Transforming Candidate and Parties cardinal ratings into weak preference orderings

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PART I

INTRODUCTION:

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Introduction

- The importance of the **systems for electing our representatives** contrasts with the fact that their origin and evolution has been little studied, and that there are **some unsolved problems regarding their consistency**.

- In general terms, our current parliamentary model derives from the paradigm underlying the **Model Parliament** implemented in England at the end of the thirteen century [Simón de Monfort - Edward I]

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Introduction

- The system of election of representatives evolved, arriving to a widespread use of a system where countries were divided into multimember districts and **each citizen could cast as many votes as positions needed to be filled in each district** [Colomer, 2004; Urdánoz, 2009].

- This enabled **a well-organized group to get all the positions in a district** if that group was supported by the largest group of citizens, even if this support was well below 50% of the population.

Introduction

- Some voices began to challenge **this system that enabled a very plural society being *represented* by monocolor parliament**; minorities were left without representation, and the "majority" of representatives had often been voted by a small percentage of citizens.

- Some alternatives were proposed:
    - **Condorcet, 1793**: proposed a *limited vote* [2 votes per citizen in districts of up to 18 deputies]
    - **Saint-Just, 1793**: proposed a *single vote* being the whole France a single district.

Introduction

- In the nineteenth century numerous proposals for electoral reform were made, which can be grouped into **three different paths**:

  - One of them led to **systems based on quotas**, as the 'single transferable vote' [Gergonne, Andrae, Hare]

  - Another led to party lists **proportional representation systems** [d'Hondt, Sainte Lague, ...]

  - Another reduced the size of all the districts to one representative, leading to **majority systems** [First Past The Post].

Introduction

- In the twentieth century we witnessed the evolution of these three paths, and some small modifications were proposed that allowed correcting some defects :

  - Systems were created with **two levels [eg, Sweden]** to maximize the global proportionality votes seats.
  - **Legal thresholds [3%, 5% ...]** were established to limit parliamentary fragmentation
  - **Open lists** …
  - …

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Introduction

What happened along the twentieth century with the Paradoxes / Impossibilities that are discovered in the field of Social Choice?

Were not paradoxes found in the preference aggregation rules underlying the electoral systems?

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Introduction

- Surprisingly, **while in the Social Choice realm a large number of 'paradoxical' and impossibility results was published** [Nurmi, 2016]**, we find scarce studies reviewing this issue regarding the electoral systems:**

  - In 1949, Duncan Black published a first study that showed the electoral rules could be providing results not fitted to citizens preferences …
  - In 1978, Colman & Pountney, detected Borda's paradoxes in the UK general election.
  - Van Deemen [1993] and Kurrild-Klitgaard [2008] published results showing prevalence of Borda paradox in elections in Netherlands and Denmark.

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Introduction

- Where are we now?

Introduction

- Despite being a very important issue, the number of studies carried out is still very small, we hardly know the size of the problem.

- **In principle, any paradox found in the realm of Social Choice can be found in electoral systems**, since they use the same voting rules as a basis [van Deemen, 1993].

- However, since political preferences tend to be spatially structured, we cannot extrapolate random calculations, so **we do not know if it is a problem that occurs frequently or rarely.**

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Introduction

- Part of the difficulty in assessing it, derives from the **difficulty of obtaining the necesssary data for the analysis** [sometimes there is no data at all].

- In most of our societies, **single vote is used, so we do not know the actual citizens preference profile.**

Introduction

- The present proposal seeks **to facilitate undertaking this type of evaluation of the electoral systems, by providing an easy method to generate citizens preference profile** based on information from citizens surveys that can be relatively easily accessed/found .

Introduction

- Noteworthy, **our goal is not "determining the true preference profile of citizens" in an absolute sense, but generating a sufficiently 'likely' citizens preference profile, so we can contrast against it the allocation of seats that different electoral systems produce.**

PART II

ALGORITHM TO GENERATE PREFERENCE PROFILES

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Proposal

- We review the information available in the surveys of the Spain Center for Sociological Research [CIS]

- We have found two type of information on voters preferences in CIS surveys:

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Proposal

- In some surveys, we find the average value of voters preference [0-10] for each candidate or political party:

Pregunta 37

Le voy a citar ahora los nombres de algunos/as líderes políticos/as. Le agradecería me indicara con respecto a cad política. Puntúelos/as de 0 a 10, sabiendo que el 0 significa que 'lo/la valora muy mal' y el 10 que 'lo/la valora mu

	TOTAL	PP	PSOE	Unidos Podemos	C's	En Comú Podem	Compromís-Podemos-EUPV	ERC	CDC
Alberto Garzón									
Valora	74,8	78,8	79,2	91,1	81,7	85,5	92,7	,2	60
No conoce	10,9	9,8	10,6	4,4	7,5	3,8	3,2	6,6	21
N.S.	12,6	11,4	9,9	4,0	8	9,7	3,2	16,8	15
N.C.	1,8	0,1	0,3	0,5	1,0	1,1	0,8	1,5	1
(N)	(6.175)	(1.196)	(1.089)	(654)	(518)	(186)	(124)	(137)	(6
Media	4,50	2,56	4,84	7,26	3,93	6,59	7,18	5,12	4,
Desviación típica	2,94	2,40	2,48	2,23	2,49	2,38	2,07	2,51	2,
(N)	(4.616)	(942)	(863)	(596)	(423)	(159)	(115)	(103)	(3

Pregunta 37

Le voy a citar ahora los nombres de algur política. Puntúelos/as de 0 a 10, sabiendo

	TOTAL	PP	P
Alberto Garzón			
Valora	74,8	78,8	
No conoce	10,9	9,8	
N.S.	12,6	11,4	
N.C.	1,8	0,1	
(N)	(6.175)	(1.196)	
Media	4,50	2,56	
Desviación típica	2,94	2,40	
(N)	(4.616)	(942)	

Proposal

- In other surveys we find lexical scales that we can transform to obtain an average value [e.g., CIS. ES 3087. Madrid]

Pregunta 23

Respecto a los siguientes partidos políticos, me gustaría que me dijera si se encuentra muy cercano, cercano, ni c... o muy distante de cada uno de ellos.

| | | | | | Recuerdo de voto en las elecciones autonómicas de la Comunidad de Madrid | | | | | |
	TOTAL	PP	PSOE	Podemos	C's	IUCM-LV	UPyD	Otro partido	Voto nulo	E...
PP										
Muy cercano	3,4	17,4	-	-	1,1	-	-	-	-	
Cercano	15,7	52,7	2,3	0,5	14,6	-	20,0	24,1		
Ni cercano ni distante	16,0	22,2	5,7	4,9	41,6	-	,0	10,3	,0	
Distante	15,3	4,2	18,9	12,0		8,7	50,0	31,0	10,0	
Muy distante	43,5	1,8	,6	,0	20,2	91,3	20,0	31,0	40,0	
N.S.	2,8	1,8	0,6	1,1	-	-	-	3,4	-	
N.C.	3,3	-	-	0,5	-	-	-	-	-	
(N)	(973)	(167)	(175)	(184)	(89)	(23)	(10)	(29)	(10)	

Pregunta 23

Respecto a los siguientes partidos políticos, m... o muy distante de cada uno de ellos.

	TOTAL	PP	PS...
PP			
Muy cercano	3,4	17,4	
Cercano	15,7	52,7	
Ni cercano ni distante	16,0	22,2	
Distante	15,3	4,2	
Muy distante	43,5	1,8	
N.S.	2,8	1,8	
N.C.	3,3	-	
(N)	(973)	(167)	

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Proposal

- From the above information **one way to build the preference profile is to consider that if the average preference value of some party voters for another party P_1 is greater than for another party P_2, then EVERY ONE AND ALL OF THAT PARTY VOTERS strictly prefer P_1 to P_2:**

$$v[P_1] > v[P_2] \leftrightarrow P_1 \succ P_2$$

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Proposal

- For example: If we review the data of "sympathy for the leaders of other political parties" in the 2016 post-election survey, the average of the sympathy value expressed by UP voters is as follows:

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	6.99	4.59	3.89	3.63	2.97	2.78	1.39

- We can then model UP voters' preference as:

$$UP \succ EH \succ PNV \succ PSOE \succ CCa \succ Cs \succ PP$$

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Proposal

- If we express the vote of a UP voter as a Pairwise Comparison Matrix [PCM], we obtain:

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	-	1	1	1	1	1	1
EH-BILDU	0	-	1	1	1	1	1
EAJ-PNV	0	0	-	1	1	1	1
PSOE	0	0	0	-	1	1	1
CCa	0	0	0	0	-	1	1
C's	0	0	0	0	0	-	1
PP	0	0	0	0	0	0	-

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Proposal

- The issue with this approach is that it seems acceptable when the difference between the average rating for two policy options is very different, but it does not seem acceptable when the difference is very small.

