Spin½ 'Plane' & Simple

Abstract: **To fully characterize any spin requires identification of its primary spin axis *and* its plane of rotation. Classical presumptions obscure both for "intrinsic" spin. Here, Euclidean interval-time coordinates literally *lift the veil* of space to reveal it. Probability amplitude is also physically realized.**

Retreat by Conceit?

Several generations of unsurpassed success in Quantum Mechanics has transformed "quantum weirdness" from a set of *mysteries* to be solved, to hollow *cornerstones*, precariously built upon. Foremost among these is quantum spin. Cautions abound:

"...spin is an intrinsic property of a particle, unrelated to any sort of motion in space."[1]

"Physically, this means it is ill-defined what axis a particle is spinning about"[2]

"…any attempt to visualize it [spin½] classically will badly miss the point."[3]

"[Quantum spin] has nothing to do with motion in space...but is somewhat analogous to classical spin"[4]

"the spin…of a fundamental fermion…with no classical analog, is…abstract…with no possibility of intuitive visualization."[5]

Is this resignation to a fundamental reality or complacency, accompanying conceit? To avoid the latter, all feasible models must be exhausted. To be sure, fermion spin is *abstractly* modeled with spinors[6], but physics isn't physics unless it's about the physical! A real, physical model is presented here.

Top Performer

Consider some facts about "spin½" which, for massive fermions, may be conveniently considered in a rest frame. Having *real* angular momentum, with measurable spin components, a *primary spin vector* (σ) is presumed to exist. The goal is to identify its axis.

1. The primary spin axis is *not* a classical axis. Such a spin vector would decompose to ordinary spatial spin components and this is not the case (Fig. 1).
2. The primary spin axis is *not* orthogonal to flat space. That would yield zero-magnitude spatial spin components, contrary to those observed.
3. The primary spin axis makes *equal* angles with *every* spatial direction, as revealed by equal size spin components (ħ/2), measured in any direction.[7]
4. As a form of angular momentum, intrinsic spin is a conserved quantity.[8]
5. Spin½ has a 720° horizon. It takes *two* classical rotations to return a fermion to its *exact* original state[7] (accounting for probability amplitude).
6. The signs (±) of intrinsic spin components correlate probabilistically. Having measured one, the sign of any other component relates to the angle between them. [9]

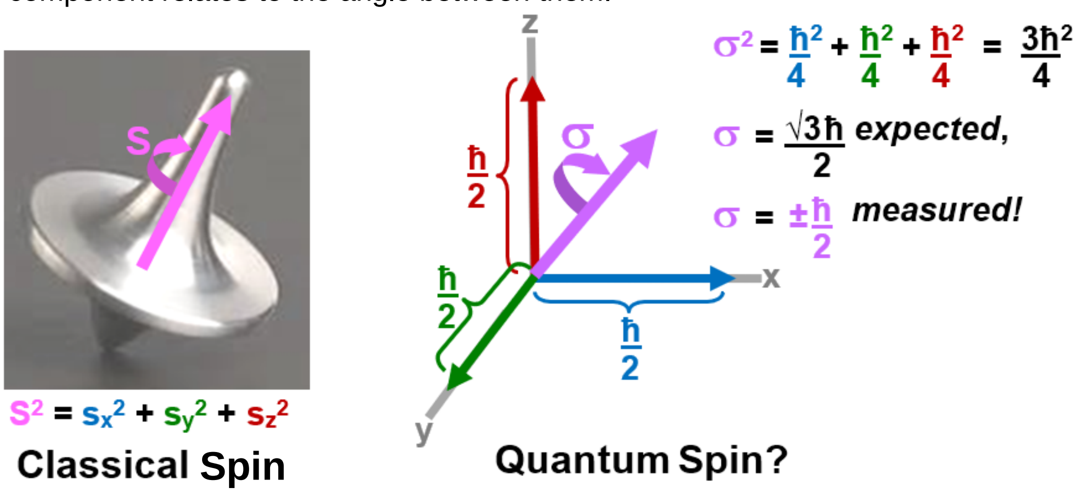


Fig. 1. Left: Classical spin vector (**S**) decomposes to coordinate projections. Right: If spin components ħ/2 are found on three spatial coordinates, a resultant spin vector (σ) of $\sqrt{3}\hbar/2$ is *expected*. Instead, ħ/2 (or –ħ/2) is *measured* there as well! (ħ is the reduced Planck constant h/2π.)

