Signal Observability at Hyper Light Velocity

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***Abstract*—In [1] an observer at rest and perpendicular to moving source will detect a difference in the time of arrival of the initial τ_0 and final τ' waves as $\tau' > \tau_0$ a transverse relative time shift. The position of the observer in [1] for source velocities near the speed of light has the disadvantage of having to wait a long time to detect τ'. The observer cannot detect τ' for velocities equal to or greater than the speed of light. This work shows that moving the observer to a strategic position downstream from the τ' signal has the advantage of introducing observability of τ'. The transverse relative time shift formulas are derived for the stationary observer perpendicular to and also on the path axis for velocities less than and much greater than c. Also, a method to calculate transverse relative length is proposed.**

***Index Terms*—Extinction shift principle, transverse relative time shift, transverse relative length, hyper light velocity.**

LIST OF ACRONYMS

- ***c*** Speed of light in meters per second, assumed $3\text{x}10^8$ m
- d_s Time is seconds from the birth signal to the death signal as observed by the stationary observer
- ***K*** Area formed by birth and death signal centers moving with velocity ***v*** and the fixed observer
- l_E Transverse relative length in meters
- L_s Time in seconds from birth signal to death signal as observed by the signal source with velocity **v**
- **m** Meters
- **s** Seconds
- ***S*** Source location
- ***t*** Standard time in seconds for all clocks
- τ_0 Time in seconds related to birth signal
- τ' Time in seconds related to death signal
- ***v*** Velocity of signal source in meters per second

I. Introduction

This work describes the geometry between time, space, and light signals when the source is moving at constant velocity. All clocks everywhere read the same time ***t*** in seconds (s) and have the same clock rate, Δt. When Δt = 1s all clocks will have advanced in measurement by 1 second. Time ***t*** progresses with increasing magnitude in physical processes. The source ***S*** travels along the **X** axis with constant velocity ***v*** in meters per second (m/s). ***S*** moving with positive velocity (+***v***) indicates that ***S*** is moving from left to right along the **X** axis; hence, after some time Δt, ***S*** will have moved X meters to the right (1).

$$\mathbf{X} = (+\Delta t)(+v)\text{m}. \tag{1}$$

Negative velocity (-***v***) indicates that ***S*** is moving from right to left along the **X** axis; hence, after some time Δt, ***S*** will have moved X meters to the left, $\mathbf{X} = (+\Delta t)(-v)$m.

The Observer (☺) is in the **X**, **Y** plane. ***S*** emits the birth signal at location S_0, the coordinate origin, at time t_0. Time t_0 marks the time on all clocks when ***S*** is at the coordinate origin and emits the birth signal W_0. ***S*** emits the death signal W_1 at τ_0 and location S_1, on the **X** axis (2) where τ_0 is in seconds.

$$\tau_0 = t_0 + \Delta t \tag{2}$$

Time t_0 does not mark the beginning of time. The time t_0 marks the time on all clocks everywhere when ***S*** is at the origin and emits the birth signal. The Observer is in a fixed position in the **X**, **Y** plane. The Observer's clock rate is the same as ***S***. When ***S*** clock reads 3 seconds the Observer's clock reads 3 seconds. There is a time delay for the birth signal W_0 to pass the Observer and there is an additional time delay for the death signal W_1 to pass the Observer. The observer does not alter the path of the signal. The next subsection will show how to calculate the signal's W_0 and W_1 position in the **X**, **Y** plane when ***S*** has velocity $v = 0$ m/s. The following subsection will show how to calculate the signal's position in the **X**, **Y** plane when ***S*** has velocity $v = c$ m/s along the X axis where ***c*** is the velocity magnitude of the speed of light, $c = 3\text{x}10^8$ m. Armed with these tools the Observer in Section II, can see why it has limited of signal observability. The geometry for transverse relative time shift is in Section III where $v < c$. Section IV is the geometry for signal observability when $v = c$. Section V is the geometry for signal observability for $v = 2c$ and greater. Section VI shows how to find the location of the Observer using Heron's formula and equations for two intersection circles. Section VII proposes a method to calculate observed *transverse relative length* for objects traveling up to and beyond c. Section VIII concludes with some observation. Section IX is added to explain the significance of Area K_3.

A. *Geometry to calculate a signals position when S has velocity v = 0 m/s.*

Signal Source, **S,** in Figure 1 is located at the origin of the X, Y axis when t_0 = 0s. **S** has velocity v_s = 0 m/s. **S** is not moving with respect to the coordinates and Observers 1, 2, 3, and 4. Signal W_0 is emitted by **S** at location S_0 the coordinate origin when t_0 = 0s. It is assumed that the signal velocity $W_0(v_0)$ is $c = 3x10^8$ m/s in the radial direction away from the signal source center. W_0 is a spherical wave expanding away from S with velocity c m/s while maintaining S for its center. The intersection of W_0 with the Observer and the **X**, **Y** axis is a circle when t_0 > 0s.

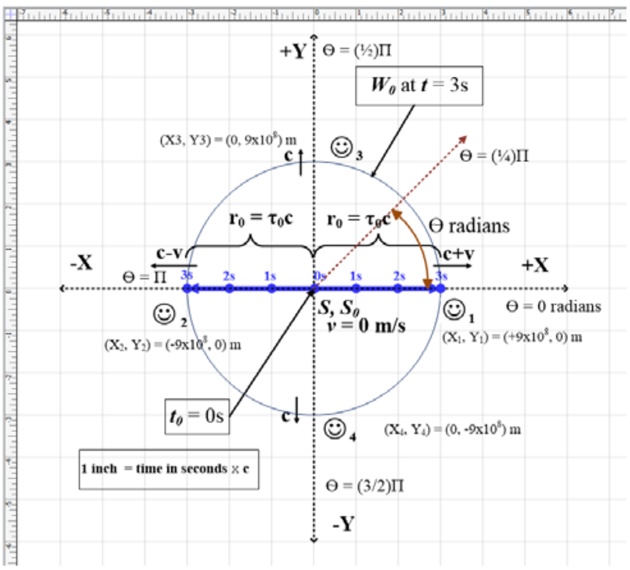


Fig. 1: Geometry to calculate signal position when ***S*** has velocity ***v***= 0 m/s.

$W_0(v_0, \Theta_0)$ is a vector having both magnitude (v_0) in m/s and direction (Θ_0) where Θ_0 is in radians. The signal progresses through some time τ_0 with velocity $W_0(v_0)$ to arrive at the coordinate $W_0(X,Y)$ (3). The distance r_0 (3) between the coordinate origin and $W_0(X,Y)$ is

$$r_0 = \tau_0 v_0 \qquad (3)$$

$$r_0 = (3s)(3x10^8 \text{ m/s})$$

$$r_0 = 9x10^8 \text{ m}$$

Signal arrives at Observer 1, Observer 3 and Observer 4 in τ_0 = 3s.

W_0 arrives at Observer 1, Observer 2, Observer 3 and Observer 4 in τ_0 = 3s.

Since the signal source ***S*** is fixed with respect to the coordinate origin and all observers there is no preferred reference for the angle Θ. The reference for Θ is selected to be the position of Observer 1 where the velocity v_0 of the signal is positive, notice that the signal W_0 time vector τ_0 is positive in all directions. In Figure 1, Θ is the angle from the +**X** axis. Positive Θ is in the counter clockwise direction from the +**X** axis where Θ = 0 radians. Θ ranges from 0 to 2Π radians. The coordinates of $W_0(X,Y)$ when time is τ_0 using r_0 (3) and Θ can be calculated from (4) and(5).

$$W_0(X,Y) = W_0(r_0 \cos(\Theta), r_0 \sin(\Theta)) \qquad (4)$$

$$W_0(X,Y) = W_0(\tau_0 v_0 \cos(\Theta), \tau_0 v_0 \sin(\Theta)) \qquad (5)$$

Table 1 include calculations for W_0 (X, Y) coordinates when t = 3s and v = 0 m/s for selected Θ. Using (5) when τ_0 = 3s and $v = c$ m/s the coordinates of the Observers are

$$W_0(X_1,Y_1) = W_0((3s)(3x10^8)\cos(0), (3s)(3x10^8)\sin(0))$$

$$W_0(X_1,Y_1) = W_0(+9x10^8, 0) \text{ m}$$

$$W_0(X_2,Y_2) = W_0((3s)(3x10^8)\cos(\Pi), (3s)(3x10^8)\sin(\Pi))$$

$$W_0(X_2,Y_2) = W_0(-9x10^8,0) \text{ m}$$

$$W_0(X_3,Y_3)=W_0((3s)(3x10^8)\cos(½ \Pi), (3s)(3x10^8)\sin(½\Pi))$$

$$W_0(X_3,Y_3) = W_0(0,+9x10^8) \text{ m}$$

$$W_0(X_4,Y_4)=W_0((3s)(3x10^8)\cos(3/2\Pi),(3s)(3x10^8)\sin(3/2\Pi))$$

$$W_0(X_4,Y_4) = W_0(0,-9x10^8) \text{ m}$$

Observer	Angle Θ (Radians)	X = cos(Θ)	Y = sin(Θ)	W_0 (meters) t=3s $r = \tau_0 c$	Θ (Radians)	W_0(X) = W_0cos(Θ) (m)	W_0(Y) = W_0sin(Θ) (m)
Observer 1	0	1.0000	0.0000	9.000E+08	0	9.000E+08	0.000E+00
	(1/4)Π	0.7071	0.7071	9.000E+08	(1/4)Π	6.364E+08	6.364E+08
Observer 3	(1/2)Π	0.0000	1.0000	9.000E+08	(1/2)Π	0.000E+00	9.000E+08
	(3/4)Π	-0.7071	0.7071	9.000E+08	(3/4)Π	-6.364E+08	6.364E+08
Observer 2	Π	-1.0000	0.0000	9.000E+08	Π	-9.000E+08	0.000E+00
	(5/4)Π	-0.7071	-0.7071	9.000E+08	(5/4)Π	-6.364E+08	-6.364E+08
Observer 4	(3/2)Π	0.0000	-1.0000	9.000E+08	(3/2)Π	0.000E+00	-9.000E+08
	(7/4)Π	0.7071	-0.7071	9.000E+08	(7/8)Π	6.364E+08	-6.364E+08

Table 1: Calculated coordinates for W_0 for selected Θ when **t** = 3s, v = 0 m/s.

The next subsection will locate W_0 when S is moving on the X-axis with velocity v m/s.

B. *Geometry to calculate a signals position when S has velocity v = c m/s.*

Now let ***S***, in Figure 2, have velocity v_s on the **X** axis where $v_s = c$ m/s. ***S*** emits a signal W_0 when X = 0 m and t_0 = 0s. W_0 travels with velocity of $c = 3x10^8$m/s in the radial direction. *The center of the sphere* W_0 *coincides with the traveling source* ***S*** *position on the* ***X*** *axis.*

The position of ***S***, ***S(X)***, using (3) is (6)

$$S(X) = \tau_0 v_s. \qquad (6)$$

When ***S*** is restricted to traveling with v_s on the X axis S_0 is the position of ***S*** on the coordinate origin and S_1 is the position of ***S*** at some time τ_0. The reference for Θ is selected to be v_s.

From (6) when τ_0 is 3s and v_s is c m/s

$$S_0(X) = \tau_0 v_s \qquad (6)$$

$$S_0\,(\mathbf{X}) = (0\text{ s})(\boldsymbol{c}\text{ m/s})$$

$$S_0\,(\mathbf{X}) = 0\text{ m}$$

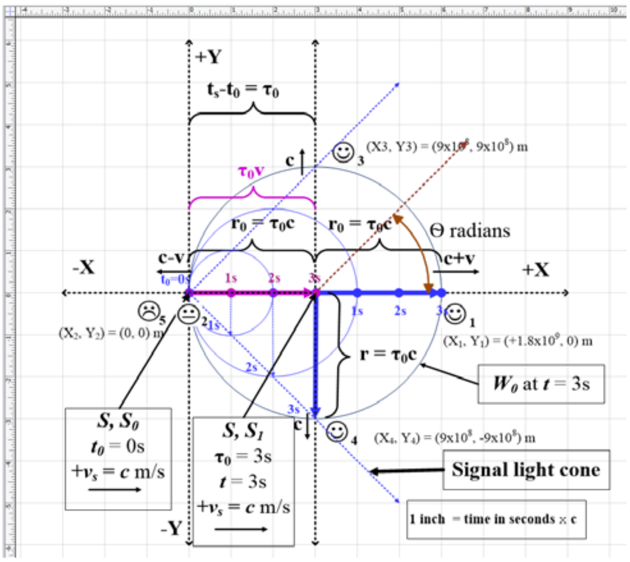


Fig. 2: Geometry to calculate signal position when $\boldsymbol{S}$ has velocity $\boldsymbol{v_s} = \boldsymbol{c}$ m/s.

$$S_1\,(\mathbf{X}) = \tau_0 v_s \tag{6}$$

$$S_1\,(\mathbf{X}) = (3\text{ s})(3\text{x}10^8\text{ m/s})$$

$$S_1\,(\mathbf{X}) = 9\text{x}10^8\text{ m.}$$

The coordinates of $\boldsymbol{W_0}$ must now be modified to include the velocity component of $\boldsymbol{S(X)}$. Adding (6) to (5) give the new coordinates $\boldsymbol{W_0}(\mathbf{X,Y})$ (7) which now include the effects from $\boldsymbol{S}$ moving with constant velocity $\boldsymbol{v_s}$ along the X-axis.

$$W_0\,(\text{X,Y}) = W_0((\tau_0 v_0 + \tau_0 v_s)\cos(\Theta),\ \tau_0 v_0 \sin(\Theta)) \tag{7}$$

The new coordinates $\boldsymbol{W_0}$ (X,Y) for Observers 1,2,3, and 4 using (7) are calculated in Table 2 when $\boldsymbol{S}$ has velocity $\boldsymbol{v_s}$ = $\boldsymbol{c}$ m/s.

Observer	Angle Θ (Radians)	S(v) on X-axis v_s(m/s)	$\tau_0 v_s$ (m)	W0 (Y) (m) t=3s r = τ_0c	W0(X) =($\tau_0$vs+τ_0c)cos(Θ) (m)	W0(Y) =(τ_0c)sin(Θ) (m)
Observer 1	0	3.000E+08	9.000E+08	9.000E+08	1.800E+09	0.000E+00
Observer 3	(1/2)∏	3.000E+08	9.000E+08	9.000E+08	9.000E+08	9.000E+08
Observer 2	∏	3.000E+08	9.000E+08	9.000E+08	0.000E+00	0.000E+00
Observer 4	(3/2)∏	3.000E+08	9.000E+08	9.000E+08	9.000E+08	-9.000E+08

Table 2: Coordinates $\boldsymbol{W_0}$ (X,Y) for Observers 1,2,3, and 4 when $\boldsymbol{S}$ has velocity $\boldsymbol{v_s}$ = $\boldsymbol{c}$ m/s.

From Figure 2 Observer 2 may not see the signal for a long time if off of the X axis and Observer 5 with –X coordinates will not be able to detect $\boldsymbol{W_0}$ at all.

An important observation is that the signal $\boldsymbol{W_0}$ reaches Observer 1 at twice the distance $\boldsymbol{S}$ has traveled from the coordinate origin.

When $\boldsymbol{S}$ has velocity $\boldsymbol{v_s}$ = -c m/s, Table 3, Observer 2 is at the coordinate origin and Observer 1 is where $\boldsymbol{W_0}$ $2\boldsymbol{\tau_0 v_s}$. $\boldsymbol{\Theta}$ measures the angle in radians from the velocity vector $\boldsymbol{v_s}$ along the –X axis in a clockwise rotation in the X, Y plane.

Observer	Angle Θ (Radians)	S(v) on X-axis v_s(m/s)	$\tau_0 v_s$ (m)	W0 (Y) (m) t=3s r = τ_0c	W0(X) =($\tau_0$vs+τ_0c)cos(Θ) (m)	W0(Y) =(τ_0c)sin(Θ) (m)
Observer 1	0	-3.000E+08	-9.000E+08	9.000E+08	0.000E+00	0.000E+00
Observer 3	(1/2)∏	-3.000E+08	-9.000E+08	9.000E+08	-9.000E+08	9.000E+08
Observer 2	∏	-3.000E+08	-9.000E+08	9.000E+08	-1.800E+09	0.000E+00
Observer 4	(3/2)∏	-3.000E+08	-9.000E+08	9.000E+08	-9.000E+08	-9.000E+08

Table 3: Coordinates $\boldsymbol{W_0}$ (X,Y) for Observers 1,2,3, and 4 when $\boldsymbol{S}$ has velocity $\boldsymbol{v_s}$ = -$\boldsymbol{c}$ m/s.

The next section will investigate signal observability when the Observer at rest is placed a distance from the nearest point on the path of a moving source with velocity $\boldsymbol{v}$ and two signals are emitted by the Source.

II. Observer Position with Limited Signal Observatility

In [1] it is shown that a resting observer placed a distance from the nearest point on the path of a moving source with velocity $\boldsymbol{v}$ will observe the arrival of the birth signal $\boldsymbol{\tau_0}$ and death signal $\boldsymbol{\tau'}$ as $\boldsymbol{\tau'} > \boldsymbol{\tau_0}$ a transverse relative time shift. Where in Figure 3 $\boldsymbol{\tau_0}$ is the time (s) in seconds of flight of the birth signal moving at velocity $\mathbf{v}$ perpendicular to the stationary observer and $\boldsymbol{\tau'}$ is the time of flight of the death signal to the observer. The birth signal is initiated at $\boldsymbol{t_0}$ = 0 s.

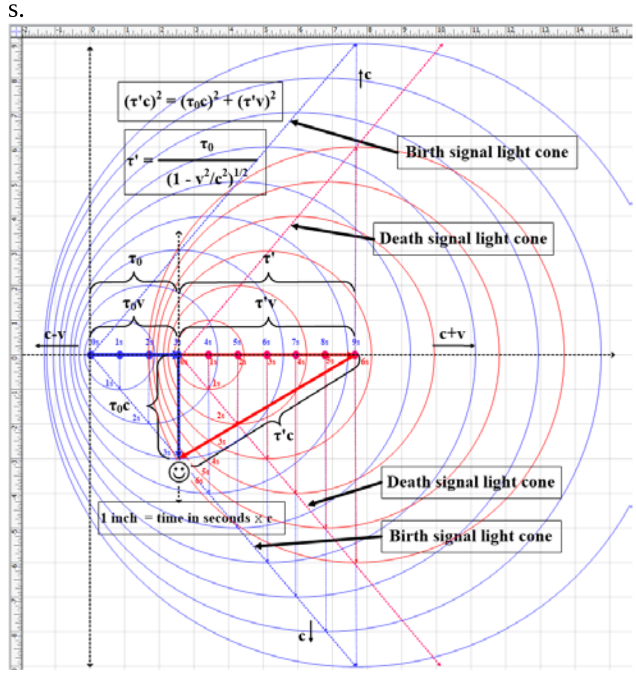


Fig. 3: Transverse Relative Time Shift. [1]

From geometry in Figure 3

$$(\boldsymbol{\tau' c})^2 = (\boldsymbol{\tau_0 c})^2 + (\boldsymbol{\tau' v})^2 \,. \tag{8}$$

From [1] $\boldsymbol{\tau'}$ is

$$\boldsymbol{\tau'} = \frac{\boldsymbol{\tau_0}}{\sqrt{1 - v^2/c^2}} \,. \tag{9}$$

The position of the observer outside the death signal light cone with the geometry in (8) and (9) poses the dilemma that as $\boldsymbol{v}$ approaches $\boldsymbol{c}$, $\boldsymbol{\tau'}$ approaches infinity. This infinity physically represents zero (0) observability.

The geometry in this work is in a 2 dimensional plane. The signals although spherical in nature are represented as circles in this plane. The observer location is restricted to the plane. The results of this work are general in nature and can be extended to three dimensions.

It is shown in Section II that moving the observer to the strategic location on the death signal light cone will provide observability, remove the infinity as $\boldsymbol{v}$ approaches $\boldsymbol{c}$, and allow for the observer to properly calculate the transverse relative time shift for $\boldsymbol{v} < \boldsymbol{c}$. Section III will explore the geometry of $\boldsymbol{v} = \boldsymbol{c}$. Section IV will evaluate $\boldsymbol{v} = 2\boldsymbol{c}$, $10\boldsymbol{c}$ and $100\boldsymbol{c}$. Section V will probe the use of Herons Formula [2] and the intersection of two circles using the area-based method to locate the coordinates of the observer. Section VI will propose a method to calculate observed *transverse relative length* for objects traveling up to and beyond $\boldsymbol{c}$. Section VII will conclude with some observations.

