Proof of Goldbach's conjecture

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January 6, 2019

Abstract

Using two simple expressions for prime numbers, three cases occurr, all of which can be shown to only yield even numbers. From the resulting expressions, all even numbers greater than 12 can be derived.

1 Strong Goldbach conjecture

Every prime number $p, q \in \mathbb{N}$ can be displayed as a sequence of either

$$p = 3 + \ldots + 3 + 2, \tag{1}$$

the number of factors of 3s being odd, respectively

$$q = 3 + \ldots + 3 + 1, \tag{2}$$

the number of terms of 3s being even. Which is the same as

$$q = 3 + \ldots + 4, \tag{3}$$

here the sum of 3s being odd as in (1). Comprehensively:

$$p = 3k + 2 \tag{4}$$

and

$$q = 3l + 4, \tag{5}$$

with $k, l \in \mathbb{N}$, k and l being odd. This leads for the combinations of $p + q$ to the following four cases. Case 1:

$$p + p = 3k + 2 + 3m + 2, m \in \mathbb{N} \tag{6}$$

and m odd.
Case 2:

$$p + q = 3k + 2 + 3l + 4, \tag{7}$$

Case 3:

$$q + q = 3l + 4 + 3o + 4, o \in \mathbb{N} \tag{8}$$

and o being odd.
Case 4:

$$q + p = p + q, \tag{9}$$

as Addition is commutative. We determine the expression Case 1: $3k$ as the product of two odd numbers, is always odd, $3m$ also always odd, and two even constants. The sum of two odd numbers is always even, plus the even constants yielding an always even expression or $2n, n \in \mathbb{N}$.

$$p + p = 3k + 2 + 3m + 2 = 2n, \tag{10}$$

Case 2: analogue.

$$p + q = 3k + 2 + 3l + 4 = 2n \tag{11}$$

Case 3: analogue.

$$q + q = 3l + 4 + 3o + 4 = 2n \tag{12}$$

So obviously all three cases are even. Also, one can easily see that from the above cases all even numbers > 12 can be combined using either of the three equations (10), (11), (12). Thus follows Goldbach's conjecture

$$s + t = 2n, s, t \in \mathbb{N}, \tag{13}$$

and being prime numbers.

2 Weak Goldbach conjecture

The weak Goldbach can be derived from the strong Goldbach conjecture using another prime or $p + q + r$, the result always being odd. we add r

$$p + q + r = 3k + 2 + 3l + 2 + 3m + 2, \tag{14}$$

obviously always odd, as there are three odd terms which the sum of is always odd.
Proof: trivial.

3 Corollary

Also, if one generalizes, all even sums of primes must be even,
and all sums of primes having an odd number of terms must be odd.
Proof: trivial.