The Emperor Has No Clothes: A Classical Interpretation of Quantum Mechanics

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I. Prolegomena

For a book that has this level of ambition, we will need some *prolegomena*. That's a Greek word. It just means: introduction, or *prologue*. It is also the title of a rather philosophical work, but I will you *google* that. The introduction to a book on physics is usually mathematical, and this book is *not* an exception. No differential equations, however. I promise. 😉 And before we get into some of elementary math, let us first tackle a more philosophical question.

What does it mean to understand an equation?

The audience of this book is pretty much people like me: people who want to truly understand quantum physics and who are, therefore, not afraid to learn about a few equations. Why are you not afraid? Because you know these equations – mathematics, that is – are the *language* scientists have come to use to describe reality—or their *idea* of reality, at least. Dirac once said God must be a mathematician, because He used very advanced mathematics to construct the Universe. Dirac was not a believer. Hence, he meant to say this: we can only understand the Universe if we understand the equations.

The next question then is: what does it mean to *understand* an equation? As we started talking about Paul Dirac, let us see what it meant to him. He was the first of those first-generation quantum physicists to bring all of quantum math together in one comprehensive volume: his 1930 *Principles of Quantum Mechanics* is, in fact, still in use as a textbook.[1] Hence, he should be the ideal person to ask, right? He said he understood an equation if he could predict the properties of its solutions without actually solving it.

I thought about that for quite a while, and I now realize Dirac was, obviously, talking more as a mathematician than as a physicist then. He was probably also thinking of some differential equation – his wave equation for the electron, perhaps – rather than, say, an equation like $E = mc^2$, or the two *de Broglie* relations $E = h \cdot f$ and $\lambda = h/p$.

We may mention other differential equations: Schrödinger developed one to explain electron orbitals—– and others built on that (think of the Klein-Gordon equation, for example). Heisenberg came up with one and, as mentioned, Dirac's equation is there too! The fact that we have a fair number of so-called fundamental differential equations – each serving a rather specific purpose (explaining an electron orbital, or the motion of an electron in free space, for example) – made me shy away from them. Not because they are difficult to understand but because I now think they are actually *not* fundamental to our understanding of the nature of Nature.

[1] If people question that, I usually joke that *I* am still using it. 😉

Why? The equations that describe reality do not have any solutions. They just describe reality. Think of these *de Broglie* equations, for example. Or the E = mc^2 relation. They don't have any solutions. They just represent some relation between two variables we can, somehow, *imagine*. That is why I think of these equations as being more real than those differential equations. Having said that, I should hastily add that we can actually *not* directly verify if these *de Broglie* relations are true. Why not? Because we cannot *directly* observe the frequency (*f*) of the matter-wave, and we cannot *directly* observe its wavelength (λ) either. Having said that, we all believe these relations to be true. Why? Because they emerge from other equations we *can* actually verify, through making simple observations or – more usually – through rather advanced experiments, such as electron interference experiments[2], or one of the other variation of a Stern-Gerlach experiment.

The same remark can be made for the E = mc^2 equation: this equation relates two *physical* concepts – mass and energy, so that's stuff we can *measure* – in a proportionality relation. This proportionality relation tells us that the ratio E/m is always equal to some constant—for any particle or system, really: electrons, photons, whatever you can think of. This relation also tells us that – for some weird reason that we'll actually explain in this book – this proportionality constant is equal to the square of the speed of light. Just like *de Broglie* equations, it is a relation one cannot prove *directly*, but we believe it to be true because it emerges out of the equations of relativity theory.

To be precise, Einstein's mass-energy equivalence relation emerges from the *Lorentz* transformation rules for measuring position and time in the stationary versus the moving reference frame. The question is: what *is* that equivalence? What *mechanism* or explanation can we find? We will come back to that: we believe the equation represents an oscillation in two dimensions—an oscillation in some *plane* of oscillation, that is.

What's the point? The point is: if you google the Dirac equation – or Schrödinger's equation or whatever other wave equation – and look at it, then you'll have to agree that the functions, variables and operators in that equation are *not* so comprehensible. Why not? Because they do not correspond to anything we can *imagine*.[3] In contrast, E, *f*, λ and even Planck's quantum of action (h) are things that make sense intuitively. We can associate energy with a battery, for example, and we think of a frequency, we may want to think of the flapping wings of a hummingbird, and if we think of a wavelength, we may want to think of water sloshing around. What about Planck's constant?

Imaging what Planck's quantum might actually *be*, is somewhat more difficult, but it is not impossible. It is *not* a mathematical constant: it is a physical constant. So it has *physical* dimensions. To be precise, it is the product of a force over some distance and some time. Alternatively, we can also write it as the product of some energy and some time interval, or as the product of some momentum and some distance. We can imagine forces, energy, momentum[4], and distance and time. Hence, we get some

[2] While Feynman thought the electron interference experiment could not be carried out *in practice*, nanotechnology has made that possible now. See: http://www.iop.org/news/13/mar/page_59670.html.

[3] I should correct myself here: one of the advantages of the physical (or geometric) interpretation of the wave*function* that we will be offering in this book, is that we can also think of a geometric interpretation of a wave *equation* such as Schrödinger's, which explains the electron orbitals. We believe Schrödinger's equation describes an energy propagation mechanism, in very much the same way as Maxwell's wave equations describe a propagation mechanics for electromagnetic waves. See: http://vixra.org/abs/1812.0202.

[4] We didn't give an intuitive example of momentum. What's momentum? It's the product of mass and velocity. Think of a speeding truck ramming into your house. That thing has a lot of momentum as compared to, say, some insect that happens to fly into face you while biking.

inkling of what Planck's quantum of action might actually describe. We will come back to that: we'll want you to associate Planck's constant – a *Wirkung*, in German[5] – with the idea of a *cycle* of some oscillation.

So we will get there. We will want to focus on equations like E = h·*f*, λ = h/p and E = E = mc^2 because, yes, we can effectively *understand* them. In fact, in this book we want to show that – with these three equations – you can explain almost everything. What's almost everything? Almost everything: the spin and the magnetic moment of an electron, photon absorption and emission, interference, electron orbitals, etcetera. After you have finished the book, you should write me and tell me what I did *not* explain. 😊 To motivate you, I'll insert the single-most important graph of the whole book already. I will also make some clues about it so you can already start thinking for yourself.

The illustration below presents the essence of the *Zitterbewegung* model of an electron. We believe it offers a classical interpretation of all of the quantum-mechanical phenomena that you'll usually see explained in terms of hocus-pocus and blah-blah. If this makes you think of an Archimedes' screw, then that's good because it is, effectively, *exactly* that shape: a combination of linear and rotational motion. We effectively think of the electron as a pointlike charge that combines two motions: rotational and linear.

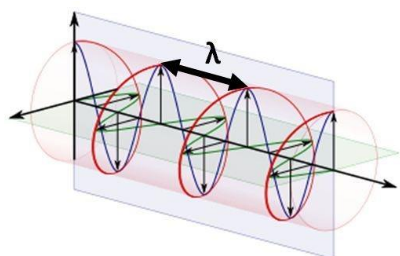

Figure 1: The *Zitterbewegung* of an electron

We should warn you, however: there is no reason whatsoever why the *plane* of the oscillation – the plane of *rotation* of the pointlike charge, that is – would be perpendicular to the direction of propagation of the electron as a whole. In fact, we think that plane of oscillation moves about itself. But that's not the point here. Look at that length λ: that's the *de Broglie* wavelength.[6] Look at the *radius* of the circular motion: that's the Compton radius of an electron. Moreover, we'll show the two are related through the velocity of our electron β = *v*/*c*. In case the Greek letter (*beta*) makes you afraid already, don't think like that: β is just the velocity *expressed in its natural unit*, which is the speed of light (*c*). So there is nothing special about it. Talking natural units, if we measure distance and time in so-called equivalent units (so the *numerical* value of *c* is effectively equal to 1), and if we also chose a force unit such that the *numerical* value of Planck's famous constant h is also equal to 1[7], then one can show that the Compton radius of an electron is equal to *a* = 1/m.

[5] In English, the dimension of h is referred to as the dimension of (physical) *action*: newton·meter·second, or joule·seconds (1 J = 1 N·s). We feel the English term doesn't quite catch the *physicality* of h—not as nicely as the German term, that is. But we have to make do with it.

[6] We will re-fine this statement later but, in essence, this interpretation is what it is.

[7] If you are like me, then you may struggle a bit with the various systems of so-called natural units. Should we equate h or its so-called *reduced* value ħ = h/2π with one, and why? We'll answer those questions in this book. For the time being, you should just *understand* what a natural unit is and what it does. For example, if we keep the second, but we define the distance unit to be

Of course, mass will now also be expressed in some natural unit[8]. The point is: we get a wonderfully simple *geometric* interpretation of what an electron actually *is*, and it's described by two variables only: that radius a = 1/m and that wavelength λ. Furthermore, these two variables are related through the following simple equation:

$$\lambda = 2\pi \cdot \beta \cdot a = \beta \cdot \lambda_C$$

The *de Broglie* wavelength is just some fraction of the circumference of the circle that's being described by our pointlike charge ($\lambda_C = 2\pi \cdot a$), and that fraction happens to be equal to the velocity of our electron (β). In addition, we get the radius a as the inverse of the electron mass m. Could it possibly be simpler?

In addition, you will probably wonder: what's the *nature* of that mass? We will show it's 'mass without mass', as John Wheeler referred to it in the 1960s: it is just the *equivalent* mass of the energy in this circular oscillation—so that's the *Zitterbewegung* of the pointlike charge, which itself has no rest mass whatsoever and which can, therefore, move about at the speed of light.

Is it *that* simple? Yes. Can we *show* it is that simple? We can. That's what this book is about. It's not about differential equations. Having said that, we will need *some* math, and that's what this first chapter will give you. Let's go for it. 😉

The force law and relativity

I said no differential equations. Except one. It's one we really can't avoid: Newton's force law. That's an easy one. So easy that you've probably never thought of it as a differential equation. But it has a derivative in it, so it's a differential equation:

$$\boldsymbol{F} = \mathrm{m} \cdot \boldsymbol{a} = \mathrm{m} \cdot \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \mathrm{m} \cdot \frac{\mathrm{d}(\mathrm{d}\boldsymbol{x}/\mathrm{d}t)}{\mathrm{d}t} = \mathrm{m} \cdot \frac{\mathrm{d}(\mathrm{d}\boldsymbol{x}/\mathrm{d}t)}{\mathrm{d}t} = \mathrm{m} \cdot \frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2}$$

You know this equation from your high school classes, but you may just have limited yourself to thinking about a force as the product of some mass and an acceleration factor: 1 newton is the force that gives a mass of 1 kg an acceleration of 1 meter per second, *per second*, that is. The latter is the dimension of the $\mathrm{d}^2x/\mathrm{d}t^2$ factor: m/s^2. We may say this law *defines* mass as a measure of inertia: some strange resistance to a *change* in their state of motion. Newton's equation tells us that objects tend to keep doing what they're doing, and that requires a *force* to change their state. That's an important concept – in both classical as well as in quantum mechanics, so we want you to think of that, *always*.

I should make another quick note here: besides being a *differential* equation, Newton's law is also a *vector* equation: the force *F*, and the position vector ***x*** (and its derivatives ***v*** and ***a***) are vectors. They have a magnitude but also some direction. That's why we write them in boldface. We hope you're familiar with that: in an illustration, you might see an arrow on top of the symbol—but so it's the same thing. OK. Let's move on and do some more thinking.

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equal to 299 792 458 m, then we have natural distance and time units, because the *numerical* value of the speed of light will turn out to be equal to one. It's as simple as that.

[8] You can *google* what natural unit *exactly*, but don't get distracted. The point is: Newton's law defines mass as a measure of inertia: an object will resist a change in its state of motion, which is why it requires a force to do so. Hence, if you re-define the force unit, and your distance and time units, then you'll get a new mass unit.

Because relativistic speeds – velocities that are a substantial fraction of *c*, that is – are not uncommon when we are discussing elementary particles – as opposed to the large-scale objects we are used to – we will also want you to understand the basics of relativity. All you should know about it is that we can no longer treat mass as some *constant*. Mass increases with velocity. When I have to explain that to someone, I usually do it in a rather light-hearted – and hopefully not too sacrilegious – way. So let me give you that story here and then you can think about it for yourself.

You know that mass has to increase because of the *absolute* speed of light, right? If it wouldn't increase, a force would be able to accelerate an object to an infinite speed. Now, infinity is a nice mathematical concept but you'll agree that, in *reality*, it's kind of a weird thing, right? So we may want to think that there is, effectively, some absolute speed *cap* in the Universe. Now, if you would be God, and you'd have to regulate the Universe by putting a cap on speed, how would you do that?

First, you would probably want to benchmark speed against the fastest thing in the Universe, which are those photons. They have no rest mass and so they can effectively travel at the speed of light: *c*. So that's the fastest thing in the Universe now: it's the speed of a *signal*, really.[9] So now you want to put a speed limiter on everything else, so it can only travel at some *fraction* of the speed of light. That fraction (*v*/*c*) is just a ratio between 0 and 1. Of course, because you're God, you do *not* want to police around so you want something mechanical: you want to *burden* everything with an intricate friction device, so as to make sure the friction goes up *progressively* as *v*/*c* goes to 0 to 1. You do *not* want something linear because you want the friction to become infinite as *v*/*c* goes to 1, so that's when *v* approaches *c*. So that's one thing you have figured out in your design.

Now, of course you also want a device that can cope with everything: electrons, bicycles, spaceships, solar systems, whatever you can think of. The speed limit applies to all. But then you don't need too much force to accelerate a proton as compared to, say, that new spaceship that was just built on planet X. So you think about brakes and engines and all that but, after a while, you realize it's probably better to just ask your best engineers to finalize your design. You all sit together and you explain your problem and the design requirements. One of them, *Newton*, will tell you that, when applying a force to an object, its acceleration will be proportional to its mass. So he goes to the blackboard and writes it down like this: $\boldsymbol{F} = \mathrm{m}\cdot\boldsymbol{a}$. Of course, you tell him you know that already, and that this is *exactly* your problem: even the smallest force can accelerate the heaviest object to crazy speeds. You just need to apply the force *long enough*.

Now *Lorentz* gets up and points to the mass factor in the formula: m should go up with speed, he says. And it should go up progressively – as per God's design. Lorentz is always well prepared, so he has a print-out ready, and puts it on the blackboard. There is an easy formula that does the trick, he says. Here, the red graph is for m = 1/2, the blue one for m = 1, and the green one for m = 3. In the beginning, nothing much happens: the thing picks up speed but its mass doesn't considerably – because you do

[9] If you know anything about quantum mechanics, you will know that the *phase* velocity of a composite wave packet may be superluminal. In fact, it usually *is*. However, this phase velocity is just a mathematical concept. It is not something *real* that is traveling through space. In other words, it can*not* carry any information. Only the *shape* of the wave can carry information—some *signal*, that is. And the shape of the wave travels with the *group* velocity of the wave packet, which is always smaller than *c*. This is probably confusing you, but I just wanted to be correct—especially because I must assume you have already done quite a lot of homework when you are reading this book.

want to allow everyone to move their stuff around, right? But when it gets a bit crazy, then the friction kicks in, and *very* progressively so as the speed gets closer to the speed of light.

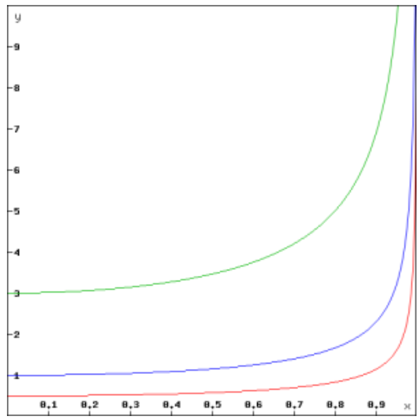

Figure 2: How to put a speed cap on bicycles, spaceships and galaxies

Now you stare at this for a few seconds, but he tells Lorentz he doesn't want to discriminate: it looks like we're putting more aggressive brakes on the green thing than on the blue or the red thing, right? But Lorentz says that is *not* the case. There is no discrimination here: his factor is the same per *unit* mass. The graphs show the *product* of the mass and his Lorentz factor, which is actually represented by the blue line—because that's the one for m = 1. So, yes, the green thing will actually have better brakes, but that's just commensurate with its mass.

You look again, and you think that makes sense. But then you hesitate, of course. You don't want to change the Laws of the Universe, as that would be messy. It would surely upset Newton, because he is pretty fussy about you tampering with stuff. So you look at both and you say: what's the implications for the force law? Newton nods in agreement: yes, what about it? We don't want to change it because there are a zillion devices that work on it now. We can't do a total recall.

Lorentz is still at the blackboard. He says: it's not a problem. We're going to use the same force law. We're just going to distinguish *two* mass concepts: the mass at rest, and the mass at some velocity *v*. Just put a subscript – m_v – and then you use that. And then he writes these two formulas:

$$\boldsymbol{F} = \mathrm{m}_v \cdot \boldsymbol{a} = \frac{\mathrm{d}(\mathrm{m}_v \cdot \boldsymbol{v})}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

$$\mathrm{m}_v = \gamma \cdot \mathrm{m}_v = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot \mathrm{m}_v$$

Now Newton stares at them, and he takes a few minutes rather than a few seconds. You think he is going to turn it down, because his formula is… Well… Newton's formula, right? But… No. Something weird happens: Newton nods and agrees! He gets up, shakes hands with Lorentz and says: excellent job! Perfect fix! So you're delighted and you tell Lorentz he can pick and choose his men and build it.

Newton walks out, and Lorentz stays behind. You see some worry on this face, and so you ask: what's up? You're not happy with your own thing? He sighs and says: my formula is the *only* thing that can do the trick because, yes, you want it to be progressive. It needs to be something based on the idea of the mass *unit*. But this mechanical thing has some weird implications. You ask: what implications? Now

Lorentz starts a discussion on a guy you've never heard about – Albert Einstein – and he starts mumbling about time dilation and length contraction. He says Newton's formula came with Galilean relativity, and that we'll need a new concept of relativity. But you want to move on by now, and so you tell Lorentz to hire that Einstein and just get on with it.

So that's what we'll do. We'll just get on with it. 😊 We need to look at some wavefunction math now. Before we do so, I want to offer you some more do-it-yourself tasks—if only to make these rather deep matters as digestible as possible.

Think for yourself: We wrote that the Lorentz formula is the only one that can do the trick. Of course, there is no *proof* that other formulas would not work and, in any case, our Universe is what it is, so the Lorentz factor is what it is. However, it is an interesting exercise to try some other formulas. The $\sqrt{1-(v/c)^2}$ factor makes us think of the formula for a circle: $y = \sqrt{1-\mathrm{x}^2}$, and so you might think some similar formula might also do the trick. Try it. It doesn't.[10]

We also wrote – rather jokingly – that infinity is a nice mathematical concept but that it is weird to think of what it could possible mean *in reality*. This is actually a rather deep philosophical statement. You should think through Zeno's paradoxes. Differential calculus shows that the idea that we can keep splitting some interval in time or in space in smaller and smaller bits – going on forever (so that's, funny enough[11], the idea of a *limit* in math) – is *not* incompatible with Achilles overtaking the tortoise, or the idea of an arrow being somewhere while flying through space, but it is good to think through those paradoxes. We need math to describe reality – whatever *idea* we have about it – but Planck's quantum of action, and the finite speed of light, seems to tell us our mathematical ideas are what they are: idealized notions to describe something finite.[12]

Wave math in a nutshell

We have introduced some math already – the force law, basically – but we need to add more. We need to add some wave math. The idea of a wave – or an oscillation – is central to our understanding of how things might work at the atomic level, so we're going to explain all of the wavefunction math you'll need—not only to understand this book, but to understand *everything* about quantum math. Don't worry too much. It is just basic circle math—and some funny notations. Look at the illustration below. It's the green circle that matters. Look at it carefully: think of the green dot going around and around as the argument θ ticks away with time. The original is, in fact, an *animated* GIF that you can easily google[13] and you may want to stare at it for a while so as to appreciate the *dynamics*.

[10] I offer some thoughts on that on the https://readingfeynman.org/e-book/ page of my blog. If you have no time, then just continue reading. I'll show the graph of the *inverse* Lorentz factor, as a function of the β = *v*/*c ratio*, is, in fact, just a simple circular arc – which is as it should be in light of the functional shape of the two formulas.

[11] Think about what I'd call that funny: the mathematical definition of a *limit* involves the idea of infinity. So that's a pretty clear example of an *contradictio in terminis*, no? 😊

[12] The rather philosophical discussion on the mathematical consistency of Dirac's delta function is a nice example of a paradox in quantum mechanics. We will not entertain such discussions in this book, however. Not because we don't like them – on the contrary – but because they have little *practical* value in trying to move towards some understanding of it all.

[13] Source: https://en.wikipedia.org/wiki/Sine#/media/File:Circle_cos_sin.gif. The illustration is public domain content from Wikimedia Commons.

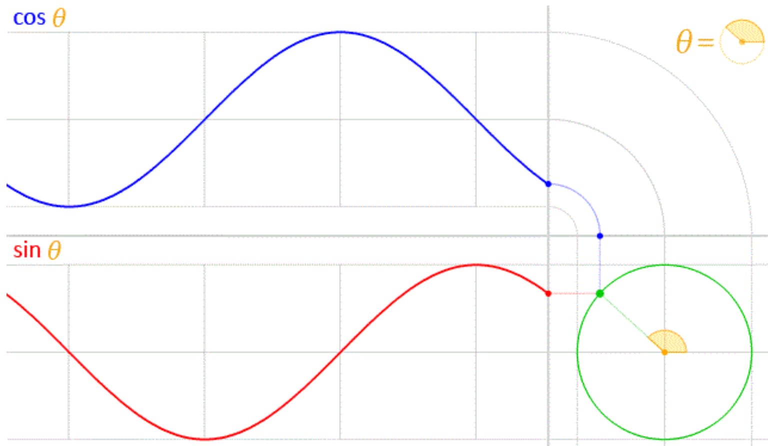

Figure 3: Rotational motion

We will write θ as θ = ω·t. Greek letters always scare but they should not. They are just letters—like *a*, *b*, *c*. Scientists usually use a Greek letter to distinguish a constant from some variable.[14] The *omega* (ω) is just an angular frequency—aka radial or circular frequency. Don't be scared by the *omega* symbol: it is just a regular frequency – but expressed in a somewhat different unit. If *f* is expressed in cycles (or oscillations) per second (*Hertz*[15]), then we will want to associate π/2 *radians* with one cycle, and we have our formula for ω:

$$\omega = 2\pi \cdot f = \frac{2\pi}{\mathrm{T}}$$

Do take your time to think about this: the *period* (T) is the inverse of the *frequency* (*f*). If the frequency (*f*) is, say, ten cycles per second, then the time that's needed for the system to come back to its original state (so that's the cycle time T) will be one tenth of a second: T = 1/*f*. Now, if you want to truly *understand* what this book is all about, then you have to get into the *physicality* of this thing! Do you remember the *Rotor* or the *Gravitron*? It was an amusement ride: a large barrel, whose rotation (about 30 revolutions per second) created a centrifugal effect. The force was about three time the force of gravity and, hence, one the barrel had attained full speed, the floor could be retracted—leaving the riders stuck to the wall of the drum.

The point is: it takes a force to keep something in orbit. We're not going into the detail of what force that might be (that will come later) but you should now be able to appreciate the physical dimension of Planck's constant: it is a product of (1) a force (as we will show, we think of this force as keeping some charge in an orbit), (2) a length (the circumference), and (3) a cycle time. Hence, its *physical* dimension is N·m·s (newton-meter-second). That's the dimension of angular momentum. It's also the dimension of a

[14] That is a simplification, of course. There is no rule here. In the E = ħ·ω = ħ·*f* expression, Planck's quantum (ħ or h) is the only constant. It relates the energy and the frequency of an electron or a photon through what we will refer to as the *form factor*. It is a sort of generalized notion of the idea of a proportionality factor. I have no formal definition of this. It is just an intuitive thing which, hopefully, will help you to understand quantum mechanics in a more *intuitive* way.

[15] It is quite significant that the frequency unit is named after Heinrich Rudolf Hertz. He died at a young age (36) – but, among his many other achievements, he managed to *prove* the reality of electromagnetic waves. His first radio transmitter – built in 1887 – was a dipole radiator which transmitted radio waves at a frequency around 50 MHz – pretty much the frequency range that is used for radio and TV transmission today. Of course, we are talking transmission so he built a receiver too, using a *spark micrometer*. Fascinating stuff for DIY physicists!

poorly understood physical quantity which, in German, is referred to as a *Wirkung*. The English translation – physical *action* – doesn't quite catch that meaning, I think—but we just have to make do with it. It's usually denoted by *S*, and for angular momentum we'll usually write it as L. We will write the following equations for an electron:

$$S = F \cdot \lambda \cdot \mathrm{T}$$

$$\mathrm{L} = I \cdot \omega$$

The *F* is a force, the λ is a length (the *Compton* wavelength, to be precise, which we'll effectively interpret as a circumference), and T is a cycle time. The *I* in the angular momentum formula is the moment of inertia, aka the angular mass or the rotational inertia. The ω is the angular frequency, which we have come across already: $\omega = 2\pi \cdot f = 2\pi/\mathrm{T}$. Now, the title of this book makes it clear, we are going to offer an interpretation of quantum mechanics based on the *Zitterbewegung* idea. We will *not* go too much in detail here, but we do want to sketch some of the basics here. *Zitter* is German for shaking or trembling, and the *Zitterbewegung* refers to a presumed local oscillatory motion—which we now believe to be *true*, whatever that means. Erwin Schrödinger stumbled upon it when he was exploring *solutions* to Dirac's wave equation for free electrons. It is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

The reference to the 'law of scattering of light by an electron' is a reference to *Compton* scattering. Compton scattering is a weird process: a photon hits an electron, basically, and its energy is sort of absorbed – temporarily – before the electron emits another photon, whose wavelength will be different, and the difference in the energy of the incoming and the outgoing photon gives the electron some linear momentum. Needless to say, the formula for the photon energies is the $E = h \cdot f = h \cdot \omega/2\pi = \hbar \cdot \omega$ equation[16], so that's just the Planck-Einstein relation, or *de Broglie's* first equation. Their wavelength can be calculated in two ways:

[16] The *reduced* Planck constant (ħ) is just h/2π. You will get a feel for when to use h and ħ when we get into the nitty-gritty of our models. Generally speaking, when we speak of Planck's quantum of action – physical action, that is – we have h. However, the angular momentum that we can *associate* with this physical concept will be expressed in units of ħ. It is a rather simple mathematical thing, but we will not explain it here, because we don't want to confuse the reader by frontloading too much!

1. We have the general $\lambda = c\cdot T = c/f$ equation: the wavelength of a wave is the product of its velocity and the cycle time. Just think about it: makes sense, right? The time that's needed for a wave to travel over a distance that is equal to one wavelength is just the cycle time T.

2. We also have the second *de Broglie* equation: λ = h/p. But what's the momentum (p) of a photon? Its energy has an equivalent mass, so Einstein's mass-energy equivalence relation tells us its mass is $m = E/c^2$. We can then use the classical formula for the momentum $p = m\cdot v$ (mass times velocity). Because $v = c$ here (our photon travels at the speed of light), we can write:

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}\cdot c} = \frac{\mathrm{h}\cdot c^2}{\mathrm{E}\cdot c} = \frac{\mathrm{h}\cdot c}{\mathrm{E}} = \frac{\mathrm{h}\cdot c}{\mathrm{h}\cdot f} = \frac{c}{f} = c\cdot \mathrm{T}$$

What's the use of this? Nothing particular. We just want to give you a bit of a feel of how these fundamental equations – Einstein's mass-energy equivalence relation, the *de Broglie* equation and some basic wave formulas – are all perfectly compatible.

Let us go back to the *Zitterbewegung*. *Compton* scattering experiments give us an effective *diameter* of an electron. This diameter is, of course, twice its radius, which we will refer to as the Compton radius[17], which we'll denote by a. This scattering *radius* has been determined through experiments but – rather remarkably, one would think – it has been found to correspond to the following ratio of *natural units*:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda}{2\pi} \approx 0.386 \ldots \times 10^{-12}\ \mathrm{m}$$

The m is the electron mass, and we do think of it as some natural unit. Think of it as one of Nature's *constants*. Why is this so? Why would a – or the associated *circumference* $\lambda = 2\pi\cdot a = h/mc \approx 2.426\times10^{-12}$ m – be equal to that rather particular combination of natural constants? The *Zitterbewegung* hypothesis explains it. Think of the green dot as a *pointlike* charge, spinning around at the speed of light. The speed of light, c is, a *tangential* velocity here, and we have a formula for that in terms of the *radius* and the angular velocity (or angular *frequency*) of the rotation.

$$c = a\cdot\omega \Leftrightarrow a = c/\omega$$

Now look what happens if we use the Planck-Einstein relation once more, to substitute ω for $\omega = E/\hbar$. We get that formula above:

$$a = \frac{c}{\omega} = \frac{\hbar c}{\mathrm{E}} = \frac{\hbar c}{\mathrm{m}c^2} = \frac{\hbar}{\mathrm{m}c}$$

Not impressed? You should be, so if you are not, stop reading. There is a lot of magic here but – if you are interested in a *physical* interpretation of these formulas, then your basic question should be this: what is that $E = mc^2$ equation telling us? We will come back to that in a moment, so just hang in here for a while. We'll give you a delightful *physical* interpretation of Einstein's mass-energy equivalence in a moment but, first, I want to give you some more of a feel for the models we're going to develop. Let's go back to these formulas for physical action and angular momentum, which we wrote as:

$$S = F\cdot\lambda\cdot \mathrm{T}$$

[17] Physicists usually speak of the Compton wavelength, which is equal to $2\pi\cdot a$.

$$L = I \cdot \omega$$

From your high school classes in physics, you should remember we can calculate an energy as a force over some distance.[18] Hence, if we have the energy and a length, we can calculate the force. So now we can calculate the amount of physical action that we should associate with one cycle of this *Zitterbewegung* oscillation. We get this:

$$S = F \cdot \lambda \cdot \mathrm{T} = \frac{\mathrm{E}}{\lambda} \cdot \lambda \cdot \frac{1}{f} = \mathrm{E} \cdot \frac{h}{\mathrm{E}} = h$$

We get Planck's constant: the *quantum* of physical action. You may shrug your shoulders and say: what about it? It is a very fundamental idea. The formula above gives us a physical interpretation of Planck's constant: Planck's quantum of action is the *physical* action of what we refer to as an elementary *cycle*. We have a cycle like this for the electron, but we also have a cycle like this for a photon. Let us show you how that works. All we have to do is to combine that formula for the wavelength of a photon

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}c}{\mathrm{E}} \Leftrightarrow \mathrm{h} = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda$$

Now you'll wonder: why is $h = p \cdot \lambda$ some unit of physical action here? Because, besides writing action as the product of force, distance and time, there are two other ways to express physical action, and they're *equivalent*:

1. The product of force and time gives us the physical dimension of (linear) momentum (N·s). Hence, if we multiply some linear momentum with a length, we also get some *Wirkung*.
2. As mentioned above, force times distance is energy, so if we multiply that with time, we also get a certain amount of physical action. In fact, from the postulate that we should associate a photon with an amount of action that is equal to Planck's constant, we get the Planck-Einstein relation, as shown below:

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f} \Leftrightarrow \mathrm{E} = \mathrm{h}f$$

Now, I realize this is getting somewhat tedious, but then I should remind you bought this book because you want to know what these things really *mean*, and I'd say: take a break, and then start reading again. 😉

I want to show one more thing here. We talked about angular momentum, and you'll know that fermions – like that electron that we're looking at here – are referred to as spin-1/2 particles. Why? Because we have that mysterious property, which is referred to as *spin*, in classical as well as in quantum mechanics. It is *not* mysterious in classical mechanics, but all that you've read so far will tell you that it *is* mysterious in QM.