III. Transverse Relative Time Shift

C. *Observer perpendicular to signal path at $\boldsymbol{\tau' c}$ and $\boldsymbol{v}<\boldsymbol{c}$.*

Locating the stationary happy observer (☺) on the death signal at τ' and perpendicular to the center of the signal allows the observer to see the signal when $\boldsymbol{v} < \boldsymbol{c}$, $\boldsymbol{v} = \frac{\sqrt{3}}{2}\boldsymbol{c}$. In Figure 4 the happy observer (☺) receives the birth signal $\boldsymbol{\tau_0}$ at t = 6.4066s and the death signal $\boldsymbol{\tau'}$ when $\boldsymbol{\tau'}$ = 6s and t = 9s. Also, calculated values in this work are rounded to 4 decimal places.

From geometry (Figure4)

$$(\boldsymbol{\tau_0 c})^2 = (\boldsymbol{d_s v})^2 + (\boldsymbol{\tau' c})^2 \tag{10}$$

$$\boldsymbol{d_s v} + \boldsymbol{\tau_0 v} = \boldsymbol{L_s v} + \boldsymbol{\tau' v} \tag{11}$$

$$\boldsymbol{d_s v} = \boldsymbol{L_s v} + \boldsymbol{\tau' v} - \boldsymbol{\tau_0 v} \tag{12}$$

$$\boldsymbol{d_s} = \boldsymbol{d_s v} / \boldsymbol{v} \,. \tag{13}$$

Where $\boldsymbol{\tau_0}$ is the time interval from the birth signal to the happy observer, $\boldsymbol{\tau'}$ is the time interval from the death signal to the happy observer, $\boldsymbol{d_s}$ is the time the observer sees the birth signal to the time the observer sees the death signal and $\boldsymbol{L_s}$ is the time interval between the source birth and death signals moving with +$\boldsymbol{v}$, left to right, and –$\boldsymbol{v}$, right to left.

Time variables ($\boldsymbol{\tau_0}$, $\boldsymbol{\tau'}$, $\boldsymbol{d_s}$, $\boldsymbol{L_s}$) are always positive. Time multiplied by the velocity of the source ($\boldsymbol{\tau_0 v}$, $\boldsymbol{\tau' v}$, $\boldsymbol{d_s v}$, $\boldsymbol{L_s v}$) is measured in meters and is positive (+$\boldsymbol{v}$) or negative (-$\boldsymbol{v}$).

The signal radiates radially outward from the moving source with velocity of light +$\boldsymbol{c}$ on the sphere with radius of the birth signal $\boldsymbol{\tau_0 c}$ and death signal $\boldsymbol{\tau' c}$ in meters (m). The center of the light signal spheres coincide with the signal source. $\boldsymbol{\tau_0 c}$ and $\boldsymbol{\tau' c}$ are positive (m) with respect to the source.

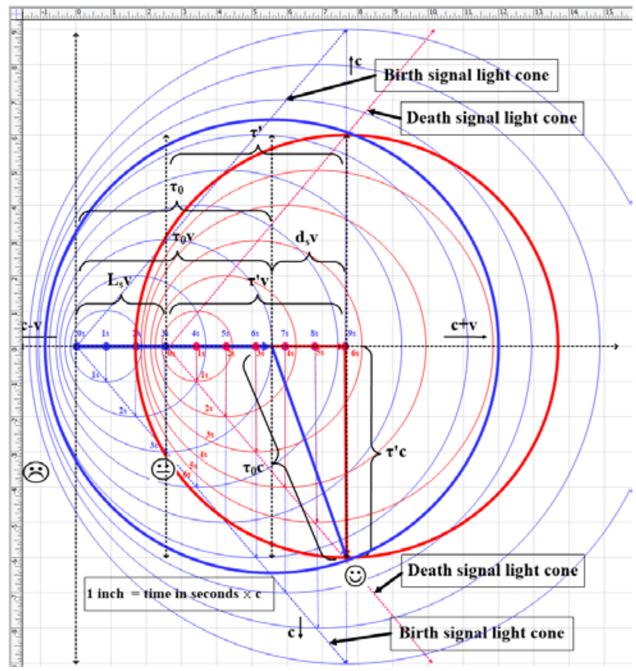


Fig. 4: Geometry for Transverse Relative Time Shift when observer is perpendicular to signal path at $\boldsymbol{\tau' c}$, +$\boldsymbol{v}$ and $\boldsymbol{v} < \boldsymbol{c}$.

Combining (10) and (12) yields (14). Set (14) to zero, (15), and solve for $\boldsymbol{\tau'}$ (16) and then $\boldsymbol{\tau_0}$ (17). The symbolic language program Maxima [3] was used to find (16) (17).

$$(\boldsymbol{\tau_0 c})^2 = (\boldsymbol{L_s v} + \boldsymbol{\tau' v} - \boldsymbol{\tau_0 v})^2 + (\boldsymbol{\tau' c})^2 \tag{14}$$

$$0 = (\boldsymbol{L_s v} + \boldsymbol{\tau' v} - \boldsymbol{\tau_0 v})^2 + (\boldsymbol{\tau' c})^2 - (\boldsymbol{\tau_0 c})^2 \tag{15}$$

$$\tau' = \frac{c\sqrt{(2L_s\tau_0 - L_s{}^2)v^2 + c^2\tau_0{}^2} + (\tau_0 - L_s)v^2}{v^2 + c^2} \tag{16a}$$

$$\tau_0 = -\frac{c\sqrt{(2L_s\tau' + L_s{}^2)v^2 + c^2\tau'^2} + (-\tau' - L_s)v^2}{v^2 - c^2} \,. \tag{17a}$$

$$\tau' = -\frac{c\sqrt{(2L_s\tau_0 - L_s{}^2)v^2 + c^2\tau_0{}^2} + (L_s - \tau_0)v^2}{v^2 + c^2} \tag{16b}$$

$$\tau_0 = \frac{c\sqrt{(2L_s\tau\prime + L_s 2)v^2 + c^2\tau\prime 2} + (\tau\prime + L_s)v^2}{v^2 - c^2} \,. \tag{17b}$$

Solution (16b) and (17b) yield results for τ_0 and $\tau' < 0$s when v <= c and are not used in this work.

Solving (17a) with $\tau' = 6$s, $c = 3\text{x}10^8$ m/s, $L_s = 3.000$s and $v = \frac{\sqrt{3}}{2}c$ yields $\tau_0 = 6.4066$s. The happy observe starts a stop watch at receipt of the birth signal (t=6.4066s). The happy observer stops the stop watch at receipt of the death signal (t=9s). The stop watch measures (counts) $d_s = 2.5934$s (13). The ratio $\tau'/\tau_0 = 0.9365$. The ratio $d_s/L_s = 0.8645$. The neutral observer (😐) using (9) measures the ratio $\tau'/\tau_0 = 2.00$ when $v = \frac{\sqrt{3}}{2}c$. The sad observer (☹) may not see the death signal for v near c due to being left of the birth signal light cone and the neutral observer.

D. *Observer on signal path at $\tau_0 c$, +v and v<c.*

Now locate the happy observer within the death signal light cone on the signal source path where **t** = 6.4066s from geometry (Figure 5)

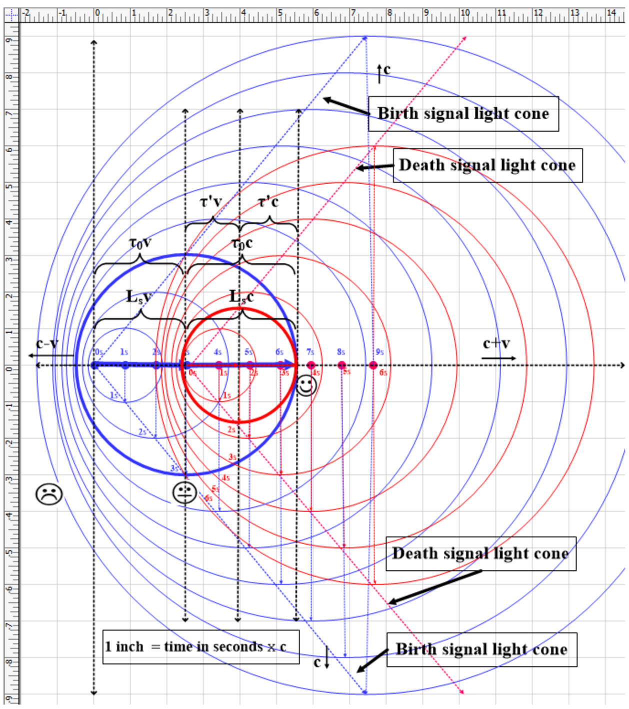


Fig. 5: Geometry for Transverse Relative Time Shift when observer is on signal path at $\tau_0 c$, +v, and $v < c$.

$$\tau_0 v + \tau_0 c = L_s v + \tau' v + \tau' c \tag{8}$$

$$\tau_0(v + c) = L_s v + \tau'(v + c) \tag{19}$$

$$\tau' = (\tau_0(v + c) - L_s v)/(v + c) \tag{20}$$

$$\tau' = \tau_0 - L_s v/(v + c)\,. \tag{21}$$

Solving (21) the happy observer measures $\tau' = 1.6077$s, and the ratio $\tau'/\tau_0 = 0.5359$ when $c = 3\text{x}10^8$ m/s, $L_s = \tau_0 = 3.000$s and $v = \frac{\sqrt{3}}{2}c$.

E. *Observer on signal path at $\tau_0 c$, -v and v<c.*

Now locate the happy observer within the death signal light cone on the signal source path where **t** = 6.4066s and $-v$. From geometry (Figure 6)

$$\tau_0 v - (\tau_0 c) = L_s v + \tau' v - (\tau' c) \tag{22}$$

$$\tau_0 (v - c) = L_s v + \tau'(v - c) \tag{23}$$

$$-\tau_0 (c - v) = L_s v - \tau'(c - v) \tag{24}$$

$$\tau_0 (c - v) = -L_s v + \tau'(c - v) \tag{25}$$

$$(\tau_0(c - v) + L_s v)/(c - v) = \tau'(c - v)/(c-v) \tag{26}$$

$$\tau' = \tau_0 + L_s v/(c - v)\,. \tag{27}$$

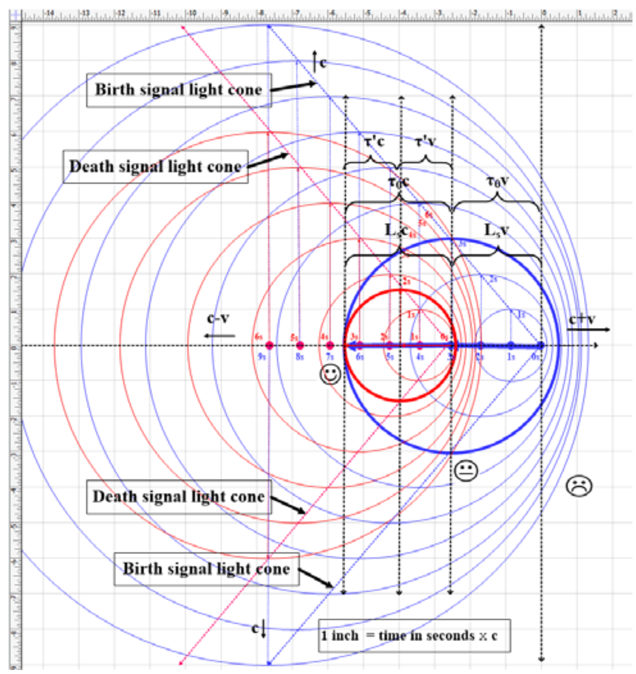


Fig. 6: Geometry for Transverse Relative Time Shift when observer is on signal path at $\tau_0 c$, -v, and $v < c$.

The introduction of the negative sign in $-(\tau' c)$ indicates the spherical signal is moving from right to left. Solving (27) the happy observer measures $\tau' = 1.6077$s, and the ratio $\tau'/\tau_0 = 0.5359$ when $c = 3\text{x}10^8$ m/s, $L_s = \tau_0 = 3.000$s and $-v$ with magnitude $v = \frac{\sqrt{3}}{2}c$.

It is interesting to note that on the death signal light cone and on the signal center path there are general real solutions where the observer can measure a difference in time between the received birth and death signal. The next section explores the geometry for $v = c$.

IV. GEOMETRY FOR $V = C$

A. *Oberver perpendicular to signal path at τ'c and v=c.*

From geometry in Figure 7, when $\boldsymbol{v}=\boldsymbol{c}$, locating the stationary happy observer on the death signal at $\boldsymbol{\tau'}$ and perpendicular to the signal center allows the observer to see the signal. The birth signal (blue) is emitted when t=0s. The death signal (red) is emitted when $\boldsymbol{L_s}$ = 3s (**t**=3s). The happy observer using (17a) sees the birth signal when $\boldsymbol{\tau_0}$ = 6.5s. The happy observation time interval $\boldsymbol{d_s}$ begins when **t** = 6.5s and ends when it receives the death signal $\boldsymbol{\tau'}$ = 6s (**t**=9s). The happy observer using (13) measures $\boldsymbol{d_s}$ = 2.5s. The ratio $\boldsymbol{\tau'/\tau_0}$ = 0.9231. The ratio $\boldsymbol{d_s/L_s}$ = 0.8333. The neutral (😐) positioned perpendicular to **t** = 3s will not observe the death signal when $\boldsymbol{v} = \boldsymbol{c}$. The sad observer (☹) positioned perpendicular to or left of **t** = 0s will not observe the birth or death signal moving with (+$\boldsymbol{v}$).

(17a) is undefined when $\boldsymbol{v} = \boldsymbol{c}$. To calculate the approximate value of τ_0 in (17a) +$\boldsymbol{v}$ was set to 0.999995c and -$\boldsymbol{v}$ was set to -1.000005$\boldsymbol{c}$. On the death signal light cone (16a) (17a) provide approximate values for $\boldsymbol{\tau'}$ and $\boldsymbol{\tau_0}$ when $\boldsymbol{v}$ is near $\boldsymbol{c}$.

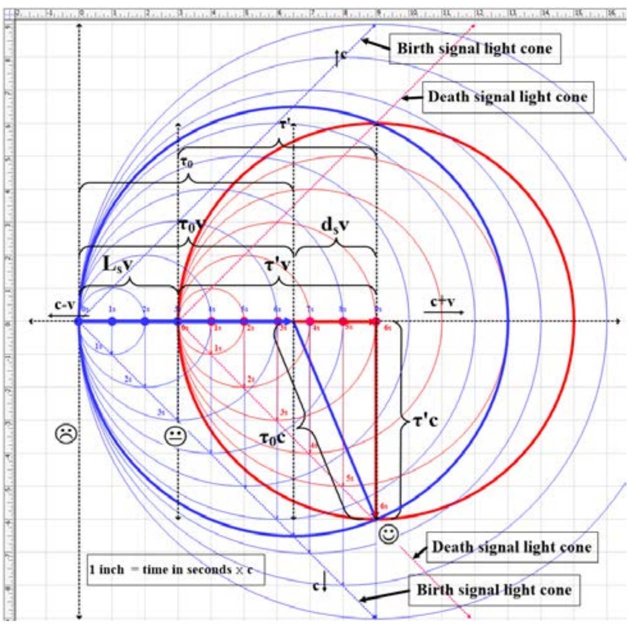


Fig. 7: Geometry for Transverse Relative Time Shift when observer is perpendicular to signal path at $\boldsymbol{\tau'c}$ and $\boldsymbol{v} = \boldsymbol{c}$.

From geometry in Figure 7 with $\boldsymbol{\tau_0 c}$ and the observer rotated upward toward the signal axis will cause the relocated observer to detect the $\boldsymbol{\tau'c}$ signal sooner, which moves the red circle to the left and reduces its radius. This scheme for relocating the observer will allow for the calculation of $\boldsymbol{\tau_0}$ when $\boldsymbol{v} = \boldsymbol{c}$. The geometry, equations, and steps to calculate the location of the relocated observer who is not plagued by the divide by zero when $\boldsymbol{v} = \boldsymbol{c}$ is in Section V.

B. *Observer on signal path at* $\boldsymbol{\tau_0 c}$ *and* $\boldsymbol{v}=\boldsymbol{c}$.

Now locate the happy observer within the death signal light cone on the signal source path where **t** = 6s.

From geometry (Figure 8) and solving (21) the happy observer measures $\boldsymbol{\tau'}$ = 1.5s, and the ratio $\boldsymbol{\tau'/\tau_0}$ = 0.5 when $\boldsymbol{c}$ = 3x10^8 m/s, $\boldsymbol{L_s} = \boldsymbol{\tau_0}$ = 3s and $\boldsymbol{v} = \boldsymbol{c}$. The next section explores the geometry for $\boldsymbol{v}$ = 2$\boldsymbol{c}$.

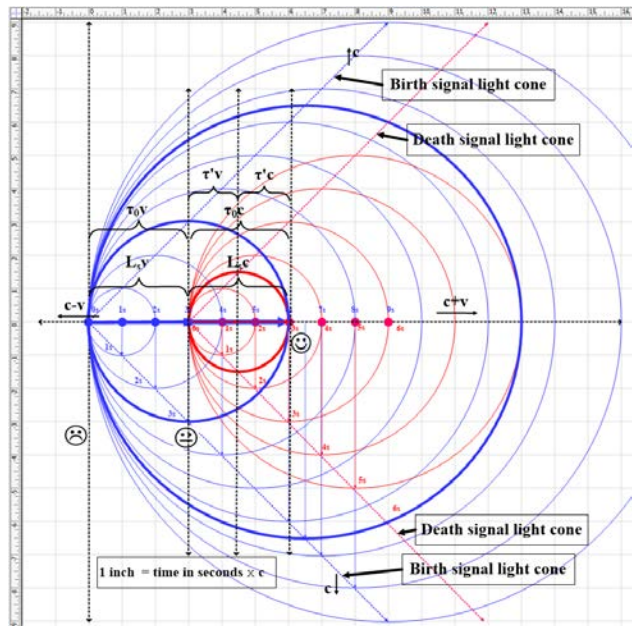


Fig. 8: Geometry for Transverse Relative Time Shift when observer is on signal path at $\boldsymbol{\tau_0 c}$ and $\boldsymbol{v} = \boldsymbol{c}$.

V. GEOMETRY FOR $V = 2C$ PLUS

A. *Observer perpendicular to signal path at* $\boldsymbol{\tau'c}$ *and* $\boldsymbol{v}$=2$\boldsymbol{c}$.

From geometry in Figure 9, when $\boldsymbol{v}$=2$\boldsymbol{c}$, locating the stationary happy observer on the death signal at $\boldsymbol{\tau'}$ and perpendicular to the signal center allows the observer to see the signal.

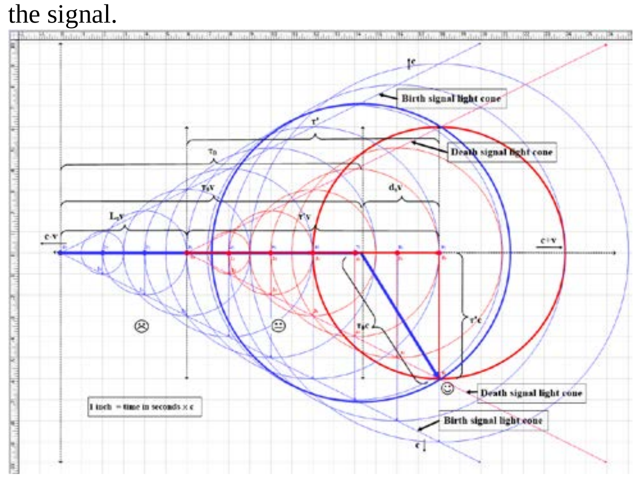


Fig. 9: Geometry for Transverse Relative Time Shift when observer is perpendicular to signal path at $\boldsymbol{\tau'c}$ and $\boldsymbol{v}$ = 2$\boldsymbol{c}$.