- For example, if we check the preference of PSOE voters regarding the leaders of CDC, ERC and PP, they are practically equal:

	PSOE	CP	CCa	C's	ECP	EM	UP	EAJ-PNV	CDC	ERC	PP	EH-BILDU
PSOE	6.77	4.67	4.53	4.23	3.51	3.40	3.07	2.50	2.09	2.08	2.07	2.00

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Proposal

- Data shows PSOE voters are mostly indifferent between these parties.

- How can we model this quasi-indifference of the voters regarding two parties?

Proposal

- We have previously stated that

$$v[P_1] > v[P_2] \leftrightarrow P_1 \succ P_2$$

- But in some [exceptional] cases we can find that two options are equally preferred, which in turn would imply:

$$v[P_1] = v[P_2] \leftrightarrow P_1 \sim P_2$$

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Proposal

- From a binary paradigm it has been argued that the two previous equations are excluding that is:

$$v[P_1] > v[P_2] \leftrightarrow P_1 \succ P_2$$
$$\vee$$
$$v[P_1] = v[P_2] \leftrightarrow P_1 \sim P_2$$

- **This raises the problem of how to model preference orderings when the different valuation between the eligible options is almost zero.**

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Proposal

- Herein we propose an algorithm based on **considering that when the difference between the evaluation of two options is very small, it implies both a degree of strict preference and a degree of indifference:**

- We establish that **a preferential difference lower than 1 implies a degree of indifference**. We then have three possible situations:

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Proposal

- **Strict Indifference**

$$v(A) - v(B) = 0 \rightarrow A \sim B$$

- **Strict Preference**

$$v(A) - v(B) \geq 1 \rightarrow A \succ B$$

- **Partial Indifference and Partial Preference**

$$1 > v(A) - v(B) > 0 \rightarrow \left(1 - \left(v(A) - v(B)\right)\right)(A \sim B)$$
$$\wedge \ (v(A) - v(B))(A \succ B)$$

Proposal

- The underlying paradigm is not adapting the order of preference according to the intensity of preference, but according to the degree to which the statement "X is preferred to Y" is fulfilled for each pair of eligible options.

- It is considered that **if the intensity differential of stated preference is greater than 1, then the statement is completely true.**

- But **if the intensity differential of stated preference is lower than 1, then the assertion is partially false.**

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Proposal

- That is, **if the average preference differential between two parties is less than 1, we assume that there is a percentage of voters with a strict preference between both parties, but that there is also a percentage of voters who are strictly indifferent among them.**

- This last percentage increases up to 100% of voters maintaining indifference [when the difference of Valuation approaches zero] while the first one increases up to 100% of the voters holding strict preference [if the difference of Valuation approaches1].

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Proposal

- Following this approach, we can easily obtain the PCM in two steps. Firstly, we draw a V Matrix expressing the difference of valuation between each pair of options:

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	-	2.40	3.10	3.36	4.02	4.21	5.60
EH-BILDU		-	0,70	0,96	1,62	1,81	3,20
EAJ-PNV			-	0,26	0,92	1,11	2,50
PSOE				-	0,66	0,85	2,24
CCa					-	0,19	1,58
C's						-	1.39
PP							-

Proposal

- Building on V, we compute PCM using below formulas:

- Yellow cells: Equation 01:

$$Iij = \min\left[\max\left[Vij; \frac{[1 - Vij]}{2} + Vij\right]; 1\right]$$

- Grey cells: Equation 02 [complement]

$$Iji = 1 - Iij$$

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	-						
EH-BILDU		-					
EAJ-PNV			-				
PSOE				-			
CCa					-		
C's						-	
PP							-

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Proposal

- Using the above formulas, we obtain the PCM:

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	-	1,00	1,00	1,00	1,00	1,00	1,00
EH-BILDU	0,00	-	0,85	0,98	1,00	1,00	1,00
EAJ-PNV	0,00	0,15	-	0,63	0,96	1,00	1,00
PSOE	0,00	0,02	0,37	-	0,83	0,93	1,00
CCa	0,00	0,00	0,04	0,17	-	0,60	1,00
C's	0,00	0,00	0,00	0,08	0,41	-	1,00
PP	0,00	0,00	0,00	0,00	0,00	0,00	-

- Since we had arranged the options from the best to the worst valued, in the upper right half there are always higher or [in case of strict indifference] equal values than in the lower left half.

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Proposal

- To see the difference between both approaches, it is useful to compare the PCM generated for a UP typical voter, considering both binary and fuzzy preferences:

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	-	1	1	1	1	1	1
EH-BILDU	0	-	1	1	1	1	1
EAJ-PNV	0	0	-	1	1	1	1
PSOE	0	0	0	-	1	1	1
CCa	0	0	0	0	-	1	1
C's	0	0	0	0	0	-	1
PP	0	0	0	0	0	0	-

	UP	EH-BILDU	EAJ-PNV	PSOE	CCa	C's	PP
UP	-	1	1	1	1	1	1
EH-BILDU	0	-	0,85	0,98	1	1	1
EAJ-PNV	0	0,15	-	0,63	0,96	1	1
PSOE	0	0,02	0,37	-	0,83	0,93	1
CCa	0	0	0,04	0,17	-	0,60	1
C's	0	0	0	0,08	0,41	-	1
PP	0	0	0	0	0	0	-

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Proposal

- Herein proposed algorithm **allows us to assess that often the preference of a group of voters between two parties can be similar, that is, it can imply a degree of indifference**.

- It is important to emphasize that this prodecure **is not based on considering interpersonal comparisons of utility but on the concepts of degree of truth and complementary value.**

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Proposal

- Once the procedure has been explained, we review two examples of real elections.

PART III

REVIEW OF TWO ELECTIONS

evaluation real elections

- First, we review **Community of Madrid elections 2015**

- We use **two contrasting criteria**:

  - We check whether the **seat assignment to parties satisfies the Condorcet winner criterion**.

  - We check the **correlation between allocation of seats and citizens collective preference for parties ...** [Miller & Stokes, 1963; Alvira, 2017]. Since we adapt this last criterion slightly, we explain it briefly below.

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evaluation real elections

- To model the collective preference, we first place the voters according to their self-location in each relevant dimension ...

LEFT.RIGHT [Es 3087.p. 34]

VOTERS SELF PLACEMENT [%]

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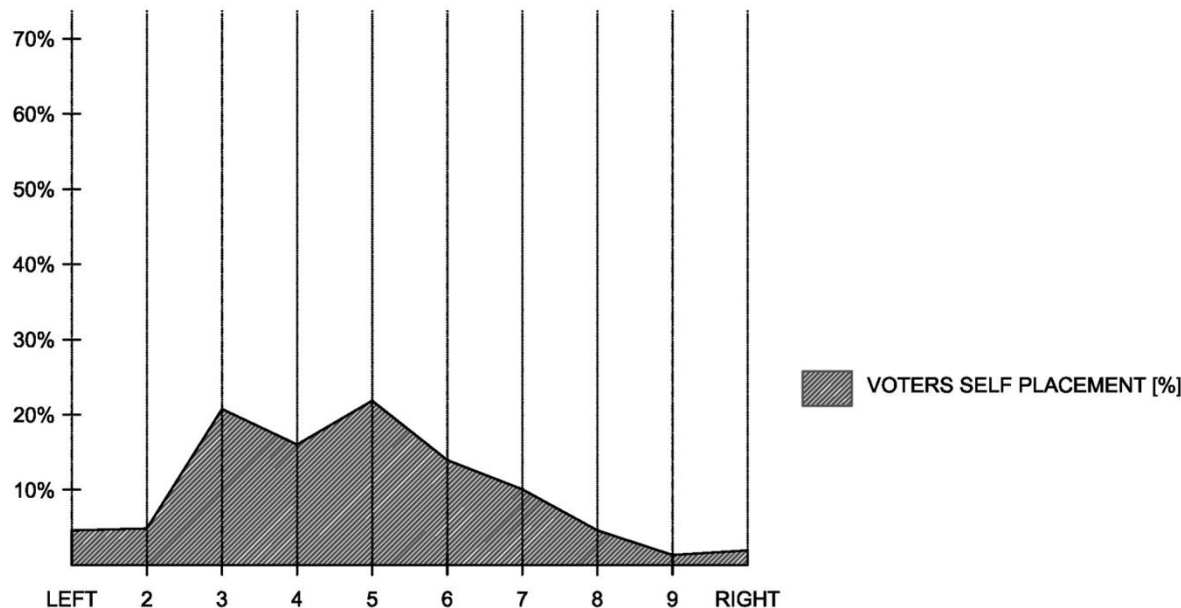
evaluation real elections