Quantum-mechanical spin is expressed – and, more importantly, also *measured* in real-life experiments (such as the Stern-Gerlach experiment, with which you should be familiar) – in units of ħ/2, but so we are told that we should *not* try to think of it as a classical property—as something that has some physical

[18] From time to time, I write some posts for my kids on my physics blog. If you want to review the concept of energy as a force over a distance, you can check here: https://readingfeynman.org/2014/05/23/a-post-for-my-kids-on-energy/.

meaning. It's just that weird number, right? No. It's not. The *Zitterbewegung* hypothesis gives us a perfectly classical explanation for it. We can just use the classical $L = I \cdot \omega$ expression and substitute I and ω for the angular mass and the angular frequency. To calculate the angular mass, we need a form factor. Are we talking some hoop or some disk or a shell, perhaps? The model we'll develop implies the energy is in the oscillation and, therefore, it implies that the effective mass of the electron will be spread over a circular disk. We should, therefore, use the 1/2 form factor for the moment of inertia: $I = ma^2/2$. So now we can write it all out:

$$\mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{\hbar}{mc} = \frac{\hbar}{2}$$

So, yes, an electron is a spin-1/2 particle. 😊 Can we do the same for the photon? That's a boson, right? So it should carry an angular momentum that's equal to ħ. A full unit of ħ: no half unit. We get a delightfully simple *physical* interpretation of that magnitude when thinking of the photon in a model that we refer to as the *one-cycle photon*. It is illustrated below: we have a rotating *electric* field vector ***E***. Sorry for the confusion with the energy E here, but you should be able to figure out what is what from the context. Note that we use **boldface** for ***E***, because it is a vector: it has a magnitude (*E*) but it also a *direction*. If not, the idea of a rotating field vector (you will recognize this from illustrations of *circularly polarized* light) would not make any sense.

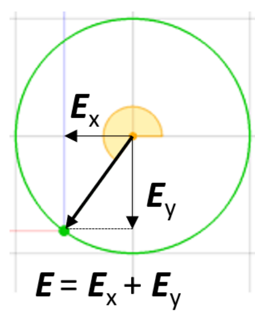

Figure 4: The photon model: circular polarization

So, yes, we can think of the photon as an oscillation that is traveling through space and time and whose *cycle* packs one unit of angular momentum (ħ) or – which amounts to the same, one unit of physical action (h). As mentioned above, it is a very fundamental idea, really: we interpret Planck's quantum of action as the *physical* action of what we refer to as an elementary *cycle* and, yes, we do not only have cycle like this for the electron, but we also have one for the photon. As we will show in this book, this model allows to actually *calculate* the actual field and the forces which, in turn, allows us to explain a number of previously mysterious quantities and numbers, such as the fine-structure constant, to just give one example that should surely be of interest to you. Most importantly, the models do give us a specific geometric idea of what an electron and a photon actually *are*, and that's the kind of intuitive and natural understanding you want to have.

I should now start the book but, unfortunately, I will first give you some more math. I need to explain Euler's function—some basic complex algebra, that is. Don't be afraid: it is *not* difficult. Why not? Because this too we will approach geometrically. Don't worry too much. It won't be long and, as mentioned, please take a break from time to time. This is *not* book you want to read in one go. 😊

Euler's function

Have a look at **Figure 3** once again: the rotational motion of the green dot. If you're going to remember only one message from this book, then we want it to be this: the mathematical *and* physical idea of a cycle, and its integrity—as expressed in Planck's quantum of action (h). So we have that circular motion. We will want to describe it in terms of the math that you should be familiar with when you've already read something about quantum mechanics (which we assume you have). That math uses complex numbers. To be precise, we will be using *Euler's function* all of the time, which is illustrated below.

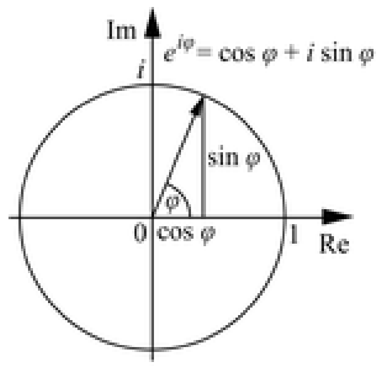

Figure 5: Euler's formula

Don't panic. Just keep staring at it for a while, and understanding should come intuitively. Think of that circular motion as a *superposition* of a sine and a cosine. In case you forgot, the sine and the cosine are basically the same function but with a *phase* difference of 90 degrees—so that is an angle that is equal to π/2 *radians*. Again, you may want to remind yourself that the radian is just the distance unit: the *circumference* of the unit circle has a length that is equal to 2π distance units. Of course, our disk will not always be described by the unit circle. In fact, it will never be, because a radius of 1 *meter* is humongous. Our circle will have a radius of, say, that Compton radius, so that's the *pico*meter scale: $a \approx 0.386\times10^{-12}$ m. All we need to do is multiply our sine and cosine with that *amplitude*. We write:

$$\boldsymbol{r} = a\cdot e^{i\theta} = x + i\cdot y = a\cdot\cos(\omega\cdot t) + i\cdot a\cdot\sin(\omega\cdot t) = (x, y)$$

That's a lot to swallow, so let us give you some pointers. The imaginary unit (i) gives us the *imaginary* axis. It is just as real a dimension as the… Well… The real dimension. In fact, the choice of real and imaginary for the two axes is quite unfortunate because it gives one the impression the imaginary dimension is, somehow, not real. Forget that idea: an object comes with its own space, so to speak, and the *Zitterbewegung* hypothesis tells us we should be thinking of elementary particles as oscillating in two dimensions—some *plane* of oscillation, in other words: a plane assumes *two* dimensions, and it is very stupid to think of one as being less real than the other. So that explains – or should explain – the $x + i\cdot y = a\cdot\cos(\omega\cdot t) + i\cdot a\cdot\sin(\omega\cdot t) = (x, y)$ in the equation(s) above. What about the $\boldsymbol{r} = a\cdot e^{i\theta}$ notation? The $\boldsymbol{r}$ is a vector, as you can see from the **boldface** that we're using, so that should not be an issue.

So all that's left to explain now is that weird $e^{i\theta} = \cos(\theta) + i\cdot\sin(\theta)$ identity. The e is Euler's number: it's an irrational number – you've seen it: $e = 2.71828…$ – and it's a *mathematical* constant, just like π. What's the difference between a *mathematical* constant and a *physical* constant? A mathematical constant has no *physical* dimension. It is just a number. Just like π, we can *visualize* it in various ways. You can check a math course on that, or Wikipedia. We can draw functions such as $1/x$ or $\ln(x)$ and then e will be some area under a curve or – quite simply – the point for which ln(x) = 1.[19] Feynman wrote a rather interesting explanation of Euler's function in terms of an *algebraic* construction of the sine and cosine functions.[20] However, this is a book on physics – not on math – and so I won't dwell on it. Having said that, I would really suggest you try to develop a *feel* for Euler's function – and all of the math that's related to algebra with complex numbers. For example, you should appreciate that a multiplication by the imaginary unit corresponds to a rotation by 90 degrees in the *counterclockwise* direction. For a clockwise rotation, we need to multiply by –i.

It is an important point because, as we will see, these two *degrees of freedom* in the mathematical description correspond to the direction of *spin* in our *physical* interpretation of the wavefunction. In fact, apart from our remarks on understanding the *physicality* of Planck's quantum of action (the association with an elementary cycle of the matter-wave, or of a photon), this is the other major *breakthrough* in our understanding of the reality of the wavefunction that we want to highlight in this book. We will come back to it soon enough. Let's conclude this math excursion by saying a few words about the *argument* of the (elementary) wavefunction.

An electron – of any matter-particle – will have no momentum in its own space, so to speak: its momentum p is zero, and the argument of the wavefunction therefore reduces to:

$$\theta = \omega \cdot \mathrm{t} = 2\pi \cdot f \cdot \mathrm{t} = 2\pi \cdot \frac{\mathrm{t}}{\mathrm{T}}$$

Look at what happens here. We re-scale time: instead of measuring it in seconds, we measure it in units of the cycle time T. This cycle time is given by T = h/E, so we get it from the energy of the particle that we're looking at, and h, of course. Hence, we might say that each particle comes with its own internal clock—a clock that has its own *clock speed*. What clock speed? It's given by the energy of our particle. We then just multiply by 2π to get an argument expressed in *radians*. Of course, the energy of our particle in its own frame of reference is equal to the *rest* energy, which we may denote by using a subscript: $E = E_0$. So the argument of the elementary wavefunction is this:

$$\theta = \frac{\mathrm{E}_0}{\hbar} t = 2\pi \cdot \frac{\mathrm{t}}{\mathrm{T}}$$

Now, you will – of course – have seen the much more complicated formula for the elementary wavefunction[21]:

$$\psi = a \cdot e^{\frac{i}{\hbar}(\mathrm{E \cdot t} - \mathbf{p \cdot x})} = a \cdot \cos\left(-\frac{\mathrm{E} \cdot t}{\hbar} - \frac{\mathbf{p} \cdot \mathbf{x}}{\hbar}\right) + i \cdot a \cdot \sin\left(\frac{\mathrm{E} \cdot t}{\hbar} - \frac{\mathbf{p} \cdot \mathbf{x}}{\hbar}\right)$$

So what about that? Well… Believe it or not, that expression is just a relativistic correction, because we will usually see our electron *moving* from here to there, and they *may* do so at pretty high speeds, at which point we should effectively use a relativistically correct formula. Having said that, if speeds are *non*-relativistic – which is usually the case – then you can forget about this correction and just think of the clock speed as a simple angular frequency given by that ω = 2π/T = E/ħ expression.

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[19] The latter example is a bit of a tautology, but useful still.

[20] You can check my blog post on that (https://readingfeynman.org/2014/06/24/eulers-formula-revisited/) or, perhaps, check out Feynman's argument online (http://www.feynmanlectures.caltech.edu/I_22.html).

[21] The argument of the wavefunction usually has a *minus* sign, but that's just a matter of convention, as we'll explain shortly.

Not convinced? Let me write it out for you. If we write the energy of our particle as it moves about as E_v, then we can apply the Lorentz formulas for transforming the time and space coordinates back to the *rest* frame of reference, as shown below:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}vt\right) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)t = \sqrt{1-\frac{v^2}{c^2}}\cdot\frac{\mathrm{E}_0}{\hbar}t$$

We get what we said we would get: the clock is just ticking away in the particle's own time, which is *our* time multiplied by the inverse Lorentz factor, which is show below. However, do not worry about this too much. We will come back to it. The important thing is that you grasp the idea of the particle's own clock, and the *nature* of the argument of the wavefunction: it really *is* something like a simple stopwatch. 😊

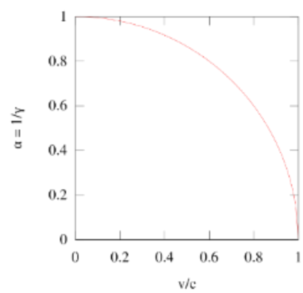

Figure 6: The inverse Lorentz factor as a function of (relative) velocity (*v*/*c*)

The graph below might or might not help you to make sense of it. [If it doesn't, don't worry, because we will come back to it.] In the *Zitterbewegung* interpretation of an electron, the trajectory of the pointlike charge will be *something like* what's illustrated below. By the way, if it makes you think of an Archimedes' screw, it should, because it's the same shape!

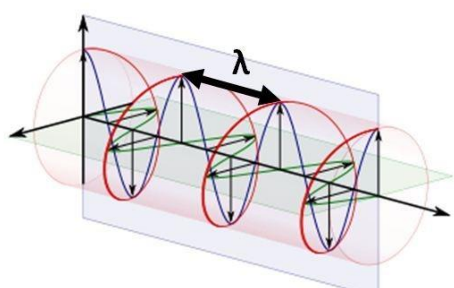

Figure 7: Is this a moving electron?

However, we say *something like it* because there is no reason whatsoever for assuming that the plane of oscillation will be perpendicular to the direction of motion of our electron. In fact, as we have no clue whatsoever here, the common-sense assumption is that its plane of oscillation (or rotation) will itself move about in a rather random way. That's consistent with another idea we will want to introduce: Planck's quantum of action is a physical constant ($h \approx 6.626 \times 10^{-34}$ N·m·s) – and we associate it with the idea of an elementary *cycle* of an equally elementary two-dimensional oscillation in this book – but we may want to think of it as a vector quantity: something with a magnitude (6.626×10^{-34} N·m·s) but with some *direction* as well. Why? The *physical* dimension of Planck's constant is the same as that of angular momentum, and the idea of a rotational cycle and angular momentum are, obviously, closely related, as illustrated below: the angular momentum (**L**) is a vector. Hence, we should probably think of Planck's constant as a vector too, so we should write it in boldface: **h**.[22]

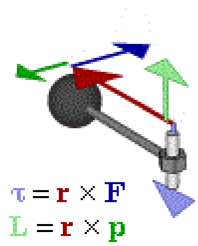

Figure 8: Torque ($\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$) and angular momentum ($\mathbf{L} = \mathbf{r}\times\mathbf{p}$) as vector (cross) products[23]

We won't waste too much time on this, but just make a mental note of it: associate one rotational cycle with some *vector* **h**, and think of the plane of oscillation as being random for the time being. At the same time, you should also make another mental note: a rotating electric charge has a magnetic moment, whose magnitude is going to be equal to the current (*I*) and the surface of the loop, as illustrated below (*S*).[24]

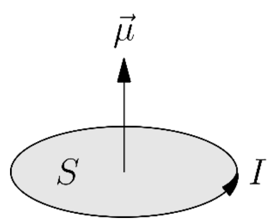

Figure 9: The magnetic moment of a rotating charge[25]

The magnetic moment will cause the plane of oscillation to line up in an external magnetic field—such as the magnetic field of a Stern-Gerlach apparatus, which is used to determine the *magnitude* of the magnetic moment of elementary particles. However, in the absence of an external magnetic field, there is no reason whatsoever to assume the plane of oscillation would have some fixed orientation. It may

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[22] We should note here that **L** is a vector because it's the vector cross-product of two other vectors: the position or radius vector **r** and the momentum vector **p**. In that sense, it is a bit of an artificial vector: it is known as an *axial* vector, or a *pseudo*-vector—as opposed to *polar* (or real) vectors. Both are vectors – in a mathematical sense, that is – but they are *different* objects: the sign of a polar vector gets reversed when coordinate axes are reversed. In contrast, that does *not* happen to an axial vector. We may, therefore, actually think of them as being more *real* than the so-called real (polar) vectors we need to multiply (using the vector cross-product formula) to get the axial vector!

[23] This is another *animated* gif (https://upload.wikimedia.org/wikipedia/commons/0/09/Torque_animation.gif) in the Wikimedia Commons public domain. Hence, you may want to watch the dynamics rather than this static image.

[24] We use *I* and *S* also for the moment of inertia and physical action, which are entirely different physical concepts. The context of the formula should make clear what is what. We do not like to multiply the symbols. Instead, we'd rather re-use them and associate them with different meanings depending on the context. This is a choice made by the author based on philosophical considerations: Wittgenstein would *not* agree!

[25] https://commons.wikimedia.org/wiki/File:Magnetic_moment.svg.

just wobble around. We have no real proof of what we're going to say now, but we like to think that the quantum-mechanical uncertainty is *not* in the magnitude of **h**, but in its direction!

However, that's just speculation and we should get back to that illustration of our moving electron (**Figure 7**). We showed a sort of wavelength there (λ). What is it? If this would be an electromagnetic wave[26], it would be its wavelength, but here we're just thinking of that rotating pointlike charge moving along some *center* that itself is moving in… Well… The direction of propagation of the whole thing. The λ here is just the distance between the crests (or the troughs) of the two-dimensional wave. What's that distance? Let's calculate it.

You should, first, note that the velocity of our electron – let us denote by v – is likely to be non-relativistic, in contrast to the tangential velocity of the pointlike *charge* in its rotation motion. That tangential velocity is equal to c. From the graph, it's obvious that λ will be equal to the product of v and the cycle time T.[27] So we get the following formula:

$$\lambda = v \cdot \mathrm{T} = \frac{v}{f} = v \cdot \frac{\mathrm{h}}{\mathrm{E}} = v \cdot \frac{\mathrm{h}}{\mathrm{m}c^2} = \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \beta \cdot \lambda_C$$

This is an interesting formula: it says that length λ is some fraction (between 0 and 1 as v goes from 0 to c) of the Compton wavelength $\lambda_C = 2\pi \cdot a = 2\pi\hbar/mc$. Note, once again, that λ_C is just the circumference of the circular orbit of our charge, and the v/c fraction is just the (relative) velocity β, so that's the velocity of our electron expressed in its natural unit (c). So it's *not* the *de Broglie* wavelength $\lambda = h/p = h/mv$, unless v is equal to c. We'll talk about that more in detail when discussing the electron model.

What's the dimension of a pointlike charge?

If you have understood anything of what we wrote above, you will understand we are introducing a rather subtle distinction between the pointlike *charge* and the electron. The electron is the *Zitterbewegung* as a whole: it is the *oscillation* of the pointlike charge. The charge itself has no rest mass – so it can effectively move at the speed of light along this weird circular orbit (about which we will write more later) – but the electron *as a whole* has some mass. In fact, that is the whole point of our *Zitterbewegung* model: we explain the rest mass of an electron by introducing a *rest matter oscillation*.

As Dirac noted, it is impossible to directly verify the model because of the extreme frequency ($f_e = \omega_e/2\pi = E/h \approx 0.123\times10^{-21}$ Hz) and the sub-atomic distance scale ($a = r_C = \hbar/mc \approx 386\times10^{-15}$ m). It is, therefore, a *logical* model only: it gives us the right values for the angular momentum ($L = \hbar/2$), the magnetic moment ($\mu = (q_e/2m)\cdot\hbar$, and the gyromagnetic factor ($g = 2$).[28] You may think that should end the discussion but it doesn't: we will show there is a way – and probably more than one – to *indirectly* verify the model. We're basically talking the theory and measurements in regard to the anomalous magnetic moment here.

[26] The illustration is actually that of a circularly polarized electric field propagating through space. The magnetic field vector is *not* shown. The structural similarity between an electromagnetic wave (or its electric field vector, at least) and the matter-wave is quite deep, and exploring it has provided us with a lot of intuitive thoughts, which we'll further explore in this book.
[27] You may, perhaps, have to think about this for a while, but just take a piece of paper and a pen and you'll be able to work it out. It's just logic. We do want you to think this through for yourself.
[28] We have not introduced these quantities yet, but we will do so later. We just wanted to flag our model also yields other *observables*.

In regard to this, we should ask ourselves: is an electron really some disk-like structure and – if it is – what's shape, *exactly*? That's a complicated discussion but we can already start it here by asking a more basic question: is our pointlike charge dimensionless, or does it have some radius itself? What do we mean by *pointlike*?

We have no *definite* answer to this – none of the answers in this book are definite – but we should make a few sensible remarks here. We do *not* believe in dimensionless objects: a pointlike charge must also occupy some space in… Well… In space. 😊 So what space would that be? This is where the classical electron radius comes in. The classical electron radius is also known as the Lorentz radius or – something that will ring more of a bell – the *Thomson* radius. Just like the *Compton* radius, we have an experimental value for this radius, which corresponds to:

$$r_{\mathrm{e}} = \frac{\mathrm{e}^2}{\mathrm{m}c^2} = \alpha \cdot a = \alpha \frac{\hbar}{\mathrm{m}c} \approx 2.818 \ldots \times 10^{-15}\ \mathrm{m}$$

What *is* this radius? We also get it from scattering experiments but – in contrast to Compton scattering – we have a totally *elastic* scattering here: the wavelength of the photon does not change. The photon just bounces back: as such, the photon doesn't seem to *mess* with the electron.[29] In short, the photon does seem to bounce off some *core*, and that core is (much) smaller than our *Zitterbewegung* electron. Hence, we think it bounces off that pointlike charge, and that the Thomson radius is the radius of our pointlike charge. Can we prove that? No. It is just a sensible interpretation of what might or might not be happening.

How much smaller is it? We see that wonderful ratio in our formula above: $\alpha \approx 1/137$. It's the fine-structure constant which – don't worry – we'll devote a lot of pages to: we claim to have an explanation for this so-called 'God-given number.'[30] But here we are still introducing some basics so let us not get ahead of ourselves. So what's the point?

The point is: pointlike does not necessarily mean *dimensionless*. In fact, as mentioned, common sense tells us something that has no dimension cannot exist. So our pointlike charge is a pointlike charge but *pointlike* just means we have no further clue on what its internal structure might be – if it has any internal structure. As far as we are concerned, we like to think of it as a perfect object – something *physical* but with a perfect mathematical shape: a tiny little sphere, probably.

What's its *nature*? Where does it come from? What are the implications? Good questions. I don't have the answer to them. The chapter on the fine-structure constant and the classical electron radius will offer some remarks – based on re-arranging and combining various formulas so as to possibly produce some new meaning, most notably on the question on what the *nature* of electric charge might be – but these are definitely not definite. 😊 We hope it will encourage the reader to actually engage in some further thinking himself.

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[29] Physicists will shout wolf here and say this process should also be analyzed in terms of *interactions*: a photon *absorption* and a photon *emission*, that is. We readily admit that we skip those niceties for the time being.

[30] There are a few references here but the one that is best known is from the little booklet that Feynman published a few years before he died (QED: The Strange Theory of Light and Matter), in which the following is written: "You might say the hand of God wrote that number, and that we don't know how He pushed his pencil." One should note that Feynman did *not* actually write that book: it is a transcript of some lectures for a popular audience, and we are *not* so impressed with it. In fact, we think it is one of those popular books that is somewhat misleading because of over-simplification.

The take-away here is that our *Zitterbewegung* electron is *not* a perfect disk: it has some *thickness*. Should we describe it as a thick disk, or as a thin disk? [In case you wonder, I am borrowing some concepts from the description of galaxies here.[31]] I have no answer to that. However, some pretty advanced – but non-mainstream – research is underway in this area. These more advanced approaches take into account that the forces and fields are rather large – in light of the small distance scale – and, hence, general relativity comes into play: Cartesian coordinate frames are no longer relevant. We have some curvature of space, and that makes the analysis rather difficult. We will come back to this.

So, we're done with the prerequisites – the *prolegomena*, as I called it. Let's now talk about this book—and what you might get out of it.[32]e

II. What's new in this book?

Lots of stuff, of course. But then I have to say this, because this book needs to sell, right? Well… Yes and no. Sales are my publisher's concern. Not mine. I do think this book is truly innovative. I hope the remarks below – which may come across as being rather un-organized – will manage to convince you this book is, effectively, very different from whatever else you might have read – or you'd want to read.

No differential equations—only solutions!

Scientists are obsessed by differential equations. Dirac was not any different. His very first degree is that of an electrical engineer, and he always wanted to do what Maxwell did to explain electromagnetism, and that is to derive a concise set of differential equations – if possible, only one equation, really – that would explain everything. Don't get me wrong: I totally *get* the obsession with differential equations: once you have the differential equation(s) for a system, you have described all of its dynamics – and you can leave the nitty-gritty of actually *solving* them to the mathematicians – or, if you can't solve it analytically – to a computer.

So it will come as a big surprise to you that we will actually avoid differential equations in this book. It is one of the many reasons why this book is different from all others. We do *not* want to focus on the rather abstract business of differential equations, but on *understanding* their solutions. Hence, we will focus on the wavefunctions that are a solution to this or that wave *equation* and try to understand what they might represent or describe.

Why? For two reasons. First, unlike most physicists, we do think the wavefunction corresponds to something *real*. For us, it's just not some weird mathematical object on which you should *then* operate with this or that operator to get this or that probability for this or that *observable*. We think the wavefunction effectively *describes* the object—an electron or a photon, to be specific. That's what this book is all about, in fact.

There is a second, subtler, reason: we effectively do *not* believe one wave equation can describe everything. When Dirac first talks about the *Zitterbewegung* of an electron, he talks of it as a motion that is 'superposed' on the regular motion. As this book is going to offer an interpretation of quantum mechanics based on this idea, we should probably give you the full quote here, and some context. *Zitter* is German for shaking or trembling, and the *Zitterbewegung* refers to a presumed local oscillatory motion—which I now believe to be *true*, whatever that means. Erwin Schrödinger stumbled upon when

[31] I have not done any research in this regard, so I will let you *google* for yourself.
[32] Of course, we hope you *already* got something out of this by reading the first chapter!

he was, effectively, exploring *solutions* to Dirac's wave equation for free electrons. He, therefore, got a shared Nobel Prize for Physics with Paul Dirac – for the "the discovery of new productive forms of atomic theory" – and it is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Wave equations should correspond to something real too. For example, we can interpret Schrödinger's wave function as an energy diffusion equation. See: Jean Louis Van Belle, A Geometric Interpretation of Schrödinger's Equation, 12 December 2018, http://vixra.org/abs/1812.0202. In fact, we regret few physicists try to understand what a differential equation, like Schrödinger's equation, might actually represent. They should do more of an effort to think of them as vector equations—modeling some relation between physical vectors, that is. Having said that, we are not going to have to deal much with differential equations in this book—not only because it requires a more advanced understanding of math but, more importantly, because we think it's better to focus on trying to understand their solutions, because these solutions have a more direct correspondence with some physical object or reality: an electron, or a photon, or an electron orbital—these are the very specific examples we'll be dealing with in this book.

We've used the same magical word two or three times now: what is it that a formula – an equation, or a set of equations (like the $E = h \cdot f$ and $\lambda = h/p$ equations, which should be thought of as a *set* of two equations), or some implicit or explicit function – is trying to ***describe***? What do these formulas *represent*, really? That's been my motivation to study physics, and so I want to share the answers I've found in this book.

Indeed, I've always wanted to know what equations in physics describe: I wanted to know what *reality* they refer to. What *is* this *de Broglie* wavelength (λ), for example? What is it that is oscillating at frequency $f = E/h$? And what *is* that wavefunction, *exactly*? We're not God, so we can never be sure. However, this book offers an *interpretation* of quantum mechanics that will give you very precise answers to those questions. It is then up to you to decide whether or not these answers make sense to you.

Any other book on quantum mechanics will tell you that can*not* be done. We disagree. For starters, one can – and should – exploit the quintessential difference between an equation in physics and a merely mathematical equation: the variables in a law or an equation in physics have some *physical* dimension—like newton-meter (the unit of energy) or a second or whatever other *physical* dimension. A second, or a newton-meter, are things we can imagine. What's a newton-meter? Think of lifting an apple to a height of one meter. It becomes somewhat more difficult with units such a *tesla* (the strength of a magnetic field), but it can be done, if only because we can deconstruct that unit and relate it to the unit we use to measure the strength of an electric field (newton (N) per coulomb (C), so that's force per unit charge). The magnetic field is produced by a *moving* electric charge. We, therefore, also have the unit of *velocity* (m/s) in the tesla (T). To be precise, 1 T = 1 N/(C·m/s). We can effectively imagine – sort of, at least – what a charge of 1 C traveling at a speed of 1 m/s represents. So, yes, a thorough *dimensional* analysis of some key equations brings a lot of *intuitive* understanding free of cost, so to speak. 😊

However, more is needed. As mentioned, truly *understanding* some equation requires some *intuitive* or *natural* understanding of whatever it is that is being described by the equation. As such, the equations become a *language* which we need to master to arrive at some deeper understanding. To put it differently, *through* the equations, we must sort of *see* how reality looks like. Now, what I see doesn't have much in terms of color vision – nor is there much other familiarity with what I see in daily life – but I do see *something*, and so I'll try to communicate *what exactly* in this book.

This involves some speculation – or some *interpretation*, I would say. And I am *not* talking the Copenhagen interpretation now. It's also none of the other – even weirder – interpretations of quantum mechanics (think of the many-worlds hypothesis, for example). If we have to give a name to the interpretation that is being offered here, I would say it is just a *classical* explanation of quantum theory. So what is it all about?

No Copenhagen interpretation: quantum mechanics is a theory—not a procedure

What we are claiming in this book is that quantum electrodynamics – as a theory, and in its current shape and form – is incomplete: it is all about electrons and photons – and the interactions between the two – but the theory lacks a good description of what electrons and photons actually *are*. All of the weirdness of Nature is, therefore, in this weird description of the fields: perturbation theory, gauge theories, Feynman diagrams, quantum field theory, etcetera. This complexity in the mathematical framework does not match the intuition that, if the theory has a simple circle group structure[33], one should not be calculating a zillion integrals all over space over 891 4-loop Feynman diagrams to explain the magnetic moment of an electron in a Penning trap.[34] Hence, this book offers a new *geometric* model of both the electron as well as the photon.

Seriously? Yes. Seriously. We offer an electron model which combines the idea of a pointlike *charge* and Wheeler's idea of mass without mass: the mass of the electron is the equivalent mass of the energy in the oscillation of the pointlike charge. Don't worry. It will soon be clear what we mean by this. 😊

We will also offer a photon model. In essence, our photon is just what electromagnetic theory suggests it is: an oscillation of the electromagnetic field. No charge. So what's new? What's new in our photon model is that we are going to calculate the exact *magnitude* of the oscillation. We refer to this model as the one-cycle photon.

[33] QED is an Abelian gauge theory with the symmetry group U(1). This sounds extremely complicated but you can interpret this rather simply: it means its mathematical structure is basically the same as that of classical electromagnetics.

[34] We refer to the latest theoretical explanation of the anomalous magnetic moment here: Stefano Laporta, *High-precision calculation of the 4-loop contribution to the electron g-2 in QED*, 10 July 2017, https://arxiv.org/abs/1704.06996.

Anything else? Yes. We will relate our electron and photon model in a more refined version of the Bohr-Rutherford model of an atom—so we have the interactions as well. Plus some other stuff. I hope you are intrigued by what I wrote above. If not, you should stop reading. 😊

Understanding Einstein's mass-energy equivalence relation

What's the other stuff? Tons. As we're talking of what a true *understanding* of equations might actually mean, let us take the example of Einstein's E = mc^2 formula, for example—especially because it's related to the above-mentioned idea of mass without mass. The E = mc^2 might well be the most famous formula in physics but I think it is fair to say that most – if not all – physicists would struggle to explain what it means, *exactly*—despite its apparent simplicity! We all know we have an *equivalence* here: the energy that keeps the protons and neutrons in a nucleus together will give the nucleus some extra mass—in *addition* to the combined mass of the individual protons and neutrons, that is.[35] But what does it *mean*, really?

We will offer a metaphor – in the very first chapter on the quantum-mechanical wavefunction – that may or may not help you to think it through. We're not saying it's an easy explanation – but I promise it is going to be more intuitive or natural than anything you've read before. Let me give you some clues already, so you can start thinking about it. The energy in an oscillation – think of an electric circuit, or a mass on a spring – will be proportional to the square of (i) the amplitude of the oscillation (which we'll write as *a*) and (ii) the frequency of the oscillation (which we'll write as ω because it is quite convenient to work with an *angular* frequency[36]). So we will have some proportionality coefficient k and we can write the energy as:

$$\mathrm{E} = \mathrm{k}a^2\omega^2$$

For example, you may remember the formula for the energy of a harmonic oscillator. Think of a mass on a perfect (read: frictionless) spring. The proportionality constant k is equal to m/2 here, so the formula is this:

$$\mathrm{E} = \frac{1}{2}\mathrm{m}a^2\omega^2$$

Now think of some device that combines the energy of *two* oscillators. We'll show (in one of the first very chapters of this book) that it's not that difficult: not in theory, and not in practice. All we need to do is to make sure the two oscillations are perpendicular one to another, so they are *independent*. The mass on the spring (the green dot below) will now go round and round, with some *constant* tangential velocity. This tangential velocity is equal to the *angular* velocity – which is just the angular frequency[37] – times the *radius* of the circular path: $v = a\cdot\omega$. This is shown below.

[35] It is this *binding* energy that is being released in a nuclear explosion. There is no proton or neutron matter that gets converted into energy.

[36] An angular frequency is expressed in radians (*rad*) per second, rather than cycles (or oscillations) per second (*hertz*). One oscillation corresponds to 2π *rad*. The radian is, of course, nothing but the *distance* unit – one meter in SI units – but used to measure an *angle*. Hence, we write: ω = 2π·*f* = 2π/T, with T the *period* of the oscillation (T is the time which corresponds to one *cycle*). I recommend reading one of my posts on wave math so as to make sure you get the basic concepts: https://readingfeynman.org/2015/09/08/a-post-for-my-kids-on-the-math-of-waves/.

[37] Think of the angular frequency as the angular velocity if the radius is one. Remember that the *radian* (rad) is the *distance* unit for measuring angles.

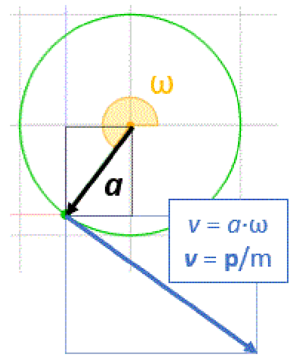

Figure 10: Angular frequency (ω) and tangential velocity ($v = a \cdot \omega$)

It is, therefore, very tempting to think of *c* as some tangential velocity too. Why? Think of the structural similarity of these equations:

1. $E = ma^2\omega^2/2$ (energy of an oscillator)
2. $E = mc^2$ (relativistic energy)
3. $E = mv^2/2$ (kinetic energy)

All we do is here is to *assume* that the kinetic energy must be matched by some potential energy[38], which works if we have *two* oscillators working in tandem.[39] The 1/2 factor in the first equation then disappears, and we can then boldly equate the $E = mc^2$, $E - ma^2\cdot\omega^2$ and $E = mv^2$ equations. So we just equate the energy of our two-dimensional oscillator with the energy of whatever it is that we're looking at (think of an electron here), and then the mass m *has to be* equal to the proportionality constant k.