Putting a Noether Spin on It

Conservation laws are of two types.[10] "Exact" laws are never expected to fail while "approximate" laws hold true within restrictions. For example, *conservation of mass-energy* is exact but divides into two approximate laws: *conservation of mass* and *conservation of energy*, which each hold, barring interchanges (exhibiting $E = mc^2$).

By Noether's theorem, each conservation law is now associated with a symmetry (i.e. *transformational invariance*).[11] For example, *conservation of mass-energy* is associated with temporal invariance. The mass-energy of an isolated system is unaltered by temporal displacement. It is "time invariant" (Fig. 2).

Two *omissions* in the exact laws should seem conspicuous:

1. Laws exist for both temporal *and* spatial **translation** but only about spatial axes for **rotation**.

2. A *non-spatial,* primary "intrinsic" spin axis (σ) *should* list with the spatial axes for angular momentum.

Exact Law	Noether Symmetry Invariance		Number of Dimensions		
Conservation of mass-energy	Time invariance	Lorentz invariance symmetry	1	**translation** along time	4D
Conservation of linear momentum	Translation symmetry		3	**translation** along x, y, z	
Conservation of angular momentum	Rotation invariance		3	**rotation** about x, y, z, σ	3D?
CPT symmetry	Lorentz invariance		1+1+1	charge (q → -q) + position (r → -r) + time (t → -t) inversions	
Conservation of electric charge	Gauge invariance		$1\otimes4$	scalar field (1D) in 4D	
Conservation of color charge	SU(3) Gauge invariance		3	r, g, b	
Conservation of weak isospin	$SU(2)_L$ Gauge invariance		1	weak charge	
Conservation of probability	Probability invariance		$1\otimes4$	Σ probabilities = 1 in x,y,z	

Fig. 2 It is inconsistent to purport 4D of translation (rows 1+ 2) and yet pretend only 3D are available for rotation (row 3). Though widely acknowledged to be non-spatial,[1-5] an additional spin dimension should be recognized by adding the primary fermion spin axis, σ.

The 'Plane' Truth

To understand intrinsic (or any) spin, one must identify not only the *primary spin axis* but the *plane of rotation*. Classically, knowing one entails the other, but this is not a given in 4D. A spin vector is defined perpendicular to its plane of rotation in a Euclidean space but that is *not* what spacetime provides.