- … secondly, we draw the collective preference for each point of the dimension, assuming preferences are single peaked, so the collective preference increases from the extremes to the median voter [Black, 1948]
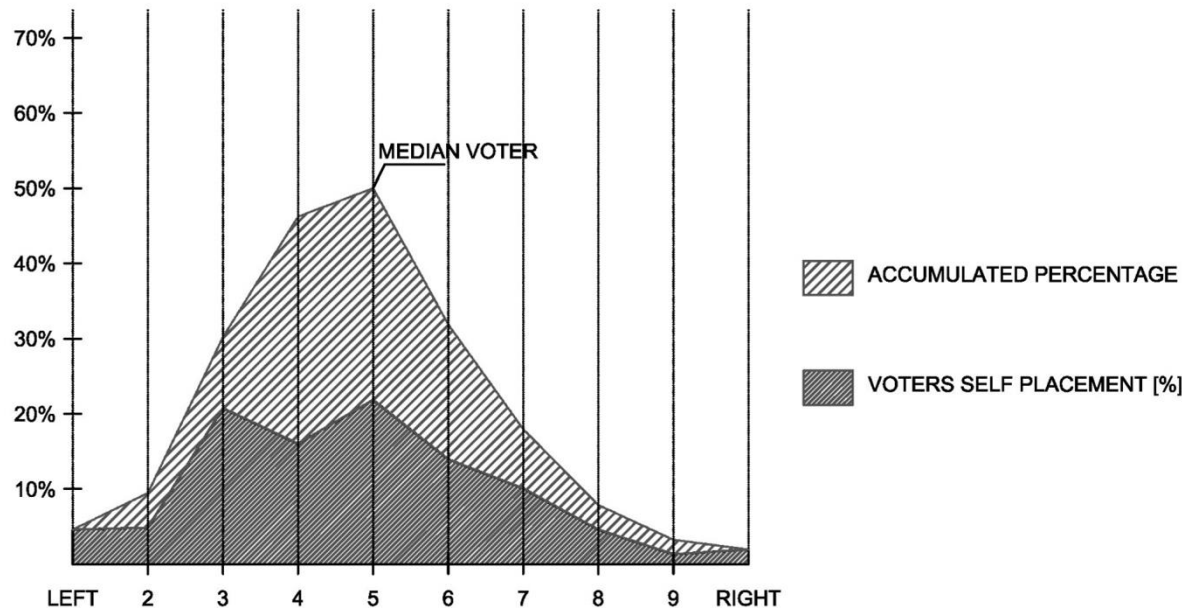
evaluation real elections

- Finally, we locate the parties in the dimension, and compare the percentage of seats that each one receives with the collective preference of the citizenship for the location of said party in the ideological space.
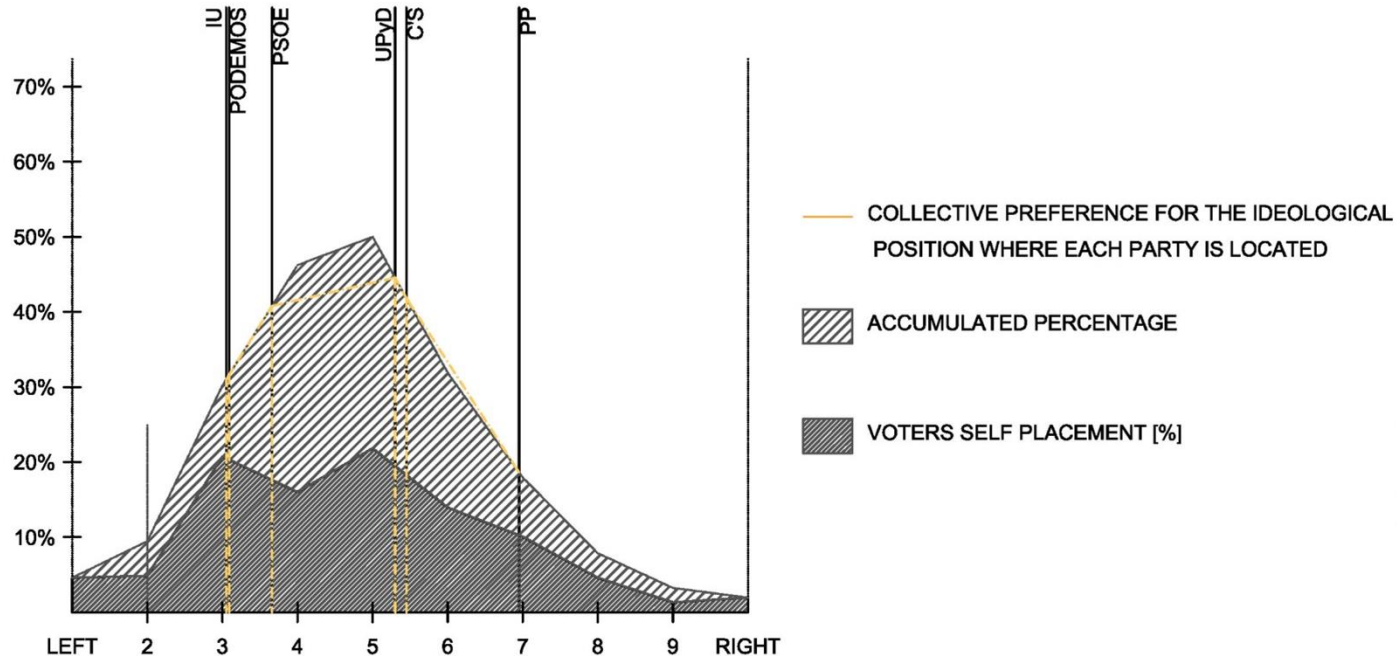
evaluation real elections

- Once the criterion has been explained, let us review the elections. We also briefly review the current electoral system in the Community of Madrid.

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evaluation: Madrid 2015.

- The *Ley 11/1986, de 16 de diciembre, Electoral de la Comunidad de Madrid* defines the electoral system [Art 18]:

  1. The electoral district is the Community of Madrid.

  2. For the distribution of seats, only the lists that have obtained at least 5% of total valid votes are taken into account.

  3. The allocation of seats shall be made in the manner established by the Ley Orgánica de Régimen Electoral General [LOREG] for the allocation of seats in the Congress at provincies level [i.e., using d'Hondt rule].

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evaluation: Madrid 2015.