The $c^2 = a^2\cdot\omega^2$ hypothesis gives us the frequency as well as the amplitude of what we will refer to as *the rest energy oscillation*. It is that what gives mass to our electron: its *rest* mass is nothing but the equivalent mass of the energy of the pointlike charge in its two-dimensional oscillation. Does this make *any* sense at all? It does. Let me show you why. We should get ω from the Planck-Einstein relation, which is just the first *de Broglie* relation: $E = h\cdot f \Leftrightarrow \omega = E/\hbar$. Hence, we can write:

$$\mathrm{E} = \mathrm{m}a^2\omega^2 = \mathrm{m}a^2\frac{\mathrm{E}^2}{\hbar^2} \Leftrightarrow \hbar^2 = \mathrm{m}a^2\mathrm{E} = \mathrm{m}a^2\mathrm{m}c^2 = m^2a^2c^2$$

$$\Leftrightarrow a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

We get the Compton radius of an electron. The Compton radius is the effective photon scattering radius of an electron, so it is effectively the size of an electron! This is a most wonderful result, and it convinced us we've got something real here in terms of an interpretation!

[38] In one of our very first blog posts on this, we referred to the potential energy as the hidden energy, or the missing energy, or the *imaginary* energy. See: https://readingfeynman.org/2016/05/23/the-missing-energy/. However, we now think that term is too ambiguous.

[39] We will show two oscillators can work in tandem if their *phase* differs by 90 degrees—which is in line with the requirement for their motion to be perpendicular to each other. In other words, we have a sine and a cosine—as these are the same function, but with a phase difference of 90 degrees. So we have Euler's function—the wavefunction!

You will probably shake your head and have all kinds of questions (which we'll try to answer soon[40]) but the more fundamental question among those should be this one: what is the *nature*, then, of this *equivalent* mass? The answer is surprisingly simple: it is just a measure of its *inertia*, so that is just its resistance to acceleration. Nothing more. Nothing less.

You may think that the explanation above is *not* very intuitive. If you do some homework on it, then you should throw these $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ formulas of harmonic oscillators at me[41] – noting that they introduce *two* (or more) degrees of freedom. In contrast, $c^2 = E/m$ for *any* particle, *always*. In fact, that's *exactly* the point: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges as *the* defining property of spacetime here—as some kind of *elasticity*, so to speak (as opposed to its quality as the velocity of travel of an electromagnetic signal, which we get out of Maxwell's equations).

Of course, you can now ask the next question: what is the nature of this *inertia* to acceleration or a change in direction? Here we can refer you to an experience you are surely familiar with: if you have a bicycle wheel in your hand – just holding it by its axle – and it is *not* spinning, then it is fairly easy to move it here or there. In contrast, when it is rapidly spinning, it will have a *moment of inertia* that comes into play—and that will complicate whatever you will want to do with is: you will *feel* its inertia to motion. Is the *nature* of what we referred to as the rest matter oscillation really the same? Yes—at least that's what I like to believe. 😉

Reinventing intuition and imagination

The little digression above makes it clear that what's intuitive or natural to me will not necessarily come across as intuitive or natural to you. Indeed, a lot of the formulas in this book feel somewhat intuitive to me but that is only because I have been staring at them for many years now. They were not intuitive – *not at all*, really – before I had diligently worked my way through them. As I learned how to relate the key equations in physics by fits and starts, some new picture emerged in a cognitive process that was characterized by the rare but essential experience of the *Aha-Erlebnis*—a sudden insight into the question that has troubled me for all of my life: what *are* those quantum-mechanical amplitudes? What does that quantum-mechanical wavefunction represent, *exactly*? I believe I have the answer to that question now—and the objective of this book is to share it with you.

I cannot promise that this book will be an easy read. It's *not*—even if I will be avoiding differential equations.[42] However, I am confident you will get some kind of feel for what the equations and formulas actually *mean* because all of the formulas I use represent *something we can imagine in terms of three-*

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[40] We'll do so in the next chapter. One question you should have is whether or not the formula for the energy oscillator is relativistically correct. We dealt with that question, head-on, in one of very first papers on the topic. See: *The Wavefunction as an Energy Propagation Mechanism*, http://vixra.org/abs/1806.0106.

[41] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the inertia, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

[42] There may be one or two very simple ones, but I am actually *not* going to talk about Schrödinger's or Dirac's equation. We have done that in various papers which the reader can consult on Phil Gibbs' viXra.org site (http://vixra.org/author/jean_louis_van_belle).

dimensional space and one-dimensional time—something we can understand in *our* Universe. With "our Universe", I really mean our world, which is not to be equated with some abstract mathematical space defined in terms of strings and hidden dimensions. I believe a true understanding of physics implies an understanding in terms of the geometry and the *physicality* of the situation at hand.

You'll say: of course! But it not so obvious. Indeed, let me contrast what I am trying to do with the rather limited ambition of mainstream physicists, which may be summed up in the following rather famous quote of the equally famous Richard Feynman:

> "Because atomic behavior is so unlike ordinary experience, it is very difficult to get used to, and it appears peculiar and mysterious to everyone—both to the novice and to the experienced physicist. *Even the experts do not understand it the way they would like to*, and it is perfectly reasonable that they should not, because all of direct, human experience and of human intuition applies to large objects. We know how large objects will act, but things on a small scale just do not act that way. So we have to learn about them in a sort of abstract or imaginative fashion and not by connection with our direct experience."[43]

I started this search for some truth long time ago because I could *not* accept the idea that I would never be able to understand quantum mechanics the way I would like to understand it. Of course, I did not expect things to be intuitive right from the start. Let me give you another example here. I must assume you understand the basics of relativity theory. If you are reading a book on quantum mechanics, you should—and if you do, you will agree relativity theory is *not* intuitive: relativistic mass, time dilation and length contraction are *not* what we observe in our daily lives, and the idea that simultaneous events may not appear as simultaneous to another observer isn't very intuitive either. However, once we accept a *signal* cannot travel faster than the speed of light, we can *derive* what must be true, and the more we play with the formulas, the more we sort of get an inkling of what the likes of Einstein and Minkowski must have *imagined* when they talked about relativistic spacetime to whatever audience they were talking to, and some kind of understanding of spacetime in terms of geometry and the physicality of the situation does follow in the end.

Now try to imagine what Feynman's path integral formulation of quantum mechanics wants us to believe.[44] According to this interpretation, one should abandon the classical idea of a single, unique classical trajectory for a particle. Instead, one should get used to the idea that a particle sort of travels simultaneously over an infinite number of quantum-mechanically possible trajectories. Each of these paths is associated with a complex number which is referred to as an *amplitude*. One then has to sum all of these amplitudes, and the *absolute* square of this complex sum then gives us the *probability* of our particle actually going from here to there. To be precise, it gives us a probability *density*, and when we say the absolute square, we mean the square of the *modulus* of the complex sum.

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[43] See: *The Feynman Lectures on Physics*, Volume III, Chapter 1, Atomic mechanics. The italics are mine. Richard Feynman is one of the most famous post-World War II physicists—, i.e. the *second* generation of quantum physicists), and his *Lectures* are a common reference in *mainstream* physics. They are accessible online (www.feynmanlectures.caltech.edu) and that is why we will often use them whenever we will want to refer to a textbook explanation of something. It allows the interested reader to put in an additional effort: he or she can look up the standard argument, so as to appreciate the subtle but fundamental *difference* in the approach that we are going to take in this book.

[44] I am choosing this example because the path integral formulation of quantum mechanics does effectively inform most of the other formalisms of quantum mechanics.

It is really hard to *imagine* what is going on here, right? Hence, one has to be *very* imaginative or abstract here. Why is it so hard? Impossible, I'd say. Because no one tells us *why* we should use this procedure. In fact, mainstream physicists tell us we should not even *try* to understand. The dominating Copenhagen interpretation of quantum mechanics tells us that Nature is just some kind of black box, and the best we can do is to think of some input-output relations to describe what goes in and what comes out. You don't believe this? Take any online course in quantum mechanics, and the first thing the professor will teach you is how to *describe* the Mach-Zehnder interference experiment, and that description is quite similar to the input-output matrix algebra that economists need to master as part of learning the tricks of their trade.[45] I will come back to this experiment in one of the final chapters, and I will show there is an alternative interpretation. It just requires a more imaginative description of the idea of a photon—a description that is more *real*, I'd say.

Hidden variables and black boxes: going where Bell tells us not to go

If you already a thing or two about quantum physics – which I assume you do – you will say: "What about Bell's No-Go Theorem, which tells us there are no *hidden variables* that can explain the interference in some kind of *classical* way?" My answer to that is like Einstein's when younger physicists would point out that his objections to quantum mechanics violated this or that axiom or theorem in quantum mechanics: "*Das ist mir wurscht.*" That means: I don't care. Bell's Theorem is what it is: a mathematical theorem. Hence, it respects the GIGO principle: garbage in, garbage out. So we will just boldly go to where Bell's Theorem says we can't go. In fact, John Stewart Bell himself – one of the third-generation physicists, we may say – did not like his own 'proof' and thought that some "radical conceptual renewal"[46] might disprove his conclusions. We should also remember Bell kept exploring alternative theories – including Bohm's *pilot wave* theory, which is a hidden variables theory – until his death at a relatively young age.

Hence, Albert Einstein was surely not the only who did not like the black box idea. In fact, all of the founding fathers of quantum mechanics ended up becoming pretty skeptical about the theory they had created.[47] Quantum physics – in its current *mainstream* rendering of it – only survived because second-generation physicists such as Freeman Dyson, Julian Schwinger, Richard Feynman and – to name a somewhat less familiar Nobel Prize name – Sinichiro Tomonaga, kept it alive by inventing a weird mathematical framework which we may summarize by referring to it as perturbation and renormalization theory. These distinguished scientists all received Nobel Prizes for it, so there is a vested interest now in further nurturing the mystery culture around quantum mechanics alive: no academic will want to hurt his or her career by exclaiming the Emperor has no clothes!

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[45] How do I know? Because I studied economics before getting into physics and math.

[46] See: John Stewart Bell, *Speakable and unspeakable in quantum mechanics*, pp. 169–172, Cambridge University Press, 1987. J.S. Bell died from a cerebral hemorrhage in 1990 – the year he was nominated for the Nobel Prize in Physics. He was just 62 years old then.

[47] See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf). We can also quote from Dirac's last paper on quantum mechanics, His last paper (1984), entitled "The inadequacies of quantum field theory," contains his final judgment on quantum field theory, *The Inadequacies of Quantum Field Theory*, which he published in 1984: "These rules of renormalization give, surprisingly, excessively good agreement with experiments. Most physicists say that these working rules are, therefore, correct. I feel that is not an adequate reason. Just because the results happen to be in agreement with observation does not prove that one's theory is correct." That is a pretty strong statement to make—and most people would actually dismiss such statement: we should be happy with a theory that's in agreement with observation, right? However, this is not a statement from your average physicist: it is a statement by a genius. Hence, we may want to think about it.

Radical conceptual renewal: the double-life of –1

By now, you should be tired of my ranting, and you'll want to know: what is the kind of "radical conceptual renewal" that I am offering here, *exactly*? What do we offer instead of Feynman's path integral math? Again, I am *not* saying it is an easy matter, and so I cannot elaborate on everything in the introduction here—especially because I actually did already reveal some basic tenets of my new physics above (cf. what I wrote about the interpretation of (rest) mass as a rest matter oscillation, the introduction of a consistent electron and photon model, etcetera). However, I do want to share why I think I can do what others could *not* do: I found a deep conceptual flaw in the early quantum-mechanical mathematical framework.

It sounds terribly arrogant but we do think the early theorists made a small mistake: they did not fully exploit the power of Euler's ubiquitous $\psi = a \cdot e^{i\theta}$ function. Schrödinger and Dirac may have been *too* obsessed by their differential equation – as opposed to the wavefunction that is its solution. They didn't integrate spin—not from the outset, that is. The mistake is illustrated below.

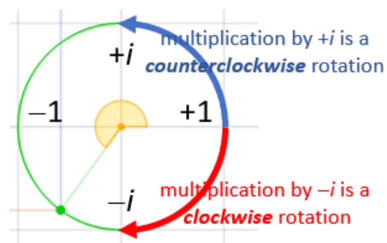

Figure 11: The meaning of $+i$ and $-i$

This looks like kids' stuff, right? I hope I didn't the simplistic illustration above didn't put you off, because it is actually a very subtle thing. Quantum physicists will tell you they don't *really* think of the elementary wavefunction as representing anything real but, in fact, they do. Of course! And, if you insist, they will tell you, rather reluctantly because they are not so sure about what is what, that it might represent some theoretical spin-zero particle. Now, we all know spin-zero particles do not exist. All *real* particles – electrons, photons, anything – have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. It is, therefore, completely odd that the plus (+) or the minus (–) sign of the imaginary unit (i) in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include the spin *direction* in the mathematical description. Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta} = a \cdot e^{-i \cdot (\omega t - kx)}$ and $a \cdot e^{+i \cdot \theta} = a \cdot e^{+i \cdot (\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). One would expect that the professors would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[48]

This, then, leads to the *false* argument that the wavefunction of spin-½ particles have a 720-degree symmetry. Again, you should not worry if you don't get anything of what I write here – because I will come back to it – but the gist of the matter is the following: *because* they think the elementary

[48] In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*). We quote this example for the same reason as why we use Feynman's *Lectures* as a standard reference: it is an authoritative course, and it's available online so the reader can check and explore for himself.

wavefunction describes some theoretical *zero*-spin particle, physicists treat -1 as a *common phase factor*: they think we can just multiply a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with -1 and we're going to get the same *states*. We find it rather obvious that that is not necessarily the case: -1 is *not* necessarily a common phase factor. We should think of -1 as a complex number itself: the phase factor may be $+\pi$ or, alternatively, $-\pi$. To put it simply, when going from +1 to -1, it matters how you get there – and vice versa – as illustrated below.[49]

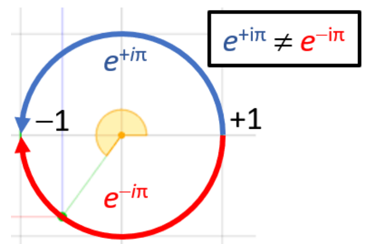

Figure 12: $e^{+i\pi} \neq e^{-i\pi}$

I know this sounds like a bad start for a book that promises to provide some *intuitive* understanding of quantum mechanics but – as mentioned above – I did *not* promise such understanding would come easily. I only promised this this book would be very different from anything else that you've read about quantum physics.

What's the point? It is this: if we exploit the full descriptive power of Euler's function, then all weird symmetries disappear – and we just talk standard 360-degree symmetries in space. Also, weird mathematical conditions – such as the *Hermiticity* of quantum-mechanical operators – can easily be explained as embodying some common-sense *physical* law. In this particular case (*Hermitian* operators), we are talking *physical* reversibility: when we see something happening *at the elementary particle level*, then we need to be able to play the movie backwards. Physicists refer to it as CPT-symmetry, but that's what it is really: *physical* reversibility.[50]

If you still wonder why this should be important, this is why: all physicists – and popular writers on physics – will tell you that the wavefunction of a particle – say, an electron – has this weird 720-degree symmetry, which we cannot really *imagine*. Of course, we have these professors doing the *Dirac belt trick* on YouTube – and many other wonderful animations[51] but, still, these visualizations all assume some weird relation between the object and the subject. To put it differently, it is fair to say that we cannot *really* imagine an object with a 720-degree symmetry, and so that's why the Copenhagen

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[49] The quantum-mechanical argument is technical, and I did *not* reproduce it in this book. I encourage the reader to glance through it, though. See: Jean Louis Van Belle, *Euler's Wavefunction: The Double Life of – 1*, http://vixra.org/abs/1810.0339. Note that the $e^{+i\pi} \neq e^{-i\pi}$ expression is *horror* to any mathematician! Hence, if you're a mathematician, you should switch off. If you're an amateur physicist, you should be excited, because it actually *is* the secret key to unlocking the so-called mystery of quantum mechanics. Remember Aquinas' warning: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions. As arrogant as it sounds, we think we've found the small error in the beginning. 😉

[50] My blog (www.readingfeynman.org) has probably more than a dozen posts on this, which we didn't reproduce into this book either – because a lot of it is quite nitty-gritty: interesting, and important, but nitty-gritty. The interested reader can use the *search* function to find the posts – if and when he or she would like to dig further.

[51] You may have come across the animations of Jason Hise. He is a professional game programmer whom I've been in touch with. I think he makes the best ones. You can find them on Wikipedia.

interpretation tells us we should just be content with the above-mentioned *procedural* approach to 'understanding' quantum mechanics.

Now, that procedural approach is, in my *not* so humble view, no understanding at all! Hence, what we want to do in this book, is to show we should not distinguish between so-called symmetric and anti-symmetric wavefunctions: all wavefunctions have standard 360-degree symmetries and, therefore, represent equally standard three-dimensional objects in relativistic spacetime.

The form factor

The argument above revolves around *geometry*, and this brings me to a second mistake of the early quantum physicists: a total neglect of what I refer to as the *form factor* in physics. Why would an electron be some perfect *sphere*, or some perfect *disk*? We will argue it is *not*. It is – most probably – some regular geometric shape – Dr. Burinskii's Dirac-Kerr-Newman model of an electron, for example, suggests it's an oblate spheroid – but so that's *not* necessarily a perfect sphere, or a perfect disk. Once you acknowledge the form factor, the so-called anomalous magnetic moment – which is touted as the ultimate *precision* test of mainstream quantum-mechanical theory – is not-so-anomalous anymore. We predict it is only a matter of time before some physicist will show classical physics explain it perfectly well.[52]

Planck's constant as a vector

The mistake is actually more general than what I wrote above. Physicists think of the key constants in *Nature* as some number. Most notably, they think of Planck's quantum of action ($h \approx 6.626\times10^{-34}$ N·m·s) as some (scalar) number. Why would it be? It is – obviously – some vector quantity or – let me be precise – some *matrix* quantity: *h* is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time (one direction only). Somehow, those dimensions disappeared in the analysis. Vector equations became *flat*: vector quantities became magnitudes. Schrödinger's equation should be rewritten as a vector or matrix equation. In contrast, we do think of Planck's quantum of action as some vector. We are, therefore, tempted to think that the uncertainty – or the probabilistic nature of Nature, so to speak[53] – is not in its *magnitude*: it's in its *direction*.

Classical clothes and Occam's Razor

So that's what we offer instead of Feynman's path integral interpretation: a real model of the electron and the photon. We then don't need perturbation theory, gauge theories, Feynman diagrams, quantum field theory, etcetera. As you will see, we'll have very classical theory instead. An *update*, basically, of what was around before Heisenberg told everyone to just give up and not even *try* to understand. This updated classical theory is based on what we will loosely refer to as the idea of the *integrity* of a cycle.

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[52] Prof. Dr. Alexander Burinskii is working on it as we are writing this. In his email of today (8 February 2019), he wrote me he has the α factor, but lacks the 2π factor and corrections. I am elated already: he has the α factor! So he's got 99.85% of the so-called anomaly (a factor of 2π is always some logical error that can easily be fixed). I feel like toasting to this future Nobel Prize. The Nobel Prize Committee should prepare for the eventuality. Why? Because this eventuality feels more like a certainty. 😉

[53] A fair amount of so-called thought experiments in quantum mechanics – and we are not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.

This concept is an ontological concept: we argue the very *idea* of a cycle implies that we *count* them: we do not think in terms of half, quarter or whatever other *fraction* of a cycle. This, we argue, gives us a new analytical framework to re-analyze the quantum math. In other words, we think we re-established a one-to-one relation between the (mathematical) *description* of physical phenomena, and the phenomena themselves. It's Occam's Razor, really. Occam's Razor Principle says we should reduce complexity and search for mathematical parsimony. Hence, if possible, we should use all of the degrees of freedom in the mathematical expression when describing reality. Orthodox quantum mechanics clearly doesn't do so. As mentioned above, spin-zero particles don't exist. We should, therefore, incorporate spin in the description (the wavefunction, that is) right from the start. If we do so, we suddenly find all makes sense.

Relating amplitudes and probabilities through energy or mass densities

What about those amplitudes and the absolute square? The squaring can be related to the universal principle we mentioned above already: the energy in any oscillation is proportional to the square of its amplitude—and please do *not* think of the quantum-mechanical concept of an amplitude here: we're just talking the (maximum) displacement of the point or the object that's oscillating. I shouldn't elaborate this because, again, I must assume you have some basic knowledge of physics already when you are reading this.[54] One should then combine this with the following easy question: what is the probability of the propeller of a plane being here or there when it's rotating? You'll agree that probability must be proportional to the mass density, right? Hence, Einstein's mass-energy equivalence relation tells us probabilities will be proportional to mass or energy densities. Does this make sense? Yes? No? A little bit? Don't worry. We'll come back to this. This is, after all, just the introduction to this book.

Figure 13: Where is the propeller, *exactly*?

QED versus QCD: black-and-white versus color vision

So, yes, I should probably start the book now but, as I have already introduced some of the basic discoveries in this book. Indeed, I've been getting ahead of myself. We should go step by step. Let me first acknowledge where I came from. Before I do so, however, I should make a final remark. All we want to do here in this book is to provide a geometric – or *physical*, I should say – understanding of the QED sector of the Standard Model. We're not getting into the nucleus itself, that is. In other words, we won't be discussing quarks and gluons—or quantum *chromo*dynamics (QCD) as it's known. Why not? It's not

[54] If not, please do go through a good textbook on general physics and electromagnetic theory – such as Feynman's Volume I and II.

(only) because QED is a big enough piece in its own. The more substantial reason is that I feel the *innate* nature of man to *generalize* did not contribute to greater clarity—in my not so humble opinion, that is.[55] I think it makes perfect sense to think that each sector of the Standard Model requires its own mathematical approach.

Let me briefly summarize this idea in totally non-scientific language. We may say that mass comes in one 'color' only: it is just some scalar number. Hence, Einstein's geometric approach to gravity makes total sense. In contrast, the electromagnetic force is based on the idea of an electric charge, which comes in two 'colors', so to speak: black or white, or + or –, that is. Maxwell's equation seemed to cover it all until it was discovered the nature of Nature – sorry for the wordplay – might be discrete and probabilistic. However, that's fine. We should be able to modify the classical theory to take that into account. There is no need to invent an entirely new mathematical framework (I am talking quantum field and gauge theories here).

Now, the strong force comes in three colors, and the rules for mixing them, so to speak, are very particular. So that's color television.[56] 😉 It is, therefore, only natural that its analysis requires a wholly different approach. Hence, I would think the new mathematical framework should be reserved for that sector. To put it differently, I really don't like the reference of Aitchison and Hey[57] to gauge theories as 'the electron-figure'. The electron figure is a pretty classical idea to me. Hence, I do hope one day some alien will show us that the application of the Dyson-Feynman-Schwinger-Tomonaga 'electron-figure' to what goes on inside of the nucleus of an atom was, perhaps, not all that useful.

The Higgs field: mass as a scalar field. Of course! What else would it be?

What about the Higgs particle – or the *Higgs* field? I won't talk about that either, but I'll just make a short remark: the Higgs field is a *scalar* field. A scalar field associates some number (a *scalar*) with some position in spacetime. So mass is some number. None of what we write contradicts that. In fact, if we think of mass as the *equivalent* mass of the energy in some oscillation, then it should not come as a surprise that we can think of mass as a scalar field.

If I have not lost you by now, please follow me to the acknowledgments section, in which I want to explain where *I* come from. Because I am *not* an alien. Would I want to be one? Probably not. Why not? Because it's not nice to have all of the answers, right? 😉

III. History and acknowledgments

You may want to skip this section but I wouldn't do it. The story will effectively help you to appreciate the sort of intuition that brought us where we are right now and, hence, it will probably help you to

[55] If you are familiar with Feynman's *Lectures* already, you'll agree that his rather weird excursion on the general difference between the equally general idea of a boson and a fermion (Volume III, Chapter 4) does not have any practical value: it just confuses the picture. It is much easier to first try to understand the specifics of electrons and photons, and their interactions, before generalizing to whatever other boson- or fermion-like particle we assume to be out there.

[56] Our eyes actually do respond to three types of color only—using three types of cone receptors, and color television works the same!

[57] I.J.R. Aitchison and A.J.G. Hey's *Gauge Theories in Particle Physics* is, for QCD, what Feynman's *Lectures* are for QED: a standard textbook. As mentioned above, we feel QCD is an entirely different ballgame altogether, so to speak, and we will, therefore, not touch on it here.

understand what – despite all simplifications – is still a rather abstruse matter. So… Well… Where do we come from?

This journey – a long search for *understanding*, really – started about thirty-five years ago. I was just a teenager then—reading popular physics books. Gribbin's *In Search of Schrödinger's Cat* is just one of the many that left me unsatisfied in my quest for knowledge.[58] However, my dad never pushed me and so I went the easy route: humanities, and economics—plus some philosophy and a research degree afterwards.

Those rather awkward qualifications (for an author on physics, that is) have served me well—not only because I had a great career abroad, but also because I now realize that physics, as a science, is in a rather sorry state: the academic search for understanding has become a race to get the next non-sensical but conformist theory published. In contrast, my search was fueled by a discontent with the orthodox view that we will never be able to understand quantum mechanics "the way we would like to understand it", as Richard Feynman puts it. That is a great advantage. In case you wonder why, I think the cartoon below – which was sent to me by Dr. Giorgio Vassallo, about whom I'll say a few words in a moment – probably explains that better than any words can do. Being independent comes with great freedom. No teaching assignments, and complete freedom in terms of what to dig into.

Talking Feynman, I must admit his meandering *Lectures* are the foundation of my current knowledge, and also the reference point from where I started to think for myself. I had been studying them on and off – an original print edition that I had found in a bookshop in Old Delhi – but it was really the 2012 Higgs-Englert experiments in CERN's LHC accelerator, and the award of the Nobel prize to these two scientists, that made me accelerate my studies. It coincided with my return from Afghanistan – where I had served for five years – and, hence, I could afford to reorient myself.

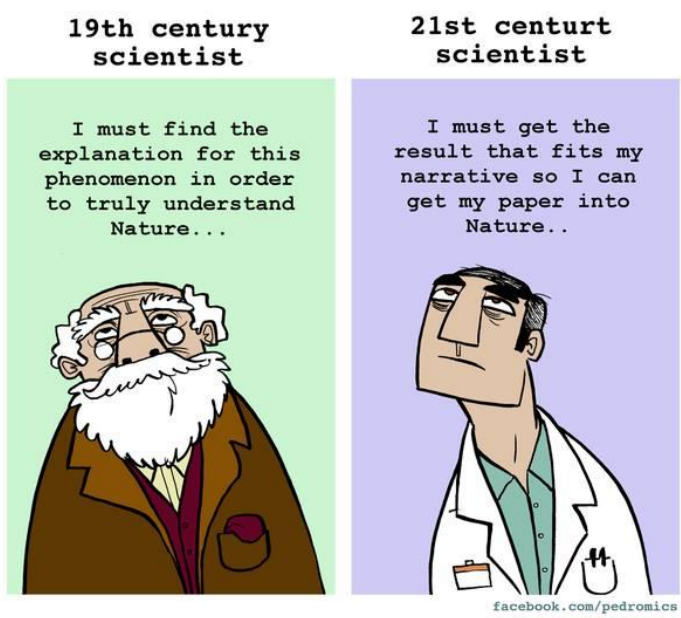

Figure 14: Academic freedom versus academic freedom

[58] I went into a bookstore recently and – to my surprise – this book is still around!

I started a blog (www.readingfeynman.org) as I started struggling through it all—and that helped me greatly. I fondly recall that, back in 2015, Dr. Lloyd N. Trefethen from the Oxford Math Institute reacted to a post in which I had pointed out a flaw in one of Richard Feynman's arguments. It was on a topic that had nothing to do with quantum mechanics – the rather mundane topic of electromagnetic shielding, to be precise – but his acknowledgement that Feynman's argument was, effectively, flawed and that he and his colleagues had solved the issue in 2014 only (Chapman, Hewett and Trefethen, *The Mathematics of the Faraday Cage*) was an eye-opener for me. Trefethen concluded his email as follows: "Most texts on physics and electromagnetism, weirdly, don't treat shielding at all, neither correctly nor incorrectly. This seems a real oddity of history given how important shielding is to technology." This resulted in a firm determination to *not* take any formula for granted – even if they have been written by Richard Feynman! With the benefit of hindsight, I might say this episode provided me with the guts to question orthodox quantum theory.

The informed reader will now wonder: what do I mean with orthodox quantum theory? I should be precise here, and I will. It is the modern theory of quantum electrodynamics (QED) as established by Dyson, Schwinger, Feynman, Tomonaga and other post-World War II physicists. It's the explanation of the behavior of electrons and photons – and their interactions – in terms of Feynman diagrams and propagators. I instinctively felt their theory might be incomplete because it lacks a good description of what electrons and photons actually *are*. Hence, all of the weirdness of quantum mechanics is now in this weird description of the fields – as reflected in the path integral formulation of quantum mechanics. Whatever an electron or a photon might be, we cannot *really* believe that it sort of travels along an infinite number of possible spacetime trajectories all over space *simultaneously*, can we?

I also found what Brian Hayes refers to as "the tennis match between experiment and theory" – the measurement (experiment) or calculation (theory) of the so-called anomalous magnetic moment – a rather weird business: the complexity in the mathematical framework just doesn't match the intuition that, if the theory of QED has a simple circle group structure, one should not be calculating a zillion integrals all over space over 891 4-loop Feynman diagrams to explain the magnetic moment of an electron in a Penning trap. There must be some *form factor* coming out of a decent electron model that can explain it, right?

Of course, all of the above sounds very arrogant, and it is. However, I always felt I was in good company, because I realized that not only Einstein but the whole first generation of quantum phycisists (Schrödinger, Dirac, Pauli and Heisenberg) had become skeptical about the theory they had created—if only because perturbation theory yielded those weird diverging higher-order terms. Dirac wrote the following about that in 1975: "I must say that I am very dissatisfied with the situation because this so-called 'good theory' [perturbation and renormalization theory] involves neglecting infinities. […] This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it!" The Wikipedia article on Dirac, from which I am quoting here, notes that "his refusal to accept renormalization resulted in his work on the subject moving increasingly out of the mainstream."

With the benefit of hindsight, I think it's not overly brutal to say that the likes of Dyson, Schwinger, Feynman – the whole younger generation of mainly American scientists who dominated the *discourse* at the time – lacked a true *general*: they kept soldiering on by inventing renormalization and other mathematical techniques to ensure those weird divergences cancel out, but they had no direction. As mentioned above, these distinguished scientists all received Nobel Prizes for their 'discoveries', so there is a vested interest now in keeping the mystery alive: no academic will want to hurt his or her career by claiming Dyson, Schwinger, Feynman or Tomonaga were wrong![59] In fact, it's probably only independent researchers like me who can just say what many might be thinking: the Emperor has No Clothes!

However, once again I am getting ahead of myself here. We will get into the meat of the matter soon. Before doing so, let me just add some remarks and acknowledge all the people who supported me in this rather lonely search. First, whom am I writing for? I am writing for people like me: amateur physicists. *Not-so-dummies*, that is. People who don't shy away from calculations. People who understand a simple differential equation[60], some complex algebra and classical electromagnetism – all of which are, indeed, necessary, to understand anything at all in this field. However, don't be afraid: I have good news for you too: I have come to the conclusion that we do *not* need to understand anything about gauges or propagators or Feynman diagrams to understand quantum electrodynamics.

Indeed, rather than "using his renormalized QED to calculate the one loop electron vertex function in an external magnetic field", Schwinger should, perhaps, have listened to Oppenheimer's predecessor on the Manhattan project, Gregory Breit, who wrote a number of letters to both fellow scientists as well as the editors of the *Physical Review* journal suggesting that the origin of the so-called discrepancy might be due to an "intrinsic magnetic moment of the electron of the order of $\alpha\mu_B$." In other words, I do *not* think Breit was acting schizophrenic when complaining about the attitude of Kusch and Lamb when they got the 1955 Nobel Prize for Physics for their work on the anomalous magnetic moment. I think he was just making a very sensible suggestion—and that is that one should probably first try investing in a good theory of the electron before embarking on mindless quantum field calculations.

