A geometric interpretation of the *de Broglie* wavelength

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Preliminary note: This paper is one of the chapters of my upcoming book (the *Zitterbewegung* interpretation of quantum mechanics).[1] Because this paper is so short, I have no table of contents.

The illustration below presents the presumed *Zitterbewegung* of an electron as we would see it when it moves through space.

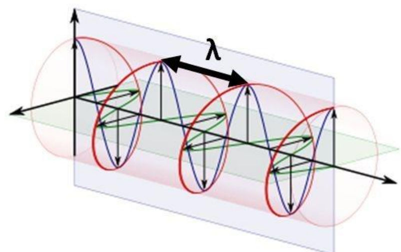

Figure 1: The *Zitterbewegung* of an electron

If this makes you think of an Archimedes' screw, then that's good because it is, effectively, the same geometric shape. We should warn you immediately: there is no reason whatsoever why the *plane* of the oscillation – the plane of *rotation* of the pointlike charge, that is – would be perpendicular to the direction of propagation of the electron as a while. In fact, we think that plane of oscillation moves about itself. We just want you to make a mental note of that as we now are going to present a rather particular geometric property of the *Zitterbewegung* (*zbw*) motion: the Compton radius must decrease as the velocity of our electron increases. The idea is visualized in the illustration below (for which credit goes to an Italian group of *zbw* theorists[2]):

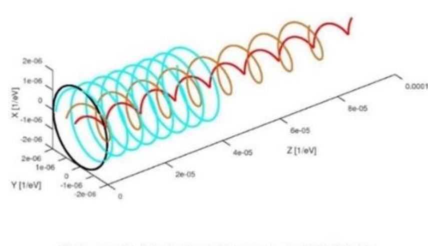

Figure 2: The Compton radius must decrease with increasing velocity

[1] See: http://vixra.org/abs/1901.0105.

[2] Vassallo, G., Di Tommaso, A. O., and Celani, F, *The Zitterbewegung interpretation of quantum mechanics as theoretical framework for ultra-dense deuterium and low energy nuclear reactions*, in: Journal of Condensed Matter Nuclear Science, 2017, Vol 24, pp. 32-41. Don't worry about the rather weird distance scale (1×10^{-6} eV^{-1}). Time and distance can be expressed in *inverse* energy units when using so-called *natural units* ($c = \hbar = 1$). We are not very fond of this because we think it does *not* necessarily clarify or simplify relations. Just note that 1×10^{-9} eV^{-1} = 1 GeV^{-1} $\approx 0.1975\times10^{-15}$ m. As you can see, the *zbw* radius is of the order of 2×10^{-6} eV^{-1} in the diagram, so that's about 0.4×10^{-12} m, which is what we calculated: $a \approx 0.386\times10^{-12}$ m.

Can the velocity go to c? In the limit, yes. This is very interesting, because we can see that the circumference of the oscillation becomes a wavelength in the process. This relates the geometry of our *zbw* electron to the geometry of the photon model we've developed.[3]

What happens here is quite easy to understand – intuitively, that is. If the tangential velocity remains equal to c, and the pointlike charge has to cover some horizontal distance as well, then the circumference of its rotational motion *must* decrease so it can cover the extra distance. But let us analyze it the way we should analyze it, and that's by using our formulas. Let us first think about our formula for the *zbw* radius a:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi}$$

The λ_C is the *Compton* wavelength, so that's the circumference of the circular motion.[4] How can it decrease? If the electron moves, it will have some kinetic energy, which we must add to the *rest energy*. Hence, the mass m in the denominator (mc) increases and, because ħ and c are physical constants, a must decrease. How does that work with the frequency? The frequency is proportional to the energy ($\mathrm{E} = \hbar\cdot\omega = \mathrm{h}\cdot f = \mathrm{h}/\mathrm{T}$) so the frequency – in whatever way you want to measure it – will *increase*. Hence, the *cycle* time T must *decrease*. We write:

$$\theta = \omega t = \frac{\mathrm{E}}{\hbar}t = \frac{\gamma \mathrm{E}_0}{\hbar}t = 2\pi\cdot\frac{\mathrm{t}}{\mathrm{T}}$$

So our Archimedes' screw gets stretched, so to speak. Let us think about what happens here. We got the following formula for this λ wavelength, which is like the distance between two crests or two troughs of the wave[5]:

$$\lambda = v\cdot\mathrm{T} = \frac{v}{f} = v\cdot\frac{\mathrm{h}}{\mathrm{E}} = v\cdot\frac{\mathrm{h}}{\mathrm{m}c^2} = \frac{v}{c}\cdot\frac{\mathrm{h}}{\mathrm{m}c} = \beta\cdot\lambda_C$$