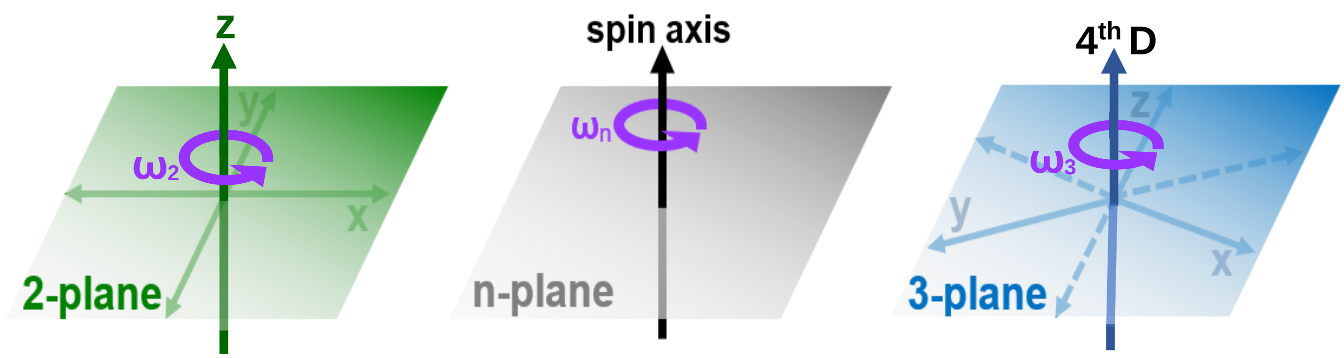


Fig. 3. An orthogonal spin axis makes no projections on a flat plane of any dimensionality.

Put Space in Its Place

To accommodate survival (e.g. bow hunting), our brains are hardwired for space and time, which have become default coordinates. But Minkowski spacetime has a hyperbolic geometry,[12] which yields a distorted view in many respects.

"Such maps necessarily distort metric relations and one has to compensate for this distortion."[13]

Adapting the balloon analogy of cosmic expansion[14] to contain a central Big Bang event, gives rise to a *curved-space, radial-time* model. Time emanates from the center as a 4D temporal field, enclosed by spatial 3-spheres, representing simultaneities in the rest frame of the cosmos. A radius corresponds to the age of the universe (Fig. 4). All locations on a 3-sphere, find Euclidean coordinates, with time normal to space and intervals tangent to it. A more detailed explanation[14] and illustrations[15-18] are provided.

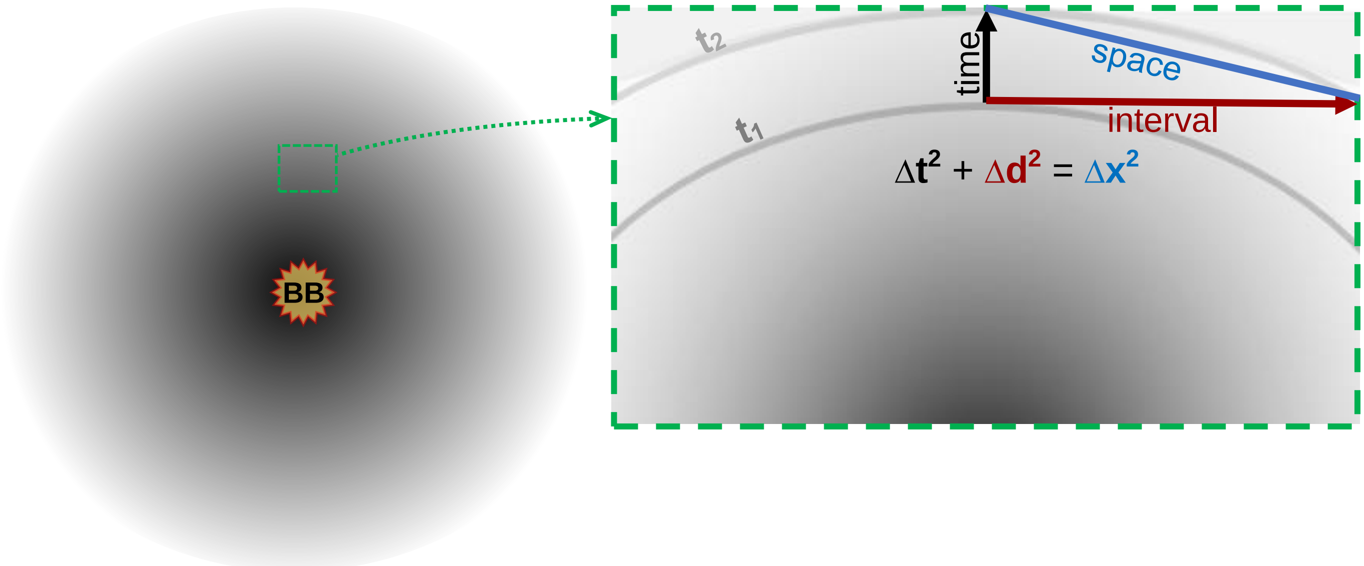


Fig. 4 Left: A temporal 4-field, centered on the Big Bang (BB) yields a curved-space, radial-time model. Right: The indicated region, between earlier ($\mathbf{t_1}$) and later ($\mathbf{t_2}$) simultaneities, illustrates Euclidean, interval-time coordinates, allowing for spatial flatness.