My search naturally led me to the *Zitterbewegung* hypothesis. *Zitter* is German for shaking or trembling. It refers to a presumed local oscillatory motion—which I now believe to be *true*, whatever that means. Erwin Schrödinger found this *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons, and I will quote Dirac's instructive summary of Schrödinger's discovery later, so I won't elaborate here. I'll just note that it took me quite a while to figure out that some *non*-mainstream physicists had actually continued to further explore this concept. To be precise, the writings of David Hestenes from the Arizona State University of Arizona who – back in 1990 – proposed a whole new interpretation of quantum mechanics based on the *Zitterbewegung* concept (Hestenes, 1990, *The Zitterbewegung Interpretation of Quantum Mechanics*) made me realize there was sort of a parallel universe of research out there – but it is *not* being promoted by the likes of MIT, Caltech or Harvard University – and, even more importantly, their friends who review and select articles for scientific journals.

I reached out to Hestenes, but he is 85 by now – and I don't have his private email, so I never got any reply to the one or two emails I sent him on his ASU address. In contrast, Dr. Giorgio Vassallo – one of the researchers of an Italian group centered around Dr. Francesco Celani – who followed up on the Schrödinger-Hestenes *zbw* model of an electron – politely directed me towards Dr. Alex Burinskii (I should have put a *Prof. Dr.* title in front of every name mentioned above, because they all are professors *and* doctors in science). Dr. Vassallo and Dr. Burinskii have both been invaluable – not necessarily because they would want to be associated with any of the ideas that are being expressed here – but

[59] All of them have died now, except Freeman Dyson, who is 95 years old now!

[60] As mentioned above, we will avoid them in general, but we do need one or two. It will be kids' stuff. Don't worry.

because they gave me the benefit of the doubt in their occasional but consistent communications. Hence, I would like to thank them here for reacting and encouraging me for at least *trying* to understand.

To be honest, I think Mr. Burinskii deserves a Nobel Price, but he will probably never get one. Why? Because it would question not one but *two* previously awarded Nobel Prizes (1955 and 1965). I feel validated because, in his latest communication, Dr. Burinskii wrote me to say he takes my idea of trying to corroborate his Dirac-Kerr-Newman electron model by inserting it into models that involve some kind of slow orbital motion of the electron – as it does in the Penning trap – seriously.[61]

I should now *really* start the book. 😊 However, before I do so, I should wrap up the acknowledgments section, so let us do that here. I have also been in touch with Prof. Dr. John P. Ralston, who wrote one of a very rare number of texts that, at the very least, *tries* to address some of the honest questions of amateur physicists and philosophers upfront. I was not convinced by his interpretation of quantum mechanics, but I loved the self-criticism of the profession: “Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don’t really believe, and regularly violate in research.” We exchanged some messages, but then concluded that our respective interpretations of the wavefunction are very different and, hence, that we should not “waste any electrons” (his expression) on trying to convince each other. In the same vein, I should mention some other seemingly random exchanges – such as those with the staff and fellow students when going through the MIT’s edX course on quantum mechanics which – I admit – I did not fully complete because, while I don’t mind calculations in general, I do mind *mindless* calculations.

I am also *very* grateful to my brother, Prof. Dr. Jean Paul Van Belle, for totally unrelated discussions on his key topic of research (which is information systems and artificial intelligence), which included discussions on Roger Penrose’s books—mainly *The Emperor’s New Mind* and *The Road to Reality*. These discussions actually provided the inspiration for the title of this book: *The Emperor Has No Clothes: the Sorry State of Quantum Physics*. We will go for another mountainbike or mountain-climbing adventure when this project is over.[62]

Among other academics, I would like to single out Dr. Ines Urdaneta. Her independent research is very similar to ours. She has, therefore, provided much-needed moral support and external validation. I also warmly thank Jason Hise, whose wonderful animations of 720-degree symmetries did *not* convince me that electrons – as spin-1/2 particles – actually *have* such symmetries – but whose communications stimulated my thinking on the subject-object relation in quantum mechanics.

Finally, I would like to thank all of my friends—especially my university friends here in Belgium, and I will also single out Soumaya Hasni, who has provided me with a whole new fan club here here in Brussels. I also had my family, which kept me sane. I would like to thank, in particular, my children – Hannah and Vincent – and my wife, Maria, for having given me the emotional, intellectual and financial space to pursue this little intellectual adventure.

[61] As mentioned above, in his most recent email, he wrote he has the α factor already (the fine-structure constant). I feel his work is done already, but I know he’s a perfectionist. So he’ll give us the 2π factor and the corrections as well. 😊

[62] We just have to! All of the late-night writing made me put on a lot of weight! 😊

So, now we should really start the book. Its structure is simple. In the first chapters, I'll just introduce the most basic math – oscillator math and Euler's function, basically – and then we'll take it from there. I will regularly refer to a series of papers I published on what I refer to as the *Los Alamos Site for Spacetime Rebels*.[63] You can find these papers at http://vixra.org/author/jean_louis_van_belle and you may want to quickly glance at the titles and see what they cover. You can then go to them in case you get stuck in this book: the papers will often give you more detail. In any case, I'll often refer to them anyway.

This brings me to the final point in my introduction. This is just a first rough version of this book. It is rather short – *cryptic*, I'd say. As such, you might give up after a few pages and say: this may be a *classical* interpretation of quantum physics, but it is *not* an easy one. To those, I'll say two things:

1. It may not be easy, but it is definitely *easier* than whatever else you'll read when exploring the more serious stuff.
2. To get my degree in philosophy, I had to study Wittgenstein's *Tractatus Logico-Philosophicus*. I hated that booklet – not because it is dense but because it is nonsense. Wittgenstein I wasn't even aware of the scientific revolution that was taking place while he was writing it. Still, it became a bestseller. Why? Because it was so abstruse *it made people think for themselves*.

Hence, I hope this book will do the same: it should make you think for yourself. The first version of this book is going to be dense but – hopefully – you will find it is full of sense. At the same time, you will also find there is a lot of overlap between the various chapters as we wanted them to be logically independent. Hence, the reader should not hesitate to skip some material here and there as there is a good chance the same idea or principle will be revisited in a subsequent chapter. If not, the advantage of a book is that one can always re-read a previous chapter. 😉

IV. The two-dimensional oscillator

Before going into the nitty-gritty of our interpretation of the wavefunction, we need to recap what I personally consider to be the nicest result of all of my forays into physics: an intuitive explanation of Einstein's $E = mc^2$ equation. As it underpins our interpretation of the wavefunction, I need to walk you through this as good as I can. I won't apologize for introducing a fair amount of equations here because it is just a very gentle warm-up for what follows. We'll start with a metaphor, which should you give some *feel* for the equations that will follow.

The V-2 metaphor: Ducati versus Harley

You know that the energy of any oscillation will always be proportional to its amplitude (let us denote that by a).[64] However, we also know that the energy in the oscillation will also be proportional to its frequency (let us denote the frequency by ω). Hence, we will have some proportionality coefficient k and we can write something like this:

$$\mathrm{E} = \mathrm{k}a^2\omega^2$$

[63] The site is managed by Phil Gibbs. I would like to acknowledge and thank him here for providing a space for independent thinkers. If you check out its origin, you'll understand my reference to Los Alamos and rebel thinkers. 😉

[64] If you don't, you should go through the basic physics: you can find them on the Web or in any textbook. Otherwise, just try to hang in and continue to read: it might be a bit of a bumpy ride but, at this stage, you don't need to understand each and every detail, so don't worry *too* much! The idea here is to give you a more of a *feel* for the equations.

For example, you may remember the formula for the energy of a harmonic oscillator:

$$E = \frac{1}{2} m a^2 \omega^2$$

Think of a mass on a spring or, somewhat less boring, perhaps, a piston in a frictionless one-cylinder engine with permanently closed valves.[65] We'll get to the formulas but let us first be creative.

If we combine *two* oscillators in a 90-degree angle – think of two springs or two pistons attached to some crankshaft as illustrated below – then we get some *perpetuum mobile* which stores *twice* that energy. Think of a V-2 engine with the pistons at a 90-degree angle, as illustrated below. The 90° angle makes it possible to perfectly balance the counterweight and the pistons, thereby ensuring smooth travel always. With permanently closed valves, the air inside the cylinder compresses and decompresses as the pistons move up and down. It provides, therefore, a restoring force. As such, it will store potential energy, just like a spring. In fact, the motion of the pistons will also reflect that of a mass on a spring: it is described by a sinusoidal function, with the zero point at the center of each cylinder. We can, therefore, think of the moving pistons as harmonic oscillators, just like mechanical springs.

Indeed, instead of two cylinders with pistons, one may also think of connecting two springs with a crankshaft. In fact, that would be the typical physicists' thing to do, but we wanted to be more creative. In fact, to make all of this somewhat less boring, I should, perhaps, admit that this metaphor has a rather mundane origin: I was doing some research on motorbikes and, as part of that process, comparing the efficiency of the Ducati and a Harley-Davidson V-2 engines: the Ducati V-2 engine is more efficient because of the 90-degree angle between the pistons. The Harley-Davidson V-2 engine has a more characteristic *ir*regular sound because of its (relative) inefficiency. 😊

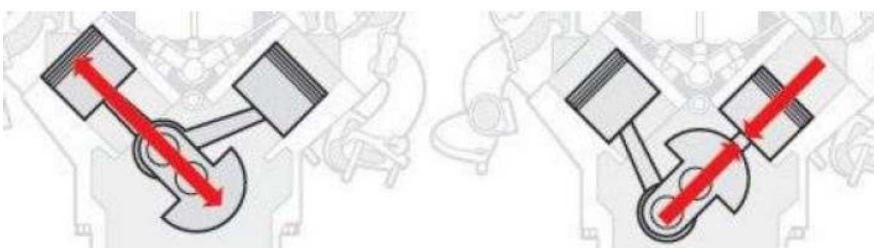

Figure 15: The V-2 metaphor

The analogy can also be extended to include two *pairs* of springs or pistons, in which case the springs or pistons in each pair would help drive each other. Whatever! The point is: we have a great *metaphor* here. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle. While transferring kinetic energy from one piston to the other, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa. More importantly, we can now just *add* the total energy of the two oscillators to get the total energy of the whole system, and so we get the $E = ma^2\omega^2$ formula.[66]

[65] The restoring force here is the air inside the cylinder, which compresses and decompresses as the piston moves up and down. We admit the example requires more imagination, but it is definitely more fun to think it through!

[66] We further developed this metaphor in: *The Wavefunction as an Energy Propagation Mechanism*, http://vixra.org/abs/1806.0106. We will also show you the basic calculations (one of) the next section(s).

The $c = a \cdot \omega$ equation and the wavefunction

We can now boldly equate c^2 and $a^2 \cdot \omega^2$ or – as shown below – think of c as a tangential velocity: $c = v = a \cdot \omega$.

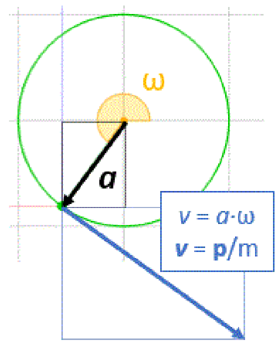

Figure 16: $E = ma^2\omega^2 = mc^2 = mv^2$?

Why would we do that? I am not sure. It is just too obvious to *not* try it. 😊 The geometry of the illustration above is *exactly* the same as that of Euler's function—the quantum-mechanical wavefunction, that is. It is illustrated below. Note the mathematical convention for measuring the phase angle (φ) is *counter*-clockwise but – as mentioned in our introduction – we think we should use that convention to incorporate *spin* in our description. We will come back to that later, but the interested reader should definitely check out our paper on this[67], as we won't say too much about it in this very first version of this book—not because we don't *want* to but because… Well… One thing at a time, right? 😊

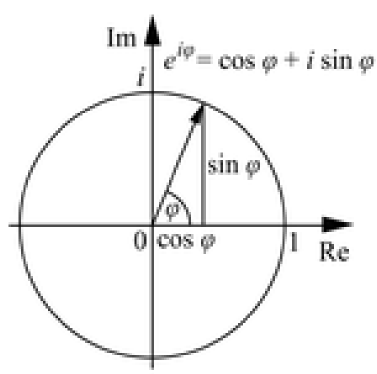

Figure 17: Euler's formula

Oscillator math

At this point, it is probably to walk through the math of this 'two-dimensional oscillator', which I used to refer to as the 'flywheel model' of matter-particles.[68] It is all pretty straightforward, and so let us just go through it to make sure you're comfortable with it.

[67] See: *Euler's wavefunction and the double life of –1*, http://vixra.org/pdf/1810.0339v2.pdf.

[68] See: https://readingfeynman.org/2017/11/19/the-flywheel-model-of-an-electron/.

If the magnitude of the oscillation is equal to a, then the motion of the piston (or the mass on a spring) will be described by $x = a\cdot\cos(\omega\cdot t + \Delta)$.[69] Needless to say, Δ is just a phase factor which defines our $t = 0$ point, and ω is the *natural* angular frequency of our oscillator. Because of the 90° angle between the two cylinders, Δ would be 0 for one oscillator, and –π/2 for the other. Hence, the motion of one piston is given by $x = a\cdot\cos(\omega\cdot t)$, while the motion of the other is given by $x = a\cdot\cos(\omega\cdot t–\pi/2) = a\cdot\sin(\omega\cdot t)$. The kinetic and potential energy of *one* oscillator (think of one piston or one spring only) can then be calculated as:

1. $\text{K.E.} = T = m\cdot v^2/2 = (1/2)\cdot m\cdot\omega^2\cdot a^2\cdot\sin^2(\omega\cdot t + \Delta)$
2. $\text{P.E.} = U = k\cdot x^2/2 = (1/2)\cdot k\cdot a^2\cdot\cos^2(\omega\cdot t + \Delta)$

The coefficient k in the potential energy formula characterizes the restoring force: $F = -k\cdot x$. From the dynamics involved, it is obvious that k must be equal to $m\cdot\omega^2$. Hence, the total energy is equal to:

$$E = T + U = (1/2)\cdot m\cdot\omega^2\cdot a^2\cdot[\sin^2(\omega\cdot t + \Delta) + \cos^2(\omega\cdot t + \Delta)] = m\cdot a^2\cdot\omega^2/2$$

The formulas above are illustrated below.

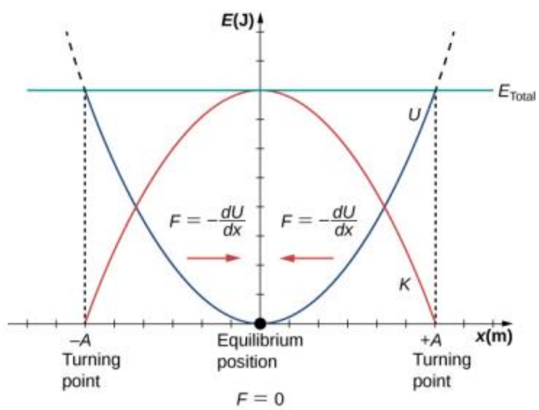

Figure 18: Kinetic (K) and potential energy (U) of an oscillator

Now, if the amplitude of the oscillation is equal to a, then we know that the sum of the kinetic and potential energy of the oscillator will be equal to $(1/2)\cdot m\cdot a^2\cdot\omega^2$. Now, if we have two oscillators – working in tandem at a 90-degree angle – then we can *add* their kinetic and potential energies. Why? Because of the 90-degree phase difference. Think of the V-2 metaphor – or of *two* springs working in tandem on the same crankshaft: it is a perpetuum mobile. Let us show you the associated math.

To facilitate the calculations, we will briefly assume that $k = m\cdot\omega^2$ and a are both equal to 1. Think of it as a normalization.[70] 😉 The motion of our first oscillator is given by the $\cos(\omega\cdot t) = \cos\theta$ function (so the phase varies with time only: $\theta = \omega\cdot t$). Its kinetic energy will be equal to $\sin^2\theta$. Hence, the

[69] If you're still thinking of those beautiful motorbikes (and their engines), then we should note that – because of the sideways motion of the connecting rods – the sinusoidal function will describe the linear motion only *approximately*. Springs connected to a crankshaft will give you the same issue. However, you can easily imagine the idealized limit situation.

[70] There is no trick here. You can check for yourself by writing it all out. In fact, we advise that – as an exercise – you re-do the calculations for $a \neq 1$ and $k = m\cdot\omega^2 \neq 1$. It's easy enough: you can treat both as a constant factor in the derivations.

(instantaneous) *change* in kinetic energy at some point in time – any point in time, really – will be equal to:

$$d(\sin^2\theta)/d\theta = 2\cdot\sin\theta\cdot d(\sin\theta)/d\theta = 2\cdot\sin\theta\cdot\cos\theta$$

Let us look at the second oscillator now. Just think of the second piston going up and down in the V-2 engine. Its motion is given by the sinθ function, which is equal to cos(θ−π /2). Hence, its kinetic energy is equal to $\sin^2(\theta-\pi/2)$, and how it *changes* – as a function of θ – will be equal to:

$$2\cdot\sin(\theta-\pi/2)\cdot\cos(\theta-\pi/2) = -2\cdot\cos\theta\cdot\sin\theta = -2\cdot\sin\theta\cdot\cos\theta$$

We have our *perpetuum mobile*! While transferring kinetic energy from one piston to the other, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa, and the *total* energy that is stored in the system is $T + U = ma^2\omega^2$. We have a great *metaphor* here. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle.

We know the wavefunction consist of a sine and a cosine: the cosine is the real component, and the sine is the imaginary component. We believe they are equally real. And we believe each of the two oscillations carries *half* of the total energy of our particle.

The relativistic oscillator

You may wonder if the math holds for relativistic speeds. If the velocity of our mass on this spring – on the *two* springs, really – becomes a sizable fraction of the speed of light, then we can no longer treat the mass as a constant factor: it will vary with velocity, and its variation is given by the Lorentz factor (γ). While we will not work out each and every detail, we will show you the basics of why our reasoning above isn't faulty – even when relativistic speeds are involved.

The relativistically correct force equation for *one* oscillator is:

$$F = dp/dt = F = -kx \text{ with } p = m_v v = \gamma m_0 v$$

The $m_v = \gamma m_0$ varies with speed because γ varies with speed:

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} = \frac{dt}{d\tau}$$

What's the dt/dτ here? Don't worry about it. We actually don't need it for what follows, but we quickly wanted to insert it so as to remind you that we no longer have a unique concept of time: there is the time in our reference frame (t) – aka as the *coordinate time* – and the time in the reference frame of the object itself (τ) – which is known as the *proper time*. We may want to use these two concepts of time in a later development, so it's good to introduce them here. But let's get on with that equation above. It actually *is* a differential equation (it involves a derivative), but you'll agree it's a very simple one. In fact, when you first learned about an equation like this, no one probably told you it's a proper differential equation. However, simple as it is, we're not going to solve it – because we don't have to. We'll just derive an energy conservation equation from it.

We do so by multiplying both sides with v = dx/dt. I am skipping a few steps (we're not going to do *all* of the work for you) but you should be able to verify the following:

$$v\frac{\mathrm{d}(\gamma \mathrm{m}_0 v)}{\mathrm{dt}} = -\mathrm{k}xv \Leftrightarrow \frac{\mathrm{d}(\mathrm{m}c^2)}{\mathrm{dt}} = -\frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2\right] \Leftrightarrow \frac{\mathrm{dE}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2 + \mathrm{m}c^2\right] = 0$$

So what's the energy concept here? We recognize the potential energy: it is the same $kx^2/2$ formula we got for the non-relativistic oscillator. No surprises: potential energy depends on position only, not on *velocity*, and there is nothing relative about position.[71] However, the $(½)m_0v^2$ term that we would get when using the non-relativistic formulation of Newton's Law is now replaced by the $mc^2 = \gamma m_0 c^2$ term. So…. What's the conclusion? Both energies vary – with position and with velocity respectively – but the equation above tells us their *sum* is some constant. In fact, that's we refer to it as an energy conservation equation. So we can imagine the same game: two oscillators working in tandem, *somehow*.[72]

[…]

All of the above should give you a funny feeling. That's good – because it gives me a weird feeling too. You should wonder: what's going on here, *really*? The following reflections may help you to work yourself through that question.

Is the speed of light a velocity or a resonant frequency?

That's a good question! We think of it as a velocity. The idea of *c* being some resonant frequency of the spacetime fabric is tempting but... Well… It's not that easy to interpret it that way. Why not? Think of the following. One of the most obvious implications of Einstein's $E = mc^2$ equation is that the ratio between the energy and the mass of *any* particle is always equal to c^2. We write:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photon}}{m_{photon}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

This should, effectively, remind you of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ formulas of harmonic oscillators – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom.[73] In contrast, $c^2 = E/m$ for *any* particle, *always*. In fact, that's *exactly* the point we are trying to make here: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light *c* emerges here as *the* defining property of spacetime.

I should, perhaps, note that Maxwell's equations tell us exactly the same thing: *c* is *the* defining property of spacetime! It's the (absolute) propagation speed of an electromagnetic *signal*. As I must assume you have a basic background in physics – and in electromagnetics in particular – you will know Maxwell's theory was relativistically correct decades before Einstein actually invented the notion of what is and isn't relativistically correct. 😊 You will know that, in fact, it is fair to say that Einstein was inspired by

[71] You may want to think about this. 😊

[72] We admit we haven't worked out the details: we'll leave it that to the more mathematically inclined reader. The metaphor is likely to require some rather sophisticated tinkering too, so we will leave that as a challenge for the more gifted under us! 😊

[73] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the inertia, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

the implications of Maxwell's equations: Einstein saw they had to be true and that, therefore, *Newtonian* or *Galilean* relativity had to be wrong.

I won't spend too much time on this. Let me just note that it is, in fact, *very* tempting to think of c as some kind of *resonant frequency*. However, the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will refer to as *the rest energy oscillation*. It is that what gives mass to our electron: its *rest* mass is nothing but the equivalent mass of the energy in its two-dimensional oscillation. As such, the only way we can interpret it, is as the velocity of the pointlike charge in its *Zitterbewegung*.

Huh? What? Yes. I mentioned the concept before. It is about time we talk about the true origin of that idea of a two-dimensional oscillation. It's not some idiot like me who thought of it first. No. It's Schrödinger. And he thought of it because he could see it in Dirac's wave equation for the (free) electron. Let me explain.

Introducing the *Zitterbewegung*

As I already mentioned, *Zitter* is German for shaking or trembling. It refers to a presumed local oscillatory motion – which I now believe to be *true*, whatever that means – which Erwin Schrödinger stumbled upon when he was exploring solutions to Dirac's wave equation for free electrons. Schrödinger shared the 1933 Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Dirac obviously refers to the phenomenon of *Compton* scattering of light by an electron and, as we shall see in the next chapter, the *Zitterbewegung* model naturally yields the Compton *radius* of an electron.[74] and – as such – effectively provides some geometric explanation of what I now believe might be *reality*. When everything is said and done, it really is what it is: a wonderfully simple explanation of Einstein's E = m·c^2 equation.

As an added benefit, we can also apply the same trick to our idea of a photon. As we will show in this book, the amplitude of the oscillation will become a *wavelength* then. We can then write:

[74] As I am sort of stitching various things together here, I realize I might not always do the best job in explaining it. The confused reader may, therefore, want to check my original paper: ***Einstein's mass-energy relation: an explanation in terms of the Zitterbewegung***, http://vixra.org/pdf/1811.0364v1.pdf.

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

Don't worry about it now. I did some substitutions here which you may or (more probably) may *not* be very familiar with. We will come back to this. Just make a mental note – for the time being – that we've got a pretty good photon model here. Sometimes physics can be *just nice*. 😊

A visualization of the wavefunction

It is now time to relate all of the above to the elementary wavefunction. You will (or should) know that it is written as:

$$\psi = a\cdot e^{-i[\mathrm{E}\cdot t - \mathbf{p}\cdot\mathbf{x}]/\hbar} = a\cdot\cos(\mathbf{p}\cdot\mathbf{x}/\hbar - \mathrm{E}\cdot t/\hbar) + i\cdot a\cdot\sin(\mathbf{p}\cdot\mathbf{x}/\hbar - \mathrm{E}\cdot t/\hbar)$$

If we assume the momentum **p** is all in the ***x***-direction, then the **p** and **x** vectors will have the same direction, and **p**·**x**/ħ reduces to p·x/ħ. Most illustrations – such as the one below – will either freeze **x** or, else, *t*. Alternatively, one can *google* web animations varying both. The point is: we do have a two-dimensional oscillation here. These two dimensions are perpendicular to the direction of propagation of the wavefunction. For example, if the wavefunction propagates in the *x*-direction, then the oscillations are along the *y*- and *z*-axis, which we may refer to as the real and imaginary axis.

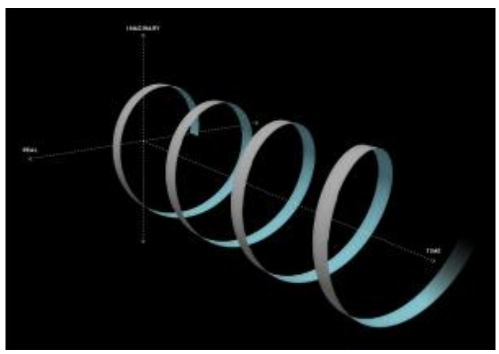

Figure 19: Geometric representation of the wavefunction

Note how the phase difference between the cosine and the sine – the real and imaginary part of our wavefunction – appear to give some spin to the whole. We have talked about this already—and will talk about it more. Still… We fear this book might not cover all details – as we are just in its very first draft – so we recommend checking our papers on how spin is actually being modeled in the current *mainstream* view of QED.[75]

Now, our model implies that the two perpendicular oscillations carry each half of the total energy of the particle. We could refer to these energies as the real and imaginary energy of the particle respectively, but we won't use that terminology because it is rather confusing.[76] Just note how the interplay between the real and the imaginary part of the wavefunction shows how energy propagates through space over time.

[75] Our paper that suggests an alternative (classical) explanation for a photon interfering with itself – *A Classical Explanation for the One-Photon Mach-Zehnder Interference Experiment* (http://vixra.org/pdf/1812.0455v1.pdf) – has a good overview of our thoughts on this.

[76] The imaginary part is as 'real' as the real part, obviously.

Huh?

Yes. You are right. The illustration above doesn't show much in terms of an explanation of how the wave actually propagates. The propagation mechanism is, effectively, quite mysterious. In fact, we may never *understand* what's really happening there. We have one lucky break though: the mechanism is not any more mysterious than the propagation mechanism of an electromagnetic wave. This is where the interpretation of Schrödinger's equation as an energy diffusion equation comes in.

The basics of that interpretation are illustrated below, but I do *not* expect the reader to have some *Aha-Erlebnis* here. It requires a deep understanding of vector equations and rather complicated vector algebra, including a geometric understanding of vector differential operators (such as the gradient, divergence and curl). Having said that, I hope the illustration below sort of shows that Maxwell's equations are pretty similar. In case you wonder what it's all about: you should recognize (fragments of) Schrödinger's equation (the first set of equations) as well as Maxwell's equations in free space(the second set of equations).[77]

$$\mathrm{Re}(\partial\psi/\partial t) = -(1/2)\cdot(\hbar/m_{eff})\cdot\mathrm{Im}(\nabla^2\psi)$$

$$\mathrm{Im}(\partial\psi/\partial t) = (1/2)\cdot(\hbar/m_{eff})\cdot\mathrm{Re}(\nabla^2\psi)$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}$$

$$\partial\mathbf{E}/\partial t = c^2\nabla\times\mathbf{B}$$

Figure 20: Wave propagation mechanisms

Incorporating spin in the wavefunction

As mentioned above, we said we'd like to incorporate spin straight from the start. The illustrations below – which are Wikipedia illustrations of a circularly polarized *electromagnetic* wave[78] – show a left- and a right-handed wave.[79]

[77] Free space means we have no (other) currents. For a more comprehensive exploration of the geometry, we refer the reader to *The Wavefunction as an Energy Propagation Mechanism* (http://vixra.org/pdf/1806.0106v1.pdf). The reader may also want to check out our *Geometric Interpretation of Schrödinger's Equation* (http://vixra.org/pdf/1812.0202v1.pdf).

[78] These illustrations show the *electric* field vector only, which is why we can use them as illustrations of the wavefunction. The *complete* image of an electromagnetic wave should involve the magnetic field vector as well. In fact, it is rather weird that illustrations such as these always conveniently forget about the magnetic field vector. We think that's a fundamental mistake, but the advantage is that we can use the illustrations to serve our purpose, and that's to *visualize* the wavefunction.

[79] What is left and right depends on your convention – and your own position in space.

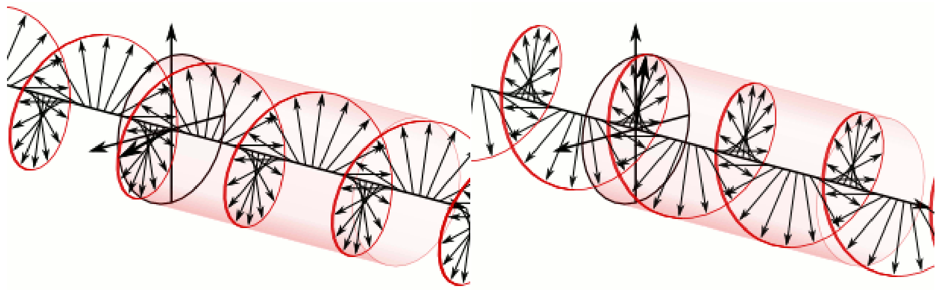

Figure 21: Left-handed and right-handed wavefunctions

Indeed, mainstream physicists do *not* think of the elementary wavefunction as representing anything real but – if they do – they would reluctantly say it might represent some theoretical spin-zero particle. I always felt that was a mistake – and this book is actually the result of a deep exploration of that intuition. Spin-zero particles do not exist. All *real* particles have spin – electrons, photons, anything – and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. Hence, it is rather odd that the plus/minus sign of the imaginary unit in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include spin in the mathematical description. Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i\cdot\theta} = a \cdot e^{-i\cdot(\omega t - kx)}$ and $a \cdot e^{+i\cdot\theta} = a \cdot e^{+i\cdot(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid).

Now, we would think physicists would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention." In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*).

Historical experience tells us theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things – think, for example, of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? Occam's Razor principle tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations, and the one obvious characteristic that would distinguish the two physical situations is the spin direction. Hence, we do *not* agree with the mainstream view that the choice is a matter of convention. Instead, we dare to suggest that the two mathematical possibilities represent identical particles with opposite spin. Combining this with the two possible directions of propagation (which are given by the +– or ++ signs in front of ω and k), we get the following table:

Table 1: Occam's Razor: mathematical possibilities versus physical realities

Spin and direction of travel	**Spin up (e.g. $J = +\hbar/2$)**	**Spin down (e.g. $J = -\hbar/2$)**
Positive *x*-direction	$\psi = a \cdot e^{-i \cdot (\omega t - kx)}$	$\psi^* = a \cdot e^{+i \cdot (\omega t - kx)}$
Negative *x*-direction	$\chi = a \cdot e^{-i \cdot (\omega t + kx)}$	$\chi^* = a \cdot e^{-i \cdot (\omega t + kx)}$

We will come back to this in a later chapter, because it has some rather devastating implications on some *core* theorems in quantum mechanics. To be precise, it rubbishes the arguments on these weird 720-degree symmetries. Think spinors and all that. However, before we can talk about that, we first need to take you through some more basic stuff.

Introducing the wavefunction – and relativity

The elementary wavefunction is written as:

$$\psi = a \cdot e^{-i[\mathrm{E} \cdot t - \mathbf{p} \cdot \mathbf{x}]/\hbar} = a \cdot \cos(\mathbf{p} \cdot \mathbf{x}/\hbar - \mathrm{E} \cdot t/\hbar) + i \cdot a \cdot \sin(\mathbf{p} \cdot \mathbf{x}/\hbar - \mathrm{E} \cdot t/\hbar)$$

When considering a particle at rest ($\mathbf{p} = \mathbf{0}$) this reduces to:

$$\psi = a \cdot e^{-i \cdot \mathrm{E} \cdot t/\hbar} = a \cdot \cos(-\mathrm{E} \cdot t/\hbar) + i \cdot a \cdot \sin(-\mathrm{E} \cdot t/\hbar) = a \cdot \cos(\mathrm{E} \cdot t/\hbar) - i \cdot a \cdot \sin(\mathrm{E} \cdot t/\hbar)$$

Let us now look at this motion in a moving reference frame. Let us consider the idea of a particle traveling in the positive *x*-direction at constant speed *v*. This idea implies a pointlike concept of position and time: we think the particle will be somewhere at some point in time. The *somewhere* in this expression does not necessarily mean that we think the particle itself will be dimensionless or pointlike. It just implies that we can associate some *center* with it. In fact, that's what we have in our *zbw* model here: we have an oscillation around some center, but the oscillation has a *physical* radius, which we referred to as the Compton radius of the electron. Of course, two extreme situations may be envisaged: $v = 0$ or $v = c$. However, let us consider the more general case. In our reference frame, we will have a position – a mathematical *point* in space, that is – which is a function of time: $x(t) = v \cdot t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t'. Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t', and the position and time in the two reference frames will be related as follows:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\left(\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{\mathrm{E}_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}\mathrm{E}_0\left(\frac{t}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{\mathrm{E}_0}{\hbar}t'$$

We have just shown that the argument of the wavefunction is relativistically invariant (E_0 is, obviously, the rest energy and, because p' = 0 in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0t'/\hbar$ in the reference frame of the electron itself).