This wavelength is *not* the *de Broglie* wavelength $\lambda_L = \mathrm{h}/\mathrm{p}$.[6] So what is it? We have *three* wavelengths now: the *Compton* wavelength λ_C (which is a circumference, actually), that weird horizontal distance λ, and the *de Broglie* wavelength λ_L. Can we make sense of that? We can. Let us first re-write the *de Broglie* wavelength:

$$\lambda_\mathrm{L} = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}v} = \frac{\mathrm{h}c^2}{\mathrm{E}v} = \frac{\mathrm{h}c}{\mathrm{E}\beta} = \frac{\mathrm{h}}{c}\cdot\frac{1}{\mathrm{m}\cdot\beta} = \frac{\mathrm{h}}{\mathrm{m}_0 c}\cdot\frac{1}{\gamma\beta}$$

What is this? We are not sure, but it might help us to see what happens to the *de Broglie* wavelength as m and v both increase as our electron picks up some momentum $\mathrm{p} = \mathrm{m}\cdot v$. Its wavelength must actually *decrease* as its (linear) momentum goes from zero to some much larger value – possibly infinity as v goes to c – but *how exactly*? The 1/γβ factor gives us the answer. That factor comes down from infinity (+∞) to zero as v goes from 0 to c or – what amounts to the same – if the relative velocity $\beta = v/c$ goes from 0 to 1. The graphs below show that works. The 1/γ factor is the circular arc that we're used to, while the 1/β function is just the regular inverse function ($y = 1/x$) over the domain $\beta = v/c$, which goes from 0 to 1 as v goes from 0 to c. Their product gives us the green curve which – as mentioned – comes down from +∞ to 0.

[3] See: http://vixra.org/abs/1901.0105.

[4] Hence, the C subscript stands for the C of Compton, not for the speed of light (c).

[5] Because it is a wave in two dimensions, we cannot really say there are crests or troughs, but the terminology might help you with the interpretation of the geometry here.

[6] The use of L as a subscript is a bit random but think of it as the L of Louis de Broglie.

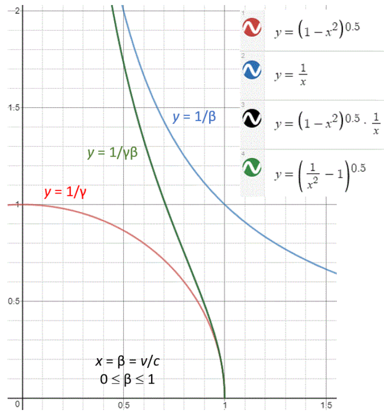

Figure 3: The 1/γ, 1/β and 1/γβ graphs

Now, we re-wrote the formula for *de Broglie* wavelength λ_L as the *product* of the 1/γβ factor and the *Compton* wavelength for *v* = 0:

$$\lambda_{\mathrm{L}} = \frac{\mathrm{h}}{\mathrm{m}_0 c} \cdot \frac{1}{\gamma\beta} = \frac{1}{\beta} \cdot \frac{\mathrm{h}}{\mathrm{m}c}$$

Hence, the *de Broglie* wavelength goes from +∞ to 0. We may wonder: when is it equal to λ_C = h/m*c*? Let's calculate that:

$$\lambda_{\mathrm{L}} = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}c} \cdot \frac{1}{\beta} = \lambda_C = \frac{\mathrm{h}}{\mathrm{m}c} \Leftrightarrow \beta = 1 \Leftrightarrow v = c$$

This is a rather weird result, and we have not yet fully interpreted its significance. Let's bring the third wavelength in: the λ = β·λ_C wavelength—which is that length between the crests or troughs of the wave.[7] We get the following two rather remarkable results:

$$\lambda_{\mathrm{L}} \cdot \lambda = \lambda_{\mathrm{L}} \cdot \beta \cdot \lambda_C = \frac{1}{\beta} \cdot \frac{\mathrm{h}}{\mathrm{m}c} \cdot \beta \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \lambda_C^2$$

$$\frac{\lambda}{\lambda_{\mathrm{L}}} = \frac{\beta \cdot \lambda_C}{\lambda} = \frac{\mathrm{p}}{\mathrm{h}} \cdot \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \frac{\mathrm{m}v^2}{\mathrm{m}c^2} = \beta^2$$

