) \quad \geq \quad \big|\big(E(a) \times E(b)\big) - \big(E(a) \times E(c)\big)\big| \tag{73}$$

The term E(a)×E(b) cancels out, we obtain

$$+1 \quad \geq \quad \big|-\big(E(a) \times E(c)\big)\big| \tag{74}$$

Rearranging inequality, it is E(a) = E(c) and thus far

$$+1-1 \quad \geq \quad \big|-\big(E(a) \times E(a)\big)\big| - 1 \tag{75}$$

or

$$+0 \quad \geq \quad \big|-\big(E(a) \times E(a)\big)\big| - 1 \tag{76}$$

We are investigating the validity of Bell's inequality under conditions where E(a) = E(c) and where $|- E(a)^2| > 1$. Thus far, dividing by ($|- E(a)^2| - 1$), while the same term is greater +0, we obtain

$$\frac{+0}{\left|-\left(E(a) \times E(a)\right)\right| - 1} \geq \frac{\left|-\left(E(a) \times E(a)\right)\right| - 1}{\left|-\left(E(a) \times E(a)\right)\right| - 1} \tag{77}$$

or

$$+0 \geq +1 \tag{78}$$

Quod erat demonstrandum.

Remark 9.

Strictly speaking, we are faced with a serious problem. The most fundamental rational and immediate insight of the theorem above is that a logical contradiction **(+0 ≥ +1)** can be derived from Bell's inequality (i.e. theorem). Thus far, Bell's inequality (i.e. theorem) is mathematically proved to be inconsistent and refuted and no logical or mathematical reason is apparent to endorse Bell's inequality (i.e. theorem) any longer. A similar single counterexample was already presented by myself at the Växjö quantum conference 2007 (not published in the conference proceedings) and later at the Växjö quantum conference 11-14 June 2012, Växjö, Sweden, and published (Barukčić, 2012) in the conference proceedings. The theorem above brings Bell's inequality/theorem to collapse logically and mathematically again without any special or additional definitions. Under certain conditions, valid experimental evidence achieved by appropriate physical experiments may provide us with valuable knowledge of objective reality. Still, we must have good reasons to believe in achieved or claimed experimental results. The means by which data are gained and mathematically analyzed may all be wrong. Data analysis with a CHSH inequality (Clauser et al., 1969) is of no help at all to provide any scientific evidence of something like a spooky action at the distance. "*A traditional measure of entanglement is constituted by violation of a Clauser–Horne–Shimony–Holt (27) (CHSH)-type Bell inequality. To accomplish this, a CHSH S value above the classical bound of S ≤ 2 needs to be experimentally obtained ... we ... violated the inequality*" (Herbst, 2015). The CSHS inequality (Barukčić, 2012; Barukčić, 2015; Barukčić, 2016a) along with Bell's inequality/theorem together with other mathematical absurdities (Barukčić, 2011; Barukčić, 2014; Barukčić, 2016b) are refuted under any circumstances which is publicly available without any access barriers.

Theorem 3.10. (*Bell's test experiments must obey the rules of special relativity theory*)

Bell's inequality/theorem tests are conducted somewhere in deep space under absolutely ideal conditions of special relativity. Bob, the co-moving observer, is located inside the spaceship Einstein one very close to the detector B and only just view steps away from the Bohm and Aharonov quantum reactor. Alice, the stationary observer is located near the detector A but equally "light years" away the Bohm and Aharonov quantum reactor and outside the spaceship Einstein one. There is a constant velocity *v* between both observers. The following figure 4 may illustrate the experimental setup.
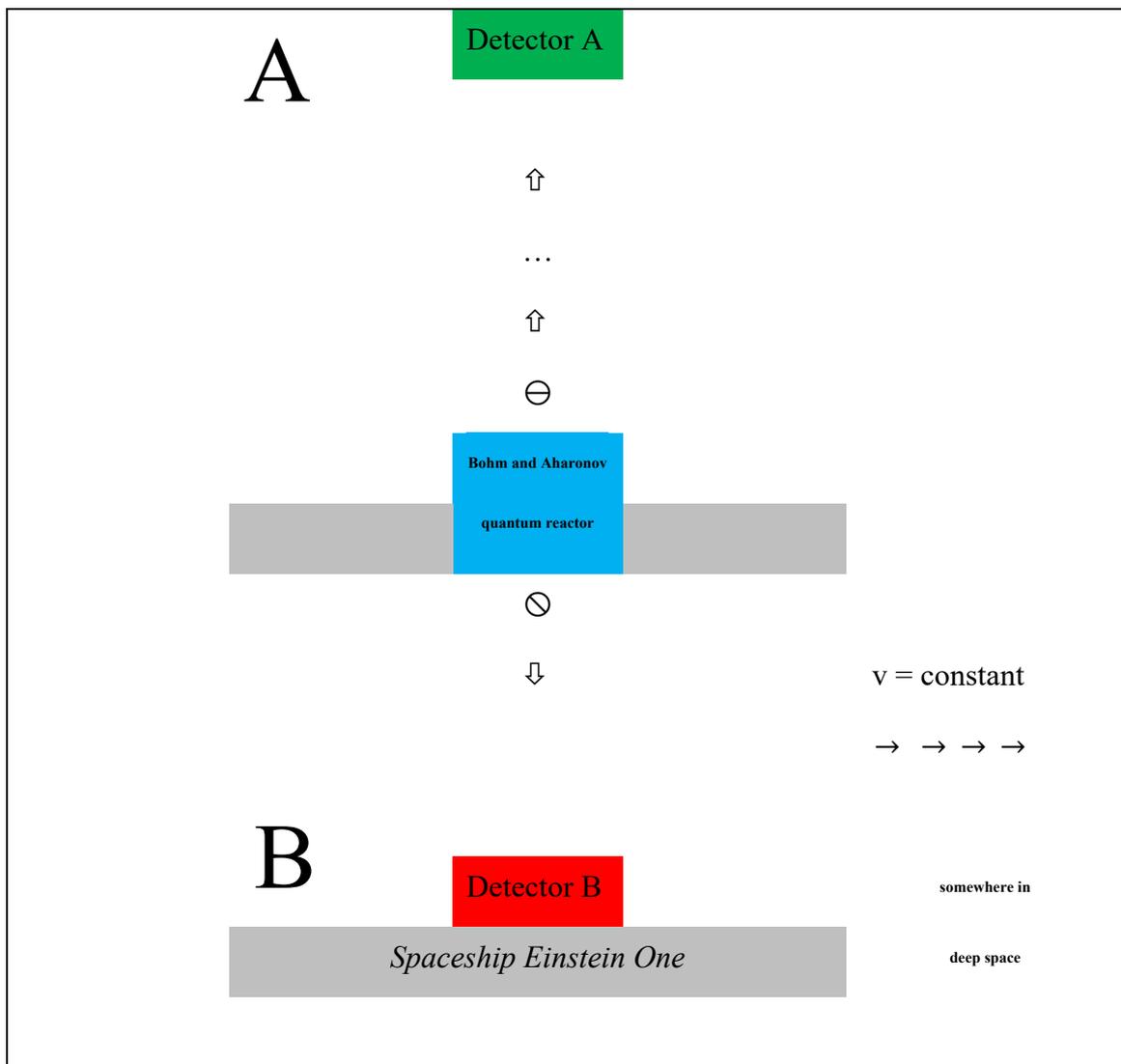
Figure 4. Bell's inequality tests under conditions of special relativity somewhere in deep space.