Alternatively, consider that the Pythagorean theorem applies uniquely to Euclidean geometry. Adopting a spacelike convention, the interval formula[19]: $\Delta \mathbf{d}^2 = \Delta \mathbf{x}^2 - \Delta \mathbf{t}^2$ rearranges simply as $\Delta \mathbf{x}^2 = \Delta \mathbf{d}^2 + \Delta \mathbf{t}^2$, which implies interval-time coordinates corresponding to the legs of a right triangle.

Time to Turn Things Around!

With that Euclidean lens, **σ** can be modeled as *chronaxial spin*, in an *interval 3-plane*. Intrinsic spin is *classical* spin about a *non-classical* axis, time. More generally, it is spin about a particle's worldline which, in its rest frame, is its *timeline*. No longer a coordinate, space instead arcs past **σ** like an umbrella over its handle (Fig. 5). A field of spin components thus projects equally in *every* spatial direction, consistent with an underlying *curved-space, radial-time* structure.

Relativity makes a 4th dimension of spin axes unsurprising. All fermions age (e.g. muons decay) so, time undeniably supports *translation*. There is thus, no basis to deny that time also supports *rotation*. An objection might be that chronaxial spin is effectively instantaneous, easily developing circumferential speeds exceeding universal limit $\boldsymbol{c}$. However, a fermion "point particle" of zero radius invokes no such restriction. In fact, instantaneous chronaxial spin provides a perfect source for quantum indeterminism.

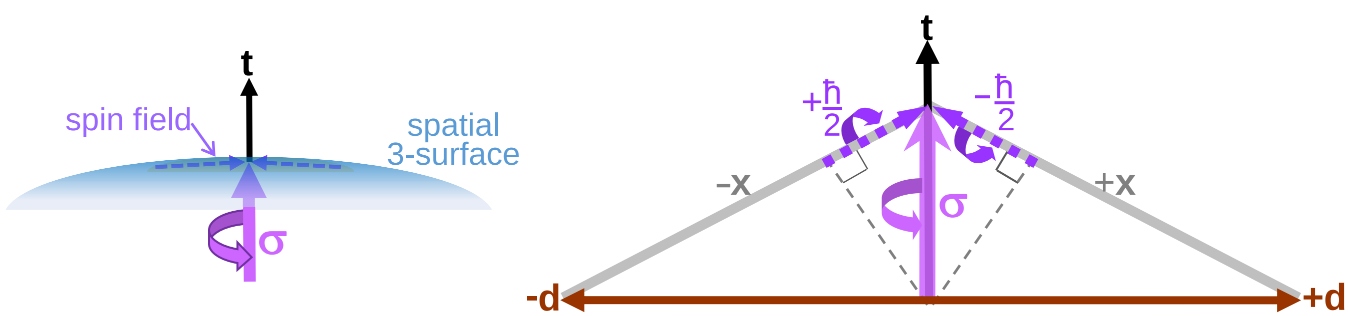


Fig. 5. Left: Arching past time, space exhibits a symmetric field of spin projections in all directions from the primary *chronaxial* spin vector (**σ**). Right: For clarity, a 2D slice in Euclidean interval-time coordinates shows spatial arc (±**x**) locally flat and highly inclined. **σ** projects symmetric ±ħ/2 components on space.