The product of the λ = β·λ_C wavelength and *de Broglie* wavelength is the square of the Compton wavelength, and their ratio is the square of the relative velocity β = *v*/*c*. – *always!* – and their ratio is equal to 1 – *always!* These two results are rather remarkable too but, despite their simplicity and apparent beauty, we are also struggling for an easy geometric interpretation. The use of *natural units* may help. Equating *c* to 1 would give us natural distance and time units, and equating *h* to 1 would give us a natural force unit—and, because of Newton's law, a natural mass unit as well. Why? Because Newton's F = m·*a* equation is relativistically correct: a force is that what gives some mass acceleration. Conversely, mass can be defined of the inertia to a change of its state of motion—because any change in motion involves a force and some acceleration. We write: m = **F**/***a***. If we re-define our distance, time and force units by equating *c* and *h* to 1, then the Compton wavelength (remember: it's a circumference, really) and the mass of our electron will have a simple inversely proportional relation:

[7] We should emphasize, once again, that our two-dimensional wave has no real crests or troughs: λ is just the distance between two points whose argument is the same—except for a phase factor equal to *n*·2π (*n* = 1, 2,…).

$$\lambda_C = \frac{1}{\gamma m_0} = \frac{1}{m}$$

We get equally simple formulas for he *de Broglie* wavelength and our λ wavelength:

$$\lambda_L = \frac{1}{\beta\gamma m_0} = \frac{1}{\beta m}$$

$$\lambda = \beta \cdot \lambda_C = \frac{\beta}{\gamma m_0} = \frac{\beta}{m}$$

This is quite deep: we have three *lengths* here – defining all of the geometry of the model – and they all depend on the *rest* mass of our object and its relative velocity *only*. Can we take this discussion any further? Perhaps, because what we have found may or may not be related to the idea that we're going to develop in the next section. However, before we move on to the next, let us quickly note the three equations – or lengths – are *not* mutually independent. They are related through that equation we found above:

$$\lambda_L \cdot \lambda = \lambda_C^2 = \frac{1}{m^2}$$

We'll let you play with that. To help you with that, you may start by noting that the $\lambda_L\lambda = 1/m^2$ reminds us of a property of an ellipse. Look at the illustration below.[8] The length of the chord – perpendicular to the major axis of an ellipse is referred to as the *latus rectum*. One half of that length is the actual *radius of curvature* of the osculating circles at the endpoints of the major axis.[9] We then have the usual distances along the major and minor axis (*a* and *b*). Now, one can show that the following formula has to be true:

$$a \cdot p = b^2$$

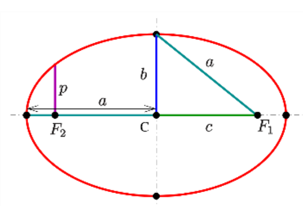

Figure 4: The *latus rectum* formula: $a \cdot p = b^2$

If you don't immediately see why this would be relevant, then… Well… Then you should look at it again. 😊

I want to add another idea here. In our previous papers[10], we suggested a couple of times that Planck's quantum of action h, which we associated with an elementary cycle, or – in its *reduced* form (ħ = h/2π) – with the fundamental unit of angular momentum, should, perhaps, be written as a vector quantity. It's a force times a circumference (or a radius or – more generally – some length) times a cycle time. A force is a vector quantity: it

[8] Source: Wikimedia Commons (By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=57428275).
[9] The endpoints are also known as the *vertices* of the ellipse. As for the concept of an osculating circles, that's the circle which, among all tangent circles at the given point, which approaches the curve most tightly. It was named *circulus osculans* – which is Latin for 'kissing circle' – by Gottfried Wilhelm Leibniz. You know him, right? Apart from being a polymath and a philosopher, he was also a great mathematician. In fact, he was the one who invented differential and integral calculus.
[10] See: http://vixra.org/author/jean_louis_van_belle

has a magnitude but it also has a *direction*. The linear momentum which appears in the second *de Broglie* relation for matter-waves is a vector quantity too—not because of the mass factor (m) but because of the velocity factor (***v***): **p** = m***v***. This makes it *very* tempting to write the second *de Broglie* relation (λ = h/p) as a vector equation:

$$\lambda = \frac{\mathbf{h}}{\mathbf{p}} = \frac{\vec{h}}{\vec{p}}$$

We would, therefore, also have to re-write the Uncertainty Principe—or the Uncertainty *Relation* as I prefer to refer to it. We are currently doing some research in this regard and it is all quite promising. For example, it provides a rather fresh perspective on the so-called random walk of an electron in free space and it may, therefore, explain Einstein's formula for it in a very different (but necessarily equivalent) way. However, we do not want to burden the reader with that at this point in time, because the mentioned research is rather immature at this point.

You may that vector equation looks weird, but it's not any different than writing Newton's force law as a vector equation:

$$\mathrm{m} = \frac{\mathbf{F}}{\boldsymbol{a}} = \frac{\vec{F}}{\vec{a}}$$

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