- In 2015, this electoral system leads to the following allocation of seats:

			SEATS	
			No.	%
PP	1,050,256	33.08%	**48**	37,21%
PSOE	807,385	25,43%	**37**	28,68%
PODEMOS	591,697	18,64%	**27**	20,93%
C's	385,836	12,15%	**17**	13,18%
IUCM - LV	132,207	4,16%	-	-
UPyD	64,643	2,04%	-	-
			129	

- Therefore, according to Law 11/1986, collective preference ordering of political parties is:

$$PP \succ PSOE \succ PODEMOS \succ Cs \succ IUCM - LV \sim UPyD$$

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evaluation: Madrid 2015. Pr.01

- **Pr. 1:** We use the CIS survey ES.3087. Q23: "How close do you feel to each political party?". We assign to each position the following value:

Very close	10
Close	7.5
Neither close nor distant	5
Distant	2.5
Very distant	0

- We calculate for each group of voters the average valuation of each party. For example, the assessment of the parties according to UPyD voters:

8.00		4.75		3,25		3,25		2,50		2,25
UPyD	>	C's	>	PP	~	PSOE	>	IUCM - LV	>	PODEMOS

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evaluation: Madrid 2015. Pr.01

- Consequently, we obtain for each party voters a PCM that implies only strict preference or strict indifference. For example, the matrix for a typical UPyD voter is:

	UPyD	C's	PP	PSOE	IUCM - LV	PODEMOS
UPyD	-	1,00	1,00	1,00	1,00	1,00
C's	0,00	-	1,00	1,00	1,00	1,00
PP	0,00	0,00	-	0,50	1,00	1,00
PSOE	0,00	0,00	0,50	-	1,00	1,00
IUCM - LV	0,00	0,00	0,00	0,00	-	1,00
PODEMOS	0,00	0,00	0,00	0,00	0,00	-

evaluation: Madrid 2015. Pr.01

- If we multiply the number of voters of each party by the PCM defined for its voters, we obtain the following PCM [we rearrange the parties according to Left –Right Dimension and we reduce the legal threshold to 3%]

	IUCM-LV	Podemos	PSOE	C's	PP
IUCM-LV	-	**2.440.327**	723.904	**1.531.289**	**1.531.289**
Podemos	591.697	-	723.904	**1.531.289**	**1.531.289**
PSOE	**2.308.120**	**2.308.120**	-	**1.531.289**	**1.563.611**
C's	1.500.735	1.500.735	1.500.735	-	**1.981.768**
PP	1.500.735	1.500.735	1.468.414	1.050.256	

- Therefore, the collective preference ordering is:

$$PSOE \succ IUCM-LV \succ PODEMOS \succ Cs \succ PP$$

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evaluation: Madrid 2015. Pr.01

- Using above matrix, we can obtain the seat allocations [in percentage] that would produce other rules. For example, using **Borda Count** [Borda, 1771] and **Prll** [Alvira, 2016]:

	BORDA	Prll
IUCM - LV	20,54%	20,39%
PODEMOS	14,44%	20,00%
PSOE	25,43%	20,78%
C's	21,38%	19,61%
PP	18,21%	19,22%

- According to these rules, the collective preference ordering of the parties is:

BORDA: $$PSOE \succ Cs \succ IUCM - LV \succ PP \succ PODEMOS$$

PRLL: $$PSOE \succ IUCM - LV \succ PODEMOS \succ Cs \succ PP$$

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evaluation: Madrid 2015. Pr.01

- On the other hand, we can calculate the allocation of seats using **Score Voting** [Heckscher, 1896], adding up the average valuations that each party voters make of the other parties and multiplying by the number of votes. We obtain the following allocation of seats [in percentage]:

IUCM - LV	PODEMOS	PSOE	C's	PP
18,09%	18,96%	22,63%	22,10%	18,22%

- Therefore, according to this rule the collective preference ordering of the parties is:

$$PSOE \succ Cs \succ PODEMOS \succ PP \succ IUCM - LV$$

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evaluation: Madrid 2015. Pr.01

- Consistency of results: **collective preference orderings**

evaluation: Madrid 2015. Pr.01

	LOREG	SCORE VOTING	BORDA Pr 1	Prll Pr 1
IU	0,00%	18,09%	20,54%	20,39%
PODEMOS	20,93%	18,96%	14,44%	20,00%
PSOE	28,68%	22,63%	25,43%	20,78%
C's	13,18%	22,10%	21,38%	19,61%
PP	37,21%	18,22%	18,21%	19,22%
	1,000	1,000	1,000	1,000

LAW 11/1986	PP	>	PSOE	>	PODEMOS	>	C's	>	IUCM - LV
SCORE VOTING	PSOE	>	C's	>	PODEMOS	>	PP	>	IUCM - LV
BORDA	PSOE	>	C's	>	IUCM - LV	>	PP	>	PODEMOS
Prll	PSOE	>	IUCM - LV	>	PODEMOS	>	C's	>	PP

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evaluation: Madrid 2015. Pr.01

- Consistency of results: **collective preference orderings**

- We see a **strong Borda Paradox: the most preferred option according to the *Law 11/1986* is the actual Condorcet Loser**

- However, it is striking that the Condorcet consistent rule [PrII] places 2nd and 3rd places away from the median voter, challenging the validity of the results …

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evaluation: Madrid 2015. Pr.01

- Consistency of results: **correlation between collective preference and allocation of seats to parties**

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evaluation: Madrid 2015. Pr.01

Left- Right Dimension
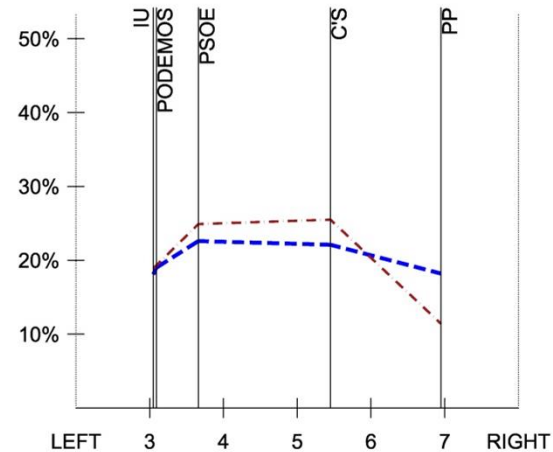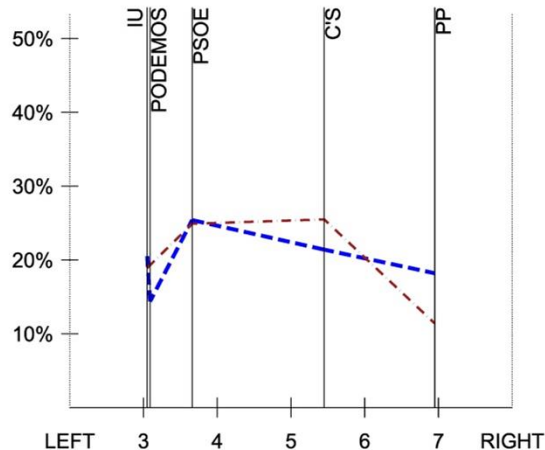
evaluation: Madrid 2015. Pr.01

Territorial Model
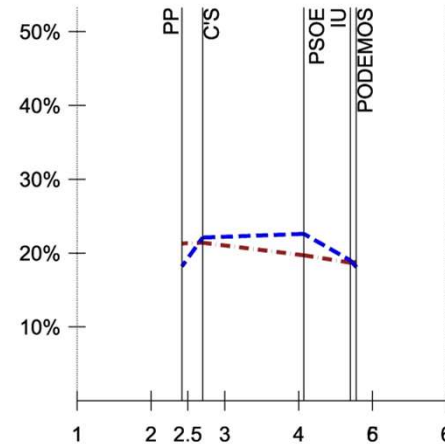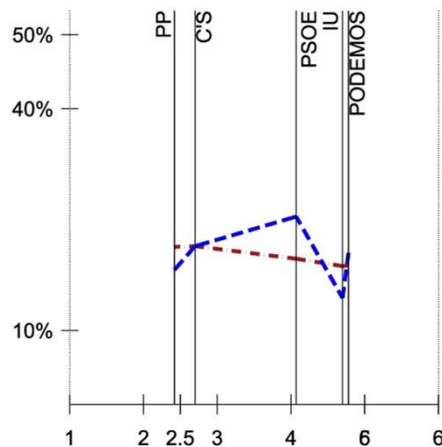
evaluation: Madrid 2015.

[correlation results are shown/reviewed later]

evaluation: Madrid 2015.