In fact, it was this rather simple discovery that made us think that of the argument of the wavefunction and – therefore – the wavefunction itself – might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution. Let us, therefore, explore it some more.

We have been interpreting the wavefunction as an implicit function here: for each *x*, we have a *t*, and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by $x(t) = v \cdot t$. We will get some linear motion. If we look at the $\psi = a\cdot\cos(\mathrm{p}\cdot x/\hbar - \mathrm{E}\cdot t/\hbar) + i\cdot a\cdot\sin(\mathrm{p}\cdot x/\hbar - \mathrm{E}\cdot t/\hbar)$ once more, we can write $\mathrm{p}\cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what *x* this phase factor $\Delta = \mathrm{p}\cdot x/\hbar$ will be equal to 2π. Hence, we write:

$$\Delta = \mathrm{p}\cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi\cdot\hbar/\mathrm{p} = h/\mathrm{p} = \lambda$$

We may think we have some meaningful interpretation of the *de Broglie* wavelength here. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.

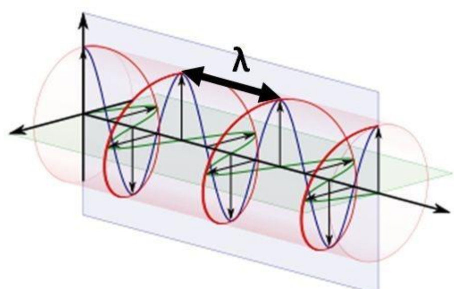

Figure 22: An interpretation of the *de Broglie* wavelength

The alert reader will note we've cut some corners here, and he's right.[80] The interpretation of what *de Broglie* wavelength actually represents is a rather tricky matter. It is surely *not* as simple as we are suggesting here. We're working on this and will come back to it in the next edition of this book.

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[80] We are actually *not* satisfied with this description. I write that it's the distance between crests of the wavefunction, but it cannot be. The analysis has to be more subtle – especially when thinking relativity: note that the *wavelength* converges to the Compton wavelength as *v* goes to *c*: $\lambda = h/p = h/mc = a$ for $v = c$. So that should give you some food for thought already. However, the interpretation of the meaning of the *de Broglie* wavelength remains a tricky matter. The standard interpretation of quantum physics (mainstream or Copenhagen) always brings some complicated argument involving uncertainty – but we do not have any uncertainty in the *Zitterbewegung* model (we can introduce uncertainty later but – at this stage – we're really looking at an electron model without uncertainty). So… Well… It requires some further thinking. At a minimum, I guess we should measure time and distance in equivalent units to say something meaningful about the $\lambda = h/p$ relation. Of course, if $v = c$, and we measure x and t in equivalent units, then we get the $\lambda = h/p$ relation from the universal $\lambda = c/f$ relation for a

It is now time to get into the meat of the matter. 😊 All of the above was just what it was: some introductory concepts – and some basic math.

wave and the Planck-Einstein relation ($E = mc^2 = hf$). We can then write: $\lambda = c/f = ch/mc^2 = h/mc = h/p$. Perhaps it's that simple. But I don't think so. The second *de Broglie* relation is not limited to the $v = c$ case. Louis de Broglie would not have gotten his PhD thesis if that were the case.

V. The wavefunction and the electron

The *Zitterbewegung* model

Euler's function is a wonderful mathematical object. We introduced it above and we must assume that the reader is fully familiar with it:

$$a \cdot e^{i\theta} = a \cdot \cos(\theta) + i \cdot \sin(\theta)$$

We can immediately visualize this using the *Zitterbewegung* model of an electron. We described the origin of the model: see the quote from Dirac's Nobel Prize speech above. The illustration below represents the circular oscillatory motion of the electron (the *Zitterbewegung*) or – possibly – of any charged particle.

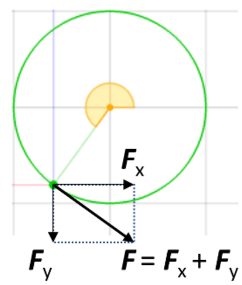


Figure 23: The *Zitterbewegung* model of an electron

It is driven by a force – which must be electromagnetic, because the force has only a charge to grab onto. We think of this charge as a pointlike object that has no rest mass. Hence, the charge spins around at the speed of light. We have a dual view of the reality of the wavefunction here.

1. On the one hand, it will describe the physical position (i.e. the *x*- and *y*-coordinates) of the pointlike charge – the green dot in the illustration, whose motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

As such, the (elementary) wavefunction may be viewed as an implicit function: it is equivalent to the $x^2 + y^2 = a^2$ equation, which describes the same circle.

2. On the other hand, the *zbw* model implies the circular motion of the pointlike charge is driven by a tangential force, which we write as:

$$\boldsymbol{F} = F_x \cdot \cos(\omega t + \pi/2) + i \cdot F_x \cdot \sin(\omega t + \pi/2) = F \cdot e^{i(\theta + \pi/2)}$$

The line of action of the force is the orbit because a force needs something to grab onto, and the only thing it can grab onto in this model is the oscillating (or rotating) charge. We think of $\boldsymbol{F}$ as a composite force: the resultant force of two perpendicular oscillations. A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree angle ensures the independence of both motions. The kinetic and potential energy of *one* harmonic oscillator add up to $E = m \cdot a^2 \cdot \omega^2/2$. If we have two, we can drop the ½ factor. We can then boldly equate the $E = mc^2$ and $E = m \cdot a^2 \cdot \omega^2$ formulas to get the *zbw* radius. We can think of this as follows. The *zbw* model – which is derived from Dirac's wave equation for free electrons – tells us the velocity of the pointlike charge is equal to *c*. If the *zbw* frequency is given by Planck's energy-frequency relation ($\omega = E/\hbar$), then we can combine Einstein's E = mc^2 formula with the radial velocity formula ($c = a\cdot\omega$) and, hence, we get the *zbw* radius, which is nothing but the (reduced) Compton wavelength – or the Compton *radius* of the electron:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The amount of *physical* action – which we will denote by *S* as per the usual convention – that is associated with one loop along the *zbw* circumference over its cycle time is equal to Planck's constant:

$$S = F \cdot \lambda_C \cdot \mathrm{T} = \frac{\mathrm{E}}{\lambda_C} \cdot \lambda_C \cdot \frac{1}{f_C} = \mathrm{E} \cdot \frac{h}{\mathrm{E}} = h$$

Planck's constant *h* is equal to 6.62607015×10^{-34} J·s. Hence, it is a small unit - but small and large are relative. In fact, because of the tiny time and distance scale, we have a rather enormous force here. We can calculate the force because the energy in the oscillator must be equal to the magnitude of the force times the length of the loop, we can calculate the magnitude of the force, which is – effectively – rather enormous in light of the sub-atomic scale:

$$\mathrm{E} = \mathrm{F}\lambda_C \Leftrightarrow F = \frac{\mathrm{E}}{\lambda_C} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\ \mathrm{N}$$

The associated current is equally humongous:

$$\mathrm{I} = \mathrm{q_e}f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

A household-level current at the sub-atomic scale? The result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. As the oscillator model implies the effective mass of the electron will be spread over the circular disk, we should use the 1/2 form factor for the moment of inertia (*I*). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

The vector notation for **μ** and **L** (boldface) in the equation above should make us think about the plane of oscillation. This question is related to the question of how we should analyze all of this is a moving reference frame. This is a complicated question. The Stern-Gerlach experiment suggests we may want to think of an oscillation plane that might be perpendicular to the direction of motion, as illustrated below.

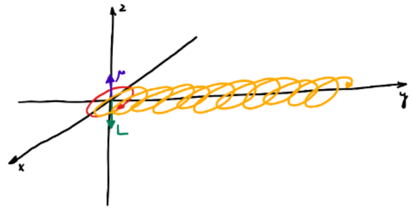


Figure 24: The *zbw* electron traveling through a Stern-Gerlach apparatus?

Of course, the Stern-Gerlach experiment assumes the application of a (non-homogenous) magnetic field. In the absence of such field, we may want to think of the plane of oscillation as something that is rotating in space itself. The idea, then, is that it sort of snaps into place when an external magnetic field is applied.

We should think some more about the nature of the force. The assumption is that the force grabs onto a pointlike charge. Hence, the force must be electromagnetic and we can write it as the product of the unit charge and the field (***E***). We write:

$$\boldsymbol{F} = q_e\boldsymbol{E}.$$

Because the force is humongous (a force of 0.0375 N is equivalent to a force that gives a mass of 37.5 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second), and the charge is tiny), we get an equally huge field strength:

$$E = \frac{F}{\mathrm{q_e}} \approx \frac{3.3743 \times 10^{-2}\ \mathrm{N}}{1.6022\ \times 10^{-19}\ \mathrm{C}} \approx 0.21 \times 10^{18}\ \mathrm{N/C}$$

Just as a yardstick to compare, we may note that the most powerful man-made accelerators may only reach field strengths of the order of 10^9 N/C (1 GV/m). Does this make sense? Can we calculate an energy density? Using the classical formula, we get:

$$u = \epsilon_0 E^2 \approx 8.854 \times 10^{-12} \cdot (0.21 \times 10^{18})^2 \frac{\mathrm{J}}{\mathrm{m}^3} = 0.36 \times 10^{24}\ \frac{\mathrm{J}}{\mathrm{m}^3} = 0.63 \times 10^{24}\ \frac{\mathrm{J}}{\mathrm{m}^3}$$

This amounts to about 7 kg per mm^3 (cubic *milli*meter). Is this a sensible value? Maybe. Maybe not. The rest mass of the electron is tiny, but then the *zbw* radius of an electron is also exceedingly small. It is very interesting to think about what might happen to the curvature of spacetime with such mass densities: perhaps our pointlike charge just goes round and round on a geodesic in its own (curved) space. We are not well-versed in general relativity and we can, therefore, only offer some general remarks here:

1. If we would pack all of the mass of an electron into a black hole, then the Schwarzschild formula gives us a radius that is equal to:

$$r_s = \frac{2\mathrm{Gm}}{c^2} \approx 1.35 \times 10^{-57}\mathrm{m}\ (meter)$$

This exceedingly small number has no relation whatsoever with the Compton radius. In fact, its scale has no relation with whatever distance one encounters in physics: it is *much* beyond the Planck scale, which is of the order of 10^{-35} meter and which, for reasons deep down in relativistic quantum mechanics, physicists consider to be the smallest possibly sensible distance scale.

2. We are intrigued, however, by suggestions that the Schwarzschild formula should not be used as it because an electron has angular momentum, a magnetic moment and other properties, perhaps, that do not apply when calculating, say, the Schwarzschild radius of the mass of a baseball. To be precise, we are particularly intrigued by models that suggest that, when incorporating the above-mentioned properties of an electron, the Compton radius might actually be the radius of an electron-sized black hole (Burinskii, 2008, 2016).

Introducing relativity – and more!

Let us look at a moving electron now. Let us consider the idea of a particle traveling in the positive *x*-direction at constant speed *v*. This idea implies a pointlike concept of position and time: we think the particle will be somewhere at some point in time. The *somewhere* in this expression does not necessarily mean that we think the particle itself will be dimensionless or pointlike. It just implies that we can associate some *center* with it. In fact, that's what we have in our *zbw* model here: we have an oscillation around some center, but the oscillation has a *physical* radius, which we referred to as the Compton radius of the electron. Of course, two extreme situations may be envisaged: $v = 0$ or $v = c$.

However, let us consider the more general case. In our reference frame, we will have a position – a mathematical *point* in space, that is – which is a function of time: $x(t) = v \cdot t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t'. Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t', and the position and time in the two reference frames will be related as follows:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows:

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \frac{E_0}{\hbar}t'$$

We have just shown that the argument of the wavefunction is relativistically invariant (E_0 is, obviously, the rest energy and, because $p' = 0$ in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself). It makes us think that of the argument of the wavefunction and – therefore – the wavefunction itself – might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution. Let us, therefore, further explore this. We have been interpreting the wavefunction as an implicit function again: for each *x*, we have a *t*, and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by $x(t) = v \cdot t$. We will get some linear motion. If we look at the $\psi = a \cdot \cos(p \cdot x/\hbar - E \cdot t/\hbar) + i \cdot a \cdot \sin(p \cdot x/\hbar - E \cdot t/\hbar)$ once more, we can write $p \cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what x this phase factor $\Delta = p \cdot x/\hbar$ will be equal to 2π. Hence, we write:

$$\Delta = p \cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi \cdot \hbar/p = h/p = \lambda$$

We now get a meaningful interpretation of the *de Broglie* wavelength. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.[81]

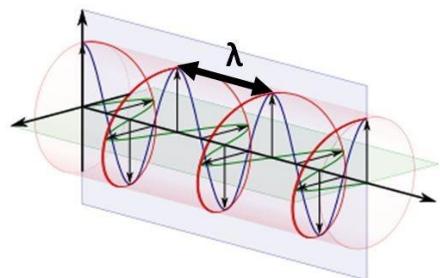


Figure 25: An interpretation of the *de Broglie* wavelength

Of course, we should probably think of the plane of oscillation as being *perpendicular* to the plane of motion – or as oscillating in space itself – but that doesn't matter. Let us explore some more. We can, obviously, re-write the argument of the wavefunction as a function of *time* only:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}vt\right) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)t = \sqrt{1-\frac{v^2}{c^2}} \cdot \frac{\mathrm{E}_0}{\hbar}t$$

We recognize the *inverse* Lorentz factor here, which goes from 1 to 0 as v goes from 0 to c, as shown below.

[81] We are actually *not* satisfied with this description. I write that it's the distance between crests of the wavefunction, but it cannot be. Note that it converges to the Compton wavelength as v goes to c: $\lambda = h/p = h/mc = a$ for $v = c$. The interpretation of the meaning of the *de Broglie* wavelength remains a tricky matter. The standard interpretation of quantum physics (mainstream or Copenhagen) always brings some complicated argument involving uncertainty – but we do not have any uncertainty in the *Zitterbewegung* model (we can introduce uncertainty later but – at this stage – we're really looking at an electron model without uncertainty). So… Well… It requires some further thinking. At a minimum, I guess we should measure time and distance in equivalent units to say something meaningful about the $\lambda = h/p$ relation. Of course, if $v = c$, and we measure x and t in equivalent units, then we get the $\lambda = h/p$ relation from the universal $\lambda = c/f$ relation for a wave and the Planck-Einstein relation ($E = mc^2 = hf$). We can then write: $\lambda = c/f = ch/mc^2 = h/mc = h/p$. Perhaps it's that simple. Any thoughts? Anyone?

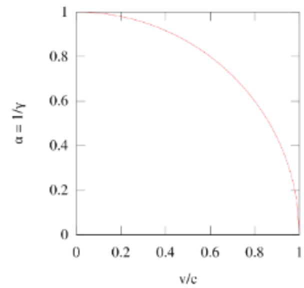


Figure 26: The inverse Lorentz factor as a function of (relative) velocity (*v*/*c*)

Note the shape of the function: it is a simple circular arc. This result should not surprise us, of course, as we also get it from the Lorentz formula:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{v^2}{c^2}t}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{1 - \frac{v^2}{c^2}} \cdot t$$

In fact, we had already introduced this formula when we were talking about the difference between coordinate time and proper time.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}} = \frac{\mathrm{dt}}{\mathrm{d}\tau}$$

We just used different symbols for it: the time in *our* reference frame (t) is the *coordinate time*, and the time in the reference frame of the object itself (τ) is referred to as the *proper time*. Why are we talking about this? What does it all mean? We want to introduce a not-so-intuitive but very important result: the Compton radius becomes a wavelength when *v* goes to *c*.[82]

Huh? Yes. Just hang in there for a while. Let us first go through a simple numerical example to think through that formula above. Let us assume that, for example, that we are able to speed up an electron to, say, about one tenth of the speed of light. Hence, the Lorentz factor will then be equal to $\gamma = 1.005$. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0: $E_v = \gamma E_0 \approx 1.005 \cdot 0.511$ MeV ≈ 0.5135 MeV. The relativistic momentum will then be equal to $m_v v = (0.5135\ \text{eV}/c^2)\cdot(0.1\cdot c) = 5.135\ \text{eV}/c$. We get:

$$\theta = \frac{E_0}{\hbar}t' = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = 0.955\frac{E_0}{\hbar}t$$

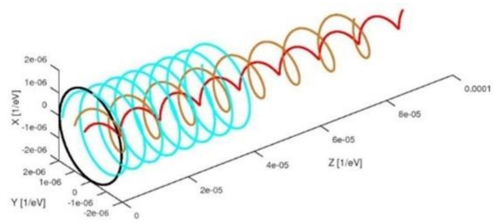
This is interesting: we get an explanation for time dilation. A more interesting question is what happens to the *radius* of the oscillation. Does it change? It must, but how should we interpret this? In the moving reference frame, we measure higher mass and, therefore, higher energy – as it includes the kinetic energy. The $c^2 = a^2\cdot\omega^2$ identity must now be written as $c^2 = a'^2\cdot\omega'^2$. Instead of the rest mass m_0 and rest

[82] To be precise, the Compton radius multiplied by 2π becomes a wavelength, so we are talking the Compton circumference , or whatever you want to call it.

energy E_0, we must now use $m_v = \gamma m_0$ and $E_v = \gamma E_0$ in the formulas for the Compton radius and the Einstein-Planck frequency, which we just write as m and E in the formula below:

$$\mathrm{m}a'^2\omega'^2 = \mathrm{m}\frac{\hbar^2}{\mathrm{m}^2c^2}\frac{\mathrm{m}^2c^4}{\hbar^2} = \mathrm{m}c^2$$

This is easy to understand intuitively: we have the mass factor in the denominator of the formula for the Compton radius, so it must increase as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, increase with velocity. It is interesting to note that we have a simple (inverse) proportionality relation here. The idea is visualized in the illustration below (for which credit goes to the modern *zbw* theorists Celani et al.): the *radius* of the circulatory motion must effectively diminish as the electron gains speed. Once again, however, we should warn the reader that he or she should also imagine the plane of oscillation to be possibly parallel to the direction of propagation, in which case the circular motion becomes elliptical.



Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

Figure 27: The Compton radius must decrease with increasing velocity

Can the velocity go to c? In the limit, yes. This is very interesting, because we can see that the circumference of the oscillation becomes a wavelength in the process! This, then, links the *zbw* electron model with our photon model, which we will explain later. We first need to talk about orbital electron motion. Before we do so, we will resume the model that we have here.

We should note that the center of the *Zitterbewegung* was plain nothingness and we must, therefore, assume some two-dimensional oscillation makes the charge go round and round. The angular frequency of the *Zitterbewegung* rotation is given by the Planck-Einstein relation ($\omega = E/\hbar$) and we get the *Zitterbewegung* radius (which is just the Compton radius $a = r_C = \hbar/mc$) by equating the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equation. The energy and, therefore, the (equivalent) mass is in the oscillation and we, therefore, should associate the momentum $p = E/c$ with the electron as a whole or, if we would really like to associate it with a single mathematical *point* in space, with the center of the oscillation – as opposed to the rotating massless *charge*.

We should note that the distinction between the pointlike *charge* and the electron is subtle, perhaps, but essential. The electron is the *Zitterbewegung* as a whole: the pointlike charge has no rest mass, but the electron as a whole does. In fact, that is the whole point of our *Zitterbewegung* model: we explain the rest mass of an electron by introducing a *rest matter oscillation*. The model cannot be verified because of the extreme frequency ($f_e = \omega_e/2\pi = E/h \approx 0.123\times10^{-21}$ Hz) and sub-atomic scale ($a = r_C = \hbar/mc \approx 386\times10^{-15}$ m). It is, therefore, a *logical* model only: it gives us the right values for the angular momentum ($L = \hbar/2$), the magnetic moment ($\mu = (q_e/2m)\cdot\hbar$, and the gyromagnetic factor ($g = 2$).

Explaining interference and diffraction

This subtle combination of the idea of a pointlike charge and an electromagnetic oscillation is interesting because it opens the door to a plain classical explanation of interference and/or diffraction. In this regard, we would link this to more recent theory and experiments that focus on how slits or holes affect wave *shapes* as electrons – or photons – go through them. The diagram below illustrates the point that we are trying to make here.[83] We do think these are *very* promising in terms of offering some kind of classical (physical) explanation for interference and/or diffraction.

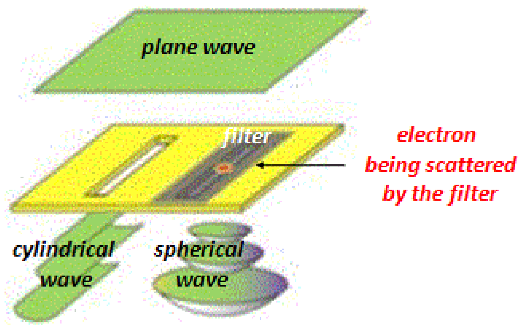


Figure 28: Physical interpretations of the electron wave

All that is left to explain – for the photon as well as the electron – is why the whole oscillation seems to stick together upon detection. We admit that's not easy to do. But – as an idea – it is definitely easier to accept this *axiom* than whatever other theory is on the market right now.

But what *is* that oscillation?

We gave an easy answer to that above: it must be electromagnetic, because the force only has a charge to grab on. It cannot be gravitational, because the pointlike charge itself has no rest mass: the mass of the electron is in the oscillation. That's Wheeler's idea of mass without mass.

However, that doesn't answer the more fundamental question: what does it all mean, *really*? We will just offer a few remarks here, which may or may not help us to further develop our understanding of the matter—literally! Let us first do some more thinking about the nature of that $E/m = c^2$ result. The idea of a two-two-dimensional oscillation is intuitive enough. It's the intuition behind the $E = ma^2\omega^2 = mc^2$ equation, really: the energy of any oscillation will be proportional to the square of (*i*) the (maximum) amplitude of the oscillation and (*ii*) the frequency of the oscillation, with the mass as the proportionality coefficient. At the same time, we should wonder: what could it possibly *mean*?

This question is difficult to answer. Is there any other idea – we mean: other than the idea of a two-dimensional oscillation – to explain the *Zitterbewegung*? We do not see anything obvious but, as mentioned, we can offer a few remarks which may or may not help the reader to develop his or her own interpretation of what might be going on *in reality*.

[83] The definition is somewhat random but we think of diffraction if there is only one slit or hole. In contrast, the idea of interference assumes two or more wave sources. The research we refer to is the work of the Italian researchers Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-doubleslit.html). The illustration was taken from the same source, but we added the explanatory tags.

The first remark is this: when everything is said and done, we should admit that the bold $c^2 = a^2 \cdot \omega^2$ assumption interprets spacetime as a relativistic aether. It is a term that is, unfortunately, taboo but, fortunately, some respected academics, such as Nobel Prize Laureate Robert Laughlin, are still defending it. This interpretation is inspired by the most obvious implication of Einstein's $E = mc^2$ equation, and that is that the ratio between the energy and the mass of *any* particle is always equal to c^2:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photon}}{m_{photon}} = \frac{E_{any\,particle}}{m_{any\,particle}} = c^2$$

As mentioned above, this reminds us of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ of harmonic oscillators – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom. [84] In contrast, $c^2 = E/m$ for *any* particle, *always*. This is the point: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges here as *the* defining property of spacetime. It is, in fact, tempting to think of it as some kind of *resonant frequency* but the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we referred to as *the rest energy oscillation*: it is that what gives mass to our electron.

Now, it's still a weird matter—literally, again. This two-oscillator model is all great, but it is hard to address its single biggest conceptual gap: what is the *nature* of the force in our electron model? So we want to go beyond what we said about its electromagnetic nature, that is. Can we do that? I am not sure. It is a very fundamental question which, perhaps, we will never be able to answer. It is what it is, right? Maybe.

Maybe not.

In case you wonder what the issue is about, I think it is nicely summarized in one of Dr. Burinskii's very first communications to me. He effectively wrote the following to me when I first contacted him on the viability on the model:

> "I know many people who considered the electron as a toroidal photon[85] and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". Editor E. Lifschitz prohibited me then to write there about *Zitterbewegung* [because of ideological reasons[86]], but there is a remnant on this notion. There was also this key problem: what keeps [the pointlike charge] in its circular orbit?"[87]

[84] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the inertia, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

[85] This is Dr. Burinskii's terminology: it does refer to the *Zitterbewegung* electron: a pointlike charge with no mass in an oscillatory motion – orbiting at the speed of light around some center.

[86] This refers to perceived censorship from the part of Dr. Burinskii. In fact, some of what he wrote me strongly suggests some of his writings have, effectively, been suppressed because – when everything is said and done – they do fundamentally question – directly or indirectly – some key assumptions of the mainstream interpretation of quantum mechanics.

[87] Email from Dr. Burinskii to the author dated 22 December 2018.

He noted that this fundamental flaw was (and still is) the main reason why had abandoned the simple *Zitterbewegung* model in favor of the much more sophisticated Kerr-Newman approaches to the (possible) geometry of an electron.

I am reluctant to make the move he made – mainly because I prefer simple math to the rather daunting math involved in Kerr-Newman geometries – and so that is why I am continuing to explore alternative explanations – such as this one. I feel there is scope here to complement the model with a third view of the wavefunction.

Huh? Yes. Let me explain. The *zbw* model offers a basic *interpretation* of the wavefunction by noting the various *aspects* of the (possible) reality that might correspond to the wavefunction. We referred to these aspects as the *dual view* of a wavefunction. That dual view consisted of (1) a description of the *position* of our pointlike particle and (2) a description of the *force* that makes it move. Both descriptions are descriptions in terms of a complex-valued function: the wavefunction itself. Hence, it would seem to be logical that we develop a *third* view now: the wavefunction as a description of the physical space that comes with the particle. How can we do that?

Perhaps Einstein provides some inspiration again. Indeed, we got that two-dimensional oscillator model (the flywheel model of an electron, as I used to call it) as a result of a *deep* exploration of the (possible) meaning of Einstein's mass-energy equivalence relation ($E = mc^2$), which comes out of *special* relativity theory. So… Well… Perhaps we should now explore some other intuition—an intuition-based Einstein's general relativity theory. What am I thinking of?

It's the following: if we can describe the particle itself by Euler's wavefunction – exploring different *aspects* of its reality, such as the position of the pointlike charge, and the nature of the force that makes it move along its circular orbit – then we should, perhaps, also explore how we can use it to describe the nature of the *space* that comes with the particle.

Huh? Yes. The $E = mc^2$ equation, which we get out of Einstein's special relativity theory, made us think about a physical interpretation of the wavefunction. I want to go further now. I feel the geometric approach to gravity suggests objects do come with their own geometry. Their own space, so to speak. A *physical* space, obviously. Not the mathematical coordinate space, which can have any geometry. Now, when we describe *physical* space (as opposed to a purely mathematical space – coordinate space, that is), we will usually describe its *nature* in terms of some *potential*, whose derivative will then give us the force acting on our particle. Hence, if the force on our particle can be described by a complex function, perhaps we should try to describe potential in terms of a complex-valued function as well.

Huh? Yes. Let me show you what I mean. In classical mechanics, a force may be defined as the (negative of the) derivative of a potential. Such potential may be gravitational or electrostatic. We write:

$$-\mathrm{d}U/\mathrm{d}x = F(x) = F_x$$

If we're considering the y-direction, then we write $-\mathrm{d}U/\mathrm{d}y = F(y) = F_y$. Now, what if we would – somehow – think of the $a \cdot e^{i\theta}$ function as some *complex-valued potential*? Let us forget about the coefficient a for a while (we can plug it back in at a later stage), so we write:

$$\mathbf{U} = e^{i\theta}$$

Let us take the derivative in regard to the variable here, which is… What? It is the angle θ. It is a *real* number, so we will *not* be calculating the usual derivative of a complex exponential, which is $d(e^z)/dz = e^z$, with *z* a complex number. Instead, we calculate:

$$-d\mathbf{U}/d\theta = -d(e^{i\theta})/d\theta = -d(\cos\theta + i\cdot\sin\theta)/d\theta = -d(\cos\theta)/d\theta - i\cdot d(\sin\theta)/d\theta$$

$$= \sin\theta - i\cdot\cos\theta = \cos(\theta-\pi/2) + i\cdot\sin(\theta-\pi/2)$$

We get the sine and cosine factors of our force formula, except the sign is right: the phase factor should be $+\pi/2$ instead of $-\pi/2$. That problem is solved if we drop the minus sign in front of the $-d\mathbf{U}/d\theta$ derivative:

$$d\mathbf{U}/d\theta = d(e^{i\theta})/d\theta = d(\cos\theta + i\cdot\sin\theta)/d\theta = d(\cos\theta)/d\theta + i\cdot d(\sin\theta)/d\theta$$

$$= -\sin\theta + i\cdot\cos\theta = \cos(\theta+\pi/2) + i\cdot\sin(\theta+\pi/2)$$

Why would we drop the minus sign? One may think it could be related to the other mathematical possibility: the rotation may be clockwise rather than counterclockwise. The mathematical formalism works out equally well, but it does not explain why we should drop the minus sign in front of the derivative. However, *if* we acknowledge there would be a minus sign if we would have adopted the convention of measuring angles clockwise rather than counterclockwise, *then* we see it's just a matter of convention, effectively.

Does this make any sense? I am not sure. The idea of a complex-valued potential may or may not provide the ultimate answer – but it sure does cater to the idea of a particle coming with its own space. The *nature* of this space is – quite simply – this new concept: a complex-valued potential.

Huh? Yes. The first reaction of the reader is predictable: this must be nonsense. I invite the reader to think about *why* he would say that – because my own initial reaction to my thoughts was the same: this is ridiculous. However, I then realized that my instinctive objection to my own thoughts was that it is somewhat hard to distinguish ontological or mathematical concepts here from what might (or might not) be reality – or *physical* concepts, I should say. In fact, the ambiguity is in the concept of a potential itself. It is *less* tangible than a force. It is like thinking of a force without thinking simultaneously about what it's going to grab onto – which is not an easy exercise. To put it differently, I have a strong feeling that my train of thought here effectively does involve some implicit tautologies. Having said that, tautologies – when made explicit – may bring new insights.[88]

This may or may not lead anywhere, but something inside of me says we should consider it. It might bring some more *integrated* view on the fundamental nature of matter. In fact, my very first paper also explored the idea of the wavefunction representing some two-dimensional *rest matter oscillation*, but I initially thought it might, effectively, be a gravitational wave[89] and – funnily enough – this paper still gets a lot of downloads. As mentioned, I have no doubt that the nature of the *force* is electromagnetic, but something inside of me also keeps suggesting that its *oscillation* might actually be an oscillation of *spacetime* itself. It's just an attractive idea: objects come with their own (physical) space.

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[88] I published a very first paper – with some comments – on viXra.org. See http://vixra.org/abs/1902.0113: *Can the idea of a complex-valued potential explain the Zitterbewegung?* However, Dr. Burinskii does not like it, because he feels these are just tautologies, indeed. We thought of taking it down, but perhaps some discussion on it can, effectively, address such feelings.
[89] See http://vixra.org/abs/1709.0390: *The Quantum-Mechanical Wavefunction as a Gravitational Wave*.

However, this is plain philosophy. We should get back to the equations.

A geometric interpretation of the *de Broglie* wavelength

As part of the *prolegomena* to this book, we discussed the concept of a wavelength in the context of the *Zitterbewegung* model. Let us copy **Figure 7** once again to focus our mind: it presented the presumed *Zitterbewegung* of an electron as we would see it when it moves through space.

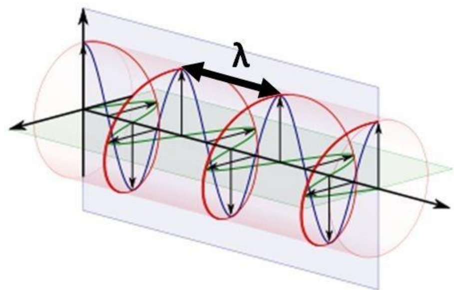

Figure 29: Is this a moving electron?