The birth signal (blue) is emitted when t=0s. The death signal (red) is emitted when $\boldsymbol{L_s}$ = 3s (**t**=3s). The happy observer using (17a) sees the birth signal when $\boldsymbol{\tau_0}$ = 7.1010s. The observation time interval $\boldsymbol{d_s}$ begins when **t** = 7.1010s and ends when the observer receives the death signal $\boldsymbol{\tau'}$ = 6s (**t**=9s). The happy observer using (13) measures $\boldsymbol{d_s}$ = 1.8990s. The ratio $\boldsymbol{\tau'/\tau_0}$ = 0.8449. The ratio $\boldsymbol{d_s/L_s}$ = 0.6330.

The neutral observer (😐) positioned between the birth and death light cones will not observe the death signal when $\boldsymbol{v} = 2\boldsymbol{c}$. The sad observer (☹) positioned perpendicular to or left (+**v**, +**c**) of **t** = 0s will not observe the birth or death signal.

B. *Oberver on signal path at $\boldsymbol{\tau_0 c}$ and **v**=2**c**.*

Now locate the happy observer within the death signal light cone on the signal source path where **t** = 4.5s from geometry (Figure 10) and solving (21) the happy observer measures $\boldsymbol{\tau'}$ = 1.0s, and the ratio $\boldsymbol{\tau'/\tau_0}$ = 0.3333 when $\boldsymbol{c}$ = $3\text{x}10^8$ m/s, $\boldsymbol{L_s} = \boldsymbol{\tau_0}$ = 3.000s and $\boldsymbol{v} = 2\boldsymbol{c}$.

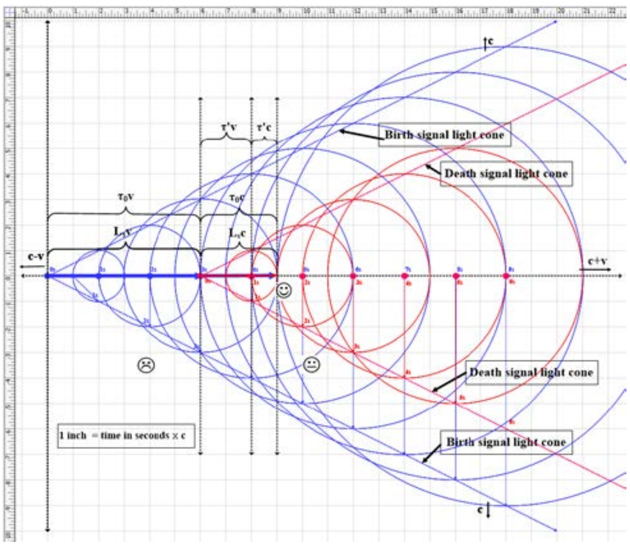


Fig. 10: Geometry for Transverse Relative Time Shift when observer is on signal path at $\boldsymbol{\tau_0 c}$ and **v** = 2**c**.

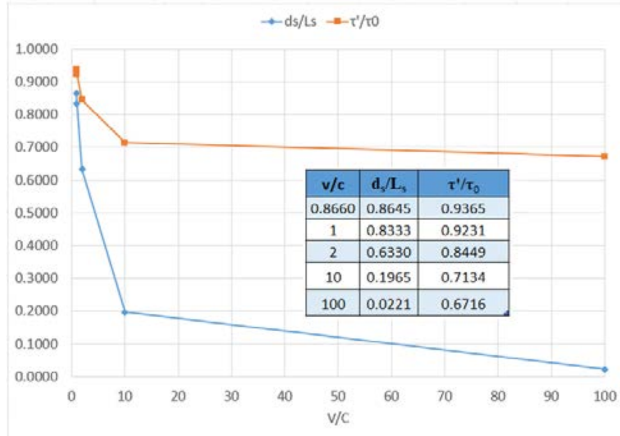
There are general real solutions where the observer can measure a difference in time between the received birth and death signal when positioned on the death signal light cone or in the death signal light cone on the signal center path. In the next section the results for **v** = 10**c** and 100**c** are explored.

C. *Results for signal with **v**=10**c** and **v**=100**c**.*

When **v** = 10**c**, $\boldsymbol{L_s}$ = 3s, and $\boldsymbol{\tau'}$ = 6s (17a) gives $\boldsymbol{\tau_0}$=8.4106s. $\boldsymbol{\tau'/\tau_0}$ = 0.7134. When **v** = -10**c**, $\boldsymbol{L_s}$ = 3s, and $\boldsymbol{\tau'}$ =6s (17b) also yields $\boldsymbol{\tau_0}$ = 8.4106s and $\boldsymbol{\tau'/\tau_0}$ = 0.7134.

When **v** = 100c, $\boldsymbol{L_s}$ = 3s, and $\boldsymbol{\tau'}$ = 6s (17a) gives $\boldsymbol{\tau_0}$=8.9338s. $\boldsymbol{\tau'/\tau_0}$ = 0.6716. When **v** = -100**c**, $\boldsymbol{L_s}$ = 3s, and $\boldsymbol{\tau'}$ =6s (17b) also yields $\boldsymbol{\tau_0}$ = 8.9338s and $\boldsymbol{\tau'/\tau_0}$ = 0.6716.

Figure 11 is a plot of $\boldsymbol{d_s/L_s}$ and $\boldsymbol{\tau'/\tau_0}$ for signal moving with velocities from $+\boldsymbol{v} = \frac{\sqrt{3}}{2}\boldsymbol{c}$ to $+\boldsymbol{v} = 100\boldsymbol{c}$ when $\boldsymbol{L_s}$ = 3s, and $\boldsymbol{\tau'}$ = 6s.



v/c	d_s/L_s	τ'/τ_0
0.8660	0.8645	0.9365
1	0.8333	0.9231
2	0.6330	0.8449
10	0.1965	0.7134
100	0.0221	0.6716

Fig. 11: Plot of $\boldsymbol{d_s/L_s}$ and $\boldsymbol{\tau'/\tau_0}$ for signal moving with velocities from $+\boldsymbol{v} = \frac{\sqrt{3}}{2}\boldsymbol{c}$ to $+\boldsymbol{v} = 100\boldsymbol{c}$ when observer is perpendicular to signal path at $\boldsymbol{\tau' c}$ and $\boldsymbol{L_s}$ = 3s, and $\boldsymbol{\tau'}$ = 6s.

VI. Finding Location of Observer using Heron's Formula and Equations for Two Intersecting Circles

A. *Oberver on death signal light cone when **v** = **c**.*

The triangle formed in Figure 12 by the centers of the birth signal **B**, death signal **A** and the location of the happy observer (**E**) form the area **K** (28) from Heron's formula [2] when **v** = **c**. Where $\mathbf{r_B}$ is the radius of the birth signal seen by the happy observer, $\mathbf{r_A}$ is the radius of the death signal seen by the happy observer, and **d** (23) is the distance between the centers of the two circles. The center of the birth signal **B** has the coordinate (X_B, Y_B). The center of the death signal **A** has the coordinate (X_A, Y_A). In [2] the distance **d** along the line from **A** to **B** is considered "positive". In this work the signal source travels from **B** to **A**, left to right, for +**v**. The signal travels from **B** to **A**, right to left, for –**v**. The birth signal is given at (t=0s) and is coincident with the coordinate origin traveling with +**v** or -**v**.

$$K = \left(\frac{1}{4}\right)\sqrt{((r_A + r_B)^2 - d^2)(d^2 - (r_A - r_B)^2)} \quad (28)$$

$$d^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2 \quad (29)$$

$$d = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} \quad (30)$$

Two solutions for the happy observer(s) located at intersecting circles are in (31) and (32).

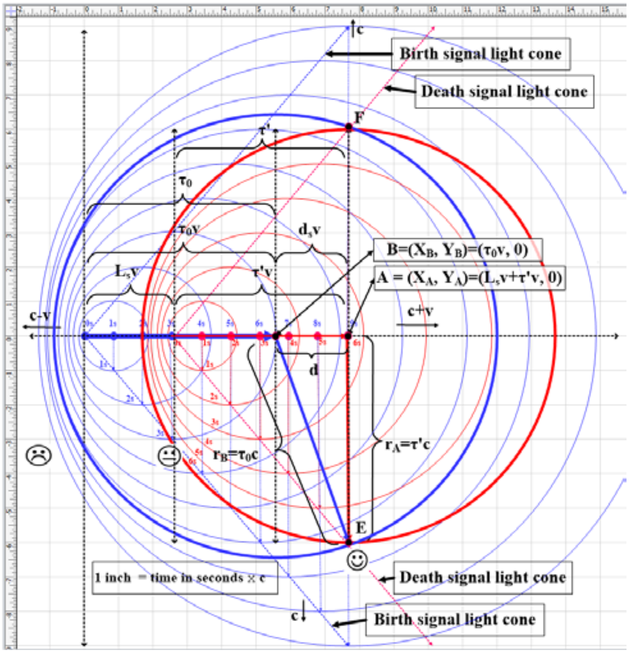


Fig. 12: Observer coordinates **E** and **F** using Heron's formula for **K** and equations for two intersecting circles [2] when **v** < **c**.

$$X_F = \left(\tfrac{1}{2}\right)(X_B + X_A) + \frac{\left(\frac{1}{2}\right)(X_B - X_A)({r_A}^2 - {r_B}^2)}{d^2} + 2(Y_B - Y_A)K/d^2 \quad (31a)$$

$$X_E = \left(\tfrac{1}{2}\right)(X_B + X_A) + \frac{\left(\frac{1}{2}\right)(X_B - X_A)({r_A}^2 - {r_B}^2)}{d^2} - 2(Y_B - Y_A)K/d^2 \quad (31b)$$

$$Y_F = \left(\tfrac{1}{2}\right)(Y_B + Y_A) + \frac{\left(\frac{1}{2}\right)(Y_B - Y_A)({r_A}^2 - {r_B}^2)}{d^2} + (-2)(X_B - X_A)K/d^2 \quad (32a)$$

$$Y_E = \left(\tfrac{1}{2}\right)(Y_B + Y_A) + \frac{\left(\frac{1}{2}\right)(Y_B - Y_A)({r_A}^2 - {r_B}^2)}{d^2} - (-2)(X_B - X_A)K/d^2 \quad (32b)$$

From geometry (Figure 12)

$$\mathbf{r_A} = \boldsymbol{\tau' c} \quad (33)$$

$$\mathbf{r_B} = \boldsymbol{\tau_0 c} \quad (34)$$

$$\mathbf{B} = (X_B, Y_B) = (\boldsymbol{\tau_0 v}, 0) \quad (35)$$

$$\mathbf{A} = (X_A, Y_A) = (\boldsymbol{L_s v} + \boldsymbol{\tau' v}, 0). \quad (36)$$

When $+\boldsymbol{v} = \frac{\sqrt{3}}{2}\boldsymbol{c}$ using [28]-[36]

F = (**X$_F$**, **Y$_F$**) = (2.3383×10^9, 1.8000×10^9) m (31a, 32a)
E = (**X$_E$**, **Y$_E$**) = (2.3383×10^9, -1.8000×10^9) m. (31b, 32b)

When $-\boldsymbol{v} = (-1)\frac{\sqrt{3}}{2}\mathbf{c}$ using [28]-[36] in Figure 12

F = (**X$_F$**, **Y$_F$**) = (-2.3383×10^9, -1.8000×10^9) m (31a, 32a)
E = (**X$_E$**, **Y$_E$**) = (-2.3383×10^9, 1.8000×10^9) m. (31b, 32b)

Time (***t***) is positive and increased from the birth signal to the death signal. A negative velocity (-***v***) yields a negative distance (–***vt)*** from the coordinate origin.

The origin marks the position and time of the birth signal and also where all clocks register (t=0s) for reference purposes. (t=0s) does not imply that there is such a thing as negative time. Negative time is not a real thing. Processes happen from one event to the next event with increasing time.

*B. Oberver inside death signal light cone when **v**=**c**.*

The observer on the death signal light cone in Figure 12 cannot use (17a) to solve for $\boldsymbol{\tau_0}$ when $\boldsymbol{v} = \boldsymbol{c}$. Figure 13 shows the geometry for the relocated observers **E** and **F** inside the death signal light cone.

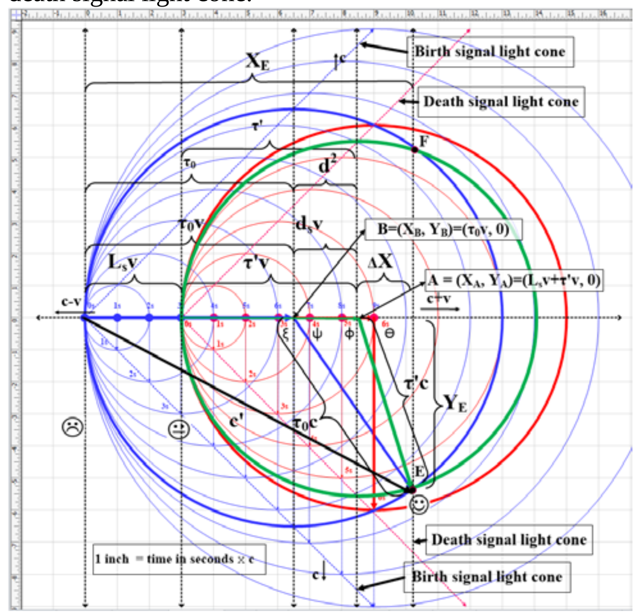


Fig. 13: Happy observer inside death signal light cone at coordinates **E** and **F** using Heron's formula for **K** and equations for two intersecting circles [2] when $\boldsymbol{v} = \boldsymbol{c}$.

The following steps show how to calculate the position of **E** and **F** inside the death signal light cone, see Figure 13.

Step 1 – Solve (29) for $\boldsymbol{d^2}$ (3.6000×10^{17}m^2) with center of death signal source at **A** when $\boldsymbol{\tau'}$ = 5.5s (t=8.5s). Now calculate $\boldsymbol{\tau' v} = \boldsymbol{\tau' c} = +1.65 \times 10^9$m when $\boldsymbol{v} = \boldsymbol{c}$.

Step 2 – Solve (28) for **K (**4.6111×10^{17}m^2).

Step 3 – Solve (31) and (32) for **E** and **F**

E = (3.1500×10^9, -1.5370×10^9) m

F = (3.1500×10^9, -1.5370×10^9) m.

Step 4 – From geometry (Figure 13)

$$X_E = L_s v + \tau' v + \Delta X \quad (37)$$

$$\Delta X = X_E - (L_s v + \tau' v) \quad (38)$$

$$\Delta X^2 = (X_E - (L_s v + \tau' v))^2 \quad (39)$$

$$(\tau' c)^2 = \Delta X^2 + (Y_E)^2 \quad (40)$$

$$(\tau_0 c)^2 = (d_s v + \Delta X)^2 + (Y_E)^2 \quad (41)$$

$$\tan(\Theta) = Y_E/\Delta X \quad (42)$$

$$\Theta = \tan^{-1}(Y_E/\Delta X) \quad (43)$$

$$\phi = \prod - \Theta \quad (44)$$

$$\cos(\phi) = \cos(\prod - \Theta). \quad (45)$$

From law of cosines

$$(\tau_0 c)^2 = (d_s v)^2 + (\tau' c)^2 - 2(d_s v)(\tau' c)\cos(\prod - \tan^{-1}(Y_E/\Delta X)). \quad (46)$$

From geometry (Figure 13)

$$\tan(\psi) = Y_E/(d_s v + \Delta X) \quad (47)$$

$$\psi = \tan^{-1}(Y_E/(d_s v + \Delta X)) \quad (48)$$

$$\xi = \prod - \psi \quad (49)$$

$$\cos(\xi) = \cos(\prod - \psi). \quad (50)$$

From law of cosines

$$(c')^2 = (\tau_0 v)^2 + (\tau_0 c)^2 - 2(\tau_0 v)(\tau_0 c)\cos(\prod - \tan^{-1}(Y_E/(d_s v + \Delta X))). \quad (51)$$

Now similar to [1] find c'

$$(c')^2 = (\tau_0 v + d_s v + \Delta X)^2 + (Y_E)^2 \quad (52)$$

$$c' = \sqrt{(\boldsymbol{\tau_0} v + \mathrm{dsv} + \Delta X)^2 + (Y_E{}^2)}\,. \quad (53)$$

C. *Keplers Law of equal areas with oberver inside death signal light cone when **v**=**c**.*

From geometry in Figure 14, Heron's Formula (28) provides the areas $\mathbf{K_1}$ (blue triangle) (54) and $\mathbf{K_2}$ (green triangle) (55) swept out by the signal source in time $\mathbf{T_1}$ (56) and $\mathbf{T_2}$ (57), traveling with velocity $\boldsymbol{v}$ and the happy observer located at the focus of signal source path. The ratio **K/T** (58) is a constant when v is constant.

When $\boldsymbol{v} = \boldsymbol{c}$

$$K = \left(\tfrac{1}{4}\right)\sqrt{((r_A + r_B)^2 - d^2)(d^2 - (r_A - r_B)^2)} \quad (28)$$

$$K_1(\boldsymbol{r_A}, \boldsymbol{r_B}, \boldsymbol{d}) = K_1(\boldsymbol{\tau_0 c}, \boldsymbol{c'}, \boldsymbol{\tau_0 v}) = 1.4986\text{x}10^{18}\ (\text{m}^2) \quad (54)$$

$$K_2(\boldsymbol{r_A}, \boldsymbol{r_B}, \boldsymbol{d}) = K_2(\boldsymbol{\tau' c}, \boldsymbol{\tau_0 c}, \boldsymbol{d_s v}) = 4.6111\text{x}10^{17}\ (\text{m}^2) \quad (55)$$

$$T_1 = \boldsymbol{\tau_0} = 6.5000\ (\text{s}) \quad (56)$$

$$T_2 = \boldsymbol{d_s} = 2.000\ (\text{s}) \quad (57)$$

$$K_1/T_1 = K_2/T_2 = 2.3056\text{x}10^{17}\ (\text{m}^2/\text{s}). \quad (58)$$

Now there is one more area K3 with its associated T3 that needs to be considered. From geometry, in Figure 15, the area of a general triangle [4] $\mathbf{K_3}$ (59) is

$$\mathbf{K_3} = (1/2)(\text{Base})(\text{Height}). \quad (59)$$

The base is $\Delta\mathbf{X}$, the height is $\mathbf{Y_E}$ substituting into (59) gives the desired $\mathbf{K_3}$ (60). Now $\mathbf{T_3}$ is found using (61) and $\mathbf{K_3/T_3}$ using (62).

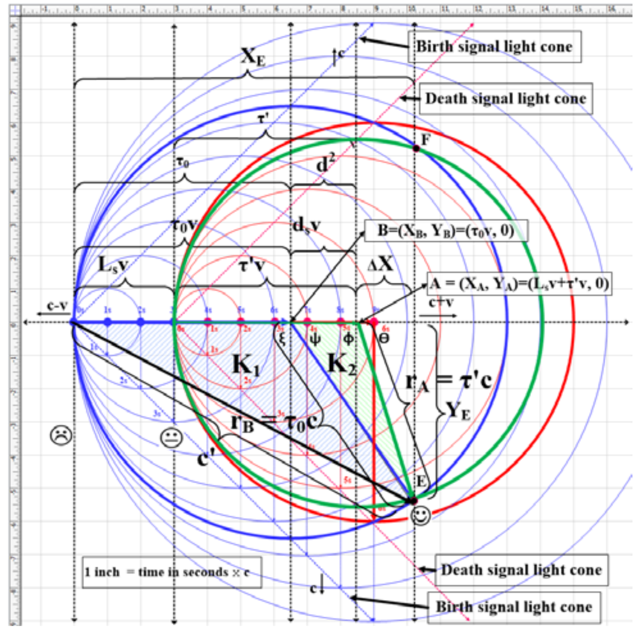


Fig. 14: Kepler's Law of equal areas with observer inside death signal light cone when v =c.

$$\mathbf{K_3} = (1/2)\,\Delta\mathbf{X}\,\mathbf{Y_E}\ (\text{m}^2) \quad (60)$$

$$\mathbf{T_3} = \Delta\mathbf{X}\,/\boldsymbol{v}\ (\text{s}) \quad (61)$$

$$\mathbf{K_3/T_3} = (1/2)\,\mathbf{Y_E}\,\boldsymbol{v}\ (\text{m}^2/\text{s}) \quad (62)$$

$$\mathbf{K_3/T_3} = 2.3056\text{x}10^{17}\ (\text{m}^2/\text{s}) \quad (63)$$

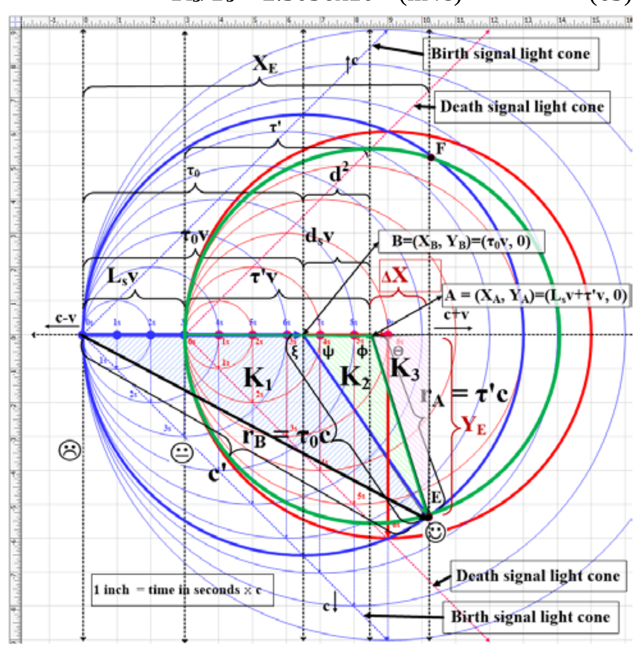


Fig. 15: $\mathbf{K_3}$ derived from area of triangle when v =c.