- Next, we review the allocation of seats using the proposed algorithm [Pr 02] for modeling the preference profile:

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evaluation: Madrid 2015. Pr.02

- Using Pr02 small preference differentials are translated into some percentage of indifference. For example, the typical PSOE voter PCM according binary [Pr 01] an fuzzy [Pr02] are:

Pr 01

	PSOE	IUCM - LV	PODEMOS	C's	UPyD	PP
PSOE	-	1,00	1,00	1,00	1,00	1,00
IUCM - LV	0,00	-	1,00	1,00	1,00	1,00
PODEMOS	0,00	0,00	-	1,00	1,00	1,00
C's	0,00	0,00	0,00	-	1,00	1,00
UPyD	0,00	0,00	0,00	0,00	-	1,00
PP	0,00	0,00	0,00	0,00	0,00	-

Pr02

	PSOE	IUCM - LV	PODEMOS	C's	UPyD	PP
PSOE	-	1,00	1,00	1,00	1,00	1,00
IUCM - LV	0,00	-	0,52	1,00	1,00	1,00
PODEMOS	0,00	0,48	-	1,00	1,00	1,00
C's	0,00	0,00	0,00	-	0,88	1,00
UPyD	0,00	0,00	0,00	0,12	-	1,00
PP	0,00	0,00	0,00	0,00	0,00	-

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evaluation: Madrid 2015. Pr.02

- And the PCMs for the whole set of voters are:

Pr 01 [binary]

	IUCM-LV	Podemos	PSOE	C's	PP
IUCM-LV	-	**2.440.327**	723.904	**1.531.289**	**1.531.289**
Podemos	591.697	-	723.904	**1.531.289**	**1.531.289**
PSOE	**2.308.120**	**2.308.120**	-	**1.531.289**	**1.563.611**
C's	1.500.735	1.500.735	1.500.735	-	**1.981.768**
PP	1.500.735	1.500.735	1.468.414	1.050.256	

Pr.02 [fuzzy]

	IUCM-LV	Podemos	PSOE	C's	PP
IUCM-LV	-	1.501.088	731.984	**1.531.289**	**1.539.369**
Podemos	**1.530.936**	-	723.904	**1.531.289**	**1.531.289**
PSOE	**2.309.040**	**2.308.120**	-	1.494.102	**1.626.309**
C's	1.500.735	1.500.735	**1.537.922**	-	**1.981.768**
PP	1.492.655	1.500.735	1.405.715	1.050.256	

59

evaluation: Madrid 2015. Pr.02

- Noteworthy, by modeling fuzzy preferences [Pr 02] **a cyclical majority relationship [Condorcet paradox] appears involving 4 options**

60

evaluation: Madrid 2015. Pr.02

- We evaluate the consistency of the results. We begin with the preference orderings

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evaluation: Madrid 2015. Pr.02

	LEY 11/1986	SCORE VOTING	BORDA Pr 2	Prll Pr 2
IUCM - LV	-	18,09%	17,49%	14,05%
PODEMOS	20,93%	18,96%	17,54%	14,33%
PSOE	28,68%	22,63%	25,49%	28,72%
C's	13,18%	22,10%	21,51%	29,12%
PP	37,21%	18,22%	17,97%	13,77%

LEY 11/1986	PP	>	PSOE	>	PODEMOS	>	C's	>	IUCM - LV
SCORE VOTING	PSOE	>	C's	>	PODEMOS	>	PP	>	IUCM - LV
BORDA	PSOE	>	C's	>	PP	>	PODEMOS	>	IUCM - LV
Prll	C's	>	PSOE	>	PODEMOS	>	IUCM - LV	>	PP

[……………………………………majority cycle ……………………………………]

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evaluation: Madrid 2015. Pr. 02

- Consistency of results: **collective preference orderings**

- Again, there is a strong **Borda Paradox: the most preferred option according to Law 11/1986 is the Condorcet Loser**

- The cyclical majority prevents us from establishing the Condorcet ordering of the other parties [PP would still be the least preferred option, since it is located below the Smith Set].

- The two closest parties to the median voter appear in 1st and 2nd position, showing more consistency than in the modeling of binary preferences, Pr. 01

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evaluation: Madrid 2015. Pr.02

- Consistency of results: **correlation between collective preference and allocation of seats to parties**

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evaluation: Madrid 2015. Pr.02

Left-Right Dimension Dcha.

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evaluation: Madrid 2015. Pr.02

Territorial Model
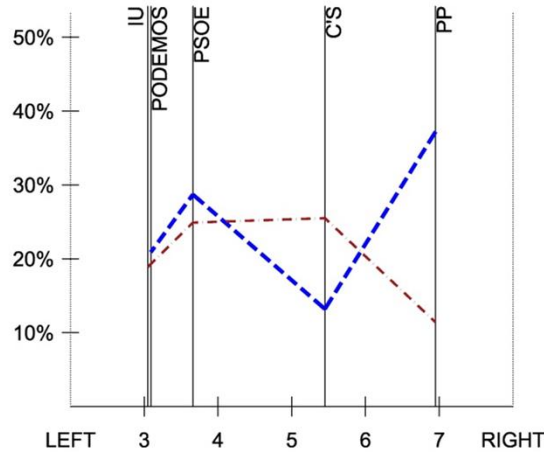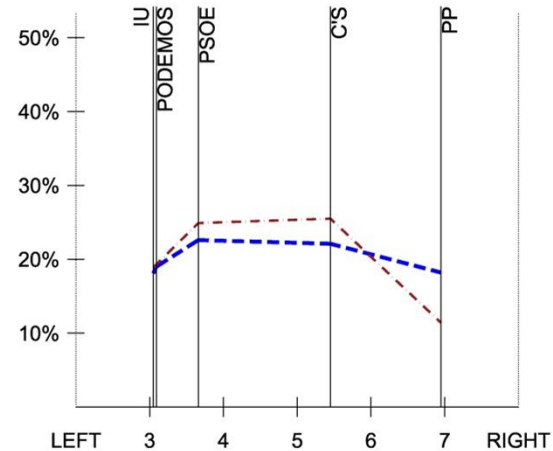
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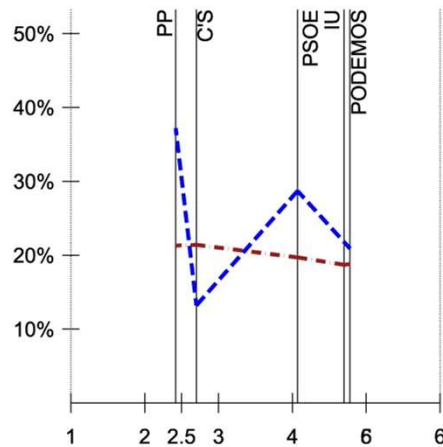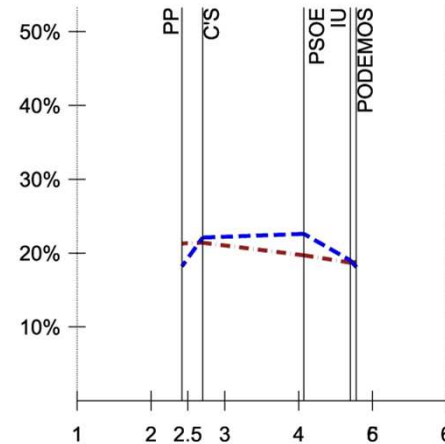
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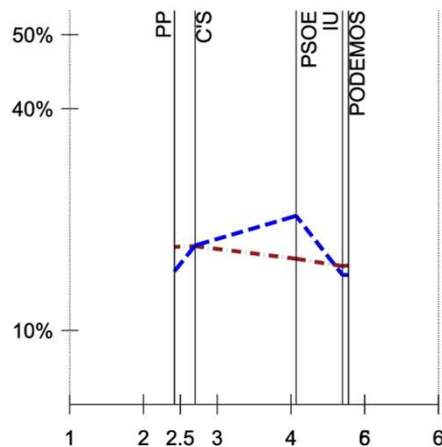
BORDA PR 2

PRLL PR 2
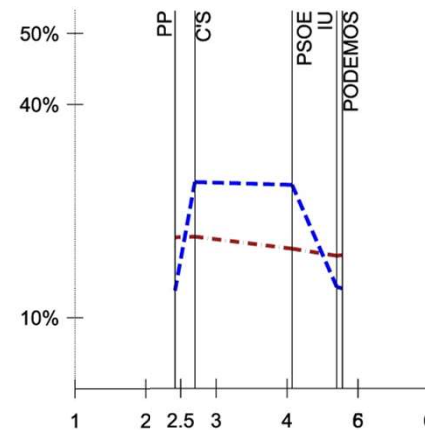
- - - CITIZENS COLLECTIVE PREFERENCE
- - - SEATS ALLOCATION TO PARTIES

evaluation: Madrid 2015.

- **Correlation between allocation of seats and collective preference:**

	LEY 11/1986	SCORE VOTING	BORDA		Prll	
			Pr. 01	Pr. 02	Pr. 01	Pr. 02
LEFT RIGHT	-0,376	0,850	0,571	0,714	0,565	0,844
TERRIT. MOD.	0,461	0,287	0,162	0,193	-0,731	0,366
MEDIA	**0,043**	**0,043**	**0,366**	**0,454**	**-0.083**	**0.605**

- **Correlation between seat allocation and Score Voting** [collective utility maximization]

	LEY 11/1986	BORDA		Prll	
		Pr. 01	Pr. 02	Pr. 01	Pr. 02
SCORE VOTING	0,121	0,707	0,932	0,347	0,986

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evaluation: Madrid 2015.

