

A closed 2-form in spinorial geometry

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Abstract

We define here a closed 2-form for any spinorial manifold.

1 The spinorial manifolds

For a riemannian manifold (M, g) , we can define the spinorial fiber bundle if the second class of Stiefel-Whitney of the manifold M vanishes. We have a Clifford multiplication over this fiber bundle, it means that we can multiply a vector and a spinor to get a new spinor.

2 The 2-form of the manifold

We can define a 2-form with the Clifford multiplication:

$$w(X, Y)(\psi) = X.Y.\psi + g(X, Y)\psi$$

It is a 2-form because of the Clifford relations:

$$XY + YX = -2g(X, Y)$$

This 2-form takes its values in the endomorphisms of the spinor fiber bundle and is closed.

3 Characteristic classes

By mean of w , we can define characteristic classes:

$$b_k = tr(w^k)$$

They are topological invariants of the manifold M .

References

- [F] T.Friedrich, "Dirac operators in Riemannian Geometry", Graduate Studies in Mathematics vol 25, AMS, 2000.
- [GHL] S.Gallot, D.Hulin, J.Lafontaine, "Riemannian Geometry", Springer, 2004.
- [J] J.Jost, "Riemannian Geometry and Geometric Analysis", Springer, 2008.