The geometry with **K/T** is similar to that of orbits of planets and satellites with focus at the observer. The orbit, ellipse, or straight line if you prefer is an open orbit with eccentricity, ***e*** >>1 (very large).

The signal source ***S*** is analogous to the orbiting satellite moving with velocity **v**. The birth and death signals W_0 communicate or carry energy to the observer. ***S***, W_0, and the Observer have zero mass. S travels in a straight line with constant velocity.

The Appendix A contains tables with calculations from equations and variable values referenced in this work. Appendix B contains larger versions of the Figures.

VII. PROPOSE METHOD TO CALCULATE OBSERVED *TRANSVERSE RELATIVE LENGTH* FOR OBJECTS TRAVELING UP TO AND BEYOND C

From geometry in Figure 16 the distance ***L*** (64) between the birth and death signals is 7.7942x10^8m.

$$L = L_S v \tag{64}$$

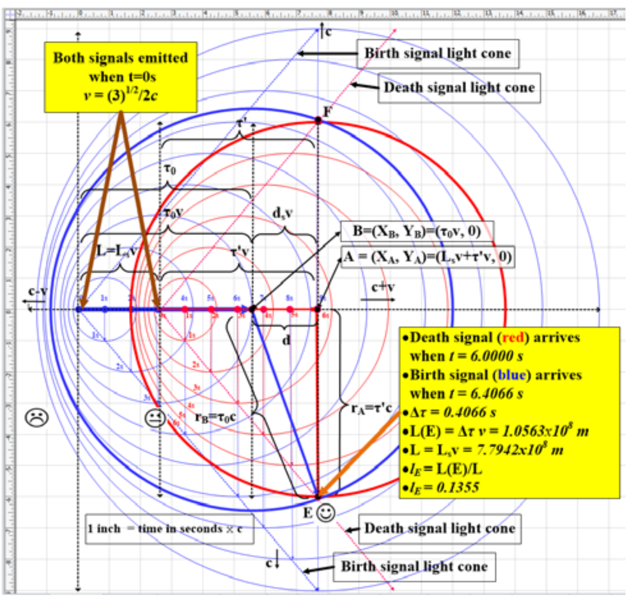


Fig. 16: Observer measures transverse relative length of rod **L** when birth and death signals are emitted at t = 0s and **v** < **c**.

If this distance were a rod, the birth signal (blue) and death signal (red) were both emitted when t=0s and $v = \frac{\sqrt{3}}{2}c$ the happy observer at location **E** would see the death signal first when **t** = 6.0000s (τ') and the birth signal when **t** = 6.4066s (τ_0). The difference in time $\Delta\tau$ =0.4066s (65) multiplied by the velocity ***v*** of the rod when $v = \frac{\sqrt{3}}{2}c$ yields the *transverse relative length* (66) $L(E)$ = 1.0563x10^8m measured by the happy observer at **E**. The ratio of the *transverse relative length* l_E divided by the distance measured between birth and death signal locations $L(E)$ is l_E (67) is 0.1355 as seen by the happy observer at location **E**.

$$\Delta\tau = \tau_0 - \tau' \tag{65}$$

$$L(E) = \Delta\tau\, v \tag{66}$$

$$l_E = L(E)/L \tag{67}$$

VIII. OBSERVATIONS

Using geometry for constant velocities, up to and exceeding the speed of light c, in Euclidean space it is shown that locating the observer downstream from the moving death signal on the signal light cone or on the signal path provides the opportunity to observe both τ', τ_0 and find τ'/τ_0. Also, a method for the fixed observer to calculate *transverse relative length* l_E is proposed. It is also show that signals advance ahead of the moving source maintaining the source as the signal center when the source is in constant linear velocity. A constant distance (m) between signals was not used for comparison for hyper velocities of ***v***. This might yield some interesting results.

IX. THE MEANING OF AREA K3

A. *Observer perpendicular to signal path at τ'c and v=c.*

The geometry, Figure 17, the time of flight of the source ***S*** traveling with velocity ***v*** (m/s) for the distance Δ**X** (m) sweeps out area $\mathbf{K_3}$ (m^2) to the Observer **E** at coordinate O(XE,YE) in $\mathbf{T_3}$ (s) (61).

$$T_3 = \Delta X / v \text{ (s)} \tag{61}$$

$$T_3 = 2\text{s}$$

Solve (61) for Δ**X** (68).

$$\Delta X = T_3 v \text{ (m)} \tag{68}$$

Now $\mathbf{X_E}$ (m) (69) is the distance traveled by ***S*** with velocity ***v*** from the birth signal (t=0s) to the point on the X axis nearest the Observer $\mathbf{E=O(X_E,0)}$.

$$X_E = \Delta t_x\, v \text{ (m)} \tag{69}$$

Δt_x is the time seconds required for ***S*** to travel $\mathbf{X_E}$ (m).

Now $\mathbf{Y_E}$ (m) (70) is the distance traveled by a signal ***W*** from a fixed observer located at $\mathbf{O(X_E,0)}$ with velocity ***c*** to the fixed Observer located at $\mathbf{O(X_E,Y_E)}$.

$$Y_E = \Delta t_y\, c \text{ (m)} \tag{70}$$

Δt_y is the time seconds required for ***W*** to travel $\mathbf{Y_E}$ (m).

The sum of $\mathbf{K_1}$, $\mathbf{K_2}$ and $\mathbf{K_3}$ is $\mathbf{K_{123}}$ which is the total area from the coordinate origin to $\mathbf{O(X_E,0)}$ and $\mathbf{O(X_E,Y_E)}$. $\mathbf{K_{123}}$ is (71)

$$K_{123} = K_1 + K_2 + K_3 \text{ (m}^2\text{)} \tag{71}$$

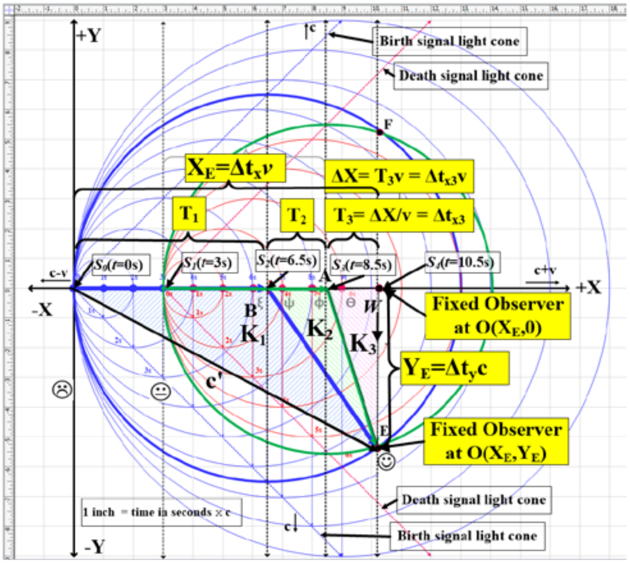


Fig. 17: $\mathbf{K_{123}} = ½\ (\boldsymbol{\Delta t_x v})\ (\boldsymbol{\Delta t_y\ c})$.

$\mathbf{T_{123}}$ is the total flight time of the S on the X axis (72)

$$\mathbf{T_{123}} = \mathbf{T_1}+\mathbf{T_2}+\mathbf{T_3} \quad (s) \qquad (72)$$

$\mathbf{K_{123}}$ from the area of a right triangle is (73)

$$\mathbf{K_{123}} = (1/2)\ \mathbf{X_E}\ \mathbf{Y_E}. \quad (m^2) \qquad (73)$$

Using the relationship that K/T is a constant (74)

$$\mathbf{K_{123}}\ /\ \mathbf{T_{123}} = (1/2)\ \mathbf{X_E}\ \mathbf{Y_E}\ /\ \boldsymbol{\Delta t_x} \quad (m^2/s) \qquad (74)$$

Substituting (69)(70) into (74) and $\mathbf{T_{123}}$ with $\boldsymbol{\Delta t_x}$ (75).

$$\mathbf{K_{123}}\ /\ \boldsymbol{\Delta t_x} = (1/2)\ (\boldsymbol{\Delta t_x\ v})\ (\boldsymbol{\Delta t_y\ c})\ /\ \boldsymbol{\Delta t_x} \ (m^2/s) \qquad (75)$$

Eliminating $\boldsymbol{\Delta t_x}$ in the denominator (76)

$$\mathbf{K_{123}} = (1/2)\ (\boldsymbol{\Delta t_x\ v})\ (\boldsymbol{\Delta t_y\ c}) \quad (m^2) \qquad (76)$$

Important note $\boldsymbol{v}$ is a velocity vector for the Source $\boldsymbol{S}$ while $\boldsymbol{c}$ is the magnitude of the speed of light which points from $\boldsymbol{W}$ a stationary source to the Observer at coordinate $\mathbf{O(X_E,Y_E)}$.

Using the geometry of area $\mathbf{K_3}$ substitute (68)(70) into (40) to get (77) where $\boldsymbol{\Delta t_{x3}} = \mathbf{T_3}$.

$$(\boldsymbol{\tau' c})^2 = \Delta \boldsymbol{X}^2 + (\boldsymbol{Y_E})^2 \quad (m^2) \qquad (40)$$

$$(\boldsymbol{\tau' c})^2 = (\boldsymbol{\Delta t_{x3} v})^2 + (\boldsymbol{\Delta t_y c})^2 \quad (m^2) \qquad (77)$$

Solve (77) for $\boldsymbol{\Delta t_{x3}}$ (82).

$$(\boldsymbol{\Delta t_{x3} v})^2 = (\boldsymbol{\tau' c})^2 - (\boldsymbol{\Delta t_y c})^2 \quad (m^2) \qquad (78)$$

$$\boldsymbol{\Delta t_{x3}}^2 \boldsymbol{v}^2 = \boldsymbol{c}^2(\boldsymbol{\tau'}^2 - \boldsymbol{\Delta t_y}^2) \quad (m^2) \qquad (79)$$

$$\boldsymbol{\Delta t_{x3}}^2 \boldsymbol{v}^2/\boldsymbol{v}^2 = \boldsymbol{c}^2(\boldsymbol{\tau'}^2 - \boldsymbol{\Delta t_y}^2)/\ \boldsymbol{v}^2 \quad (s^2) \qquad (80)$$

$$\boldsymbol{\Delta t_{x3}} = (\boldsymbol{c}^2(\boldsymbol{\tau'}^2 - \boldsymbol{\Delta t_y}^2)/\ \boldsymbol{v}^2)^{1/2} \quad (s) \qquad (81)$$

$$\boldsymbol{\Delta t_{x3}} = \boldsymbol{c}/\mathrm{v}(\boldsymbol{\tau'}^2 - \boldsymbol{\Delta t_y}^2)^{1/2} \quad (s) \qquad (82)$$

Where $\boldsymbol{\Delta t_{x3}}$ is the time of signal flight of $\boldsymbol{S}$(8.5s) to $\boldsymbol{S}$(10.5s) which is from A to coordinate $\boldsymbol{S}(\mathbf{X_E,0})$. Again $\boldsymbol{\Delta t_y}$ is the total time it takes for a signal from $\mathbf{O(X_E,0)}$ to arrive at $\mathbf{O(X_E,Y_E)}$.

It is very important to notice that the time of flight of $\boldsymbol{S}(\mathbf{T_3})$ begins when the traveling $\boldsymbol{S}$ clock reads 8.5s and the fixed Observer $O(X_E,Y_E)$ clock reads 8.5s. $\mathbf{T_3}$ is 2s. In this example $\boldsymbol{v} = \boldsymbol{c}$.

Solve (77) for $\boldsymbol{\Delta t_y}$ (85).

$$(\boldsymbol{\Delta t_y c})^2 = (\boldsymbol{\tau' c})^2 - (\boldsymbol{\Delta t_{x3} v})^2 \quad (m^2) \qquad (83)$$

$$\boldsymbol{\Delta t_y} = ((\boldsymbol{\tau' c})^2/\boldsymbol{c}^2 - (\boldsymbol{\Delta t_{x3} v})^2/\boldsymbol{c}^2)^{1/2} \quad (s) \qquad (84)$$

$$\boldsymbol{\Delta t_y} = (\boldsymbol{\tau'}^2 - (\boldsymbol{v}^2/\boldsymbol{c}^2)\boldsymbol{\Delta t_{x3}}^2)^{1/2} \quad (s) \qquad (85)$$

Why all the math? From geometry, Figure 17, the source $\boldsymbol{S}$ traveling with velocity $\boldsymbol{v}$ will be at location $\mathbf{A}$ when the fixed observer $\mathbf{E}$ sees the signal sent by $\boldsymbol{S}$ at time $\boldsymbol{t}$=3s. This is important, the area $\mathbf{K_3}$ is in the future to $\boldsymbol{S}$ when $\boldsymbol{S}$ fly's over coordinate $\mathbf{A}$! Area $\mathbf{K_3}$ is a measure of the observability opportunity for an observer at location $\mathbf{E}$ or $\mathbf{F}$.

Rotating $\mathbf{E}$ about the X axis by 2∏ radians create a circle in the $\mathbf{Y\ Z}$ plane with radius $\mathbf{Y_E}$ and center at $\mathbf{S_4}$(t=10.5s). The area A, B, and a point on this circle with radius $\mathbf{Y_E}$ is equal to $\mathbf{K_2}$ and have the same observability to signals from S moving with velocity $\boldsymbol{v}$ as Observers at $\mathbf{E}$ or $\mathbf{F}$.
In conclusion, observability is a function of the source $\boldsymbol{S}$ velocity $\boldsymbol{v}$, the position of the observer, and the area $\mathbf{K}$ swept out by $\mathbf{S}$ pointing to the observer's location off of the source trajectory in this case the X axis. Area $\mathbf{K_3}$ represents the future observability for the signal $\mathbf{S}$ by the observer $\mathbf{O}$.

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Appendix A.1-Table 1: Calculated coordinates for $\boldsymbol{W_0}$ for selected **ϴ** when t = 3s, $\boldsymbol{v}$ = 0 m/s.

Observer	Angle ϴ (Radians)	X = cos(ϴ)	Y = sin(ϴ)	W_0 (meters) t=3s r = $\tau_0 c$	ϴ (Radians)	$W_0(X) = W_0\cos(ϴ)$ (m)	$W_0(Y) = W_0\sin(ϴ)$ (m)
Observer 1	0	1.0000	0.0000	9.000E+08	0	9.000E+08	0.000E+00
	(1/4)∏	0.7071	0.7071	9.000E+08	(1/4)∏	6.364E+08	6.364E+08
Observer 3	(1/2)∏	0.0000	1.0000	9.000E+08	(1/2)∏	0.000E+00	9.000E+08
	(3/4)∏	-0.7071	0.7071	9.000E+08	(3/4)∏	-6.364E+08	6.364E+08
Observer 2	∏	-1.0000	0.0000	9.000E+08	∏	-9.000E+08	0.000E+00
	(5/4)∏	-0.7071	-0.7071	9.000E+08	(5/4)∏	-6.364E+08	-6.364E+08
Observer 4	(3/2)∏	0.0000	-1.0000	9.000E+08	(3/2)∏	0.000E+00	-9.000E+08
	(7/4)∏	0.7071	-0.7071	9.000E+08	(7/8)∏	6.364E+08	-6.364E+08

Appendix A.2 - Table 2: Coordinates $\boldsymbol{W_0}$ (X,Y) for Observers 1,2,3, and 4 when $\boldsymbol{S}$ has velocity $\boldsymbol{v_s} = \boldsymbol{c}$ m/s.

Observer	Angle ϴ (Radians)	X = cos(ϴ)	Y = sin(ϴ)	c (m/s)	S(v) on X-axis v_s(m/s)	τ_0 (s)	$\tau_0 v$ (m)	W0 (Y) (m) t=3s r = $\tau_0 c$	ϴ (Radians)	$W_0(X) = (\tau_0 v_s + \tau_0 c)\cos(ϴ)$ (m)	$W_0(Y) = (\tau_0 c)\sin(ϴ)$ (m)
Observer 1	0	1.0000	0.0000	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	0	1.800E+09	0.000E+00
	(1/4)∏	0.7071	0.7071	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	(1/4)∏	1.536E+09	6.364E+08
Observer 3	(1/2)∏	0.0000	1.0000	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	(1/2)∏	9.000E+08	9.000E+08
	(3/4)∏	-0.7071	0.7071	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	(3/4)∏	2.636E+08	6.364E+08
Observer 2	∏	-1.0000	0.0000	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	∏	0.000E+00	0.000E+00
	(5/4)∏	-0.7071	-0.7071	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	(5/4)∏	2.636E+08	-6.364E+08
Observer 4	(3/2)∏	0.0000	-1.0000	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	(3/2)∏	9.000E+08	-9.000E+08
	(7/4)∏	0.7071	-0.7071	3.000E+08	3.000E+08	3.0000	9.000E+08	9.000E+08	(7/8)∏	1.536E+09	-6.364E+08

Appendix A.3 - Table 3: Coordinates $\boldsymbol{W_0}$ (X,Y) for Observers 1,2,3, and 4 when $\boldsymbol{S}$ has velocity $\boldsymbol{v_s} = -\boldsymbol{c}$ m/s.

Observer	Angle ϴ (Radians)	X = cos(ϴ)	Y = sin(ϴ)	c (m/s)	S(v) on X-axis v_s(m/s)	τ_0 (s)	$\tau_0 v_s$ (m)	W0 (Y) (m) t=3s r = $\tau_0 c$	W0(X) $= (\tau_0 vs + \tau_0 c)\cos(ϴ)$ (m)	W0(Y) $= (\tau_0 c)\sin(ϴ)$ (m)
Observer 1	0	1.0000	0.0000	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	0.000E+00	0.000E+00
	(1/4)∏	0.7071	0.7071	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-2.636E+08	6.364E+08
Observer 3	(1/2)∏	0.0000	1.0000	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-9.000E+08	9.000E+08
	(3/4)∏	-0.7071	0.7071	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-1.536E+09	6.364E+08
Observer 2	∏	-1.0000	0.0000	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-1.800E+09	0.000E+00
	(5/4)∏	-0.7071	-0.7071	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-1.536E+09	-6.364E+08
Observer 4	(3/2)∏	0.0000	-1.0000	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-9.000E+08	-9.000E+08
	(7/4)∏	0.7071	-0.7071	3.000E+08	-3.000E+08	3.0000	-9.000E+08	9.000E+08	-2.636E+08	-6.364E+08

Appendix A.1: Transvers relative time shift (+v) when observer is inside death signal light cone.