- We see, **correlations between allocations of seats by Borda and PrII and citizens collective preference notably improves by modelling preference profile following the fuzzy procedure** [Pr 02]. **Collective utility maximization also increases** for the fuzzy procedure. Both issues sustain the **validity of herein proposed procedure, Pr 02.**

- On the other hand, **we see current LEY 11/1986 leads to an almost nil correlation between allocation of seats and citizens collective preference, as well as to almost zero collective utility. This greatly challenges current electoral system.**

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evaluation: Catalonia 2017

- Next, we review the **regional elections of the Community of Catalonia in December 2017**

evaluation: Catalonia 2017

- The current electoral law establishes the province as a circumscription, and allocation of seats through the d'Hondt rule. In the 2017 elections, results were:

CANDIDATE	VOTES		DEPUTIES	
	No.	%	No.	
CUP	195.246	4,53%	4	2,96%
ERC-CATSI	935.861	21,72%	32	23,70%
CATCOMU	326,360	7,58%	8	5,93%
PSC-PSOE	606,659	14,08%	17	12,59%
JUNTSXCAT	948,233	22,01%	34	25,19%
C's	1,109,732	25,76%	36	26,67%
PP	185.670	4,31%	4	2,96%
	4,307,761	100,00%	135	

- Therefore, the collective preference ordering of the parties is:

$$Cs \succ JxCat \succ ERC \succ PSC \succ CatComu \succ CUP \succ PP$$

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evaluation: Catalonia 2017

- For modeling the preference profile, we use CIS survey ES.3202. Q26: voters perceived distance to each political party.

evaluation: Catalonia 2017 Pr. 01

- If we consider strict preferences we obtain the following PCM:

	CUP	CatComú-Podem	ERC-CatSí	PSC-PSOE	JUNTSxCAT	C's	PP
CUP	-	2.079.340	195.246	2.444.443	521.606	2.444.443	3.051.102
CatComú-Podem	2.267.164	-	2.228.421	2.444.443	2.081.494	2.444.443	3.051.102
ERC-CatSí	4.151.258	2.118.083	-	2.444.443	1.496.210	2.444.443	3.051.102
PSC-PSOE	1.902.061	1.902.061	1.902.061	-	2.228.421	3.012.359	4.160.834
JUNTSxCAT	3.824.898	2.265.010	2.850.294	2.118.083	-	2.444.443	3.051.102
C's	1.902.061	1.902.061	1.902.061	1.334.145	1.902.061	-	4.160.834
PP	1.295.402	1.295.402	1.295.402	185.670	1.295.402	185.670	-

- Therefore, the collective preference ordering of the parties is:

$$[JxCat \succ ERC \succ PSC \succ CatComu \succ CUP] \succ Cs \succ PP$$

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evaluation: Catalonia 2017. Pr. 1

- Therefore, **following the binary modeling of preferences, a cyclical majority relationship [Condorcet Paradox] appears involving 5 options**

evaluation: Catalonia 2017. Pr. 1

- From the previous matrix, we obtain the following allocation of seats [in percentage] using **Borda and PrII rules**:

	BORDA	PrII
CUP	11,76%	15,64%
ERC-CATSI	17,21%	17,69%
CATCOMU	15,90%	18,11%
PSC-PSOE	16,55%	13,59%
JUNTSXCAT	18,14%	18,53%
C's	14,36%	11,54%
PP	6,08%	4,90%

- Therefore, the collective preference ordering of the parties are:

BORDA: $JxCAT \succ ERC \succ PSC \succ CatComu \succ Cs \succ CUP \succ PP$

PRLL: $JxCAT \succ CatComu \succ ERC \succ CUP \succ PSC \succ Cs \succ PP$

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evaluation: Catalonia 2017. Pr. 1

- On the other hand, if we add up the average valuations that each party voters make of the other parties and multiply by the number of votes [**score voting**] we obtain the following allocation of seats [in percentage]:

C's	JxCAT	ERC-CATSI	PSC-PSOE	CatComú	CUP	PP
0,141	0,204	0,228	0,133	0,123	0,120	0,052

- Therefore, the collective preference ordering of the parties is:

$$ERC \succ JxCAT \succ Cs \succ PSC \succ CatComu \succ CUP \succ PP$$

evaluation: Catalonia 2017. Pr. 1

- Consistency of results: **collective preference orderings**

evaluation: Catalonia 2017. Pr. 1

	LOREG	SCORE	BORDA PR.1	PRLL PR.1
CUP	2,96%	12,01%	11,76%	15,64%
ERC-CATSI	23,70%	22,77%	17,21%	17,69%
CATCOMU	5,93%	12,30%	15,90%	18,11%
PSC-PSOE	12,59%	13,27%	16,55%	13,59%
JUNTSXCAT	25,19%	20,41%	18,14%	18,53%
C's	26,67%	14,09%	14,36%	11,54%
PP	2,96%	5,15%	6,08%	4,90%

LOREG	C's >	JxCAT >	ERC >	PSC >	Catcomu >	CUP >	PP
SCORE VOTING	ERC >	JxCAT >	C's >	PSC >	Catcomu >	CUP >	PP
BORDA	JxCAT >	ERC >	PSC >	Catcomu >	C's >	CUP >	PP
Prll	JxCAT >	Catcomu >	ERC >	CUP >	PSC >	C's >	PP

[...majority cycle ....]

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evaluation: Catalonia 2017. Pr. 1

- Consistency of results: **collective preference orderings**

- We see an almost strong **Borda Paradox: the most preferred option according to the LOREG would be in fact the preferred option in penultimate place** [although a cycle appears, C's is located below the Smith set, being only strictly preferred to the PP].

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evaluation: Catalonia 2017. Pr. 1

- Consistency of results: **correlation between collective preference and allocation of seats to parties**

evaluation: Catalonia 2017. Pr. 1

Left- Right Dimension

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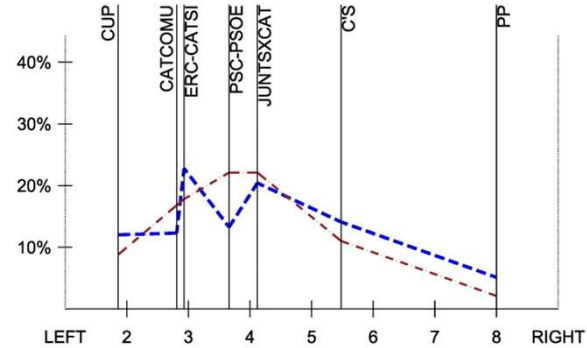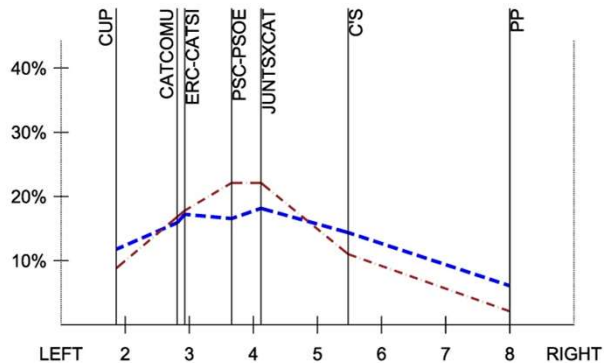
evaluation: Catalonia 2017. Pr. 1