Solid Reasoning

Just as *conservation of mass-energy* divides to approximate laws for mass and energy, *conservation of angular momentum* divides into three *approximate* laws (barring interchanges), distinguished by the dimensionality of their angular velocities ($\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2$, $\boldsymbol{\omega}_3$). Time is always excluded from an *n-plane* of rotation since its fundamental unidirectionality denies the needed oscillatory freedom.

1. *Conservation of **Linear**-Angular Momentum* - vibration, simple harmonic motion. $\boldsymbol{\omega_1} = \mathbf{0\pi f}$
2. *Conservation of **Planar**-Angular Momentum* - classical spin & orbits. $\boldsymbol{\omega_2} = \mathbf{2\pi f}$
3. *Conservation of **Solid**-Angular Momentum* - quantum spin & *orbitals* (both chronaxial). $\boldsymbol{\omega_3} = \mathbf{4\pi f}$

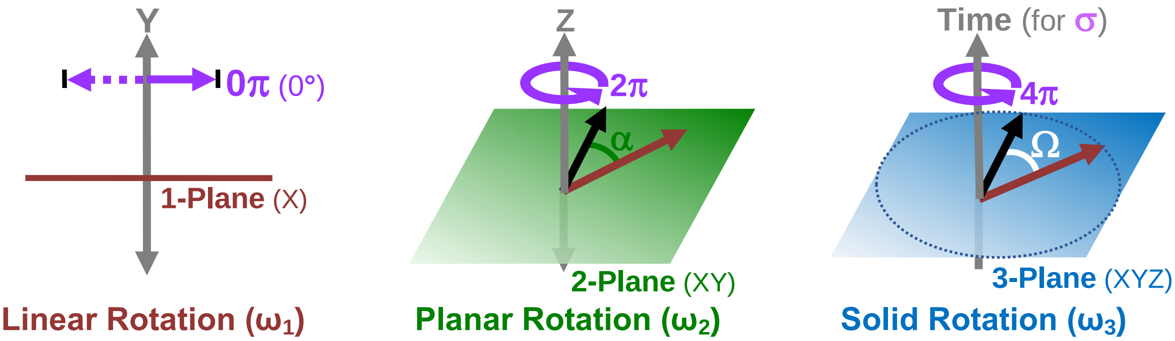
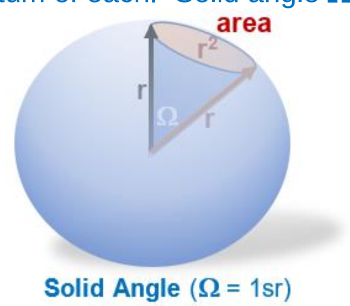


Fig. 6. Dimensional Spin Progression: Each rotation occurs in a flat n-plane about an orthogonal axis. Angular velocity ($\boldsymbol{\omega_n}$) relates to the approximately-conserved, angular momentum of each. Solid angle $\boldsymbol{\Omega}$ appears circular in a 3-plane (above right) and within a sphere (below).

Planar rotation entails 2π radians. Going up a dimension, chronaxial spin may be depicted in a 3-plane about a timeline, where a sphere's volume is flatly exposed (Fig. 6). This entails a *solid angle* of **4π** steradians (sr).[20]



Easy as Pi

"A half quantum" is an oxymoron because a "quantum" is *"the minimum amount of any physical entity involved in an interaction."*[21] Yet "spin½" implies such a halving, arising from inadequate *reduction* of the Planck constant (h).

"In applications where it is natural to use the angular frequency (i.e. …in terms of radians per second…) it is often useful to absorb a factor of 2π into the Planck constant…called the reduced Planck constant …equal to the Planck constant divided by 2π, and is denoted ħ (pronounced 'h-bar')"[22]

Division by 2π is fine for *classical* rotation, but there is *no basis* to apply this to quantum spin. Solid-angular, *chronaxial* spin must instead be reduced by 4π. Applied to fermions, spin is not "½" but quite *whole* at h/4π (i.e. ħ/2), exactly as measured (Fig. 7). Further, QED rightly boasts 12 digits of precision for the electron magnetic moment, but mysteriously remains off by a factor of two!