Claim.

Bell's inequality/theorem must obey the rules of special theory of relativity.

Proof.

Under this experimental setup Bob is performing its own measurements only view nano-seconds after the quantum objects where produced by the Bohm and Aharonov quantum reactor recorded the same immediately. After the measurement by Bob's detector B, the quantum objects (1) were no longer existent and destroyed. What is extraordinary about the ongoing debate on entanglement is the claim that entanglement can persist over long distances and spontaneous decay of entanglement as two entangled particles separate do not occur. In contrast to Bob, Alice is light years away and can perform the necessary measurements on objects (2) only light years later, when the objects send by the Bohm and Aharonov quantum reactor in direction to Alice arrive at the detector A without being disturbed somehow. When Alice performs some measurements, Bob and Einstein's spaceship one is already gone to dust, just no longer existent. However, the destruction of one particle by Bob gives rise to seemingly immediately effects. Alice itself should not be able to measure anything, but the same will. Bell's inequality/theorem requires an entanglement between *something existing and non-existing*, which is not very convincing since such a scenario is able to support something like a spooky action at a distance (Einstein, Podolsky & Rosen, 1935b) without any logical foundation.

Quod erat demonstrandum.

Remark 10.

Whatever way we may look at the quantum mechanical formalism, John Bell and his published inequalities, the above experiment can already be conducted today. On the moon, there is a mirror, able to reflect a beam of photons, even if the same are entangled and enable us to proof whether the connection of two particles which have once interacted always remain bound in a very strange and hardly understandable state is being independent of distance.

4. Discussion

The relationship between mind and matter has been approached from many different points of view and by various authors. To be precise, especially the relationship between objective reality and quantum theory is of special interest. The quantum concept of indeterminism and randomness as standing out against the old-fashioned concept of a deterministic worldview has been found to be attractive in discussing even the old conflict and the dichotomy between human mind/consciousness and matter as such. In particular, before proceeding further, it should be emphasized that a lifetime study of quantum mechanics convinced especially Bernard D'Espagnat's (1921 – 2015) (D'Espagnat, 1979), a colleague of John Bell at CERN, winner of the Templeton Prize 2009, to reconsider the notions about space, time and causality. According to D'Espagnat quantum mechanical objects cannot be thought of as 'self-existent'. To put it in a nutshell: "The doctrine that the world is made up of objects whose existence is independent of human consciousness turns out to be in conflict with quantum mechanics and with facts established by experiment." (D'Espagnat, 1979). In contrast to D'Espagnat and other similar outdated positions, Axiom I is an appropriate foundation of our science (Barukčić, 1989; Barukčić, 1997; Barukčić, 2017; Barukčić, 2016c) and of our thinking and is of use to prevent logical fallacies in everyday scientific work. The same axiom is testable by physical experiment. To date, we have reason to assume, that it is justified to rely upon *principium identitatis*. Especially in theoretical and applied sciences, human medicine and the testing of drugs and other sciences, it does not make any sense to rely on Bell's inequality and not to respect the law of contradiction and the principium identitatis. Following this chain of thoughts, it is reasonable and necessary to abandon Bell's inequality in toto. The same is mathematically inconsistent and completely useless. Bell's inequality is not able to provide anything useful on the relationship between causality, quantum theory and objective reality.

5. Conclusion

Bell's inequality is mathematically inconsistent. Bell's inequality is refuted and must be abandon completely and without any hesitation.

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