We warned you immediately: there is no reason whatsoever why the *plane* of the oscillation – the plane of *rotation* of the pointlike charge, that is – would be perpendicular to the direction of propagation of the electron as a whole. In fact, we think that plane of oscillation moves about itself, and we'll come back to that. We just want you to make a mental note of as we now are going to present a rather particular geometric property of the *Zitterbewegung* (*zbw*) motion: the Compton radius must decrease as the velocity of our electron increases. The idea is visualized in the illustration below (for which credit goes to an Italian group of *zbw* theorists[90]):

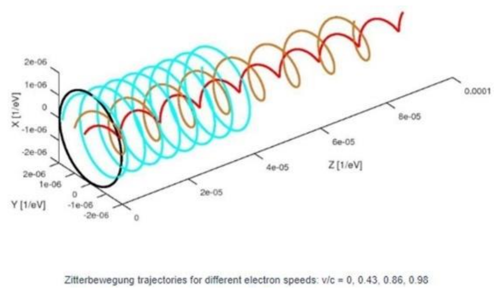

Figure 30: The Compton radius must decrease with increasing velocity

Can the velocity go to *c*? In the limit, yes. This is very interesting, because we can see that the circumference of the oscillation becomes a wavelength in the process! We'll come back to this because it relates the geometry of our *zbw* electron to the geometry of the photon model we're going to develop in a later chapter.

[90] Vassallo, G., Di Tommaso, A. O., and Celani, F, *The Zitterbewegung interpretation of quantum mechanics as theoretical framework for ultra-dense deuterium and low energy nuclear reactions*, in: Journal of Condensed Matter Nuclear Science, 2017, Vol 24, pp. 32-41. Don't worry about the rather weird distance scale (1×10^{-6} eV^{-1}). Time and distance can be expressed in *inverse* energy units when using so-called *natural units* ($c = \hbar = 1$). We are not very fond of this because we think it does *not* necessarily clarify or simplify relations. Just note that 1×10^{-9} eV^{-1} = 1 GeV^{-1} $\approx 0.1975\times10^{-15}$ m. As you can see, the *zbw* radius is of the order of 2×10^{-6} eV^{-1} in the diagram, so that's about 0.4×10^{-12} m, which is what we calculated: $a \approx 0.386\times10^{-12}$ m.

What happens here is quite easy to understand – intuitively, that is. If the tangential velocity remains equal to *c*, and the pointlike charge has to cover some horizontal distance as well, then the circumference of its rotational motion *must* decrease so it can cover the extra distance. But let us analyze it the way we should analyze it, and that's by using our formulas. Let us first think about our formula for the *zbw* radius *a*:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi}$$

The λ_C is the *Compton* wavelength, so that's the circumference of the circular motion.[91] How can it decrease? If the electron moves, it will have some kinetic energy, which we must add to the *rest energy*. Hence, the mass m in the denominator (m*c*) increases and, because ħ and c are physical constants, *a* must decrease.[92] How does that work with the frequency? The frequency is proportional to the energy ($E = \hbar\cdot\omega = h\cdot f = h/T$) so the frequency – in whatever way you want to measure it – will *increase*. Hence, the *cycle* time T must *decrease*. We write:

$$\theta = \omega t = \frac{\mathrm{E}}{\hbar}t = \frac{\gamma \mathrm{E}_0}{\hbar}t = 2\pi \cdot \frac{\mathrm{t}}{\mathrm{T}}$$

So our Archimedes' screw gets stretched, so to speak. Let us think about what happens here. We got the following formula for this λ wavelength, which is like the distance between two crests or two troughs of the wave[93]:

$$\lambda = v \cdot \mathrm{T} = \frac{v}{f} = v \cdot \frac{\mathrm{h}}{\mathrm{E}} = v \cdot \frac{\mathrm{h}}{\mathrm{m}c^2} = \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \beta \cdot \lambda_C$$

This wavelength is *not* the *de Broglie* wavelength $\lambda_L = h/p$.[94] So what is it? We have *three* wavelengths now: the *Compton* wavelength λ_C (which is a circumference, actually), that weird horizontal distance λ, and the *de Broglie* wavelength λ_L. Can we make sense of that? We can. Let us first re-write the *de Broglie* wavelength:

$$\lambda_\mathrm{L} = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}v} = \frac{\mathrm{h}c^2}{\mathrm{E}v} = \frac{\mathrm{h}c}{\mathrm{E}\beta} = \frac{\mathrm{h}}{c} \cdot \frac{1}{\mathrm{m} \cdot \beta} = \frac{\mathrm{h}}{\mathrm{m}_0 c} \cdot \frac{1}{\gamma\beta}$$

What is this? We are not sure, but it might help us to see what happens to the *de Broglie* wavelength as m and *v* both increase as our electron picks up some momentum $p = m\cdot v$. Its wavelength must actually *decrease* as its (linear) momentum goes from zero to some much larger value – possibly infinity as *v* goes to *c* – but *how exactly*? The 1/γβ factor gives us the answer. That factor comes down from infinity (+∞) to zero as *v* goes from 0 to *c* or – what amounts to the same – if the relative velocity $\beta = v/c$ goes from 0 to 1. The graphs below show that works. The 1/γ factor is the circular arc that we're used to, while the 1/β function is just the regular inverse function ($y = 1/x$) over the domain $\beta = v/c$, which goes from 0 to 1 as *v* goes from 0 to *c*. Their product gives us the green curve which – as mentioned – comes down from +∞ to 0.

[91] Hence, the *C* subscript stands for the *C* of Compton, not for the speed of light (*c*).
[92] We advise the reader to always think about proportional ($y = kx$) and inversely proportional ($y = x/k$) relations
[93] Because it is a wave in two dimensions, we cannot really say there are crests or troughs, but the terminology might help you with the interpretation of the geometry here.
[94] The use of L as a subscript is a bit random but think of it as the L of Louis de Broglie.

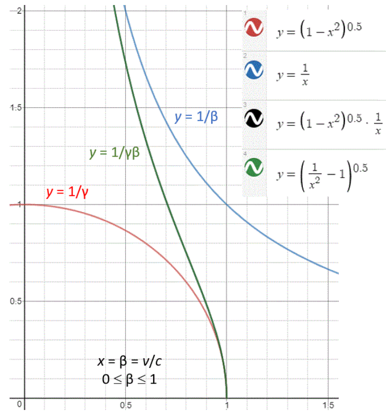

Figure 31: The 1/γ, 1/β and 1/γβ graphs

Now, we re-wrote the formula for *de Broglie* wavelength λ_L as the *product* of the 1/γβ factor and the *Compton* wavelength for $v = 0$:

$$\lambda_{\mathrm{L}} = \frac{\mathrm{h}}{\mathrm{m}_0 c} \cdot \frac{1}{\gamma\beta} = \frac{1}{\beta} \cdot \frac{\mathrm{h}}{\mathrm{m}c}$$

Hence, the *de Broglie* wavelength goes from +∞ to 0. We may wonder: when is it equal to $\lambda_C = h/mc$? Let's calculate that:

$$\lambda_{\mathrm{L}} = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\mathrm{m}c} \cdot \frac{1}{\beta} = \lambda_C = \frac{\mathrm{h}}{\mathrm{m}c} \Leftrightarrow \beta = 1 \Leftrightarrow v = c$$

This is a rather weird result, and we have not yet fully interpreted its significance. Let's bring the third wavelength in: the $\lambda = \beta \cdot \lambda_C$ wavelength—which is that length between the crests or troughs of the wave.[95] We get the following two rather remarkable results:

$$\lambda_{\mathrm{L}} \cdot \lambda = \lambda_{\mathrm{L}} \cdot \beta \cdot \lambda_C = \frac{1}{\beta} \cdot \frac{\mathrm{h}}{\mathrm{m}c} \cdot \beta \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \lambda_C^2$$

$$\frac{\lambda}{\lambda_{\mathrm{L}}} = \frac{\beta \cdot \lambda_C}{\lambda} = \frac{\mathrm{p}}{\mathrm{h}} \cdot \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \frac{\mathrm{m}v^2}{\mathrm{m}c^2} = \beta^2$$

The product of the $\lambda = \beta \cdot \lambda_C$ wavelength and *de Broglie* wavelength is the square of the Compton wavelength, and their ratio is the square of the relative velocity $\beta = v/c$. – *always!* – and their ratio is equal to 1 – *always!* These two results are rather remarkable too but, despite their simplicity and apparent beauty, we are also struggling for an easy geometric interpretation. The use of *natural units* may help. Equating c to 1 would give us natural distance and time units, and equating h to 1 would give us a natural force unit—and, because of Newton's law, a natural mass unit as well. Why? Because Newton's F = m·a equation is relativistically correct: a force is that what gives some mass acceleration. Conversely, mass can be defined of the inertia to a change of its state of motion—because any change in motion involves a force and some acceleration. We write: m = **F**/***a***. If we re-define our distance, time and

[95] We should emphasize, once again, that our two-dimensional wave has no real crests or troughs: λ is just the distance between two points whose argument is the same—except for a phase factor equal to $n \cdot 2\pi$ (n = 1, 2,…).

force units by equating *c* and *h* to 1, then the Compton wavelength (remember: it's a circumference, really) and the mass of our electron will have a simple inversely proportional relation:

$$\lambda_C = \frac{1}{\gamma \mathrm{m}_0} = \frac{1}{\mathrm{m}}$$

We get equally simple formulas for he *de Broglie* wavelength and our λ wavelength:

$$\lambda_{\mathrm{L}} = \frac{1}{\beta\gamma \mathrm{m}_0} = \frac{1}{\beta \mathrm{m}}$$

$$\lambda = \beta \cdot \lambda_C = \frac{\beta}{\gamma \mathrm{m}_0} = \frac{\beta}{\mathrm{m}}$$

This is quite deep: we have three *lengths* here – defining all of the geometry of the model – and they all depend on the *rest* mass of our object and its relative velocity *only*. Can we take this discussion any further? Perhaps, because what we have found may or may not be related to the idea that we're going to develop in the next section. However, before we move on to the next, let us quickly note the three equations – or lengths – are *not* mutually independent. They are related through that equation we found above:

$$\lambda_{\mathrm{L}} \cdot \lambda = \lambda_C^2 = \frac{1}{\mathrm{m}^2}$$

We'll let you play with that. To help you with that, you may start by noting that the $\lambda_L\lambda = 1/m^2$ reminds us of a property of an ellipse. Look at the illustration below.[96] The length of the chord – perpendicular to the major axis of an ellipse is referred to as the *latus rectum*. One half of that length is the actual *radius of curvature* of the osculating circles at the endpoints of the major axis.[97] We then have the usual distances along the major and minor axis (*a* and *b*). Now, one can show that the following formula has to be true:

$$a \cdot p = b^2$$

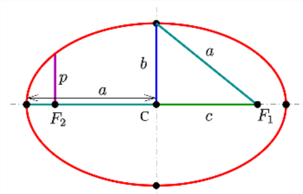

Figure 32: The *latus rectum* formula: $a \cdot p = b^2$

[96] Source: Wikimedia Commons (By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=57428275).

[97] The endpoints are also known as the *vertices* of the ellipse. As for the concept of an osculating circles, that's the circle which, among all tangent circles at the given point, which approaches the curve most tightly. It was named *circulus osculans* – which is Latin for 'kissing circle' – by Gottfried Wilhelm Leibniz. You know him, right? Apart from being a polymath and a philosopher, he was also a great mathematician. In fact, he was the one who invented differential and integral calculus.

If you don't immediately see why this would be relevant, then… Well… Then you should look at it again. 😊 And if you don't see much right now and you're tired of thinking about it, don't worry too much: we'll come back to it. 😊

Onwards!

The *de Broglie* equation as a vector equation?

We suggested that Planck's quantum of action h, which we associated with an elementary cycle, or – in its *reduced* form (ħ = h/2π) – with the fundamental unit of angular momentum, should, perhaps, be written as a vector quantity. It's a force times a circumference (or a radius or – more generally – some length) times a cycle time. A force is a vector quantity: it has a magnitude but it also has a *direction*. The linear momentum which appears in the second *de Broglie* relation for matter-waves is a vector quantity too—not because of the mass factor (m) but because of the velocity factor ($\boldsymbol{v}$): $\mathbf{p} = \mathrm{m}\boldsymbol{v}$. This makes it *very* tempting to write the second *de Broglie* relation (λ = h/p) as a vector equation:

$$\lambda = \frac{\mathbf{h}}{\mathbf{p}} = \frac{\vec{h}}{\vec{p}}$$

We would, therefore, also have to re-write the Uncertainty Principe—or the Uncertainty *Relation* as I prefer to refer to it. We are currently doing some research in this regard and it is all quite promising. For example, it provides a rather fresh perspective on the so-called random walk of an electron in free space and it may, therefore, explain Einstein's formula for it in a very different (but necessarily equivalent) way. However, we do not want to burden the reader with that at this point in time, because the mentioned research is rather immature at this point.

You may that vector equation looks weird, but it's not any different than writing Newton's force law as a vector equation:

$$\mathrm{m} = \frac{\mathbf{F}}{\boldsymbol{a}} = \frac{\vec{F}}{\vec{a}}$$

Let's move to the next: the behavior of an electron in an atom. We can describe it by the very same wavefunction, but its physical *interpretation* is somewhat different. Let's go for it! 😊

VI. The wavefunction and the atom

The illustration below depicts the geometry of a Bohr orbital. We describe such orbital by the same mathematical object – the elementary wavefunction (Euler's function) – but we do have a different geometry here.

In fact, the situation is *very* different. The Bohr model has a positively charged nucleus at its center and its electron has an effective rest mass: the radial velocity $v = a \cdot \omega$ of the electron is, therefore, some *fraction* of the speed of light ($v = \alpha \cdot c$). It also has some non-zero momentum $\mathrm{p} = \mathrm{m} \cdot v$ which we can relate to the electrostatic centripetal force using the simple classical formula $\mathrm{F} = \mathrm{p} \cdot \omega = \mathrm{m}v^2/a$. In contrast, the model of an electron in free space is based on the presumed *Zitterbewegung*, which combines the idea of a very high-frequency circulatory motion with the idea of a pointlike *charge* which – importantly – has *no inertia* and can, therefore, move at the speed of light ($v = c$).

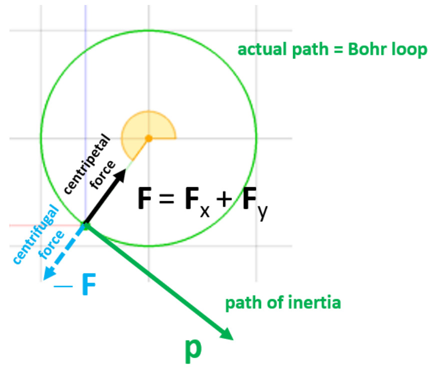

Figure 33: The position, force and momentum vector in a Bohr loop

The formulas in the Bohr-Rutherford model are derived from the quantum-mechanical that angular momentum comes in units of ħ = h/2π. We rephrased that rule as: physical action comes in unit of h. We also associated Planck's quantum of action with a *cycle*: one rotation will pack some energy over some time (the cycle time) or – what amounts to the same – some momentum over some distance (the circumference of the loop). We wrote:

$$S = \mathrm{h} = \mathrm{E}\cdot\mathrm{T} = \mathrm{L}\cdot 2\pi\cdot r_{\mathrm{B}}$$

Using the $v = \alpha\cdot c$ and $r_C = \alpha\cdot r_B$ relations[98] one can easily verify this for the momentum formulation:

$$S = \mathrm{p}\cdot 2\pi\cdot r_{\mathrm{B}} = \mathrm{m}v\cdot(r_{\mathrm{C}}/\alpha) = \mathrm{m}\alpha c\cdot\frac{2\pi\hbar}{\alpha\mathrm{m}c} = \mathrm{h}$$

We can also calculate *S* by calculating the force and then multiply the force with the distance and the time. The force is just the (centripetal) electrostatic force between the charge and the nucleus

$$\mathrm{F} = \frac{\mathrm{q_e}^2}{4\pi\varepsilon_0 r_{\mathrm{B}}^2} = \alpha\cdot\frac{\hbar c}{r_{\mathrm{B}}^2}$$

We can then recalculate *S* as:

$$S = \mathrm{F}\cdot r_{\mathrm{B}}\cdot\mathrm{T} = \alpha\cdot\frac{\hbar c}{r_{\mathrm{B}}^2}\cdot r_{\mathrm{B}}\cdot\frac{2\pi r_{\mathrm{B}}}{v} = \alpha\cdot\frac{\mathrm{h}c}{\alpha c} = \mathrm{h}$$

All is consistent. However, we should note the implied energy concept is somewhat surprising:

$$S = h = \mathrm{E}\cdot\mathrm{T} = \mathrm{E}\cdot\frac{2\pi r_{\mathrm{B}}}{v} = \mathrm{E}\cdot\frac{\frac{h}{\alpha\mathrm{m}c}}{\alpha c} \Leftrightarrow \mathrm{E} = \alpha^2\mathrm{m}c^2$$

This is *twice* the ionization energy of hydrogen (Ry = $\alpha^2 mc^2/2$), and it is also *twice* the kinetic energy ($\hbar^2/2ma^2 = \alpha^2 mc^2/2$). It is also just a fraction ($\alpha^2 \approx 0.00005325$) of the rest energy of the electron.[99] This somewhat odd result can be explained if we would actually be thinking of a two-dimensional oscillation

[98] These relations come out of the model. They are, therefore, not some new hypothesis. The α in the formula is the fine-structure constant. It pops up in (almost) all of the equations we get. As such, it does appear as some magical dimensionless number that relates almost all (physical) dimensions of the electron (radii, circumferences, energies, momenta, etcetera).
[99] The reader can check the conversion of the Rydberg energy in terms of the fine-structure constant and the rest mass (or rest energy) of the electron.

here. In that case, we would effectively write the force as **F** = **F**$_x$ + **F**$_y$ (as suggested in the illustration above) in a moment) and, hence, we should therefore *add* the kinetic and potential energy of *two* oscillators.

Let us explain and generalize these results for all electron orbitals. In other words, let us explain it in terms of the Bohr atom. The quantum of action effectively underpins the Rutherford-Bohr model of an atom. This 105-year old model[100] was designed to explain the wavelength of a photon that is emitted or absorbed by a hydrogen atom – a one-electron atom, basically – and does a superb job of it. The idea is that the energy of such photon is equal to the *difference* in energy between the various orbitals. The energy of these orbitals is usually expressed in terms of the energy of the *first* Bohr orbital, which is usually referred to as the *ground state* of (the electron in) the hydrogen atom. The *Rydberg* energy E_R is just the combined kinetic and potential energy of the electron in the first Bohr orbital and it can be expressed in terms of the fine-structure constant (α) and the rest energy ($E_0 = mc^2$) of the electron[101]:

$$\mathrm{E}_R = \frac{\alpha^2 \mathrm{m}c^2}{2} = \frac{1}{2}(\frac{\mathrm{q_e}^2}{2\varepsilon_0 \mathrm{h}c})^2 \mathrm{m}c^2 = \frac{\mathrm{q_e}^4 \mathrm{m}}{8\varepsilon_0{}^2 \mathrm{h}^2} \approx 13.6\ \mathrm{eV}$$

To be precise, the *difference* in energy between the various orbitals should be equal to:

$$\Delta\mathrm{E} = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \mathrm{E}_R$$

The Rydberg formula then becomes self-evident. The idea of the wavelength of a wave (λ), its velocity of propagation (*c*) and its frequency (*f*)[102] are related through the λ = *c*/*f* relation, and the Planck-Einstein relation (E = *h*·*f*) tells us the energy and the wavelength of a photon are related through the frequency:

$$\lambda = \frac{c}{f} = \frac{hc}{\mathrm{E}}$$

Hence, we can now write the Rydberg formula by combining the above:

$$\frac{1}{\lambda} = \frac{\mathrm{E}}{\mathrm{h}c} = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \frac{\mathrm{E}_R}{\mathrm{h}c} = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right) \cdot \frac{\alpha^2 \mathrm{m}c^2}{2\mathrm{h}c}$$

The Rydberg formula uses the fine-structure constant, but actually describes the so-called *gross* structure of the hydrogen spectrum only (illustrated below). Indeed, when the spectral lines are examined at *very* high resolution, the spectral lines are split into finer lines. This is due to the intrinsic spin of the electron. This *intrinsic* spin of the electron is to be distinguished from its orbital motion. It shows we should not be thinking of the electron as a pointlike (infinitesimally small) particle: it has a

[100] Around 1911, Rutherford had concluded that the nucleus had to be very small. Hence, Thomson's model – which assumed that electrons were held in place because they were, somehow, embedded in a uniform sphere of positive charge – was summarily dismissed. Bohr immediately used the Rutherford hypothesis to explain the emission spectrum of hydrogen atoms, which further confirmed Rutherford's conjecture, and Niels and Rutherford jointly presented the model in 1913. As Rydberg had published his formula in 1888, we have a gap of about 25 years between experiment and theory here.

[101] We should write m_0 instead of m – *everywhere*. But we are using non-relativistic formulas for the velocity and kinetic energy everywhere. Hence, we dropped the subscript.

[102] Our papers – and this book – relate mathematical and physical concepts. Hence, we prefer to think of a wavelength as a mathematical *idea* right now, as opposed to some (physical) *reality*. Our ontological viewpoint is very simple: language describes reality. Hence, math describes physics. There is an intimate relation between both but – at the same time – we should not confuse the two.

radius.[103] Hence, we speak of *spin* angular momentum versus *orbital* angular momentum. However, as we will explain, there is some *coupling* between the two motions. We will come back to this later.

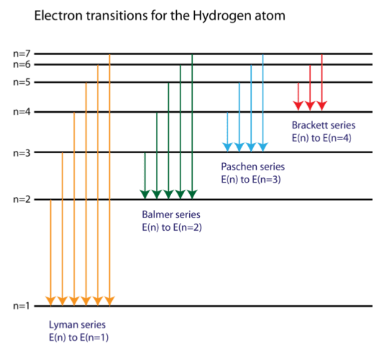

Figure 34: The gross structure of the hydrogen spectrum

The Copenhagen interpretation of quantum mechanics – which, privately, we have started to think of as the *Heisenberg Diktatur*[104] – dismisses Bohr's model. However, it is actually a proper quantum-mechanical explanation and Schrödinger's equation does not seem to add much in terms of a scientific explanation for the atomic electron orbitals. Feynman (*Lectures*, III-2-4) derives it from the momentum-space expression of the Uncertainty Principle which we may loosely state as follows: the product of the uncertainty in the momentum (Δp) and the uncertainty in the position (Δx) has an order of magnitude that is equal to Planck's quantum (*h*). His equation is the following:

$$\mathrm{p} \cdot a \approx \hbar \Leftrightarrow \mathrm{p} \approx \hbar / a$$

This allows him to write the kinetic energy of the electron as $mv^2/2 = p^2/2m = \hbar^2/2ma^2$. The potential energy is just the electrostatic energy $-e^2/a$.[105] The idea is then that the configuration must minimize the total energy $E = \hbar^2/2ma^2 - e^2/a$. The variable is the radius a and, hence, we get a by calculating the $d\mathrm{E}/da$ derivative and equating it to zero. We thus get the correct Bohr radius:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{mq_e}^2} = \frac{1}{\alpha} \cdot r_{\mathrm{Compton}} \approx 53 \times 10^{-12}\ \mathrm{m}$$

We find it useful to write the Bohr radius as the Compton radius divided by the fine-structure constant:

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103 We argue its radius is the Compton radius. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

104 No one should take offense here. It is an opinion which is rooted in our experience trying to submit articles to scientific journals as well as interactions with academics. In fact, we should tone down and *not* specifically associate the Copenhagen interpretation with Heisenberg and other founding fathers of the quantum-mechanical framework, as they were part of the group of 'founding fathers' who actually became quite skeptical about the theory they had created because of the divergences in perturbative quantum electrodynamics (QED). Todorov (2018) specifically Heisenberg, Dirac, and Pauli in this regard, and mentions that QED, as a theory, only survived because of the efforts of the second generation of quantum physicists (Feynman, Schwinger, Dyson, etcetera). See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

105 The e^2 in this formula is the squared charge of an electron (q_e^2) divided by the electric constant ($4\pi\varepsilon_0$). The formula assumes the potential is zero when the distance between the positively charged nucleus and the electron is infinite, which explains the minus sign. We also get the minus sign, of course, by noting the two charges (electron and nucleus) have equal magnitude but opposite sign. One should note that the formulas are non-relativistic. This is justified by the fact that the velocities in this model are non-relativistic (the electron velocity in the Bohr orbital is given by $v_e = \alpha \cdot c \approx 0.0073 \cdot c$. This is an enormous speed but still less than 1% of the speed of light.

$$r_B = r_C/\alpha = \hbar/\alpha mc \approx (386/0.0073)\times10^{-15}\ \text{m} \approx 53\times10^{-9}\ \text{m}.$$

We can now calculate the Rydberg energy – which is the *ionization* energy of hydrogen – by using the Bohr radius to calculate the energy $E = \hbar^2/2ma^2 - e^2/a$:

$$E_R = \frac{1}{2}\frac{\hbar^2}{m}\frac{m^2e^4}{\hbar^4} - e^2\frac{me^2}{\hbar^2} = -\frac{1}{2}\frac{me^4}{\hbar^2} \approx -13.6\ \text{eV}$$

This amount equals the kinetic energy ($\hbar^2/2ma^2 = \alpha^2mc^2/2$). The electrostatic energy itself is *twice* that value ($-e^2/r_{Bohr} = -\alpha^2mc^2$).

Feynman's Uncertainty Principe is suspiciously certain. He basically equates the uncertainty in the momentum as the momentum itself ($\Delta p = p$) and the uncertainty in the position as a precise radius. We offer an alternative interpretation. If Planck's constant is, effectively, a physical constant ($h \approx 6.626\times10^{-34}$ N·m·s), then we should interpret it as such. If physical action – some force over some distance over some time – comes in units of h, then the relevant distance here is the loop, so that is $2\pi\cdot r_{Bohr}$. We would, therefore, like to re-write Feynman's $p\cdot a \approx \hbar$ assumption as:

$$S = h = p\cdot2\pi\cdot r_{Bohr} = p\cdot\lambda$$

The λ is, of course, the circumference of the loop. The equation resembles the *de Broglie* equation $\lambda = h/p$. How should we interpret this? We can associate Planck's quantum of action with a *cycle*: let us refer to it as a Bohr loop and, yes, we think of it as a circular orbit. As such, we can write h either as the energy times the cycle time or, else, as the (linear) momentum times the loop: $h = p\cdot2\pi\cdot r_B$. The latter expression not only reflects the second *de Broglie* relation but also the quantum-mechanical rule that angular momentum should come in units of $\hbar = h/2\pi$. Indeed, the angular momentum can always be written in terms of the tangential velocity, the radius and the mass. As such, the two formulas below amount to the same:

$$L = m\cdot v\cdot r_B = p\cdot r_B = \hbar \Leftrightarrow S = p\cdot2\pi\cdot r_B = p\cdot\lambda_B = h$$

Let us continue our calculations. We get the velocity out of the expression for the kinetic energy:

$$K.E. = \frac{mv^2}{2} = \frac{\alpha^2mc^2}{2} \Leftrightarrow v = \alpha\cdot c \approx 0.0073\cdot c$$

Of course, we should also be able to express the velocity as the product of the radius and an angular frequency, which we can do as follows:

$$v = \alpha\cdot c = r_B\cdot\omega_B = \frac{\hbar}{\alpha mc}\cdot\frac{\alpha^2mc^2}{\hbar} = \alpha\cdot c \Leftrightarrow \omega_B = \frac{\alpha^2mc^2}{\hbar}$$

We then calculate the cycle time T as $T = 1/f_B = 2\pi/\omega_B$. Interestingly, the formula for f_B (or, thinking in terms of *angular* frequencies, for ω_B) reflects the first *de Broglie* relation: $f_B = E/h = \alpha^2mc^2/h$. However, we should note that α^2mc^2 is *twice* the Rydberg energy – and, unlike some physicists, we do care about a 1/2 or π factor in our model of a Bohr electron. Hence, we should have a look at this energy concept. We will do so later. Let us – just for now – roll for a moment with this $E = \alpha^2mc^2$ energy concept. It is, obviously, the energy that is associated with the loop. We wrote the quantum of action as the product of the (linear) momentum and the distance along the loop: $h = p\cdot\lambda_B = p\cdot2\pi\cdot\lambda_B$. Likewise, we can write:

$$h = \mathrm{E} \cdot \mathrm{T} = \alpha^2 \mathrm{m}c^2 \cdot \frac{2\pi \cdot r_\mathrm{B}}{v} = \alpha^2 \mathrm{m}c^2 \cdot \frac{2\pi \cdot r_\mathrm{C}}{\alpha \cdot c \cdot \alpha} = \mathrm{m}c^2 \frac{2\pi \cdot \hbar}{c \cdot \mathrm{m} \cdot c} = h$$

Let us now generalize our formulas for all of the Bohr orbitals:

Table 2: Generalized formulas for the Bohr orbitals

Orbital electron (Bohr orbitals)
$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v_n = \frac{1}{n}\alpha c$
$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}} n\hbar$
$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

The reader can easily verify these formulas – by googling them, doing the calculations himself or, preferably, just doing some substitutions here and there. Let us substitute the equation for ω_n in the L_n formula, for example:

$$\mathrm{L}_n = I \cdot \omega_n = \mathrm{m} \cdot {r_n}^2 \cdot \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \mathrm{m} \cdot \frac{n^4}{\alpha^2}\frac{\hbar^2}{\mathrm{m}^2 c^2} \cdot \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = n\hbar$$

The reader should note that these formulas are *not* so obvious as they seem. The table below shows what happens with radii, velocities, frequencies and cycle times as we move out. The velocities go down, all the way to zero for $n \to \infty$, and the corresponding cycle times increases as the *cube* of *n*. Using totally non-scientific language, we might say the numbers suggest the electron starts to lose interest in the nucleus so as to get ready to just wander about as a free electron.

Table 3: Functional behavior of radius, velocity and frequency of the Bohr orbitals

n	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
$r_n \propto n^2$	1	4	9	16	25	36	49	64	81
$v_n \propto 1/n$	1	0.500	0.333	0.250	0.200	0.167	0.143	0.125	0.111
$\omega_n \propto 1/n^3$	1	0.125	0.037	0.016	0.008	0.005	0.003	0.002	0.001
$T_n \propto n^3$	1	8	27	64	125	216	343	512	729

The important thing is the energy formula, of course, because it should explain the Rydberg formula, and it does:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{{n_2}^2}\mathrm{E}_R + \frac{1}{{n_1}^2}\mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \frac{\alpha^2 \mathrm{m}c^2}{2}$$

Let us know look at the energies once again and try to connect this model with the idea of a photon.

VII. The wavefunction and the photon

The Bohr orbitals are separated by a amount of *action* that is equal to *h*. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of *h*. Our photon will have to pack that, somehow. It will also have to pack the related energy, which is given by the Rydberg formula (see above). To focus our thinking, let us consider the transition from the second to the first level, for which the $1/1^2 - 1/2^2$ is equal 0.75. Hence, the photon energy should be equal to $(0.75)\cdot \mathrm{E}_R \approx 10.2$ eV.[106] Now, if the total action is equal to *h*, then the cycle time T can be calculated as:

$$\mathrm{E}\cdot\mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135 \times 10^{-15}\,\mathrm{eV}\cdot\mathrm{s}}{10.2\ \mathrm{eV}} \approx 0.4 \times 10^{-15}\ \mathrm{s}$$

This corresponds to a wave train with a length of $(3\times10^8$ m/s$)\cdot(0.4\times10^{-15}$ s$)$ = 122 nm. That is the size of a large molecule and it is, therefore, much more reasonable than the length of the wave trains we get when thinking of transients using the supposed Q of an atomic oscillator.[107] In fact, this length is *exactly* equal to the wavelength $\lambda = c/f = c\cdot\mathrm{T} = \mathrm{h}c/\mathrm{E}$.

What picture of the photon are we getting here? Because of the angular momentum, we will probably want to think of it as a circularly polarized wave, which we may represent by the elementary wavefunction, as shown below.[108] We will call this interpretation of the wavefunction the *one-cycle photon*: the wavefunction represents the rotating field vector itself or, remembering the $\mathrm{F} = \mathrm{q_e}\mathrm{E}$ equation, the force field.

[106] This is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. The ozone layer of our atmosphere blocks most of it.