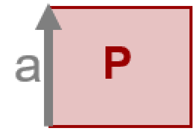
"... one famous triumph of the Quantum Electrodynamics theory is the accurate prediction of the electron g-factor. The magnetic moment of an electron is approximately twice what it should be in classical mechanics. The factor of two [g_S] implies that the electron appears to be twice as effective in producing a magnetic moment as the corresponding classical charged body. …a correction term [a_e]… takes account of …interaction…with the magnetic field"[23]

$$\mu_S = -\frac{g_S \mu_B S}{\hbar} = -\frac{\cancel{g}_S \mu_B S}{\frac{h}{\cancel{2}\pi}} \;\leftarrow 4\pi$$

Fig. 7 Denominator ħ is only *half* reduced, as h/2π. Correcting with *solid-angular* range 4π is equivalent to having a factor of 2 in the numerator. Thus, g_S does not mysteriously need to be *"twice"* the classical g-factor g_L.[24] The anomalous magnetic moment (a_e)[25] is then accommodated at *half* the conventional value in: $g_S = 1 + a_e = 1.001159652181643$.

Both **S** (electron spin angular momentum) and $\boldsymbol{\mu}_B$ (Bohr magneton)[23] incorporate ħ/2 which, in that form, is fully reduced (i.e. h/4π). Sufficiently reducing denominator ħ as well makes *fudge factor* g_s obsolete.

Probable Cause



Two related mysteries of fermion spin remain.

1. While any two spin components have equal magnitude, their signs (±) vary, correlating probabilistically with the angle separating them. An essential, but so far *abstract*, "probability amplitude" (a) is strangely considered the *square root* of that probability (P).

"The probability of an event is represented by the square of an arrow [probability amplitude]."[26]

"The [probability] amplitude arrows are fundamental to the description of the world given by quantum theory. No satisfactory reason has been given for why they are needed."[27]

"There have been many attempts to derive the Born rule from the other assumptions of quantum mechanics, with inconclusive results. ... probability is equal to the amplitude-squared"[28]

"These [probability amplitudes] are extremely abstract, and it is not at all obvious what their physical significance is."[29]

2. The probability amplitude sees a 720° horizon.

"[The] physical effects of the difference between the rotation of a spin-½ particle by 360° as compared with 720° have been experimentally observed in classic experiments in neutron interferometry."[30]

One might guess that 720° relates to the 4π, noted earlier for solid angles. But solid angles range to 4π *steradians* (square radians of *area*), while 720° refers to 4π *radians* (of arc *length*).

Experimentally, the sign of a prepared spin will correlate with that of a subsequently-measured spin, at angle θ, with probability (P) such that: $P = \cos^2(\theta/2)$. If amplitude (a) has a real representation, it will be confined in a boundary condition from which to generalize (Fig. 8).

For example, amplitude a *must* coincide with the prepared spin when the subsequently measured spin has the same axis (i.e. $\theta = 0$). More generally, as the *half-angle* specification hints, a is recognized on the angle *bisector*. Instead of amplitude as a *square root* of probability, it should be viewed as a probability in its own right, the projection of 100% self-correlation on the bisector, i.e. $a = \mathbf{1}\cos(\theta/2)$. That value is in turn, projected onto the subsequently-measured component, which results in the observed correlation probability: $P = \mathbf{1}\cos(\theta/2)\cos(\theta/2) = \cos^2(\theta/2)$.