Variable	v < c	units	v = c	units2	v = 2c	units3	v = 10c	units4	v = 100c	units5
v =	2.5981E+08	m/s	3.0000E+08	m/s	6.0000E+08	m/s	3.0000E+09	m/s	3.0000E+10	m/s
c =	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s
v/c =	0.8660		1.0000		2.0000		10.0000		100.0000	
τ_0 =	**6.4066**	s	**6.5000**	s	**7.1010**	s	**8.4106**	s	**8.9338**	s
L_s =	3.0000	s	3.0000	s	3.0000	s	3.0000	s	3.0000	s
$L = L_s v$ (39) (52)	7.7942E+08	m	9.0000E+08	m	1.8000E+09	m	9.0000E+09	m	9.0000E+10	m
$\tau_0 v$ =	1.6645E+09	m	1.9500E+09	m	4.2606E+09	m	2.5232E+10	m	2.6801E+11	m
τ' =	**5.5000**	s	**5.5000**	s	**5.5000**	s	**5.7000**	s	**5.9700**	s
τ'v =	1.4289E+09	m	1.6500E+09	m	3.3000E+09	m	1.7100E+10	m	1.7910E+11	m
$d_s v = L_s v + \tau' v - \tau_0 v$ (5)	5.4388E+08	m	6.0000E+08	m	8.3940E+08	m	8.6818E+08	m	1.0857E+09	m
$(d_s v)^2$ =	2.9581E+17	m^2	3.6000E+17	m^2	7.0459E+17	m^2	7.5373E+17	m^2	1.1788E+18	m^2
$(\tau' c)^2$ =	2.7225E+18	m^2	2.7225E+18	m^2	2.7225E+18	m^2	2.9241E+18	m^2	3.2077E+18	m^2
$(\tau_0 c)^2$ =	3.6940E+18	m^2	3.8025E+18	m^2	4.5382E+18	m^2	6.3664E+18	m^2	7.1832E+18	m^2
$d_s = (d_s v)/v$ (6)	2.0934	s	2.0000	s	1.3990	s	0.2894	s	0.0362	s
d_s/L_s=	0.6978		0.6667		0.4663		0.0965		0.0121	
τ'/τ_0=	0.8585		0.8462		0.7745		0.6777		0.6682	
$\tau_0 - \tau'$ =	0.9066	s	1.0000	s	1.6010	s	2.7106	s	2.9638	s
$X_B = \tau_0 v$ (35)	1.6645E+09	m	1.9500E+09	m	4.2606E+09	m	2.5232E+10	m	2.6801E+11	m
$X_A = L_s v + \tau' v$ (36)	2.2084E+09	m	2.5500E+09	m	5.1000E+09	m	2.6100E+10	m	2.6910E+11	m
Y_B = (35)	0.0000	m	0.0000	m	0.0000	m	0.0000	m	0.0000	m
Y_A = (36)	0.0000	m	0.0000	m	0.0000	m	0.0000	m	0.0000	m
d^2 = (29)	2.9581E+17	m^2	3.6000E+17	m^2	7.0459E+17	m^2	7.5373E+17	m^2	1.1788E+18	m^2
$r_A = \tau' c = ((\tau' c)^2)^{1/2}$ (33)	1.6500E+09	m	1.6500E+09	m	1.6500E+09	m	1.7100E+09	m	1.7910E+09	m
$r_B = \tau_0 c = ((\tau_0 c)^2)^{1/2}$(34)	1.9220E+09	m	1.9500E+09	m	2.1303E+09	m	2.5232E+09	m	2.6801E+09	m
K = (28)	4.1569E+17	m^2	4.6111E+17	m^2	6.3435E+17	m^2	3.1496E+17	m^2	6.7566E+17	m^2
X_F = (31a)	2.8295E+09	m	3.1500E+09	m	5.7618E+09	m	2.7648E+10	m	2.7039E+11	m
X_E= (31b)	2.8295E+09	m	3.1500E+09	m	5.7618E+09	m	2.7648E+10	m	2.7039E+11	m
Y_F = (32a)	1.5286E+09	m	1.5370E+09	m	1.5114E+09	m	7.2558E+08	m	1.2446E+09	m
Y_E = (32b)	-1.5286E+09	m	-1.5370E+09	m	-1.5114E+09	m	-7.2558E+08	m	-1.2446E+09	m
$\tau' c = ((\tau' c)^2)^{1/2}$	1.6500E+09	m	1.6500E+09	m	1.6500E+09	m	1.7100E+09	m	1.7910E+09	m
$\Delta\tau = \tau_0 - \tau'$ (65)	0.9066	s	1.0000	s	1.6010	s	2.7106	s	2.9638	s
L(E)= Δτ v (66)	2.3554E+08	m	3.0000E+08	m	9.6060E+08	m	8.1318E+09	m	8.8914E+10	m
L(E)/L= (67)	0.3022		0.3333		0.5337		0.9035		0.9879	
$\Delta X = X_E - (L_s v + \tau' v)$ (38)	6.2118E+08	m	6.0000E+08	m	6.6183E+08	m	1.5484E+09	m	1.2879E+09	m
$\Delta X^2 = (XE - (Lsv + \tau' v))^2$ (39)	3.8587E+17	m^2	3.6000E+17	m^2	4.3802E+17	m^2	2.3976E+18	m^2	1.6587E+18	m^2
Y_E^2=	2.3366E+18	m^2	2.3625E+18	m^2	2.2845E+18	m^2	5.2646E+17	m^2	1.5490E+18	m^2
$(\tau' c)^2 = \Delta X^2 + Y_E^2$ (40)	2.7225E+18	m^2	2.7225E+18	m^2	2.7225E+18	m^2	2.9241E+18	m^2	3.2077E+18	m^2
$(\tau_0 c)^2 = (d_s v + \Delta X)^2 + Y_E^2$ (41)	3.6940E+18	m^2	3.8025E+18	m^2	4.5382E+18	m^2	6.3664E+18	m^2	7.1832E+18	m^2
$(\tau_0 c)^2$ = (46)	3.6940E+18	m^2	3.8025E+18	m^2	4.5382E+18	m^2	6.3664E+18	m^2	7.1832E+18	m^2
$(c')^2$ = (51)	1.0343E+19	m^2	1.2285E+19	m^2	3.5483E+19	m^2	7.6496E+20	m^2	7.3111E+22	m^2
$(c')^2$ = (52)	1.0343E+19	m2	1.2285E+19	m2	3.5483E+19	m2	7.6496E+20	m2	7.3111E+22	m2
c' = (53)	3.2160E+09	m	3.5050E+09	m	5.9568E+09	m	2.7658E+10	m	2.7039E+11	m
v =	2.5981E+08	m/s	3.0000E+08	m/s	6.0000E+08	m/s	3.0000E+09	m/s	3.0000E+10	m/s
K_2 = (55)	4.1569E+17	m^2	**4.6111E+17**	m^2	6.3435E+17	m^2	3.1496E+17	m^2	6.7566E+17	m^2
K_1 = (54)	1.2722E+18	m^2	**1.4986E+18**	m^2	3.2198E+18	m^2	9.1538E+18	m^2	1.6678E+20	m^2
$K_3 = (1/2)\Delta X Y_F$ = (60)	4.7477E+17	m^2	4.6111E+17	m^2	5.0016E+17	m^2	5.6175E+17	m^2	8.0145E+17	m^2
$T_2 = d_s$ (57)	2.0934E+00	s	**2.0000E+00**	s	1.3990E+00	s	2.8939E-01	s	3.6192E-02	s
$T_1 = \tau_0$ (56)	6.4066E+00	s	**6.5000E+00**	s	7.1010E+00	s	8.4106E+00	s	8.9338E+00	s
$T_3 = \Delta X/v$ (61)	2.3909	s	**2.0000**	s	1.1031	s	0.5161	s	0.0429	s
$R_2 = (\tau' c + \tau_0 c)/2$	1.7860E+09	m	1.8000E+09	m	1.8902E+09	m	2.1166E+09	m	2.2356E+09	m
$R_1 = (c' + \tau_0 c)/2$	1.6080E+09	m	1.7525E+09	m	2.9784E+09	m	1.3829E+10	m	1.3520E+11	m
K_2/T_2 = (58)	1.9857E+17	m^2/s	**2.3056E+17**	m^2/s	4.5343E+17	m^2/s	1.0884E+18	m^2/s	1.8669E+19	m^2/s
K_1/T_1 = (58)	1.9857E+17	m^2/s	**2.3056E+17**	m^2/s	4.5343E+17	m^2/s	1.0884E+18	m^2/s	1.8669E+19	m^2/s
$K_3/T_3 = (1/2) Y_E v$ (62)	1.9857E+17	m^2/s	2.3056E+17	m^2/s	4.5343E+17	m^2/s	1.0884E+18	m^2/s	1.8669E+19	m^2/s
K_{2+3}/T_{2+3}	**1.9857E+17**	**m^2/s**	**2.3056E+17**	**m^2/s**	**4.5343E+17**	**m^2/s**	**1.0884E+18**	**m^2/s**	**1.8669E+19**	**m^2/s**
K_{1+2+3}/T_{1+2+3}	**1.9857E+17**	**m^2/s**	**2.3056E+17**	**m^2/s**	**4.5343E+17**	**m^2/s**	**1.0884E+18**	**m^2/s**	**1.8669E+19**	**m^2/s**
$\Delta t_X = X_E\ v$	**10.8909**	s	**10.5000**	s	**9.6031**	s	**9.2161**	s	**9.0129**	s
$c' = (X_E^2 + Y_E^2)^{1/2}$	**3.2160E+09**	m	**3.5050E+09**	m	**5.9568E+09**	m	**2.7658E+10**	m	**2.7039E+11**	m
$\Delta t_y = (Y_E^2)^{1/2}/c$=	5.0953E+00	s	5.1235E+00	s	5.0382E+00	s	2.4186E+00	s	4.1487E+00	s
Δt_y^2 =	2.5963E+01	s^2	2.6250E+01	s^2	2.5383E+01	s^2	5.8496E+00	s^2	1.7211E+01	s^2
τ'^2 =	3.0250E+01	s^2	3.0250E+01	s^2	3.0250E+01	s^2	3.2490E+01	s^2	3.5641E+01	s^2
$\Delta t_{x3} = c/v(\tau'^2 - \Delta t_y^2)^{1/2}$ (82)	2.3909E+00	s	2.0000E+00	s	1.1031E+00	s	5.1614E-01	s	4.2930E-02	s
$\Delta t_y = (\tau'^2 - ((v^2/c^2)\Delta t_{x3}^2)^{1/2}$ (85)	5.0953E+00	s	5.1235E+00	s	5.0382E+00	s	2.4186E+00	s	4.1487E+00	s
$\Delta t_y c = \lvert Y_E \rvert$ (70)	1.5286E+09	m	1.5370E+09	m	1.5114E+09	m	7.2558E+08	m	1.2446E+09	m
$\Delta X = T_3 v = \Delta t_{x3} v$ = (68)	6.2118E+08	m	6.0000E+08	m	6.6183E+08	m	1.5484E+09	m	1.2879E+09	m
$\Delta X/\Delta tx3 = v$	2.5981E+08	m	3.0000E+08	m	6.0000E+08	m	3.0000E+09	m	3.0000E+10	m

Appendix A.2: Transvers relative time shift (-v) when observer is inside death signal light cone.

Variable	v < c	units	v = c	units2	v = 2c	units3	v = 10c	units4	v = 100c	units5
v =	-2.5981E+08	m/s	-3.0000E+08	m/s	-6.0000E+08	m/s	-3.0000E+09	m/s	-3.0000E+10	m/s
c =	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s
v/c =	-0.8660		-1.0000		-2.0000		-10.0000		-100.0000	
τ_0 =	6.4066	s	6.5000	s	7.1010	s	8.4106	s	8.9338	s
L_s =	3.0000	s	3.0000	s	3.0000	s	3.0000	s	3.0000	s
L= L_sv (39) (52)	-7.7942E+08	m	-9.0000E+08	m	-1.8000E+09	m	-9.0000E+09	m	-9.0000E+10	m
τ_0v =	-1.6645E+09	m	-1.9500E+09	m	-4.2606E+09	m	-2.5232E+10	m	-2.6801E+11	m
τ' =	5.5000	s	5.5000	s	5.5000	s	5.7000	s	5.9700	s
τ'v =	-1.4289E+09	m	-1.6500E+09	m	-3.3000E+09	m	-1.7100E+10	m	-1.7910E+11	m
$d_s v = L_s v + \tau' v - \tau_0 v$ (5)	-5.4388E+08	m	-6.0000E+08	m	-8.3940E+08	m	-8.6818E+08	m	-1.0857E+09	m
$(d_s v)^2$ =	2.9581E+17	m^2	3.6000E+17	m^2	7.0459E+17	m^2	7.5373E+17	m^2	1.1788E+18	m^2
$(\tau' c)^2$ =	2.7225E+18	m^2	2.7225E+18	m^2	2.7225E+18	m^2	2.9241E+18	m^2	3.2077E+18	m^2
$(\tau_0 c)^2$ =	3.6940E+18	m^2	3.8025E+18	m^2	4.5382E+18	m^2	6.3664E+18	m^2	7.1832E+18	m^2
$d_s = (d_s v)/v$ (6)	2.0934	s	2.0000	s	1.3990	s	0.2894	s	0.0362	s
d_s/L_s=	0.6978		0.6667		0.4663		0.0965		0.0121	
τ'/τ_0=	0.8585		0.8462		0.7745		0.6777		0.6682	
$\tau_0 - \tau'$ =	0.9066	s	1.0000	s	1.6010	s	2.7106	s	2.9638	s
$X_B = \tau_0 v$ (35)	-1.6645E+09	m	-1.9500E+09	m	-4.2606E+09	m	-2.5232E+10	m	-2.6801E+11	m
$X_A = L_s v + \tau' v$ (36)	-2.2084E+09	m	-2.5500E+09	m	-5.1000E+09	m	-2.6100E+10	m	-2.6910E+11	m
Y_B = (35)	0.0000	m	0.0000	m	0.0000	m	0.0000	m	0.0000	m
Y_A = (36)	0.0000	m	0.0000	m	0.0000	m	0.0000	m	0.0000	m
d^2 = (29)	2.9581E+17	m^2	3.6000E+17	m^2	7.0459E+17	m^2	7.5373E+17	m^2	1.1788E+18	m^2
$r_A = \tau' c = ((\tau' c)^2)^{1/2}$ (33)	1.6500E+09	m	1.6500E+09	m	1.6500E+09	m	1.7100E+09	m	1.7910E+09	m
$r_B = \tau_0 c = ((\tau_0 c)^2)^{1/2}$(34)	1.9220E+09	m	1.9500E+09	m	2.1303E+09	m	2.5232E+09	m	2.6801E+09	m
K = (28)	4.1569E+17	m^2	4.6111E+17	m^2	6.3435E+17	m^2	3.1496E+17	m^2	6.7566E+17	m^2
X_F = (31a)	-2.8295E+09	m	-3.1500E+09	m	-5.7618E+09	m	-2.7648E+10	m	-2.7039E+11	m
X_E= (31b)	-2.8295E+09	m	-3.1500E+09	m	-5.7618E+09	m	-2.7648E+10	m	-2.7039E+11	m
Y_F = (32a)	-1.5286E+09	m	-1.5370E+09	m	-1.5114E+09	m	-7.2558E+08	m	-1.2446E+09	m
Y_E = (32b)	1.5286E+09	m	1.5370E+09	m	1.5114E+09	m	7.2558E+08	m	1.2446E+09	m
$\tau' c = ((\tau' c)^2)^{1/2}$	1.6500E+09	m	1.6500E+09	m	1.6500E+09	m	1.7100E+09	m	1.7910E+09	m
$\Delta\tau = \tau_0 - \tau'$ (65)	0.9066	s	1.0000	s	1.6010	s	2.7106	s	2.9638	s
L(E)= $\Delta\tau$ v (66)	-2.3554E+08	m	-3.0000E+08	m	-9.6060E+08	m	-8.1318E+09	m	-8.8914E+10	m
L(E)/L= (67)	0.3022		0.3333		0.5337		0.9035		0.9879	
$\Delta X = X_E - (L_s v + \tau' v)$ (38)	-6.2118E+08	m	-6.0000E+08	m	-6.6183E+08	m	-1.5484E+09	m	-1.2879E+09	m
$\Delta X^2 = (XE - (Lsv + \tau' v))^2$ (39)	3.8587E+17	m^2	3.6000E+17	m^2	4.3802E+17	m^2	2.3976E+18	m^2	1.6587E+18	m^2
$Y_E^{\ 2}$=	2.3366E+18	m^2	2.3625E+18	m^2	2.2845E+18	m^2	5.2646E+17	m^2	1.5490E+18	m^2
$(\tau' c)^2 = \Delta X^2 + Y_E^{\ 2}$ (40)	2.7225E+18	m^2	2.7225E+18	m^2	2.7225E+18	m^2	2.9241E+18	m^2	3.2077E+18	m^2
$(\tau_0 c)^2 = (d_s v + \Delta X)^2 + Y_E^{\ 2}$ (41)	3.6940E+18	m^2	3.8025E+18	m^2	4.5382E+18	m^2	6.3664E+18	m^2	7.1832E+18	m^2
$(\tau_0 c)^2$ = (46)	2.3426E+18	m^2	2.3625E+18	m^2	2.3160E+18	m^2	9.8921E+17	m^2	1.5899E+18	m^2
$(c')^2$ = (51)	2.5860E+18	m^2	2.9250E+18	m^2	9.8986E+18	m^2	5.2106E+20	m^2	7.0566E+22	m^2
$(c')^2$ = (52)	1.0343E+19	m2	1.2285E+19	m2	3.5483E+19	m2	7.6496E+20	m2	7.3111E+22	m2
c' = (53)	3.2160E+09	m	3.5050E+09	m	5.9568E+09	m	2.7658E+10	m	2.7039E+11	m
v =	-2.5981E+08	m/s	-3.0000E+08	m/s	-6.0000E+08	m/s	-3.0000E+09	m/s	-3.0000E+10	m/s
K_2 = (55)	4.1569E+17	m^2	4.6111E+17	m^2	6.3435E+17	m^2	3.1496E+17	m^2	6.7566E+17	m^2
K_1 = (54)	1.2722E+18	m^2	1.4986E+18	m^2	3.2198E+18	m^2	9.1538E+18	m^2	1.6678E+20	m^2
$K_3 = (1/2)\Delta X Y_F$ = (60)	4.7477E+17	m^2	4.6111E+17	m^2	5.0016E+17	m^2	5.6175E+17	m^2	8.0145E+17	m^2
$T_2 = d_s$ (57)	2.0934E+00	s	2.0000E+00	s	1.3990E+00	s	2.8939E-01	s	3.6192E-02	s
$T_1 = \tau_0$ (56)	6.4066E+00	s	6.5000E+00	s	7.1010E+00	s	8.4106E+00	s	8.9338E+00	s
$T_3 = \Delta X/v$ (61)	2.3909	s	2.0000	s	1.1031	s	0.5161	s	0.0429	s
$R_2 = (\tau' c + \tau_0 c)/2$	1.7860E+09	m	1.8000E+09	m	1.8902E+09	m	2.1166E+09	m	2.2356E+09	m
$R_1 = (c' + \tau_0 c)/2$	1.6080E+09	m	1.7525E+09	m	2.9784E+09	m	1.3829E+10	m	1.3520E+11	m
K_2/T_2 = (58)	1.9857E+17	m^2/s	2.3056E+17	m^2/s	4.5343E+17	m^2/s	1.0884E+18	m^2/s	1.8669E+19	m^2/s
K_1/T_1 = (58)	1.9857E+17	m^2/s	2.3056E+17	m^2/s	4.5343E+17	m^2/s	1.0884E+18	m^2/s	1.8669E+19	m^2/s
$K_3/T_3 = (1/2)Y_E v$ (62)	1.9857E+17	m^2/s	2.3056E+17	m^2/s	4.5343E+17	m^2/s	1.0884E+18	m^2/s	1.8669E+19	m^2/s

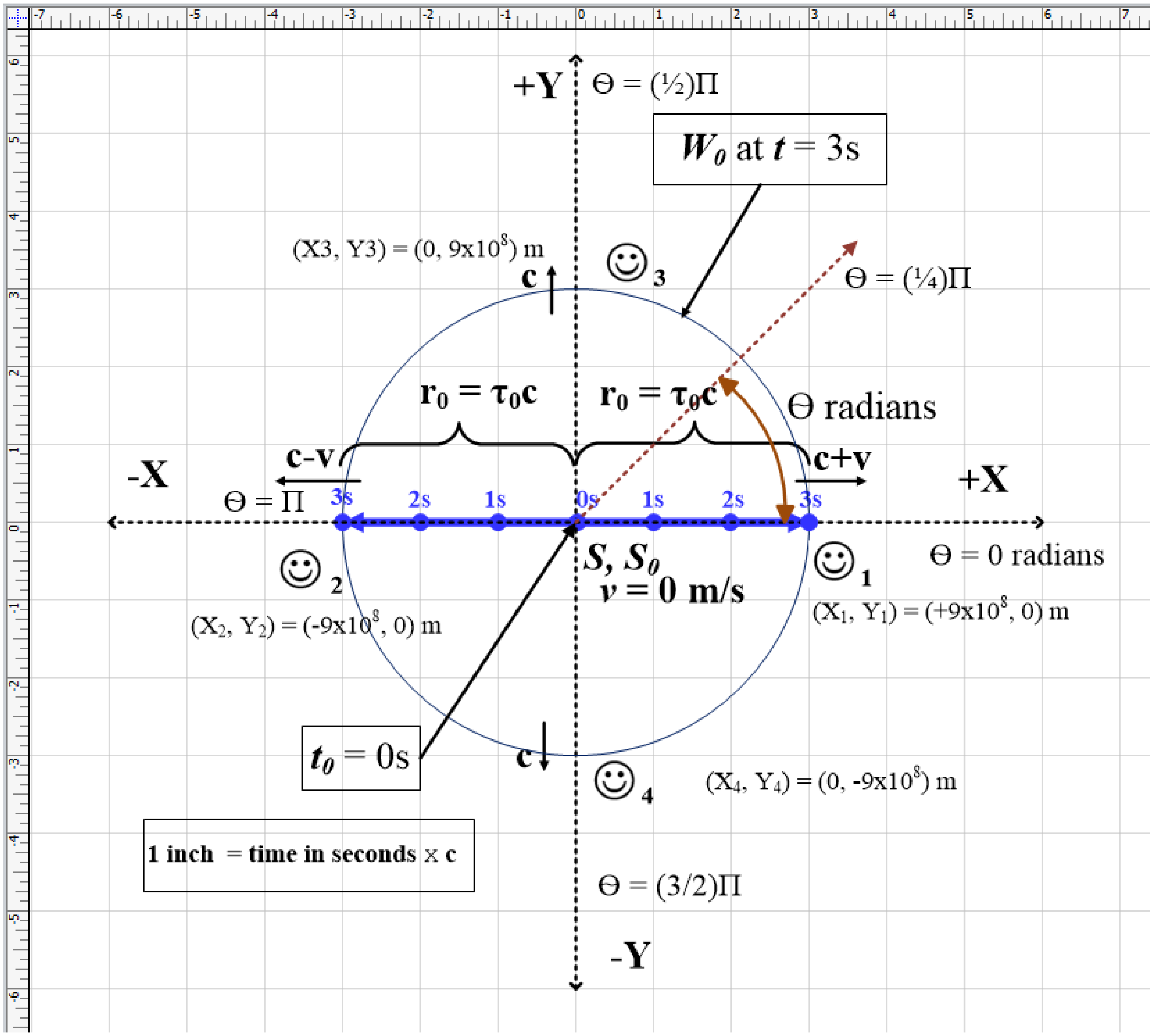
Appendix A.3: Transvers relative time shift (+***v***) when observer is on signal path.