Territorial Model

LOREG
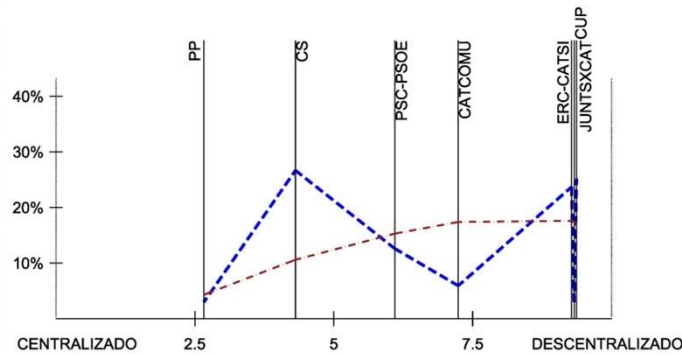
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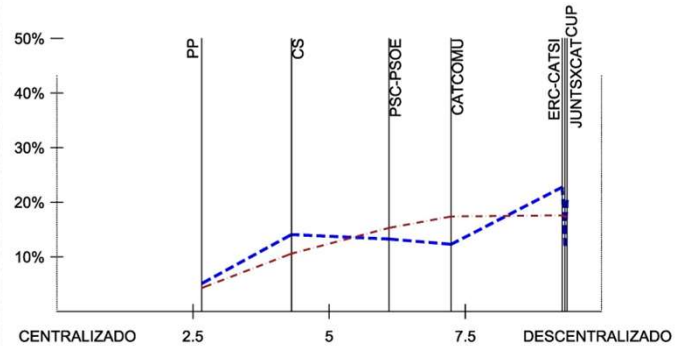
BORDA PR.01
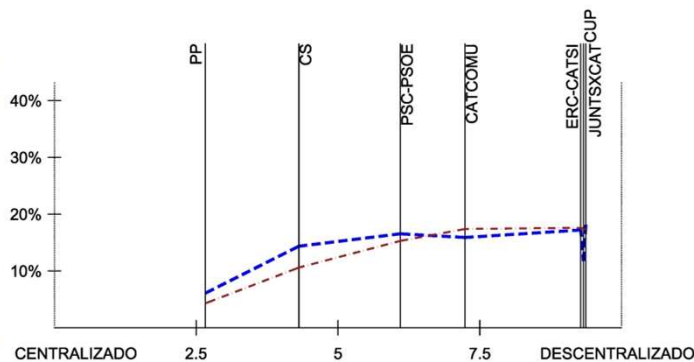
PRLL PR.01

- - - CITIZENS COLLECTIVE PREFERENCE
- - - SEATS ALLOCATION TO PARTIES

RICARDO ALVIRA **2018**
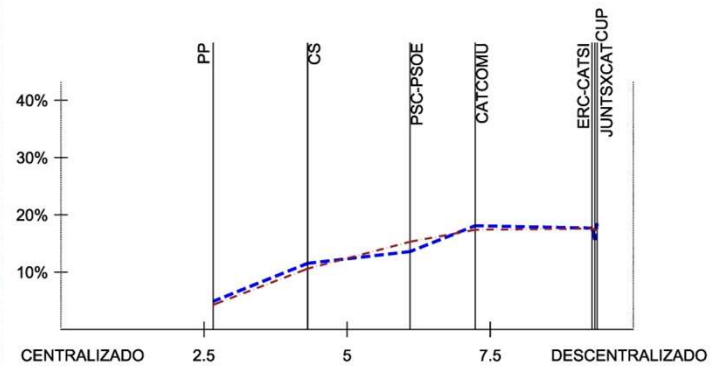
evaluation: Catalonia 2017

[correlation results are shown/reviewed later]

evaluation: Catalonia 2017

- Next, we review the allocation of seats using the proposed algorithm [Pr 02]

evaluation: Catalonia 2017. Pr. 2

- We obtain the following PCM:

	CUP	CatComú-Podem	ERC-CatSí	PSC-PSOE	JUNTSxCAT	C's	PP
CUP	-	**2.226.363**	913.157	**2.363.520**	935.898	**2.434.515**	**2.788.044**
CatComú-Podem	2.120.141	-	1.743.683	**2.238.345**	1.631.387	**2.746.249**	**3.346.433**
ERC-CatSí	**3.433.347**	**2.602.821**	-	**2.397.685**	**2.288.980**	**2.598.476**	**3.009.751**
PSC-PSOE	1.982.984	2.108.159	1.948.819	-	2.046.812	**2.924.514**	**3.734.577**
JUNTSxCAT	**3.410.606**	**2.715.117**	2.057.524	**2.299.692**	-	**2.533.905**	**3.087.610**
C's	1.911.989	1.600.255	1.748.028	1.421.990	1.812.599	-	**3.318.903**
PP	1.558.460	1.000.071	1.336.753	611.927	1.258.894	1.027.601	-

- Therefore, the collective preference ordering of the parties is:
$$ERC > JxCat > CUP > CatComu > PSC > Cs > PP$$

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evaluation: Catalonia 2017 Pr. 2

- Consistency of results: **collective preference orderings**

evaluation: Catalonia 2017 Pr. 2

	LOREG	SCORE	BORDA PR.1	PRLL PR.1
CUP	2,96%	12,01%	12,78%	16,44%
ERC-CATSI	23,70%	22,77%	17,89%	18,24%
CATCOMU	5,93%	12,30%	15,15%	16,12%
PSC-PSOE	12,59%	13,27%	16,16%	15,74%
JUNTSXCAT	25,19%	20,41%	17,64%	17,56%
C's	26,67%	14,09%	12,94%	11,32%
PP	2,96%	5,15%	7,44%	4,58%

LOREG	C's >	JxCAT >	ERC >	PSC >	Catcomu >	CUP >	PP
SCORE VOTING	ERC >	JxCAT >	C's >	PSC >	Catcomu >	CUP >	PP
BORDA	ERC >	Borda >	PSC >	Catcomu >	C's >	CUP >	PP
Prll	ERC >	Borda >	CUP >	Catcomu >	PSC >	C's >	PP

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evaluation: Catalonia 2017 Pr. 2

- We see again an **almost strong Borda Paradox: the most preferred option according to the LOREG would actually be the preferred option in penultimate place.**

evaluation: Catalonia 2017 Pr 2

- Consistency of results: **correlation between collective preference and allocation of seats to parties**
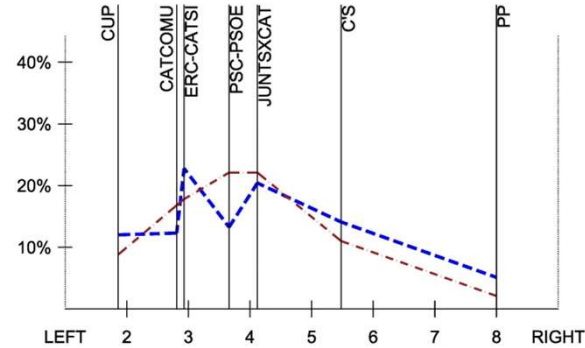
88

evaluation: Catalonia 2017 Pr 2
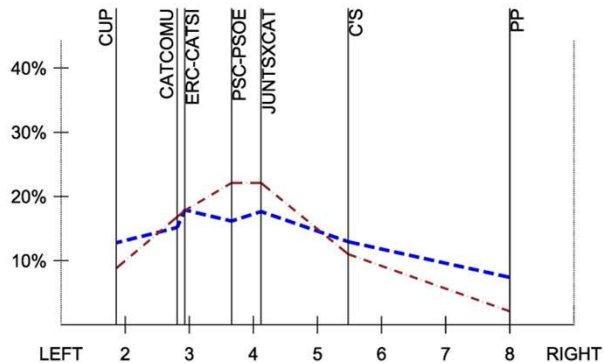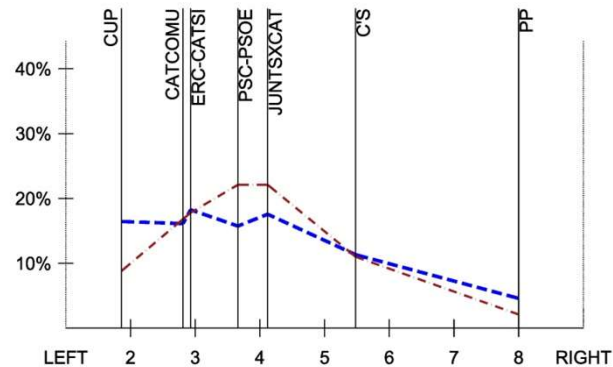
Left- Right Dimension
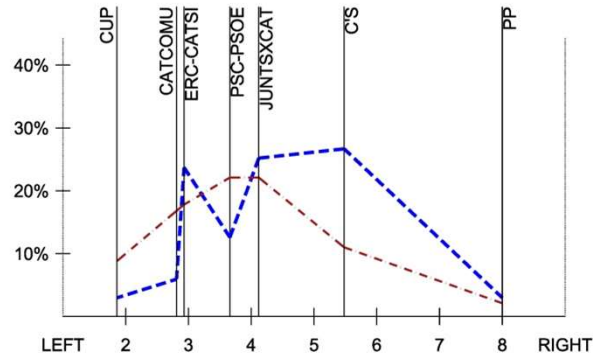
evaluation: Catalonia 2017 Pr 2