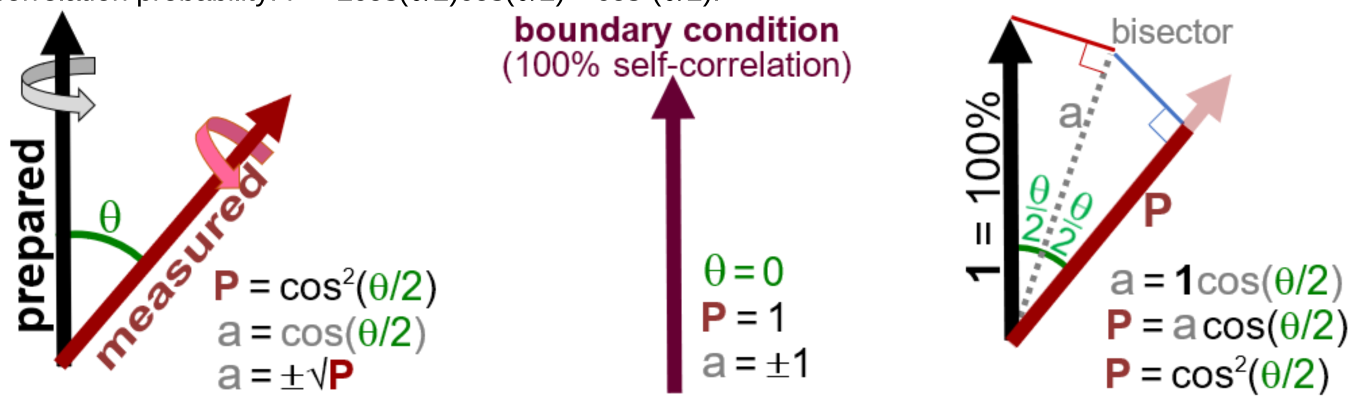


Fig. 8. Left: The half-angle correlation of prepared and subsequently measured spin components entails a 720° range for a yet, *unidentified* "probability amplitude" (a). Center: Amplitude a is pinned down in the boundary condition of **100%** self-correlation. Right: a is spin probability **1**, projected on the bisector. It is the spin correlation probability for the *unmeasured* bisector. Probability **P** is in turn, the projection of a on the subsequently-*measured* axis.

Being on the angle bisector, probability amplitude a has the mysterious property of existing, while *never* directly measurable. To do so would make it the subsequently-measured component, which immediately redefines the bisector. The amplitude is thus always out of reach, as is the half-way point of Zeno's dichotomy paradox.[31]

With equal magnitude spin components as sides, the triangle they describe is isosceles. Its altitude is the probability amplitude. Both change sign when angle θ crosses 180° (becomes convex). At 360° the value is -1, which continues back to +1 at 720°.

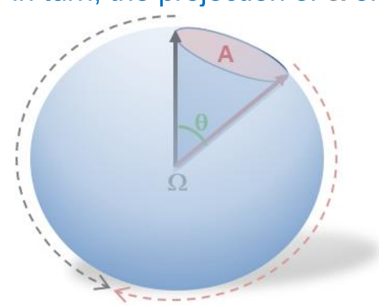


Fig. 9 An equator of solid angle Ω is described by θ, for which altitude turns negative after 180°.

Simple as Riding a Bi-Cycle

Having previously dealt with bosons,[14] interval-time coordinates here provide a Euclidean lens which clearly reveals fermion spin by axis and plane of rotation. With that, 4π seems unavoidable, whether 4π steradians of solid angular rotation, the combined ranges (2π each) of two spherical coordinates or the 4π radians of probability amplitude. It seems awkward at first, but with practice, it becomes second nature, like riding a bike.

Bohr stated, *"...however far [quantum] phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms."*[32]

Quantum spin is ***classical*** spin about a ***non-classical*** axis. It is ***chronaxial spin***.

Wheeler said, *"Behind it all is surely an idea so simple, so beautiful, that when we grasp it - in a decade, a century, or a millennium - we will all say to each other, how could it have been otherwise? How could we have been so stupid for so long?"*[33]

We don't have to be *"stupid"* about quantum spin anymore.

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