Variable	v < c	units	v = c	units2	v = 2c	units3	v = 10c	units4	v = 100c	units5
v =	2.5981E+08	m/s	3.0000E+08	m/s	6.0000E+08	m/s	3.0000E+09	m/s	3.0000E+10	m/s
c =	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s
v/c =	0.8660		1.0000		2.0000		10.0000		100.0000	
τ_0 =	**3.0000**	s	**3.0000**	s	**3.0000**	s	3.0000	s	3.0000	s
L_s =	**3.0000**	s	**3.0000**	s	**3.0000**	s	3.0000	s	3.0000	s
$L_s v$ =	7.7942E+08	m	9.0000E+08	m	1.8000E+09	m	9.0000E+09	m	9.0000E+10	m
$\tau_0 v$ =	7.7942E+08	m	9.0000E+08	m	1.8000E+09	m	9.0000E+09	m	9.0000E+10	m
τ' = (21)	**1.6077**	s	**1.5000**	s	**1.0000**	s	0.2727	s	0.0297	s
τ'v =	4.1769E+08	m	4.5000E+08	m	6.0000E+08	m	8.1818E+08	m	8.9109E+08	m
τ'/τ_0=	**0.5359**		**0.5000**		**0.3333**		0.0909		0.0099	
$\tau_0 - \tau'$ =	1.3923	s	1.5000	s	2.0000	s	2.7273	s	2.9703	s

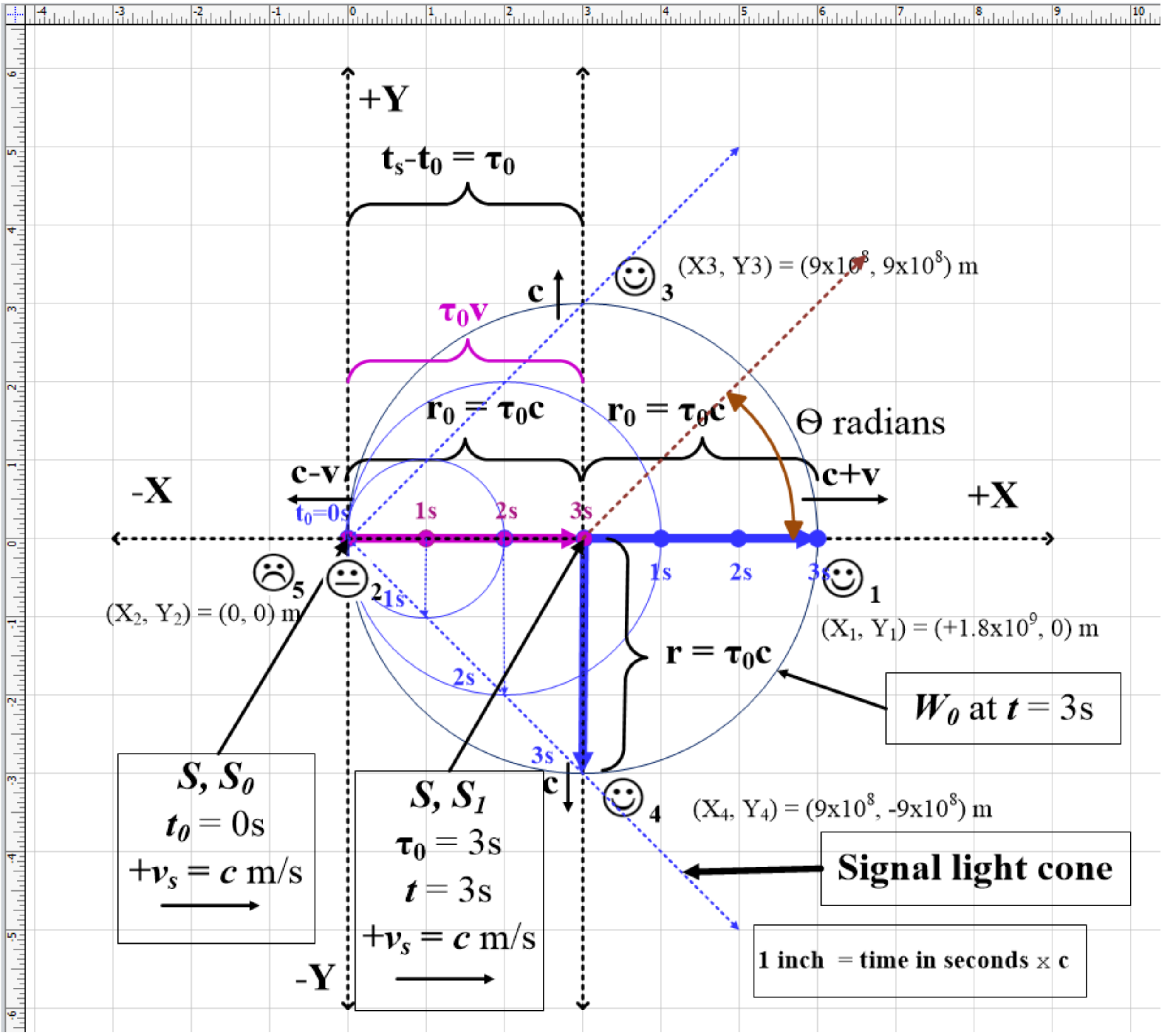
Appendix A.4: Transvers relative time shift (-***v***) when observer is on signal path.

Variable	v < c	units	v = c	units2	v = 2c	units3	v = 10c	units4	v = 100c	units5
v =	-2.5981E+08	m/s	-3.0000E+08	m/s	-6.0000E+08	m/s	-3.0000E+09	m/s	-3.0000E+10	m/s
c =	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s	3.0000E+08	m/s
v/c =	-0.8660		-1.0000		-2.0000		-10.0000		-100.0000	
τ_0 =	**3.0000**	s	3.0000	s	3.0000	s	3.0000	s	3.0000	s
L_s =	**3.0000**	s	3.0000	s	3.0000	s	3.0000	s	3.0000	s
$L_s v$ =	-7.7942E+08	m	-9.0000E+08	m	-1.8000E+09	m	-9.0000E+09	m	-9.0000E+10	m
$\tau_0 v$ =	-7.7942E+08	m	-9.0000E+08	m	-1.8000E+09	m	-9.0000E+09	m	-9.0000E+10	m
τ' = (27)	**1.6077**	s	1.5000	s	1.0000	s	0.2727	s	0.0297	s
τ'v =	-4.1769E+08	m	-4.5000E+08	m	-6.0000E+08	m	-8.1818E+08	m	-8.9109E+08	m
τ'/τ_0=	**0.5359**		0.5000		0.3333		0.0909		0.0099	
$\tau_0 - \tau'$ =	1.3923	s	1.5000	s	2.0000	s	2.7273	s	2.9703	s

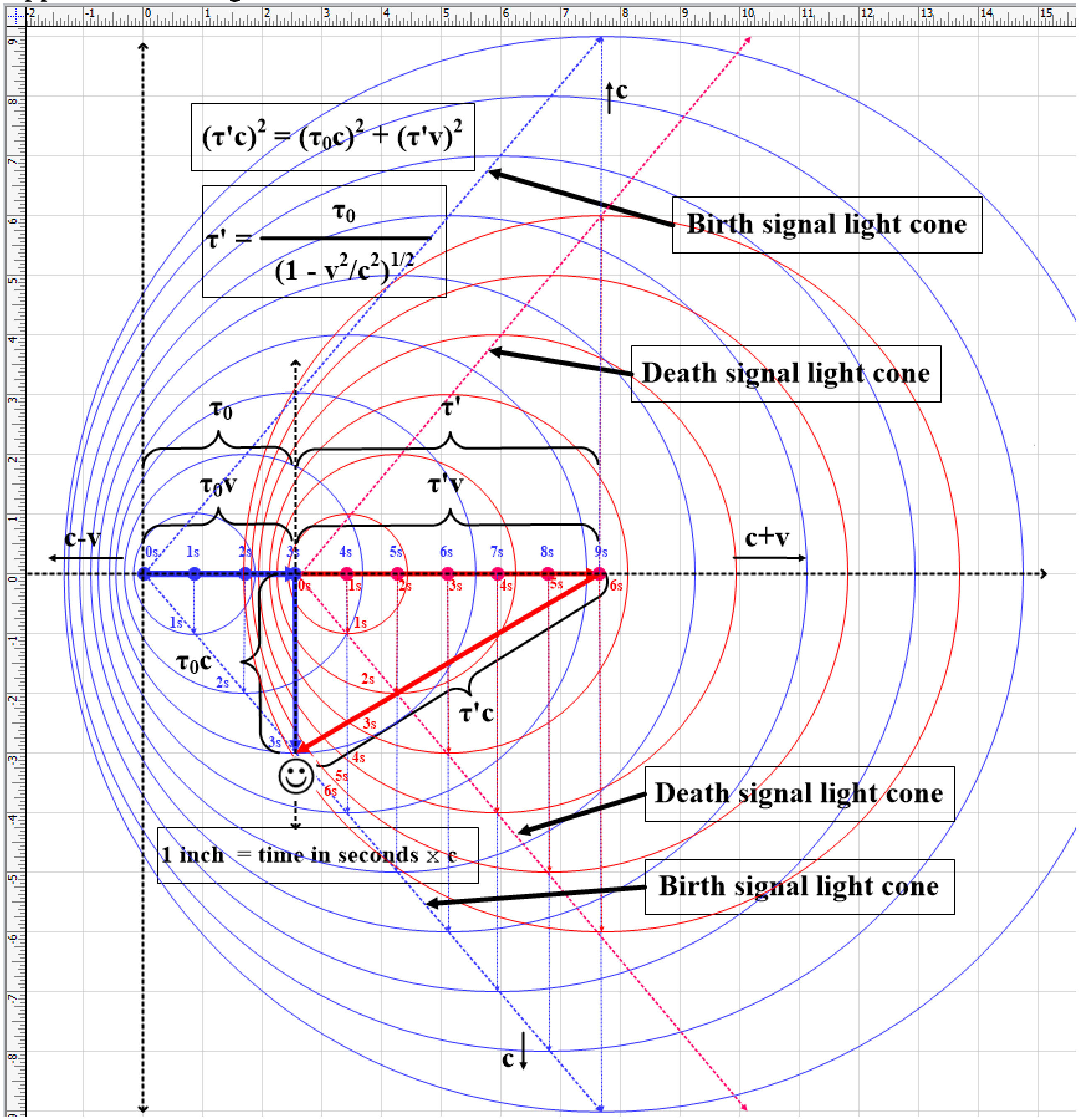
Appendix B.1 - Fig. 1: Geometry to calculate signal position when S has velocity v= 0 m/s.



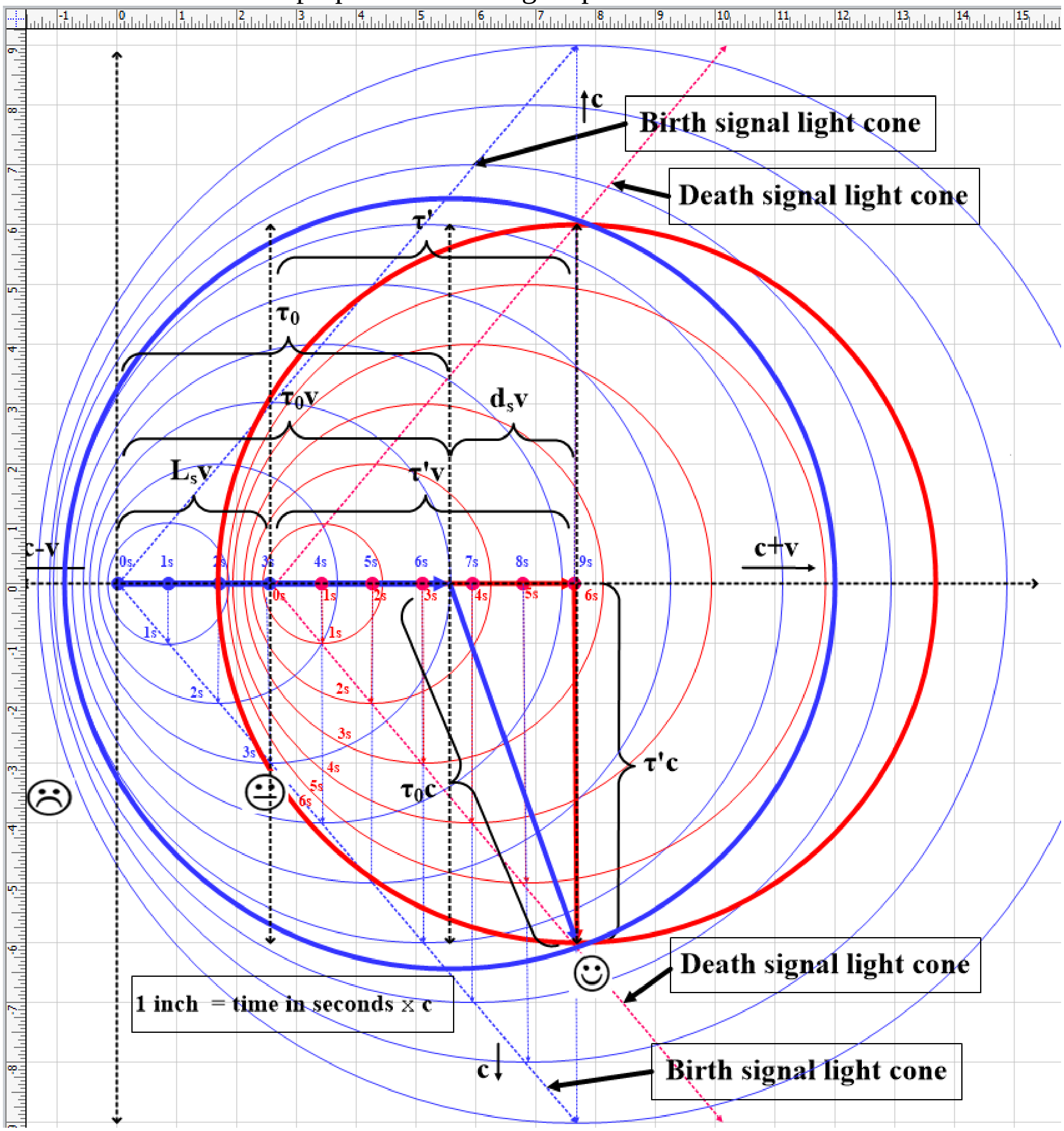
Appendix B.2 - Fig. 2: Geometry to calculate signal position when S has velocity v = c m/s.



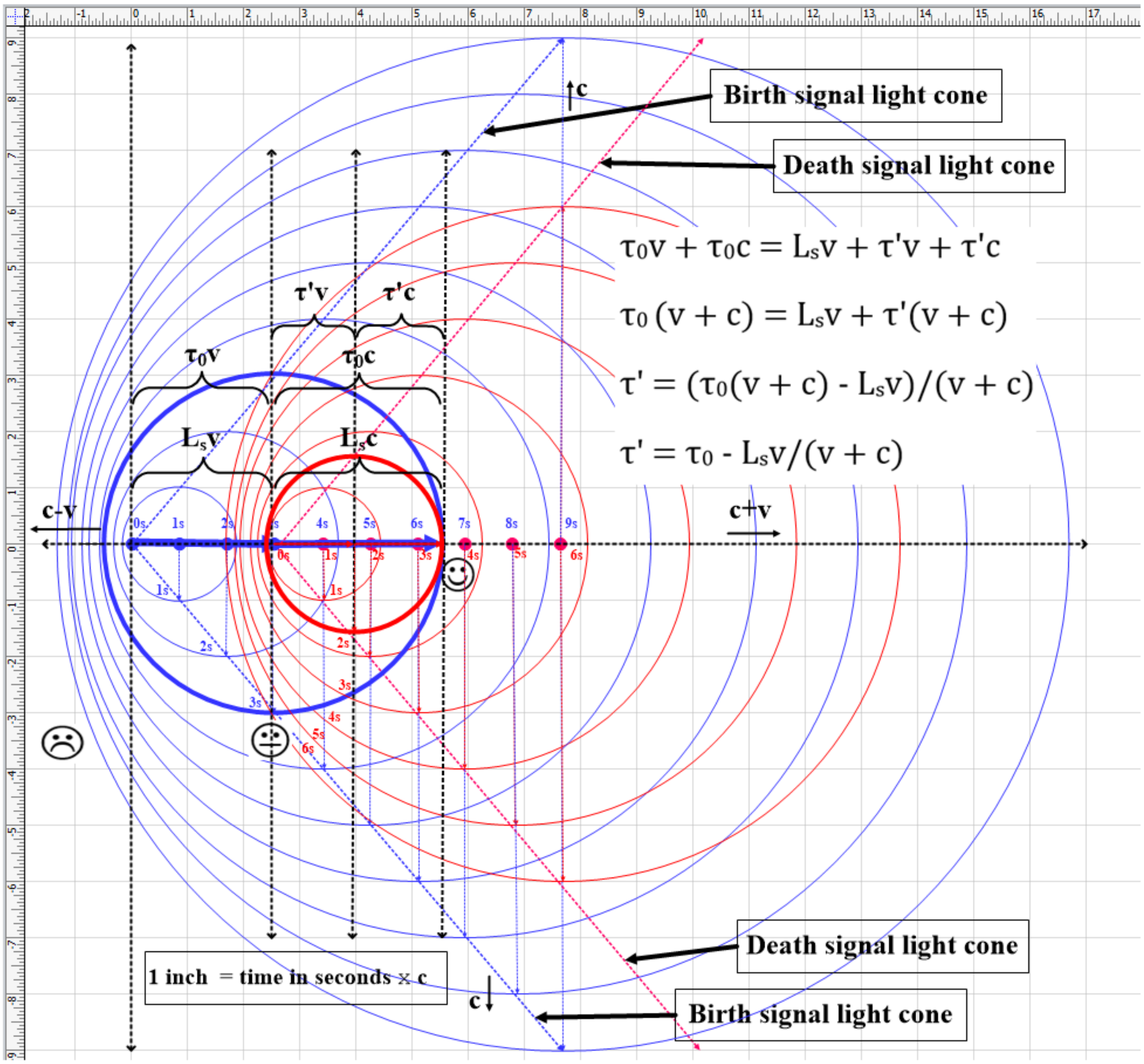
Appendix B.3 - Fig. 3: Transverse Relative Time Shift.



