Elements 5 : Three trigonometric Identities

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10-12-2018 13:18:21

Abstract. This note presents three trigonometric identities.

Formulas

If $0<x<1$,then

$$\sum_{n=0}^{\infty}\left(\tan^{-1}x^{12n+1}-\tan^{-1}x^{12n+3}+\tan^{-1}x^{12n+9}-\tan^{-1}x^{12n+11}\right)=$$

$$=\tan^{-1}\left(\frac{\sum_{n=1}^{\infty}x^{(2n-1)^2}-x^{3(2n-1)^2}}{1+\sum_{n=1}^{\infty}x^{(2n)^2}+x^{3(2n)^2}}\right) \tag{1}$$

$$\frac{\pi}{2}-\sum_{n=0}^{\infty}\left(\tan^{-1}x^{12n+3}-\tan^{-1}x^{12n+5}+\tan^{-1}x^{12n+7}-\tan^{-1}x^{12n+9}\right)=$$

$$=\tan^{-1}\left(\frac{\sum_{n=1}^{\infty}x^{(2n-1)^2}+x^{3(2n-1)^2}}{\sum_{n=1}^{\infty}x^{(2n)^2}-x^{3(2n)^2}}\right) \tag{2}$$

$$\frac{\pi}{2}-\sum_{n=0}^{\infty}\left(\tan^{-1}x^{8n+1}-\tan^{-1}x^{8n+3}+\tan^{-1}x^{8n+5}-\tan^{-1}x^{8n+7}\right)=$$

$$=\tan^{-1}\left(\frac{\sum_{n=1}^{\infty}x^{(2n-1)^2}}{\sum_{n=1}^{\infty}x^{(2n)^2}+x^{2(2n-1)^2}-x^{2(2n)^2}}\right) \tag{3}$$

References

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2. Ramanujan, S.: Notebooks (2 volumes), Tata Institute of Fundamental Research, Bombay , 1957; second ed., 2012.
3. Ramanujan, S.: The Lost Notebook and Other Unpublished Papers, Narosa, New Delhi, 1988.