Territorial Model
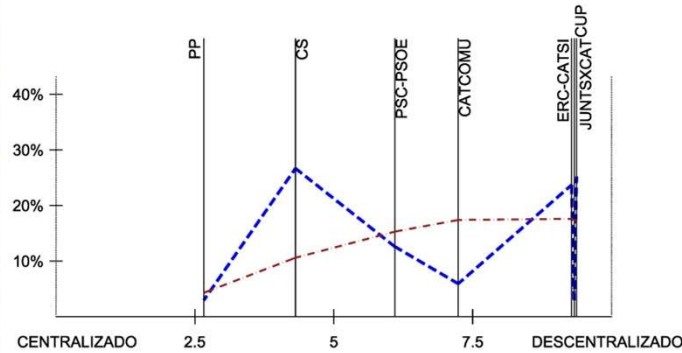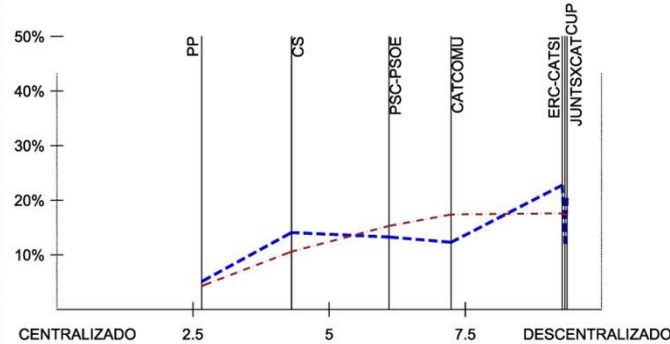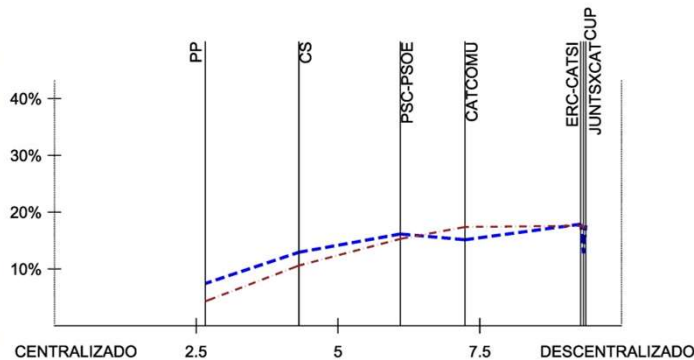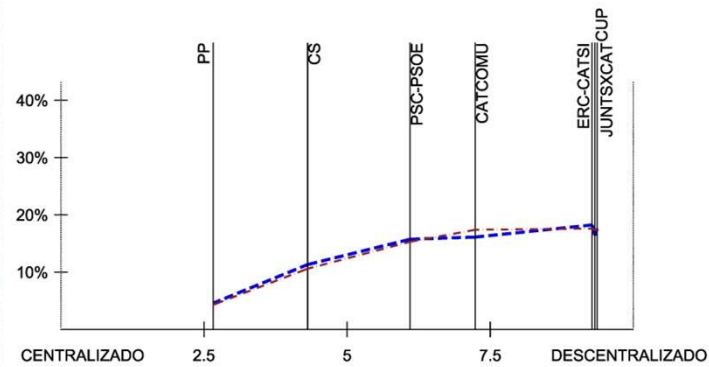
evaluation: Catalonia 2017 Pr. 2

- **Correlation between allocation of seats and collective preference:**

	LOREG	SCORE VOTING	BORDA		Prll	
			Pr.01	Pr.02	Pr. 01	Pr.02
Left Right	0,209	0,702	0,806	0,861	0,969	0,988
TERRIT. MOD.	0,520	0,744	0,951	0,944	0,776	0,808
AVERAGE	**0,364**	**0.723**	**0.878**	**0.902**	**0,872**	**0,898**

- **Correlation between seat allocation and score voting**
  [maximization of collective utility]

	LOREG	BORDA		Prll	
		Pr. 01	Pr. 02	Pr. 01	Pr. 02
SCORE VOTING	0,779	0,846	0,895	0,772	0,789

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evaluation: Catalonia 2017 Pr. 2

- We see **correlation between allocations of seats by Borda and PrII and citizens collective preference improves by modelling preference profile following the fuzzy procedure** [Pr 02]. **Collective utility maximization also increases** for the fuzzy procedure. Both issues sustain again the **validity of herein proposed procedure, Pr 02.**

- On the other hand, **we see current electoral system leads to a much lower correlation between allocation of seats and citizens collective preference, and lower collective utility, which again challenges its validity.**

PART IV

CONCLUSIONS:

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Conclusions

- While the properties and paradoxes of voting rules has been widely studied in the realm of Social Choice, we find scarce equivalent studies **referred to the systems for the election of representative chambers**.

- This scarce interest on the issue contrasts with the fundamental importance that Representative Chambers have acquired in our political systems, and may be due to the **difficulty of obtaining data to perform the analyzes.**

- In most cases, available data for reviewing past elections are not individuals' preference orderings but average preference values.

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Conclusions

- We are witnessing in Spain a situation of **increasing citizen disaffection** with current political system, **which many experts relate -at least in part- to the *biased* design of Spain current electoral system.**

- However, different experts disagree on the ultimate design the electoral system should have, leading to **several different proposals for electoral reform being made in recent times** [for a review of the latest ones, see Crespo et al, 2019]

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Conclusions

- As a way to move forward on the issue, it seems useful to evaluate the validity of both the current electoral system and its possible modifications by comparing allocation of seats and citizens collective preference. In order to do so, we need to review the **complete preference ordering of each voter**, not only first preferences expressed by single votes.

- However, this information is not usually accessible. As a way to fill this gap, **we have proposed a simple procedure to estimate citizens preference profile from usually available data**: average values which are frequently published by CIS.

Conclusions

- Using that information, we have modeled two elections recently held in Spain.

- In the two reviewed examples, **modeling the preference profile using the fuzzy procedure [Pr.02] reflects more accuratelly the citizenship collective preference and maximixes collective utility compared to a profile based on strict preference/indifference [Pr. 01].**

- **Results suggests therefore the utility of herein proposed procedure [Pr02].**

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Conclusions

- Additionally, **the evaluation has revealed some important issues**:

Conclusions

- **The ubiquity of the Borda Paradox** [there is a strong paradox in Madrid, and an almost strong one in Catalonia] **suggests current electoral systems [d'Hondt in both cases] may be creating an important ideological distance between representantives and citizens.**

- This departure is further increased by the fact regional governments are chosen by the representatives, and **could be the main cause of the current political instability and citizen discontent**, reinforcing the importance of reforming current Spanish [national and regional] electoral system[s].

Conclusions

- The **great improvement in the correlation between citizenship preference and allocation of seats to the parties** provided by the rules that take into account voters' complete preference orderings [Score Voting, Borda and Prll] compared to current electoral systems, shows that **the path should start by abandoning single vote and adopting preferential vote.**

AVERAGE CORRELATION BETWEEN CITIZENSHIP PREFERENCE AND ALLOCATION OF SEATS FOR THE TWO REVISED ELECTIONS				
	SINGLE VOTE	PREFERENTIAL VOTE		
	LOREG	SCORE VOTING	BORDA [PR 02]	Prll [PR 02]
LEFT RIGHT	-0,08	0,78	0,79	0,92
MOD. TERR.	0,49	0,52	0,57	0,59
AVERAGE	**0.20**	**0.65**	**0,68**	**0,75**

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Conclusions

- The case of Madrid 2015 is illustrative because **being a single district with 129 deputies, the allocation of seats is almost strictly proportional to single votes**, **and nevertheless a strong Borda Paradox is produced; ie, the most voted option is the least preferred.**

- **The reform needed by the Spanish electoral system should not be based on increasing the proportionality between votes and seats, but on replacing single vote by preferential vote, and using a voting rule that takes into account the entire voters preference orderings.**

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Conclusions

- Finally, it should be noted that **the frequent appearance [50% of cases] of the Condorcet Paradox** , highlights that the preference aggregation rule must be able to satisfactorily solve cyclical majority relationships .

Part V

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Thank you for attention !!

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