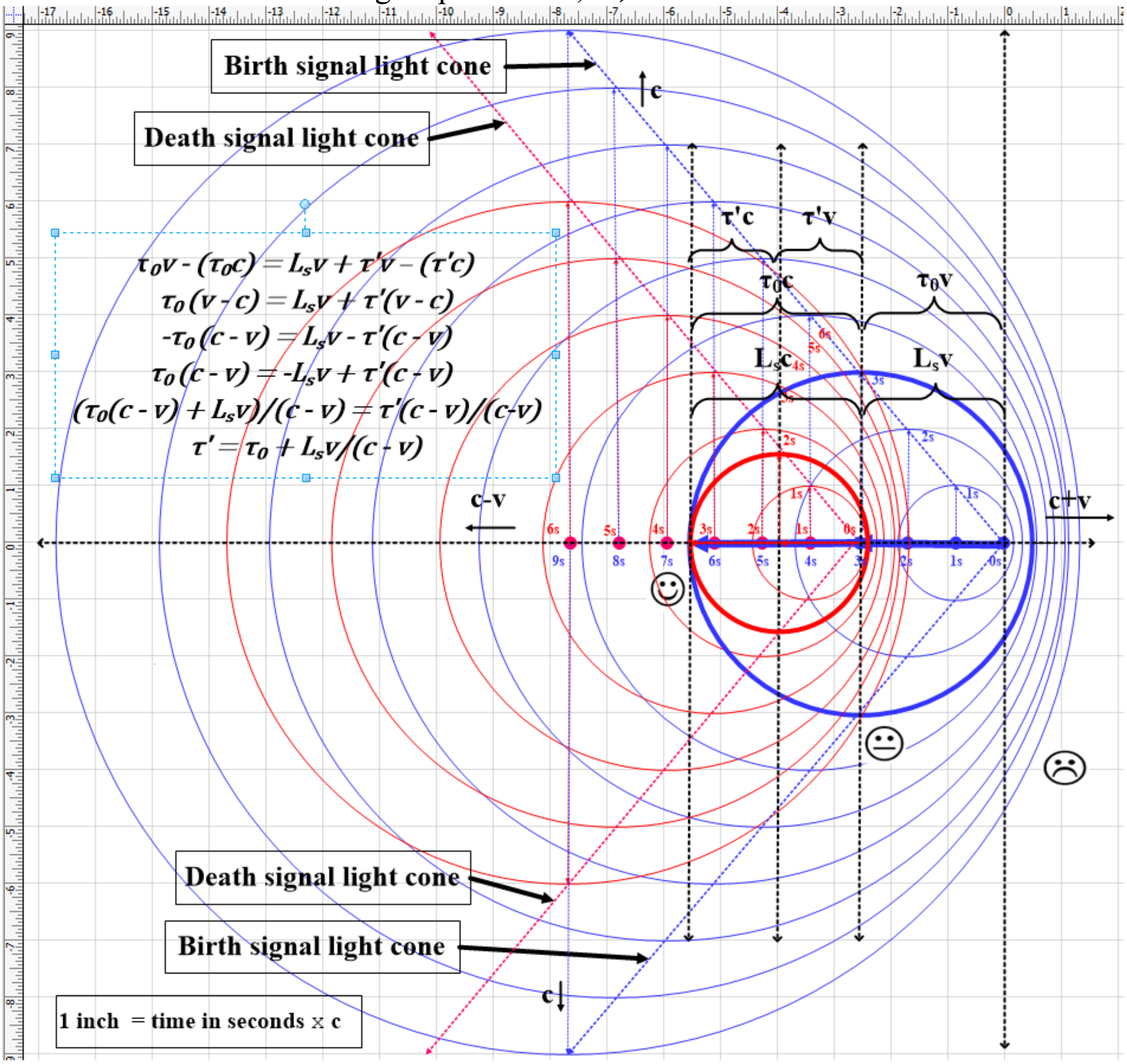
Appendix B.4 – Fig. 4: Geometry for Transverse Relative Time Shift when observer is perpendicular to signal path at $\tau'c$ and $v < c$.



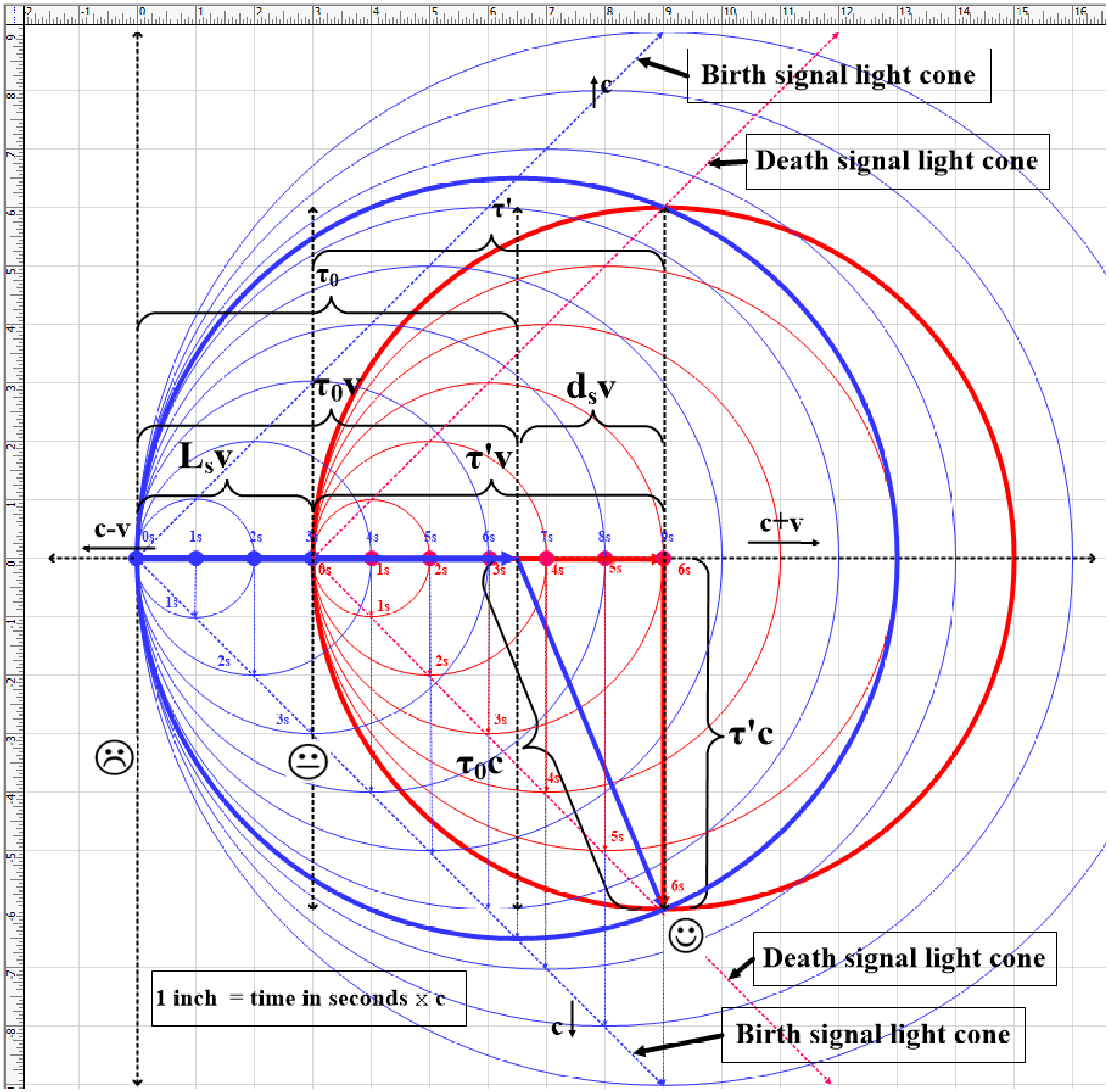
Appendix B.5 – Fig. 5: Geometry for Transverse Relative Time Shift when observer is on signal path at $\tau_0 c$ and $v < c$.



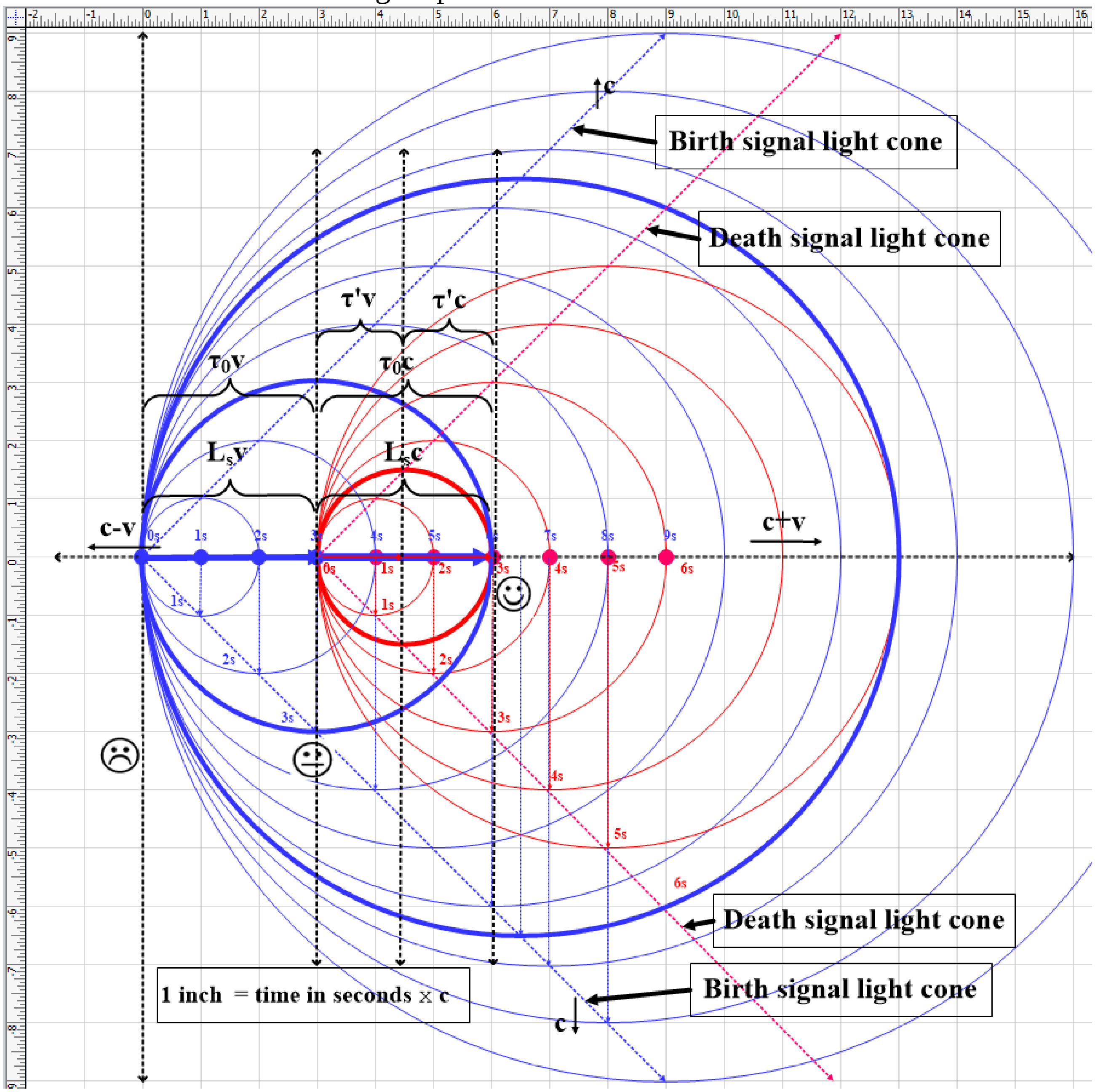
Appendix B.6 – Fig. 6: Geometry for Transverse Relative Time Shift when observer is on signal path at τ0c, -v, and v < c.



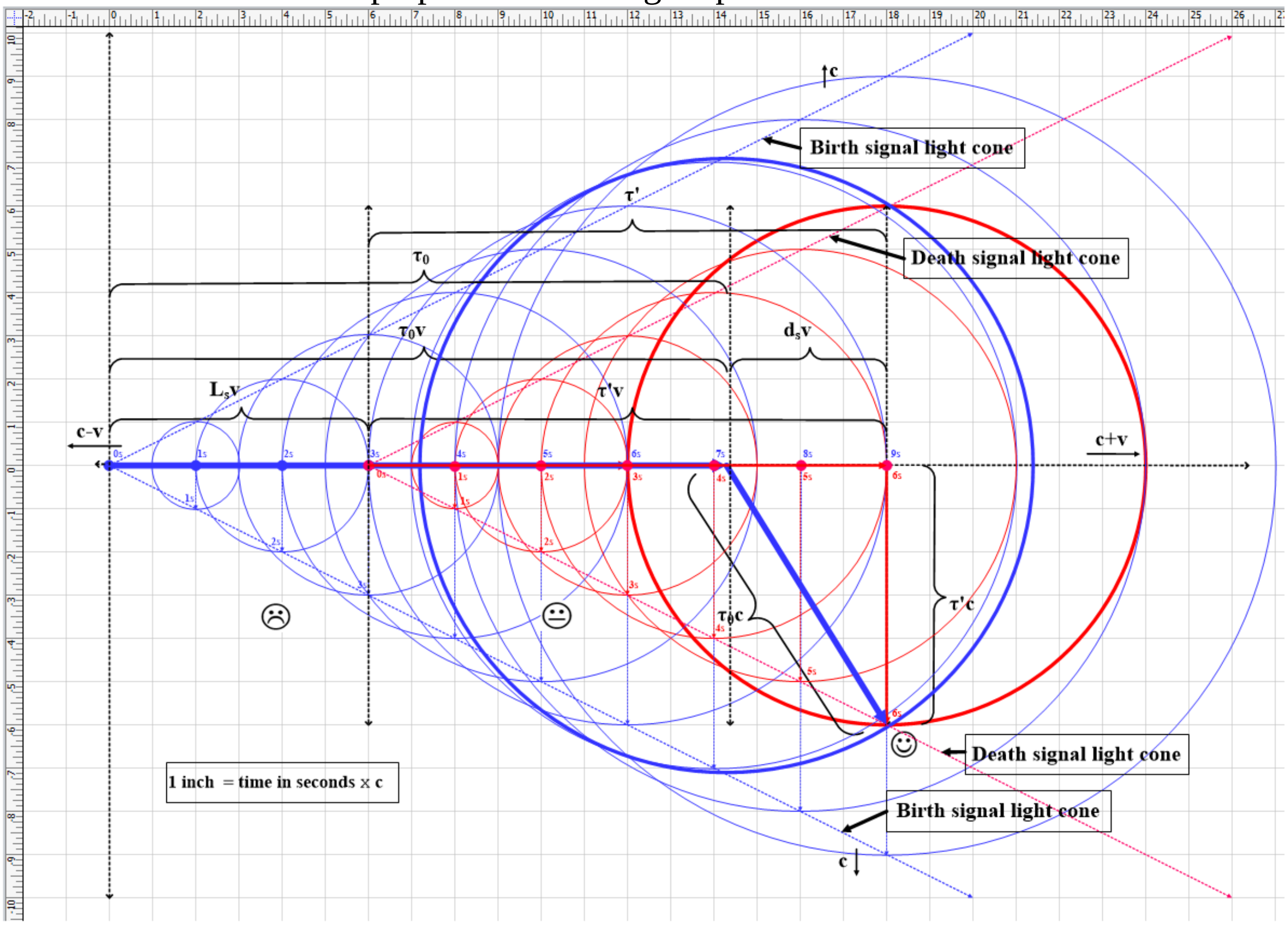
Appendix B.7 – Fig. 7: Geometry for Transverse Relative Time Shift when observer is perpendicular to signal path at $\tau'c$ and $v = c$.



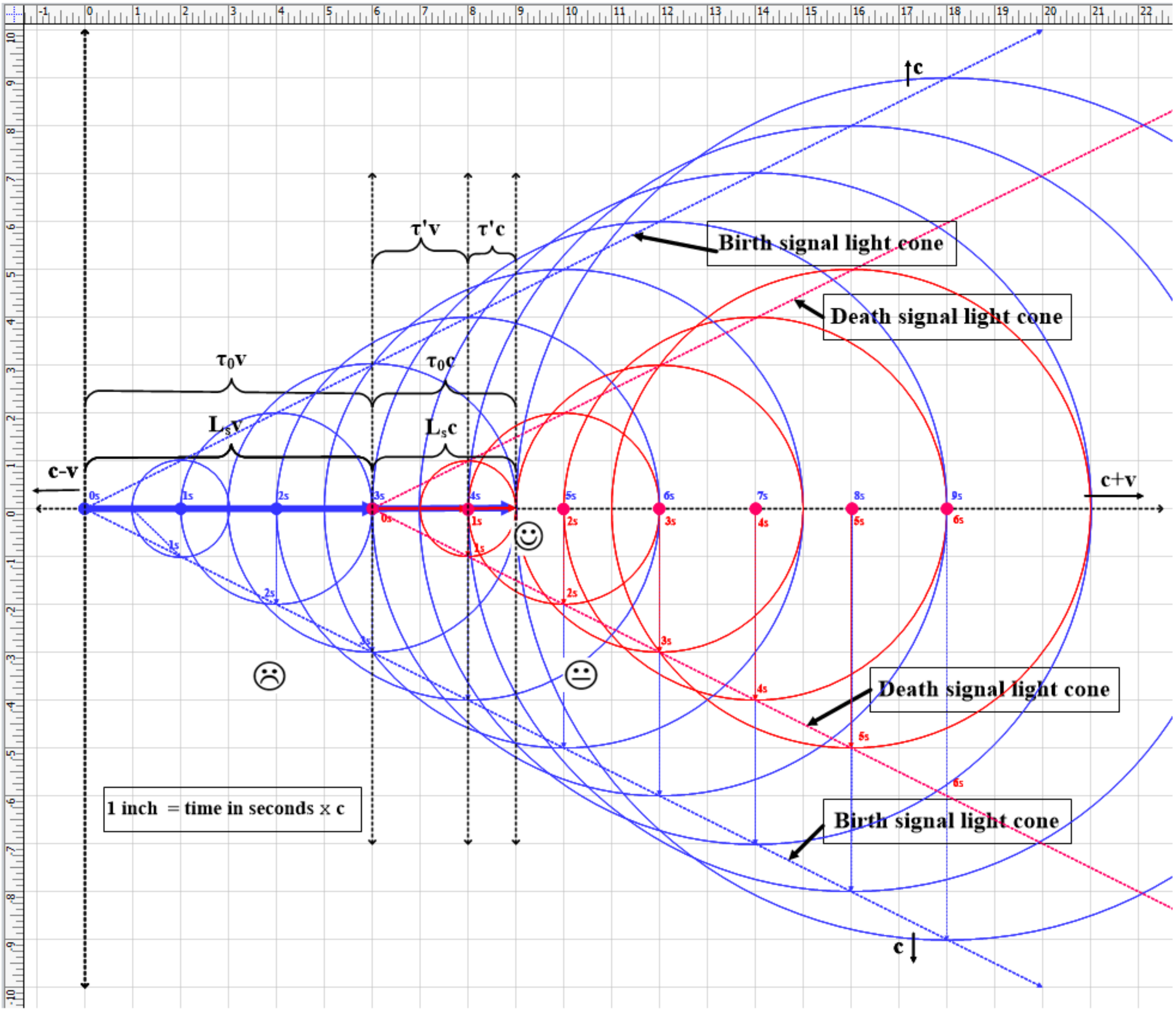
Appendix B.8 – Fig. 8: Geometry for Transverse Relative Time Shift when observer is on signal path at $\tau_0 c$ and $v = c$.