[107] In one of his famous *Lectures* (I-32-3), Feynman thinks about a sodium atom, which emits and absorbs sodium light, of course. Based on various assumptions – assumption that make sense in the context of the blackbody radiation model but *not* in the context of the Bohr model – he gets a Q of about 5×10^7. Now, the frequency of sodium light is about 500 THz (500×10^{12} oscillations per second). Hence, the *decay time* of the radiation is of the order of 10^{-8} seconds. So that means that, after 5×10^7 oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. That seems to be very short, but it still makes for 5 million oscillations and, because the wavelength of sodium light is about 600 nm (600×10^{-9} meter), we get a wave train with a considerable length: $(5\times10^6)\cdot(600\times10^{-9}$ meter$)$ = 3 meter. *Surely you're joking, Mr. Feynman!* A photon with a length of 3 meter – or longer? While one might argue that relativity theory saves us here (relativistic length contraction should cause this length to reduce to zero as the wave train zips by at the speed of light), this just doesn't feel right – especially when one takes a closer look at the assumptions behind.

[108] Note that the wave could be either left- or right-handed.

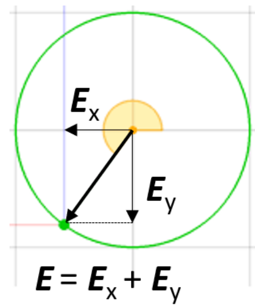


Figure 35: The one-cycle photon

It is a delightfully simple model: the photon is just one single cycle traveling through space and time, which packs one unit of angular momentum ($\hbar$) or – which amounts to the same, one unit of physical action (h). This gives us an equally delightful interpretation of the Planck-Einstein relation (f = 1/T = E/h) and we can, of course, do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the E = mc^2 mass-energy equivalence relation can be written as p = mc = E/c for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

Let us now try something more adventurous: let us try to calculate the strength of the electric field. How can we do that? Energy is some force over a distance. What distance should we use? We could think of the wavelength, of course.

However, the formulas above imply the following equation: E·λ = h·c. This suggest we should, perhaps, associate some *radius* with the wavelength of our photon. We write:

$$\mathrm{E} \cdot \frac{\lambda}{2\pi} = \mathrm{E} \cdot r = \hbar \cdot c \Leftrightarrow r = \frac{\lambda}{2\pi} = \frac{\hbar \cdot c}{\mathrm{E}}$$

A strange formula? The reader can check the physical dimensions. They all work out: we do get a distance – something that is expressed in *meter*. But why the 2π factor? We do not want to confuse the reader too much but let us quickly re-insert the graph on the presumed *Zitterbewegung* of a free electron – which is interpreted as an oscillation of a pointlike *charge* (with *zero* rest mass) moving about a center at the speed of light. Now, as the electron starts moving along some trajectory at a relativistic velocity (i.e. a velocity that is a *substantial* fraction of c), the *radius* of the oscillation will have to diminish – because the tangential velocity remains what it is: c. The geometry of the situation (see below) shows the circumference becomes a wavelength in this process.

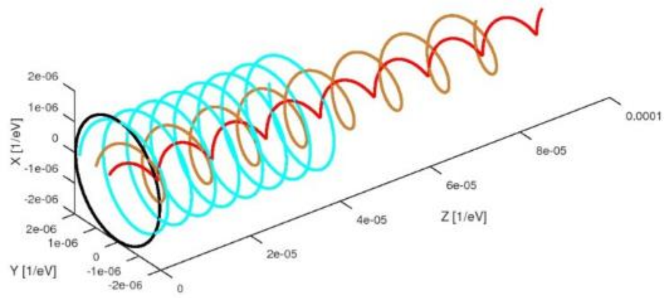


Figure 36: The Compton radius must decrease with increasing velocity

We have probably confused the reader now, but he or she should just hang on for a while. Let us just jot down the following expression and then we can think about it:

$$\mathrm{E}_\gamma = \mathrm{F}_\gamma \cdot r_\gamma = \mathrm{F}_\gamma \cdot \frac{\lambda_\gamma}{2\pi}$$

We use the γ subscript to denote we're talking the energy, force and radius in the context of a photon because – in order to justify the formula above – we will remind ourselves of one of the many meanings of the fine-structure constant here: as a coupling constant, it is defined as the ratio between (1) $k \cdot q_e^2$ and (2) E·λ. We can interpret this as follows:

1. The $k \cdot q_e^2$ in this ratio is just the product of the electric potential between two elementary charges (we should think of the proton and the electron in our hydrogen atom here) and the distance between them:

$$\mathrm{U}(r) = \frac{k \cdot \mathrm{q_e}^2}{r} = \frac{\mathrm{q_e}^2}{4\pi\varepsilon_0 r} \Leftrightarrow k \cdot \mathrm{q_e}^2 = \mathrm{U}(r) \cdot r$$

2. The fine-structure constant can then effectively be written as:

$$\alpha = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{U}(r) \cdot r}{\mathrm{E}_{photon} \cdot r_{photon}}$$

We can also write this in terms of forces times the *squared* distance:

$$\alpha = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F}_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F}_\gamma \cdot r_\gamma^2} = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma}$$

This doesn't look too bad. We use B as a subscript in the denominator to remind ourselves we are talking the Bohr energies and radii. Let us write it all out – using the generalized formulas (n = 1, 2,…) above – to demonstrate the consistency of this formula:

$$\alpha = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 \mathrm{m}c^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}}{\mathrm{E}_\gamma \cdot \frac{\hbar \cdot c}{\mathrm{E}_\gamma}} = \alpha$$

Onwards! We think the following formula for the force may make sense now:

$$\mathrm{F}_\gamma = \frac{\mathrm{E}_\gamma}{r_\gamma} = \frac{2\pi \cdot \mathrm{E}_\gamma}{\lambda_\gamma} = \frac{2\pi \cdot h \cdot f_\gamma}{\lambda_\gamma} = \frac{2\pi \cdot h \cdot c}{{\lambda_\gamma}^2}$$

The electric field (E) is the force per unit charge which, we should remind the reader, is the *coulomb* – *not* the electron charge. Dropping the subscript, we get a delightfully simple formula for the strength of the electric field vector for a photon[109]:

$$E = \frac{\frac{2\pi hc}{\lambda^2}}{1} = \frac{2\pi hc}{\lambda^2} = \frac{2\pi \mathrm{E}}{\lambda} (\frac{N}{C})$$

Let us calculate its value for our 10.2 eV photon. We should, of course, express the photon energy in SI units here:

$$E \approx \frac{2\pi \cdot 1.634 \times 10^{-18}\, J}{122 \times 10^{-9}\, m \cdot C} \approx 84 \times 10^{-12} \frac{\mathrm{N}}{\mathrm{C}}$$

This seems pretty reasonable![110] Let us make a final check on the logical consistency of this model. The energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Do we get any meaningful result when we apply that principle here? If we write the proportionality coefficient as k, we could write something like this:

$$\mathrm{E} = k \cdot a^2 \cdot \omega^2$$

It would be wonderful if this would give some meaningful result – and even more so if we could interpret the proportionality coefficient k as the mass m. Why? Because we have used the $\mathrm{E} = \mathrm{m}\cdot a^2\cdot\omega^2$ equation before: it gave us this wonderful interpretation of the *Zitterbewegung* as what we referred to as the *rest matter oscillation*. We will show, in the next section, that the idea of a two-dimensional oscillation can also be applied to the Rutherford-Bohr model. Hence, can we repeat the trick here? We can, but the amplitude of the oscillation here is the *wavelength*. We can then write:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

Sometimes physics can be *just nice*. I think we have a pretty good photon model here.

Before we move on, we need to answer an obvious question: what happens when an electron jumps several Bohr orbitals? The angular momentum between the orbitals will then differ by *several* units of $\hbar$. What happens to the photon picture in that case? It will pack the energy difference, but should it also pack several units of $\hbar$. In other words, should we still think of the photon as a one-cycle oscillation, or will the energy be spread over several cycles?

We will let the reader think about this, but our intuitive answer is: the photon is a spin-one particle and, hence, its energy should, therefore, be packed in one cycle only. This is also necessary for the

[109] The E and E symbols should not be confused. E is the magnitude of the electric field vector and E is the energy of the photon. We hope the italics (E) – and the context of the formula, of course! – will be sufficient to distinguish the electric field vector (E) from the energy (E).

[110] We got a rather non-sensical value in one of our first papers (http://vixra.org/abs/1812.0028) but that's because we used the electron charge instead of the unit charge to calculate the field.

consistency of the interpretation here: when everything is said and done, we do interpret the wavelength as a *physical* distance. To put it differently, the equation below needs to make sense:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

VIII. The two-dimensional oscillator model: additional considerations

Let us summarize what we have presented so far. We explained the rest mass of the electron in terms of its *Zitterbewegung*. This interpretation of an electron combines the idea of motion with the idea of a pointlike *charge*, which has no inertia and can, therefore, move at the speed of light. The illustration below described the presumed circular oscillatory motion of the charge (the *Zitterbewegung*). We got wonderful results. The most spectacular result is the explanation for the *rest* mass of an electron: it is the equivalent mass of what we referred to as the *rest matter oscillation*.

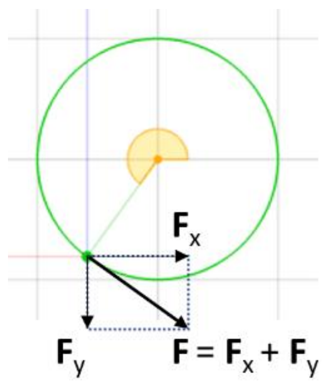


Figure 37: The *Zitterbewegung* model of an electron

The table summarizes the properties – angular momentum, magnetic moment, g-factor, etc. – we calculated:

Table 4: The properties of the free electron (spin-only)

Spin-only electron (*Zitterbewegung*)
$\mathrm{S} = \mathrm{h}$
$\mathrm{E} = \mathrm{m}c^2$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$
$v = c$
$\mathrm{L} = I \cdot \omega = \frac{\hbar}{2}$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}} \hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}} \frac{\mu}{\mathrm{L}} = 2$

The reader should keep his wits about him[111] here: the *Zitterbewegung* model should not be confused with our Bohr atom. We do not have any centripetal force here. There is no nucleus or other charge at the center of the *Zitterbewegung*. Instead of a tangential momentum vector, we have a tangential *force* vector (**F**), which we thought of as being the resultant force of two perpendicular oscillations.[112] This led us to boldly equate the E = mc^2, E = m·a^2·ω^2 and E = ħ·ω equations – which gave us all the results we wanted. The *zbw* model – which, as we have mentioned in the footnote above, is inspired by the solution(s) for Dirac's wave equation for free electrons – tells us the velocity of the pointlike *charge* is equal to *c*. Hence, *if* the *zbw* frequency would be given by Planck's energy-frequency relation (ω = E/ħ), *then* we can easily combine Einstein's E = mc^2 formula with the radial velocity formula ($c = a\cdot\omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$r_{\text{Compton}} = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_e}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The calculations relate the Bohr radius to the Compton radius through the fine-structure constant:

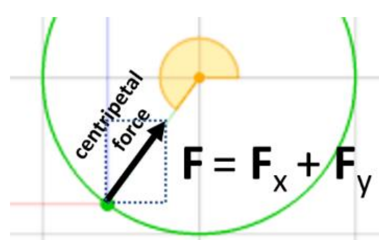
$$r_{\text{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{mq_e}^2} = \frac{1}{\alpha}\cdot r_{\text{Compton}} = \frac{\hbar}{\alpha\mathrm{m}c} \approx 53 \times 10^{-12}\ \mathrm{m}$$

The fine-structure constant also relates the respective velocities, frequencies and energies of the two oscillations. We wrote:

$$v = \alpha\cdot c = r_{\mathrm{B}}\cdot\omega_{\mathrm{B}} = \frac{\hbar}{\alpha\mathrm{m}c}\cdot\frac{\alpha^2\mathrm{m}c^2}{\hbar} = \alpha\cdot c \Leftrightarrow \omega_{\mathrm{B}} = \frac{\alpha^2\mathrm{m}c^2}{\hbar}$$

As we mentioned before, the formula for the frequency of the motion of the electron in the Bohr orbitals reflects the first *de Broglie* relation: f_B = E/h = α^2mc^2/h. Needless to say, the cycle time T is given as a function of the Bohr loop frequency by T = 1/f_B = 2π/ω_B. [In this section, we will just use the formulas for the first Bohr orbital (*n* = 1). It is easy generalize for *n* = 2, 3, 4, etc.] However, we noted that the α^2mc^2 is *twice* the Rydberg energy – and, unlike some physicists, we do care about a 1/2 or π factor in our model of a Bohr electron. Hence, we should have a look at this energy concept.

The E = α^2mc^2 energy concept is the energy that is associated with the loop. It is *twice* the kinetic energy, but it is a different energy concept altogether. In line with our interpretation of the elementary wavefunction in the context of our one-cycle photon and our free (spin-only) electron, we are thinking of the orbital motion as being driven by a two-dimensional oscillation, as illustrated below.



[111] The him could be a her, of course.
[112] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

Figure 38: The oscillator model for the Bohr orbital

We look at the centripetal force as a *resultant* force here – a vector sum of two perpendicular components: **F** = $\mathbf{F}_x + \mathbf{F}_y$. Needless to say, the **boldface** here indicates *vectors*: these force *components* have a magnitude as well as a direction. We can now develop yet another interpretation of the elementary wavefunction and think of a dual view of what is going on. Let us start with the description of the physical position (i.e. the *x*- and *y*-coordinates) of the electron. This is the green dot in the illustration of Euler's function above. Its motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot cos(\theta) + i \cdot a \cdot sin(\theta) = (x, y)$$

We can now think of this motion being driven by two perpendicular oscillations. These oscillations are associated with a kinetic and a potential energy. We illustrate this below for *one* oscillator only.

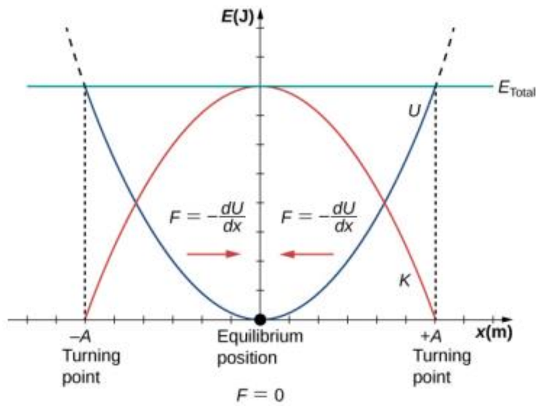


Figure 39: Kinetic (K) and potential energy (U) of an oscillator

Now, if the amplitude of the oscillation is equal to *a*, then we know that the sum of the kinetic and potential energy of the oscillator will be equal to $(1/2) \cdot m \cdot a^2 \cdot \omega^2$. In this case (the Bohr orbital), we have two oscillators, and we can add their kinetic and potential energies because of the 90-degree phase difference. Indeed, it is easy to see that the *total* kinetic energy – added over the *two* oscillators – will effectively be constant *over the cycle* and will be equal to:

$$\text{K.E.} = \frac{1}{2}\text{m} \cdot r_\text{B}{}^2 \cdot \omega^2 = \frac{1}{2}\text{m} \cdot v^2 = \frac{1}{2}\alpha^2 \cdot \text{m} \cdot c^2$$

The potential energy will be equal to the kinetic energy and we, therefore, get the desired result: the *total* energy of the loop is equal to $\text{E} = \alpha^2 \text{mc}^2$. We can now re-write the quantum of action as the product of the energy and the cycle time:

$$h = \text{E} \cdot \text{T} = \alpha^2 \text{m}c^2 \cdot \frac{2\pi \cdot r_\text{B}}{v} = \alpha^2 \text{m}c^2 \cdot \frac{2\pi \cdot r_\text{C}}{\alpha \cdot c \cdot \alpha} = \text{m}c^2 \frac{2\pi \cdot \hbar}{c \cdot \text{m} \cdot c} = h$$

Of course, we can also write it as the product of the (linear) momentum and the distance along the loop:

$$h = \text{p} \cdot \lambda_\text{B} = \text{m} \cdot v \cdot 2\pi \cdot r_\text{B} = \text{m} \cdot \alpha \cdot c \cdot 2\pi \cdot \frac{\hbar}{\alpha \text{m}c} = h$$

All makes sense. Now, we said we have a *dual* view of the meaning of the wavefunction here. What is the dual view? It is that of the force vector: we will want to write the energy as the product of a force over a distance. Hence, what is the force and what is the distance here? The Bohr model implies the circular motion of the *electron* is driven by (1) its inertia[113] and (2) a centripetal force (because of the presence of a nucleus with the opposite charge). The geometry of the situation shows we can write $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ as:

$$\boldsymbol{F} = -F\cdot\cos(\omega t) - i\cdot F\cdot\sin(\omega t) = -F\cdot e^{i\theta}$$

The nature of this force is electric, of course. Hence, we should write in in terms of the electric field vector E[114]: $\boldsymbol{F} = q_e\boldsymbol{E}$. The electric field is, of course, the force on the unit charge which, in this case, is a force between q_e (the electron) and $-q_e$ (the proton or hydrogen nucleus).[115] Let us calculate the magnitude of the force by using the fine-structure constant to check the consistency of the model:

$$F = q_e E = \frac{{q_e}^2}{4\pi\varepsilon_0 {r_B}^2} = \frac{\alpha\hbar c}{{r_B}^2} = \frac{\hbar v}{{r_B}^2} = \frac{\hbar r_B \omega_B}{{r_B}^2} = \frac{\hbar}{r_B}\frac{\alpha^2 mc^2}{\hbar} = \frac{E}{r_B}$$

This $F = q_e E = \mathrm{E}/r_B$ is confusing (E is the electric field, but E is the *energy*) but very interesting because it allows us to write the quantum of action in its usual dimensions – which is the product of a force, a distance (the radius of the oscillation, in this particular case), and a time:

$$h = F\cdot r\cdot T = \frac{E}{r_B}\cdot r_B\cdot\frac{1}{f} = \frac{E}{r_B}\cdot r_B\cdot\frac{h}{E} = h$$

Hence, we have a bunch of equivalent expressions for Planck's quantum of action – all of which help us to understand the *complementarity* of the various viewpoints:

$$h = p\cdot 2\pi\cdot r = p\cdot\lambda$$

$$h = E\cdot T = E/f$$

$$h = r\cdot T\cdot F = r\cdot T\cdot q_e E = r\cdot T\cdot E/r = E\cdot T$$

We could also combine these formulas with the classical formulas for a centripetal force – think of the $F = m\cdot r\cdot\omega^2$ and $F = m\cdot v^2/r = p\cdot v/r$ formulas here – but we will let the reader play with that.

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113 Note the difference with the *Zitterbewegung* model, which assumes a pointlike *charge* with *no inertia to motion*. Its orbital velocity is, therefore, effectively equal to the speed of light (c). This is *very* different from the Bohr model, in which the electron moves at a non-relativistic speed $v = \alpha c$ with $\alpha \approx 0.0073$. However, the two models are obviously complementary: the *Zitterbewegung* model – Dirac's electron, we might say – effectively explains the (rest) mass of the Bohr electron.

114 Symbols may be confusing. We use E for the energy, but E for the electric field vector. Likewise, I is a moment of inertia, and I is an electric current. The context is usually clear enough to make out what is what.

115 The concepts of potential, potential energy and the electric field can be quite confusing. The potential and the potential energy of a charge in a field vary with $1/r$. The electric field is the electric force – generally defined as the *Lorentz* force $\boldsymbol{F} = q\boldsymbol{E} + q(\boldsymbol{v}\times\boldsymbol{B})$ – on the unit charge. Hence, the $\boldsymbol{F} = q\boldsymbol{E}$ formula here is nothing but the $\boldsymbol{E} = \boldsymbol{F}/q$ formula. The electric field varies with $1/r^2$ and is, therefore, associated with the inverse-*square* law. It is also quite confusing that q_e is actually the (supposedly negative) electron charge and that we have to, therefore, use a minus sign for the charge of the (supposedly positive) proton charge – but then the signs always work out, of course.

The point is: there is an *energy* in this oscillation, and the energy makes sense if we think of it as a two-dimensional oscillation. We can write this two-dimensional oscillation – using Euler's formula - in various but complementary ways. We can use the position vector, the force vector, or the electric field vector:

$$\boldsymbol{F} = -F_x \cdot \cos(\omega t) - i \cdot F_x \cdot \sin(\omega t) = -F \cdot e^{i\theta}$$

$$\boldsymbol{E} = -(E/q_e) \cdot \cos(\omega t) - i \cdot (E/q_e) \cdot \sin(\omega t) = -F \cdot e^{i\theta}$$

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot cos(\theta) + i \cdot a \cdot sin(\theta) = (x, y)$$

The various viewpoints of the oscillation are complementary. They pack the same energy ($E = \alpha^2 mc^2$), and they pack one unit of physical action (h). We will leave it to the reader to generalize for the n = 2, 3, etc. orbitals. It is an easy exercise: the energy for the higher loops is equal to $E_n = \alpha^2 mc^2/n^2$ and the associated action is equal to $S = n \cdot h$. One obvious way to relate both is through the frequency of the loop. We write:

$$f_n = \frac{E_n}{S_n} = \frac{\frac{1}{n^2}\alpha^2 mc^2}{nh} = \frac{\alpha^2}{n^3 h} mc^2$$

IX. The fine-structure constant as a scaling constant

The fine-structure constant pops up as a dimensional scaling constant in the calculations above. It relates the Bohr radius to the Compton radius, for example:

$$r_{\text{Bohr}} = \frac{\hbar^2}{me^2} = \frac{4\pi\varepsilon_0 \hbar^2}{m{q_e}^2} = \frac{1}{\alpha} \cdot r_{\text{Compton}} = \frac{\hbar}{\alpha mc} \approx 53 \times 10^{-12} \text{ m}$$

But it also relates the respective velocities, frequencies and energies of the two oscillations. We may summarize these relations in the following equations:

$$v = \alpha \cdot c = r_B \cdot \omega_B = \frac{\hbar}{\alpha mc} \cdot \frac{\alpha^2 mc^2}{\hbar} = \alpha \cdot c$$

But this is not the only meaning of the fine-structure constant. We know it pops up in many other formulas as well. To name just a few:

1. It is the mysterious quantum-mechanical *coupling* constant.
2. It explains the so-called anomalous magnetic moment – which, as we will explain in a moment, might *not* be anomalous at all!
3. Last but not least, it explains the *fine structure* of the hydrogen spectrum – which is where it got its name from, of course!

Can we make some more sense of this as a result of the interpretations we have offered above? Let us start with the coupling constant because there is a lot of nonsensical writing on that.[116] We basically

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[116] Feynman's *QED: The Strange Theory of Light and Matter* (1985) refers to its (negative) *square root* as the coupling constant, and states that is "the amplitude for a real electron to emit or absorb a real photon." We take it to be just one example of an ambiguous remark by a famous physicist that is being explained by an amateur physicist. The book was not written by Richard Feynman: it is a transcription of a short series of lectures by Feynman for a popular audience. We are not impressed by the transcription.

showed that, as a coupling constant, the fine-structure continues to act as a dimensional scaling constant. We wrote:

$$\alpha = \frac{k \cdot q_e^2}{\hbar \cdot c} = \frac{F_B \cdot r_B^2}{F_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{F_B \cdot r_B^2}{F_\gamma \cdot r_\gamma^2} = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma}$$

We use B as a subscript in the denominator to remind ourselves we are talking the Bohr energies and radii. Let us use the generalized formulas (n = 1, 2,…) for the Bohr orbitals once again and write it all out:

$$\alpha = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 mc^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{mc}}{E_\gamma \cdot \frac{\hbar \cdot c}{E_\gamma}} = \alpha$$

While the formula is obvious, its interpretation is not necessarily as obvious: what *is* this product of an energy and a radius? How should we interpret this? The physical dimension of this product (in the denominator and the numerator, of course) is J·m = N·m·m = N·m^2. We get the same physical dimension if we multiply action or angular momentum with a velocity, so let us try this to check if it makes us any wiser:

$$\alpha = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma} = \frac{L_n \cdot v_n}{L_\gamma \cdot v_\gamma} = \frac{n\hbar \cdot \frac{1}{n}\alpha c}{\hbar \cdot c} = \alpha = \frac{S_n \cdot v_n}{S_\gamma \cdot v_\gamma} = \frac{nh \cdot \frac{1}{n}\alpha c}{h \cdot c} = \alpha$$

The formulas show we should, most probably, just think of them as yet another expression of the idea of a scaling constant.

Let us think of the fine-structure constant in yet one more way. We know the Compton and Bohr radius are related through the fine-structure constant. We used this formula many times already:

$$r_C = \alpha \cdot r_B$$

Let us write this out:

$$r_C = \frac{\hbar c}{mc^2} = \frac{\hbar c}{E_e}$$

$$r_B = \frac{\hbar c}{\alpha mc^2} = \frac{\hbar c}{E_B}$$

The E_e is just the (rest) energy of the electron, and E_B is the energy in the (first) Bohr orbital. Hence, we can also write the fine-structure constant as the ratio between these two energies:

$$\alpha = \frac{r_C}{r_B} = \frac{\frac{\hbar c}{E_e}}{\frac{\hbar c}{E_B}} = \frac{E_B}{E_e}$$

Because $r_n = n^2 r_B$ and $E_n = E_B/n^2$, we know that $r_n = \frac{n^2 \hbar c}{\alpha mc^2} = \frac{\hbar c}{E_n}$ and, hence, we can easily generalize for the n = 2, 3,… orbitals:

$$\alpha = \frac{r_{\mathrm{C}}}{r_n} = \frac{\frac{\hbar c}{\mathrm{E_e}}}{\frac{\hbar c}{\mathrm{E}_n}} = \frac{\mathrm{E}_n}{\mathrm{E_e}}$$

The explorations above - and the interpretation of the fine-structure constant as a scaling constant – raise an interesting question. We know there is also the idea of a classical electron radius, which is related to the Compton radius in the same way as the Compton radius to the Bohr radius:

$$r_{\mathrm{e}} = \alpha \cdot r_{\mathrm{C}} = \alpha^2 \cdot r_{\mathrm{B}}$$

We have already explained the second identity ($\alpha r_{\mathrm{C}} = \alpha^2 r_{\mathrm{B}} \Leftrightarrow r_{\mathrm{C}} = \alpha r_{\mathrm{B}}$) but what about $r_{\mathrm{e}} = \alpha r_{\mathrm{C}}$? Let us think about that in a separate section.

X. The fine-structure constant and the classical electron radius

Let us write all out and see if there is something triggering some idea:

$$r_{\mathrm{e}} = \frac{\mathrm{e}^2}{\mathrm{m}c^2} = \alpha \frac{\hbar c}{\mathrm{m}c^2}$$

We, once again, have two energies in the numerator – but they are the same! Hence, when writing the fine-structure constant as the ratio between the two radii, we get:

$$\alpha = \frac{r_e}{r_{\mathrm{C}}} = \frac{\frac{\mathrm{e}^2}{\mathrm{m}c^2}}{\frac{\hbar c}{\mathrm{m}c^2}} = \frac{\mathrm{e}^2}{\hbar c} = k\mathrm{q_e}^2 = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q_e}^2}{\hbar c}$$

We just get the usual formula for the fine-structure constant here. What does it mean in terms of interpretation? Here we should probably try to think of the meaning of e^2. There is something interesting here: the elementary charge e^2 has the same physical dimension – the *joule-meter* (J·m) – as the h·*c* = E·λ product:

$$[\mathrm{e}^2] = \left[\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q_e}^2}{\hbar c}\right] = \frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}^2}\cdot\mathrm{C}^2 = \mathrm{N}\cdot\mathrm{m}^2 = \mathrm{J}\cdot\mathrm{m}$$

What was that h·*c* = E·λ product again? We got it in the context of our photon model. To be precise, we got it by applying the *second* de Broglie equation to a photon:

$$h = \mathrm{p}\cdot\lambda = \frac{\mathrm{E}}{c}\cdot\lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

In fact, it appears we may apply this relation to any particle that is traveling at the speed of light. *Huh?* What other particle do we have? Our pointlike *charge* in the *Zitterbewegung* model of an electron: this charge has, effectively, no rest mass and, therefore, does make us think of a photon. But we should be precise here: it is the *square* of the elementary charge that that *joule-meter* dimension. We write:

$$[\mathrm{e}^2] = [\mathrm{E}]\cdot[\lambda] = [h]\cdot[c]$$

This is strange: what energy and what wavelength would we associate with this pointlike charge. I am not sure – but if we try the energy and the circumference of the loop of the *Zitterbewegung,* we get a sensible relation on the right-hand side:

$$\mathrm{E} \cdot \lambda = \mathrm{m}c^2 \cdot \frac{\hbar}{\mathrm{m}c} = h \cdot c$$

Obvious, you'll say. But, *no*, this is not obvious: we are not talking the energy and the mass of a photon here but the energy and the mass of… Well… Our pointlike charge in its *Zitter* motion.

And what about the suggestion we should be able to write something like $e^2 = E \cdot \lambda$? Well… We can start by re-writing the formula for the classical electron radius so it gives us a product of an energy and a distance:

$$\mathrm{e}^2 = r_\mathrm{e}\mathrm{m}c^2 = r_e\mathrm{E}$$

Does this make sense? Yes, it does. It gives us the formula for the fine-structure constant once again:

$$\mathrm{e}^2 = r_\mathrm{e}\mathrm{m}c^2 = \alpha r_\mathrm{C}\mathrm{E} = \alpha\frac{\hbar c}{\mathrm{m}c^2}\mathrm{E} = \alpha\frac{\hbar c}{\mathrm{m}c^2}\mathrm{E} = \alpha\hbar c \Leftrightarrow \alpha = \frac{\mathrm{e}^2}{\hbar c}$$

By now, the reader is probably tired of these gymnastics and, hence, we will stop here. What was the use? Interpretation. The formulas are not presenting anything new: we have just been substituting and re-arranging equations but we have, hopefully, succeeded in presenting a coherent picture while doing so.

XI. The fine-structure constant and the anomalous magnetic moment

Introduction

This is going to be a long and difficult digression. However, it is essential. Let us briefly remind the reader of the context. The theoretical derivation of an exact value for the anomalous magnetic moment of a real-life electron – and, importantly, its agreement with what is experimentally measured – is considered to be one of the greatest triumphs of modern quantum mechanics.

Dirac – and many others – weren't that convinced. Let me quote from Dirac's very last paper on the topic, which was written in 1984, so that's the year he died. It was entitled: "The inadequacies of quantum field theory" and, in light of the fact he died the very same year, I think it is fair to say – I am just talking as a rational human being here[117] – that it must the final judgment of a genius on these matters on the state of the post-WW II developments on the topic. The title of the paper – the *inadequacies* of quantum field theory – is, obviously, very significant, and this one line sums it all up:

> "These rules of renormalization give, surprisingly, excessively good agreement with experiments. Most physicists say that these working rules are, therefore, correct. I feel that is not an adequate reason. Just because the results happen to be in agreement with observation does not prove that one's theory is correct."

Read it again: you'll either burst out laughing – saying he must have been crazy (which he might have been - depending on your definition of crazy) or, else, that he had a lot of courage in the face of death. Indeed, why wouldn't one be happy with a theory that's in agreement with observation? Is there anything better? We think there is. That's what this book is all about. We, therefore, think it's the latter: Dirac had a lot of courage in the face of death, and so he spoke out lot. It was in vain, unfortunately, as the Wikipedia article on Dirac, from which I am quoting here, notes that "his refusal to accept re-normalization resulted in his work on the subject moving increasingly out of the mainstream."

Of course, the challenge is out there. If we want to challenge orthodox theory, then it should be possible to explain the anomalous magnetic moment based on some *form factor* that would come out of a classical electron model. We suggested this recently[118] and, hence, this chapter is basically a copy of the paper we then wrote. While we initially thought about these things from a learning perspective only – we just wanted to possibly identify a better *didactic* approach to teaching quantum mechanics – the idea seems to have taken some life on its own now.[119]

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[117] Having said that, I do take pleasure in the fact that the stuff I am working on addresses key intuitions of geniuses such as Einstein, Dirac and – more recently – John Stewart Bell, who didn't like his own No-Go Theorem.

[118] See: Jean Louis Van Belle, *Testing QED. The Other Game in Town*, 26 December 2018, http://vixra.org/pdf/1812.0419v1.pdf. Also see: Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018, http://vixra.org/pdf/1812.0233v3.pdf. These papers should, preferably, be read in conjunction with a more recent paper on how the fine-structure constant relates the various layers in the motion of an electron: *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

[119] Our physics blog attracts a fair amount of comments from fellow amateur physicists. These remarks are encouraging but do not add any credibility to the model (on the contrary, we'd say). However, we also had discussions with some researchers on Kerr-Newman and *Zitterbewegung* models. While we speak a very different language, these discussions suggest the key ideas might make some sense. Dr. Burinskii – one of the *core* researchers on the Kerr-Newman model of an electron – recently wrote me he has the α correction. So he is missing the 2π factor but that's probably just some stupid calculation error. And, of course, he should, hopefully, also manage to get some of the second- or third-order terms. Let us see where this research goes.