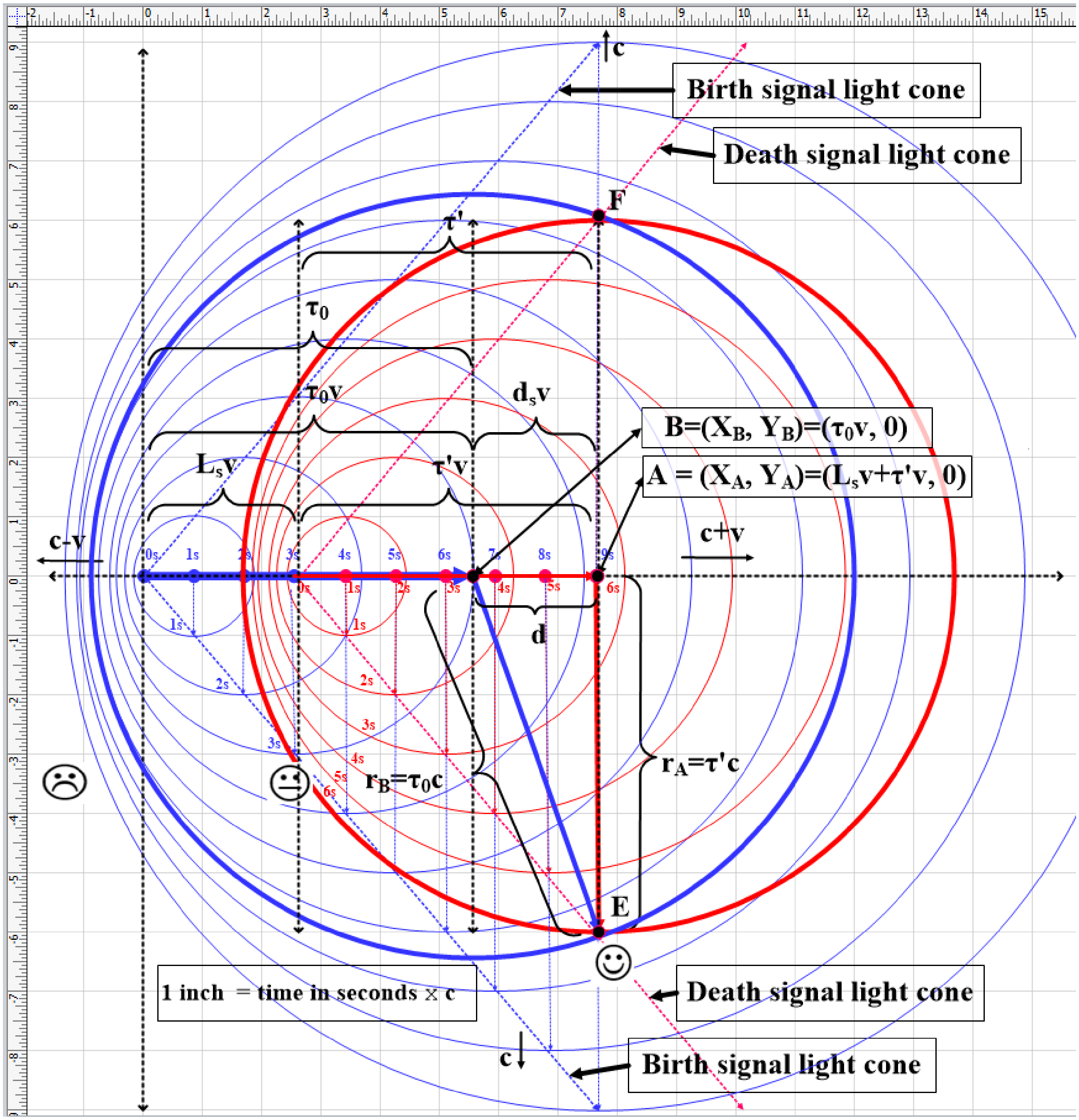
Appendix B.9 – Fig. 9: Geometry for Transverse Relative Time Shift when observer is perpendicular to signal path at $\boldsymbol{\tau' c}$ and $\boldsymbol{v} = 2\boldsymbol{c}$.



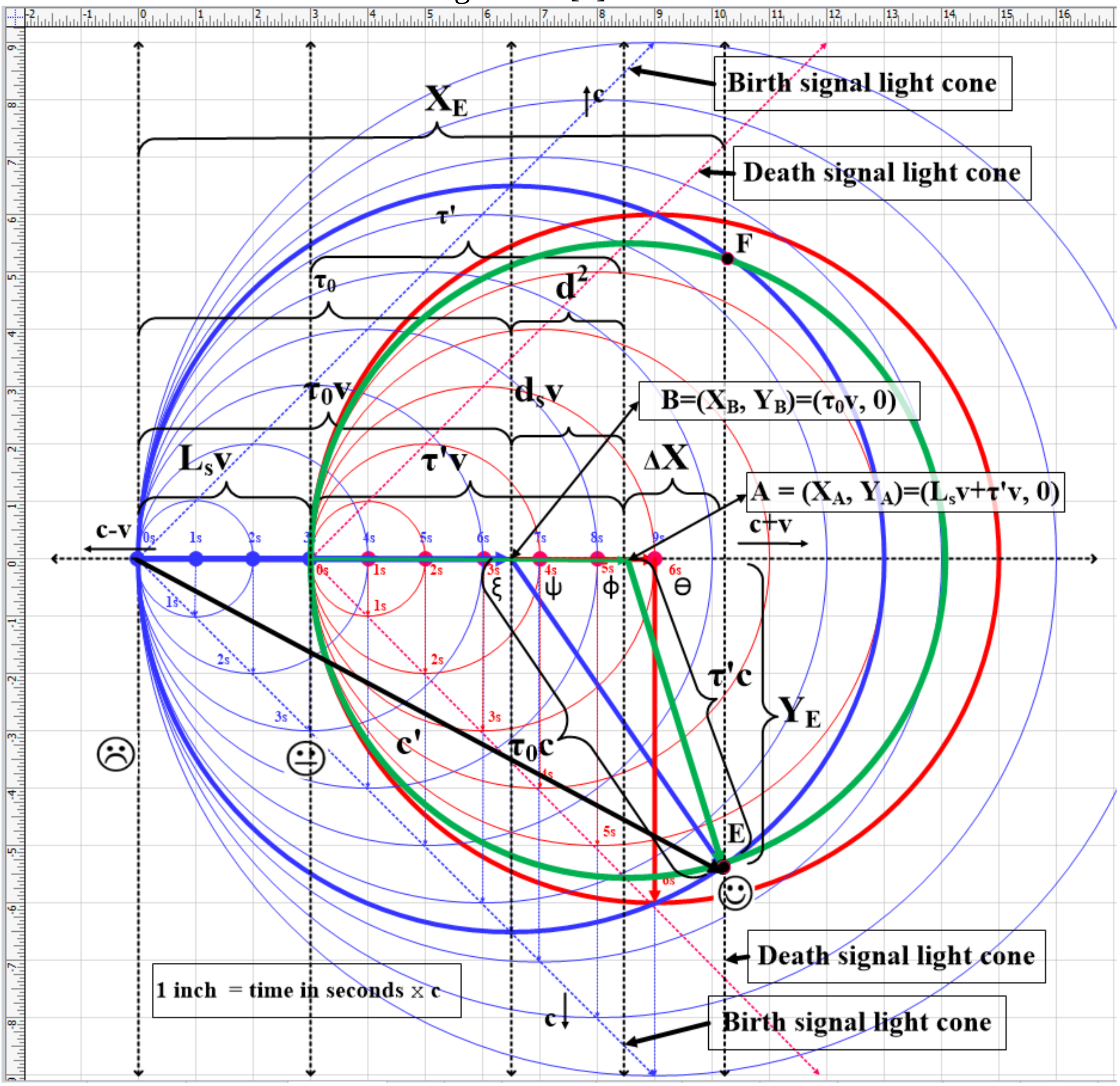
Appendix B.10 – Fig. 10: Geometry for Transverse Relative Time Shift when observer is on signal path at $\boldsymbol{\tau_0 c}$ and $\boldsymbol{v} = 2\boldsymbol{c}$.



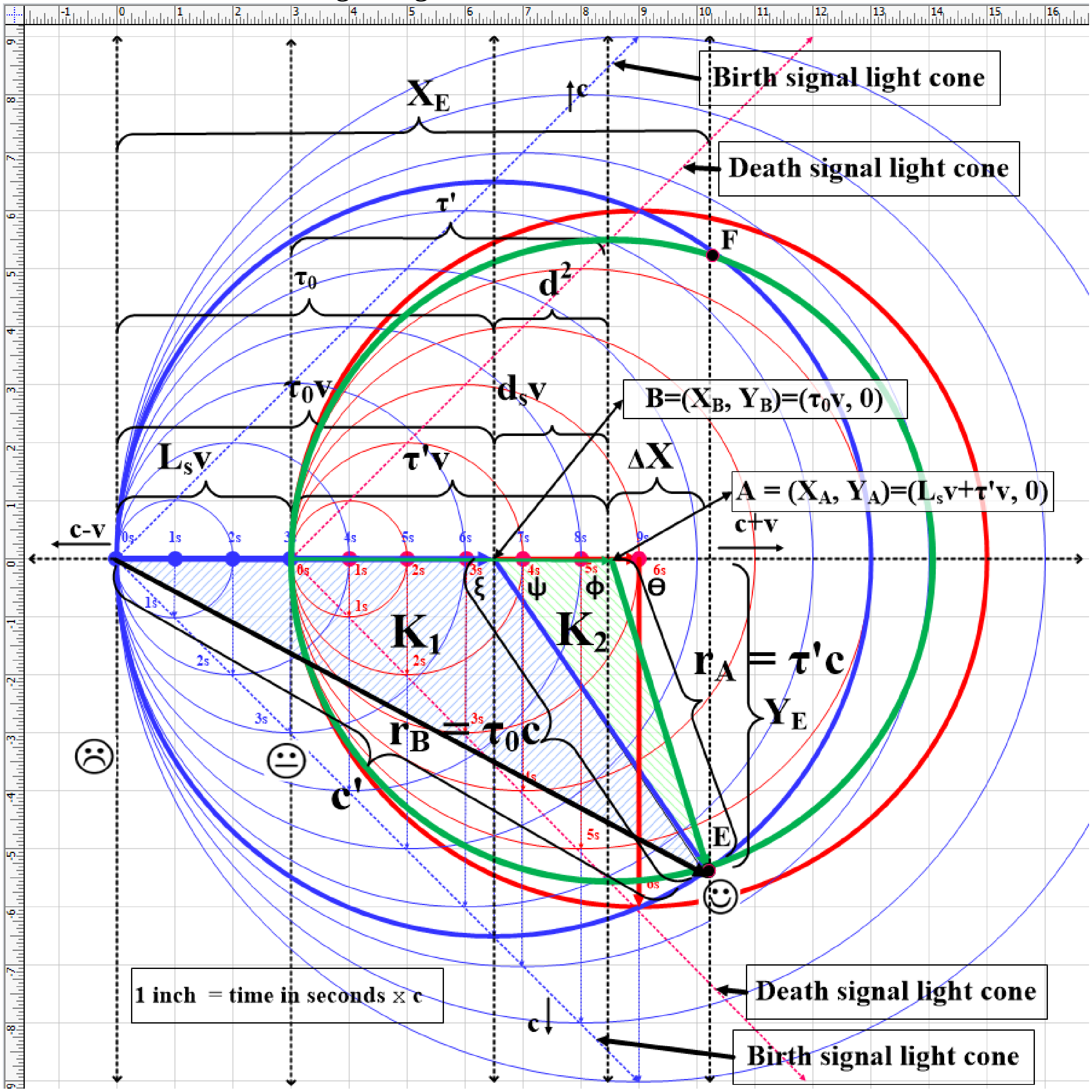
Appendix B.11 - Fig. 12: Observer coordinates **E** and **F** using Heron's formula for **K** and equations for two intersecting circles [2] when $v < c$.



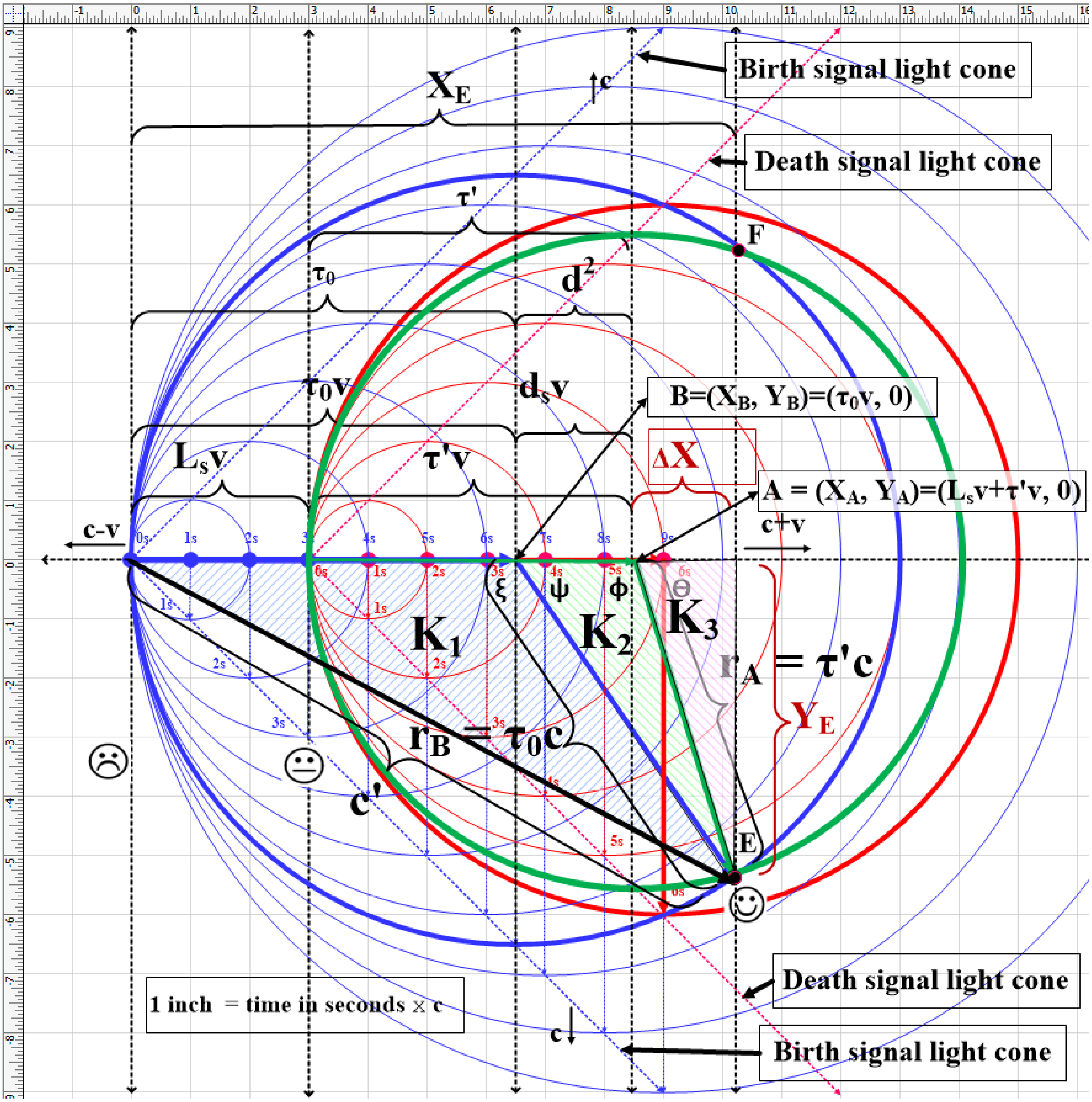
Appendix B.12 - Fig. 13: Happy observer inside death signal light cone at coordinates **E** and **F** using Heron's formula for **K** and equations for two intersecting circles [2] when v =c.



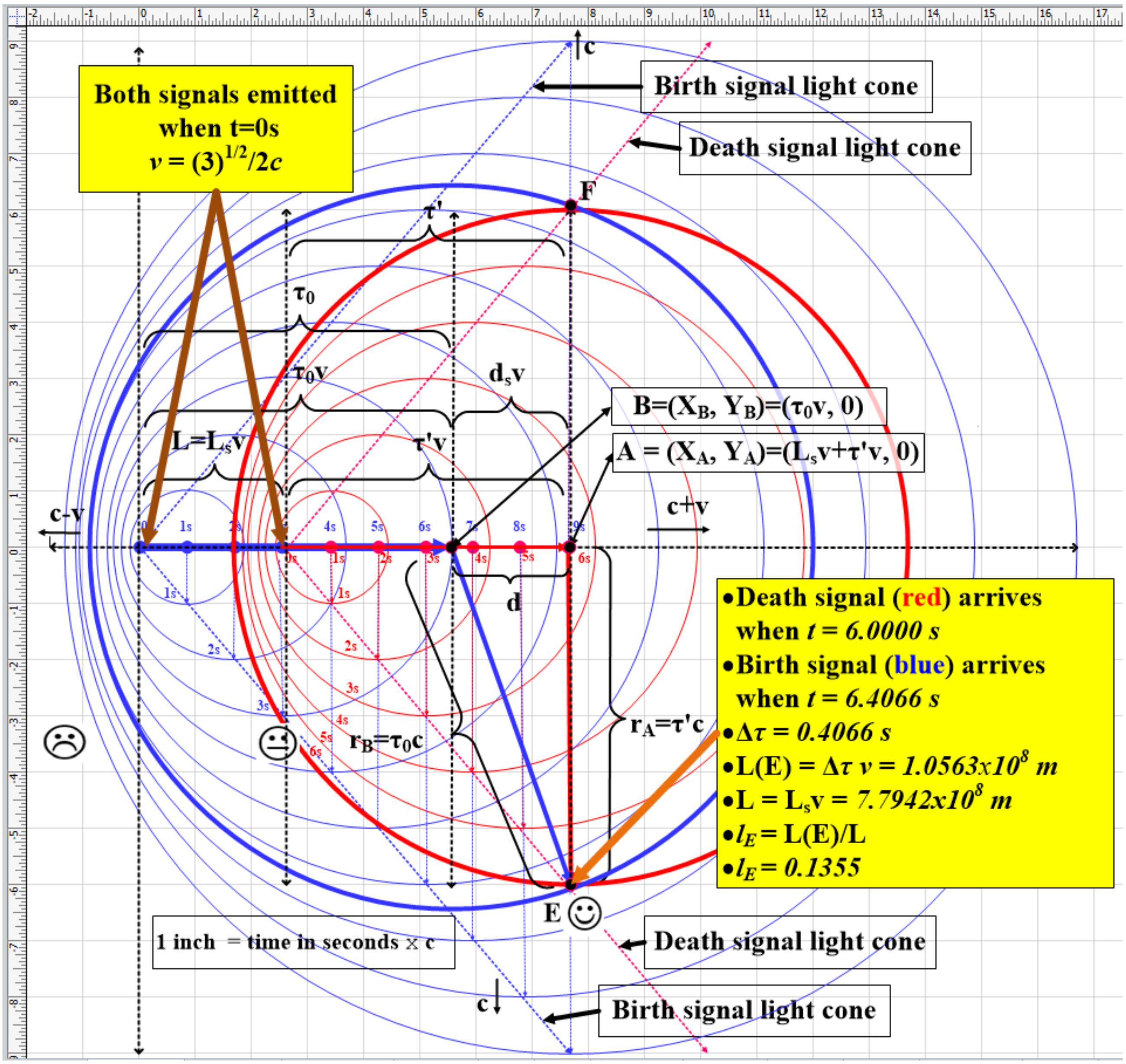
Appendix B.13 - Fig. 14: Fig. 12: Kepler's Law of equal areas with observer inside death signal light cone when v =c.



Appendix B.14 - Fig. 15: K_3 derived from area of triangle when v =c.



Appendix B.15 - Fig. 16: Observer measures transverse relative length of rod L when birth and death signals are emitted at t = 0s and v < c.



Appendix B.16 - Fig. 17: $\mathbf{K_{123}} = \frac{1}{2}\,(\boldsymbol{\Delta t_x v})\,(\boldsymbol{\Delta t_y c})$.