What is a 'classical' electron model? We use this term to refer to any theory of an electron that does *not* invoke perturbation theory. We do not like perturbation theory because of the very same reason that made the founding fathers (Heisenberg, Dirac, Pauli, ...) skeptical about the theory they had created.[120] Interestingly, Ivan Todorov – whose paper notes the above – also speaks of the theoretical value of the spin angular momentum (g_{spin} = 2) as a "dogma" and mentions two letters of Gregory Breit to Isaac Rabi, which may be interpreted as Breit defending the idea that an intrinsic magnetic moment "of the order of $\alpha\mu_B$" may not be anomalous at all.[121] Needless to say, the issue is quite controversial because a classical explanation of the anomalous magnetic moment would question some of the rationale behind the award of two Nobel Prizes for physics.

Let us be precise here. Polykarp Kusch got (half of) the 1955 Nobel Prize "for his precision determination of the magnetic moment of the electron."[122] As such, we should *not* associate him with the theory behind. Having said that, the measurement obviously corroborated the new theories of what Todorov refers to as "the younger generation" of physicists – in particular Richard Feynman, Julian Schwinger and Shinichiro Tomonaga, who got their 1965 Nobel Prize for "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles" – for the *theory*, that is.

What Brian Hayes refers to as "the tennis match between experiment and theory" [123] seems to be a game without end. The question is: is there another game in town? We think there might be one.

The new quantum physics

We will *not* explain perturbation theory here.[124] We only want to give a quick overview of its results in the context of the *theoretical* explanation of the anomalous magnetic moment. Indeed, we described the methodology of its *measurement* above and, hence, we will not repeat ourselves here. In fact, we suggest the reader directly consults the 2009 article of the Harvard University group that does these experiments.[125] We will just note that the confusion starts with the definition of the anomalous magnetic moment. It is actually *not* a magnetic moment but a *gyromagnetic ratio* (i.e. a *ratio* between a magnetic moment and an angular momentum) and it's defined as:

Something inside of me says no one would talk about Bell's Theorem anymore, because we would - effectively - have a whole new theory based on a decent electron (and photon) model. But... Well... Let's see... 😉

[120] See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

[121] For a more detailed account of the substance of these conversations, see: Silvan S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga* , p. 222-223.

[122] See: https://www.nobelprize.org/prizes/physics/1955/summary/ and https://www.nobelprize.org/prizes/physics/1965/summary/.

[123] See: Brian Hayes, *Computing Science: g-ology*, in: *American Scientist*, Vol. 92, No. 3, May-June 2004, pages 212-216. The subtitle says it all: it is an article 'on the long campaign to refine measurements and theoretical calculations of a physical constant called the *g* factor of the electron.' https://pdfs.semanticscholar.org/4c12/50f66fc1fb799610d58f25b9c1e1c2d9854c.pdf.

[124] The interested reader may consult any standard textbook on that. See, for example, Jon Mathews and R.L. Walker, *Mathematical Methods of Physics*, 1970.

[125] D. Hanneke, S. Fogwell, N. Guise, J. Dorr and G. Gabrielse, *More Accurate Measurement of the Electron Magnetic Moment and the Fine-Structure Constant*, in: Proceedings of the XXI International Conference on Atomic Physics, 2009. We prefer this article over the original 2006 article (G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *New Determination of the Fine Structure Constant from the Electron g Value and QED*, Phys. Rev. Lett. 97, 030802, 2006) because it can be freely consulted online: http://gabrielse.physics.harvard.edu/gabrielse/papers/2009/PushingTheFrontiersOfAtomicPhysics.pdf.

$$a_e = \frac{g}{2} - 1$$

The 2009 article states that the *measured* value of g is equal to 2.00231930436146(56). The 56 (between brackets) is the (un)certainty: it is equal to 0.00000000000056, i.e. 56 *parts per trillion* (ppt) and it is measured as a standard deviation.[126] Hence, a_e is equal to 0.00115965218073(28).

The so-called anomaly is the difference with the theoretical value for the *spin* angular momentum which came out of Dirac's equation for the free electron, which is equal to 2. The confusion starts here because there is no obvious explanation of why one would use the (theoretical) g-factor for the intrinsic spin of an electron (g = 2). The electron in the Penning trap that is used in these experiments is *not* a spin-only electron. It follows an orbital motion – that is one of the three or four layers in its motion, at least – and, hence, if some theoretical value for the g-factor has to be used here, then one should also consider the g-factor that is associated with the orbital motion of an electron, which is that of the Bohr orbitals (g = 1).

In any case, one would expect to see a *classical* coupling between (1) the precession, (2) the orbital angular momentum and (3) the spin angular momentum, and the situation is further complicated because of the electric fields in the Penning trap, which add another layer of motion. We illustrate the complexity of the situation below[127].

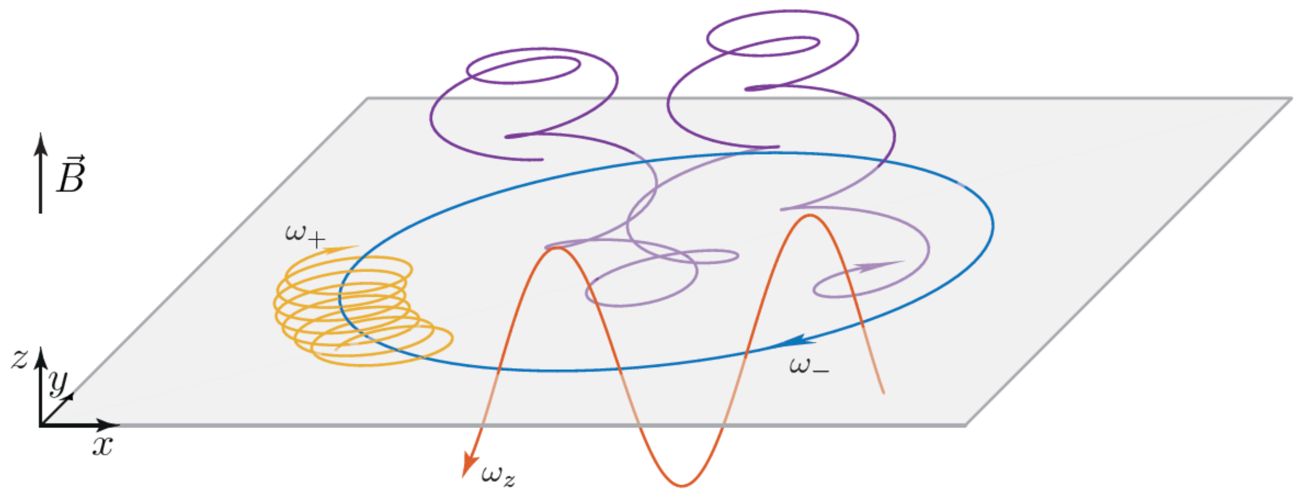

Figure 40: The three principal motions and frequencies in a Penning trap

The point we are trying to make is the following: the theoretical value for a_e (*zero*) would seem to need a better explanation. However, let us roll for a moment with the idea that – through the magic of classical coupling – that its theoretical value should be zero and that we, therefore, do have some anomaly here of the measured order of magnitude, i.e. a_e = 0.00115965218073(28). How is it being

[126] To be precise, the article gives the *measured* value for g/2, which is equal to 1.00115965218073(28).
[127] We took this illustration from an excellent article on the complexities of a Penning trap: *Cylotron frequency in a Penning trap*, Blaum Group, 28 September 2015, https://www.physi.uni-heidelberg.de/Einrichtungen/FP/anleitungen/F47.pdf. The motions are complicated because the Penning trap traps the electron using both electric as well as magnetic fields (the electric field is *not* shown in the illustration, but it is there). One should note the illustration does *not* show the intrinsic spin of the electron, which we should also consider. See our above-mentioned paper for a more detailed description of the various layers of motion.

explained? The new quantum physicists write it as (the sum of) a series of first-, second-, third-,…, n^{th}-order *corrections*:

$$a_e = \sum_n a_n \left(\frac{\alpha}{\pi}\right)^n$$

The first coefficient (a_1) is equal to 1/2 and the associated first-order correction is, therefore, equal to:

$$\alpha/2\pi \approx 0.00116141$$

Using "his renormalized QED theory", Julian Schwinger had already obtained this value back in 1947. He got it from calculating the "one loop electron vertex function in an external magnetic field." I am just quoting here from the above-mentioned article (Todorov, 2018). Julian Schwinger is, of course, one of the most prominent representatives of the second generation of quantum physicists, and he has this number on this tombstone. Hence, we surely do not want to question the depth of his understanding of this phenomenon. However, the difference that needs to be explained by the 2nd, 3rd, etc. corrections is only 0.15%, and Todorov's work shows all of these corrections can be written in terms of a sort of exponential series of α/2π and a *phi*-function ϕ(*n*) which had intrigued Euler for all of his life. We copy the formula for (the sum of) the first-, second- and third-order term of the theoretical value of a_e as calculated in 1995-1996 (*th* : 1996).[128]

$$\begin{aligned}
a_e(th:1996) &= \frac{1}{2}\frac{\alpha}{\pi} + \left[\phi(3) - 6\,\phi(1)\,\phi(2) + \phi(2) + \frac{197}{2^4\,3^2}\right]\left(\frac{\alpha}{\pi}\right)^2 \\
&+ \left[\frac{2}{3^2}\,(83\,\phi(2)\,\phi(3) - 43\,\phi(5)) - \frac{50}{3}\,\phi(1,3) + \frac{13}{5}\,\phi(2)^2\right. \qquad (19) \\
&+ \left.\frac{278}{3}\left(\frac{\phi(3)}{3^2} - 12\,\phi(1)\,\phi(2)\right) + \frac{34202}{3^3\,5}\,\phi(2) + \frac{28259}{2^5\,3^4}\right]\left(\frac{\alpha}{\pi}\right)^3 + \ldots \\
&= 1.159652201(27) \times 10^{-3}
\end{aligned}$$

We also quote Todorov's succinct summary of how this result was obtained: "Toichiro Kinoshita of Cornell University evaluated the 72 [third-order loop Feynman] diagrams numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. A year later, the last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna."

Apparently, the calculations are even more detailed now: the mentioned Laporta claims to have calculated 891 *four*-loop contributions to the anomalous magnetic moment.[129] One gets an uncanny feeling here: if one has to calculate a zillion integrals all over space using 72 third-order diagrams to calculate the 12th digit in the anomalous magnetic moment, or 891 fourth-order diagrams to get the next level of precision, then there might something wrong with the theory. Is there an alternative? We think there is, and the idea is surprisingly simple.

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[128] It is worth quoting Todorov's succinct summary of how this result was obtained: Toichiro Kinoshita of Cornell University evaluated the 72 [Feynman] diagrams [corresponding to the third-order loop] numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. later the last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna.
[129] See: Stefano Laporta, *High-precision calculation of the 4-loop contribution to the electron g-2 in QED*, as reported in: https://www.sciencedirect.com/science/article/pii/S0370269317305324.

Classical electron models

Mr. Burinskii would probably *not* wish to describe his Dirac-Kerr-Newman model of an electron as a classical electron model – and neither would he want to be considered as a classical physicist[130] – but that is what it is for us: a charge with a geometry in three-dimensional space. To be precise, it is a disk-like structure, and its *form factor* – read: the ratio between the radius and thickness of the disk – depends on various assumptions (as illustrated below) but reduces to the ratio between the Compton and Thomson radius of an electron when assuming classical (non-perturbative) theory applies. We quote from Mr. Burinskii's 2016 paper: "It turns out that the flat Compton zone free from gravity may be achieved without modification of the Einstein-Maxwell equations."

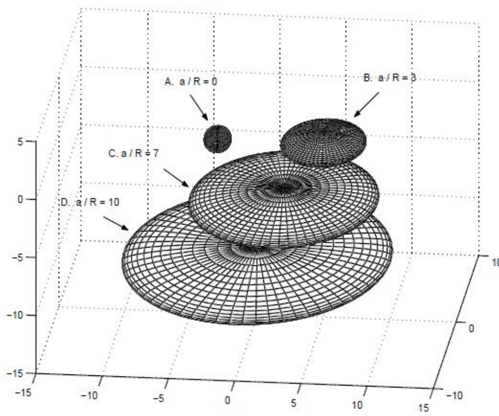

Figure 41: Alexander Burinskii's electron model

Hence, it would seem we get the fine-structure constant as the ratio of the Compton radius – i.e. the radius of the disk *R* – and the classical electron radius – i.e. the thickness of the disk *r* – out of a smart model based on Maxwell's and Einstein's equations, i.e. classical electromagnetism and general relativity theory:

$$\alpha = \frac{r}{R} = \frac{r_e}{r_C} = \frac{\mathrm{e}^2/\mathrm{m}c^2}{\hbar c/\mathrm{m}c^2} = \frac{\mathrm{e}^2}{\hbar c}$$

There is no need for smart quantum mechanics here! These results, therefore, confirm the intuitive but, admittedly, rather primitive *Zitterbewegung* model we introduced in our own papers. To illustrate the point, we would like to summarize one of the many possible interpretations of the fine-structure constant as a dimensional scaling constant here.[131]

First, we need to think about the meaning of e^2. There is something interesting here: the elementary charge e^2 has the same physical dimension – the *joule-meter* (J·m) – as the *hc* = Eλ product.

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[130] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008, https://arxiv.org/abs/hep-th/0507109. A more recent article of Mr. Burinskii (*New Path to Unification of Gravity with Particle Physics*, 2016, https://arxiv.org/abs/1701.01025, relates the model to more recent theories – most notably the "supersymmetric Higgs field" and the "Nielsen-Olesen model of dual string based on the Landau-Ginzburg (LG) field model." We admit we do not understand a word of this. As for Mr. Burinskii's general expertise (which is *quantum* physics, mainly), see his profile: https://www.researchgate.net/profile/Alexander_Burinskii.

[131] See the above-mentioned paper: Jean Louis Van Belle, *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

$$[e^2] = \left[\frac{{q_e}^2}{4\pi\epsilon_0}\right] = \frac{Nm^2C^2}{C^2} = Nm^2 = Jm$$

Now, what was that hc = Eλ product again? We get it in the context of the description of a photon. To be precise, we get it by applying one of the two *de Broglie* equations to a photon:

$$h = p\lambda = \frac{E}{c}\lambda \Longleftrightarrow \lambda = \frac{hc}{E}$$

The energy (E) and wavelength (λ) are, of course, the energy and the wavelength of our photon. However, it turns out it makes sense to apply these equations to any particle that moves at the speed of light. The reader will wonder: what other particle? Our electron has a rest mass, right? It does, but our *Zitterbewegung* model assumes this rest mass is the equivalent mass of the *rest matter oscillation*. This rest matter oscillation is a two-dimensional oscillation: a local circulatory motion, in fact. It is illustrated below.

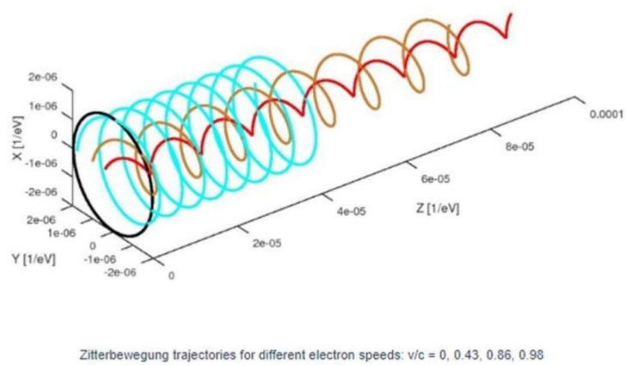

Figure 42: The *Zitterbewegung* model of an electron

The illustration above does not only show the *Zitterbewegung* itself but also another aspect of the theory. As the electron starts moving along some trajectory at a relativistic velocity (i.e. a velocity that is a substantial fraction of c), then the radius of the oscillation will have to diminish. Why? Because the tangential velocity remains what it is: c. Hence, the geometry of the situation shows that the radius of the oscillation becomes a wavelength in the process.[132] As Dirac noted in his Nobel Prize speech[133], the idea of the Zitterbewegung is very intuitive – and, therefore, very attractive – because it seems to give us a geometric (or, we might say, physical) explanation of the (reduced) Compton wavelength as the Compton scattering radius of an electron (a = ħ/mc).15 However, if we think of an actual physical interpretation, then it is quite obvious that the suggested plane of circulatory motion is not consistent with the measured direction of the magnetic moment – which, as the Stern-Gerlach experiment has shown us, is either up or down. Hence, we may want to think the plane of oscillation might be parallel to the direction of propagation, as drawn below.

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[132] We refer to the mentioned paper for a more elaborate *exposé* of the geometry.

[133] Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf.

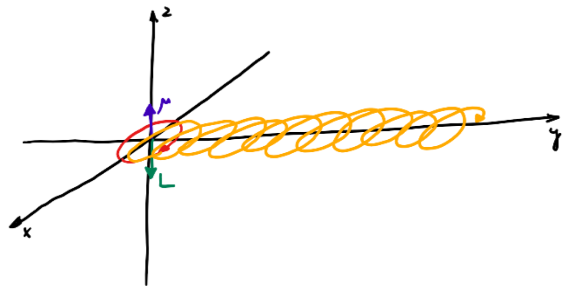

Figure 43: An alternative orientation of the *zbw* plane of rotation

We like the alternative picture of the *zbw* electron above not only because it is more consistent with the idea of the up-or-down orientation of the magnetic moment (cf. the Stern-Gerlach experiment) but also because it might provide us with a *physical* explanation of relativistic length contraction: as velocities increase, the radius of the circular motion becomes smaller (as illustrated above) which, in this model, may be interpreted as a contraction of the size of the *zbw* electron.

However, these remarks are not the point here. Let us return to our discussion of the anomalous magnetic moment.

How to test the classical electron models

Mr. Burinskii's model is very flexible. If one limits the assumptions - combining gravity and electromagnetism, we get the *Zitterbewegung* electron – a simple disk-like structure whose *form factor* is given by the fine-structure constant:

$$\alpha = \frac{r}{R} = \frac{r_e}{r_C} = \frac{\mathrm{e}^2/\mathrm{m}c^2}{\hbar c/\mathrm{m}c^2} = \frac{\mathrm{e}^2}{\hbar c}$$

When calculating the angular momentum, this form factor translates into a simple ½ factor when calculating the moment of inertia. We write $I = \mathrm{m}r^2/2$ – as opposed to the $I = \mathrm{m}r^2$ formula we would use for a pure orbital moment. This effectively gives us Dirac's theoretical value for the gyromagnetic ratio (*g*-factor) of the *spin-only* electron: g = 2. The table below summarizes the difference between the spin and orbital angular momentum.

Table 5: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr orbitals)**
$S = h$	$S_n = nh \text{ for } n = 1, 2, \dots$
$E = mc^2$	$E_n = -\frac{1}{2}\frac{\alpha^2}{n^2}mc^2 = -\frac{1}{n^2}E_R$
$r = r_C = \frac{\hbar}{mc}$	$r_n = n^2 r_B = \frac{n^2 r_C}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{mc}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{mc}{\hbar} = \frac{E}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}mc^2 = \frac{\frac{1}{n^2}\alpha^2 mc^2}{n\hbar}$
$L = I \cdot \omega = m\frac{\hbar^2}{m^2c^2}\frac{E}{\hbar} = \frac{\hbar}{2}$	$L_n = I \cdot \omega_n = n\hbar$
$\mu = I \cdot \pi r_C{}^2 = \frac{q_e}{2m}\hbar$	$\mu_n = I \cdot \pi r_n{}^2 = \frac{q_e}{2m}n\hbar$
$g = \frac{2m}{q_e}\frac{\mu}{L} = 2$	$g_n = \frac{2m}{q_e}\frac{\mu}{L} = 1$

As we mentioned in our paper[134], we will have a classical coupling between the two moments because of the Larmor precession of the electron in the Penning trap, as illustrated below. The *effective* current and the *effective* radius of the orbital motion will, therefore, *not* be equal to the values one would get from using the formulas in the right-hand column of the table above.[135]

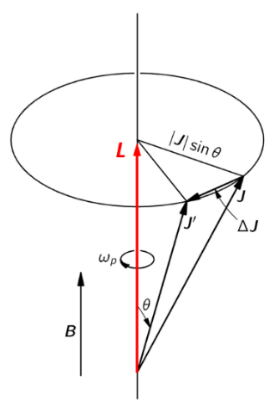

Figure 44: The precession of an orbital electron

Now, this classical coupling may or may not explain the bulk of what is actually being measured in these famous experiments measuring the (anomalous or not) magnetic moment of an electron in a Penning

[134] Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018, http://vixra.org/pdf/1812.0233v3.pdf.
[135] Note that the formulas in the right column are the formulas for the properties of the Bohr orbitals. These *resemble* the cyclotron orbitals – to some extent – but one should not confuse them: the cyclotron orbitals have no nucleus at their center. In fact, the oft-quoted description of the electron in the Penning trap as an artificial atom is quite confusing and, therefore, not very useful: the radius and kinetic energy of the electron in a magnetron is of an entirely different order of magnitude! However, we would expect the formulas to be similar.

trap. However, we would suspect there will, effectively, be a small anomaly left – which is only natural because all of the formulas above assume the electron is a perfect disk (when calculating the values for the spin-only moment), or a perfect sphere (when calculating the values for the orbital moment). However, the Dirac-Kerr-Newman model of an electron tells us that is, perhaps, *not* the case. Let us copy the illustration again.

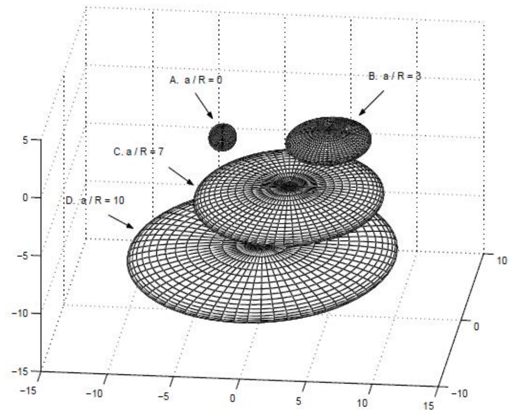

Figure 45: Burinskii's electron model

Despite all of the complexities of Mr. Burinskii's model, the shape of the electron can be characterized by a simple a/R ratio. Somewhat confusingly, the R in this formula is actually the surface area. Hence, if we re-use the r symbol for the radius of the disk, then R will be – roughly – equal to πr^2. The a is the ratio between the angular momentum (J) and the electron mass. Hence, the a/R ratio can be written as:

$$\frac{a}{R} = \frac{J}{\mathrm{m}} \frac{1}{\pi r^2}$$

We have not only the angular momentum here, but also the surface area here (πr^2) which co-determines the magnetic moment of the loop of current (I).[136] In short, all of the variables that could, potentially, explain the anomalous magnetic moment in terms of a form factor are there. Hence, the next logical step would be to validate this classical electron model by inserting it into some other model. Indeed, as Dirac noted, "the very-high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us", as a result of which "the velocity of the electron at any time equals the velocity of light" is a "prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small."[137]

[136] The symbols in the table may be somewhat confusing: I (*italicized*) is a moment of inertia, but I (non-italicized) is a current. We did not want to use new symbols because the context of the formula makes clear what it what. We should note that Mr. Burinskii does not quite agree with my rendering of his formulas: his model is somewhat more complex and, hence, I am still struggling to 'translate' it to my rather modest level of understanding of these things.

[137] Erwin Schrödinger had, effectively, already derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth to now quote all of Dirac's summary of Schrödinger's discovery in his 1933 Nobel Prize speech: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be

However, we can, of course, *insert* this *Zitterbewegung* model – or, preferably, the more flexible model of Mr. Burinskii – into models that do *not* involve micro-motion at the speed of light. What models? Models involving the slow motion of an electron around a nucleus (atomic orbitals) or – in this particular case – the motion of an electron in a Penning trap.

Theoretical implications

The reader may wonder: what's the use if there is already a satisfactory theory (perturbative theory)? The answer to this question is quite obvious. First, a classical theory would be simpler, and Occam's Razor Principle, therefore, tells us we should consider it. More generally, all physicists would agree the King of Science should respect Boltzmann's adage: "Bring forth the truth. Write it so it's clear. Defend it to your last breath." Indeed, even if the results would only remotely explain the anomaly, we would still have achieved two very significant scientific breakthroughs. First, it would show that these seemingly irrelevant micro-models can be validated externally. More importantly, it would prove that an alternative (classical) explanation of the anomalous magnetic moment would be possible.

One may, of course, wonder, further down the line, if an *augmented* classical explanation of QED would upset the theoretical approach in other sectors of the Standard Model. Indeed, as Aitchison and Hey write, the new quantum electrodynamical theory (QED) provided physicists with a model – they refer to it as the 'electron-figure' but what we are talking about are gauge theories, really[138] – to analyze the forces in the nucleus – i.e. the strong and weak force. We do not think so, because these forces are *non*-linear and are also quite different in their *nature* in other respects.

Using totally non-scientific language, we may say that mass comes in one 'color' only: it is just some scalar number. Hence, Einstein's geometric approach to it makes total sense. In contrast, the electromagnetic force is based on the idea of an electric charge, which can come in two 'colors' (+ or −), so to speak. Maxwell's equation seemed to cover it all until it was discovered the nature of Nature – sorry for the wordplay – might be discrete and probabilistic.[139] Now, the strong force comes in three colors, and the rules for mixing them, so to speak, are *very* particular. It is, therefore, only natural that its analysis requires a wholly different approach. In fact, who knows? Perhaps one day some alien will show us that the application of the 'electron-figure' to these sectors was actually *not* so useful. Don't get us wrong: we think these models are all very solid, but history has shown us that one can never exclude a scientific revolution!

We had sent our very first paper on this topic to Mr. Burinskii. He took up the suggestion and has already obtained the α factor. We are confident he'll able to figure out the 1/2π factor and the

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directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

[138] Ian J.R. Aitchison and Anthony J.G. Hey, *Gauge Theories in Particle Physics*, 4th edition, Volume I, p. 3.

[139] In the above-mentioned paper, we note it helps a lot to think of Planck's quantum of action as a *vector* quantity: the uncertainty may then be related to its *direction*, rather than its magnitude. We also note the theoretical framework might benefit from using the ± sign in the argument of the wavefunction to associate the wavefunction with a *non-zero* spin particle. We argue that the weird 720-degree symmetries which discouraged research into geometric (or *physical*) interpretations of the wavefunction might then disappear. See: Jean Louis Van Belle, Euler's Wavefunction: The Double Life of −1, 30 October 2018, http://vixra.org/pdf/1810.0339v2.pdf.

remaining corrections. Mr. Burinskii should probably be considered for the next Nobel Prize. As for me – an amateur physicist – I'll be quite happy to document the unfolding story. 😊

XII. The fine-structure constant and the fine structure

We should now explain the final and last meaning of the fine-structure constant – the one that gave it its name! Why is that the fine-structure constant explains the fine structure of the hydrogen spectrum? However, because this is actually a topic that is well covered in standard physics textbooks – we will, effectively, refer the reader to such physics textbooks. He or she should, by now, be able to apply the knowledge gained here to *translate* the quantum-mechanical explanation into something that is *not*-so-mysterious as physicists and popular writers want us to believe.

Let us – after all this – offer some more fundamental reflections on the meaning of the wavefunction. We're going to combine these reflections with a bit of a *recap* of what we've learned so far.

XIII. The meaning of the wavefunction

Thomas Aquinas starts his *de Ente et Essentia* (on Being and Essence) quoting Aristotle: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions. This philosophical warning – combined with Occam's quest for mathematical parsimony – made us think about the mathematical framework of quantum mechanics: its rules explain reality, but no one understands them. Perhaps some small mistake has been made – early on – in the *interpretation* of the math. This has been a long quest – with little support along the way (see the acknowledgments above) – but we think we have found the small mistake – and we do believe it has led to some substantial misunderstandings – or, at the very least, serious *ambiguities* in the description.

We think that the power of Euler's function – as a mathematical description of what we believe to be a real particle – has not been fully exploited. We, therefore, have a redundancy in the description. The fallacy is illustrated below. When we combine –1 with an amplitude, we should not think of it as a scalar: we should think of –1 as a complex number itself. Hence, when we are multiplying a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with –1, we are *not* necessarily multiplying them with the same thing: –1 is *not* necessarily a common phase factor. The phase factor may be +π or, alternatively, –π. To put it simply, when going from +1 to –1, it matters how you get there – and vice versa.

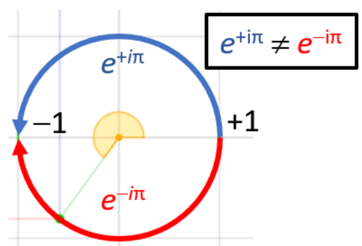


Figure 46: $e^{+i\pi} \neq e^{-i\pi}$

Let us elaborate this.

Spin-zero particles do not exist!

Quantum physicists don't think of the elementary wavefunction as representing anything real but – if they do – they would reluctantly say it might represent some theoretical spin-zero particle. Now, we all know spin-zero particles do not exist. All *real* particles have spin – electrons, photons, anything – and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. Hence, it is rather odd that the plus/minus sign of the imaginary unit in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include spin in the mathematical description. Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i\cdot\theta} = a \cdot e^{-i\cdot(\omega t - kx)}$ and $a \cdot e^{+i\cdot\theta} = a \cdot e^{+i\cdot(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). We would think physicists would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention." In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*).

Historical experience tells us theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things – think, for example, of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? Occam's Razor principle tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations, and the one obvious characteristic that would distinguish the two physical situations is the spin direction. Hence, we do *not* agree with the mainstream view that the choice is a matter of convention. Instead, we dare to suggest that the two mathematical possibilities represent identical particles with opposite spin. Combining this with the two possible directions of propagation (which are given by the +– or ++ signs in front of ω and k), we get the following table:

Table 6: Occam's Razor: mathematical possibilities versus physical realities

Spin and direction of travel	**Spin up (e.g. $J = +\hbar/2$)**	**Spin down (e.g. $J = -\hbar/2$)**
Positive *x*-direction	$\psi = a \cdot e^{-i \cdot (\omega t - kx)}$	$\psi^* = a \cdot e^{+i \cdot (\omega t - kx)}$
Negative *x*-direction	$\chi = a \cdot e^{-i \cdot (\omega t + kx)}$	$\chi^* = a \cdot e^{-i \cdot (\omega t + kx)}$

Let us think this through. Physicists tell us that wavefunctions of spin-1/2 particles (which is what we are thinking of here) have a weird 720° symmetry, but that this weird symmetry is not there for spin-1 particles. Hence, intuition tells us that it should disappear when we would use the two mathematical possibilities for describing the wavefunction of a particle to distinguish between two particles that are identical but have opposite spin. If our intuition is correct (we do not have a formal proof of this – but we do have a *heuristic disproof* (see: Euler's wavefunction, the double life of −1: http://vixra.org/abs/1810.0339), then the most important objection to a physical interpretation of the wavefunction would no longer be valid and, in our humble view, it would trigger a whole new wave (pun intended) of geometric (read: physical) interpretations of the wavefunction.

For starters, it would get rid of the desiccated idea that the complex conjugate of the (elementary) $\psi = \exp(i\theta) = \exp[i(kx-\omega t)]$ function – so that is $\psi^* = \exp(-i\theta) = \exp[i(\omega t-kt)]$ – is just another *mathematical* possibility to describe reality. In other words, it would get rid of the idea that it is just some *convention.* We insist on this point. Why? We readily acknowledge conventions are essential in any (mathematical) description of (physical) reality, so why don't we like *this* convention? It's Occam. Occam tells us the degrees of freedom in the mathematical description (and we are talking just some plain number here, like 3 or 5 or whatever) should match the degrees of freedom in our measurement of whatever we think reality might be. The idea of just settling on a mathematical convention in this particular context (a mathematical object describing a physical reality) is, for us, plain anathema.

Let us mention some (possible) implications so as to illustrate the point.

The meaning of the complex conjugate

The idea of associating the complex conjugate of a wavefunction with a particle that's identical except for its (opposite) spin might be outlandish, which is why we should first try to connect with a much simpler idea – which might or might not be more palatable: the complex conjugate of a wavefunction obviously reverses the trajectory of the particle in space and in time: $\boldsymbol{x}$ becomes $-\boldsymbol{x}$ and t becomes $-t$.

What? Yes. A true physical interpretation will present the real and imaginary part of the elementary wavefunction $a \cdot e^{i\theta}$ as *real* field vectors driven by the same function but with a phase difference of 90 degrees:

$$a \cdot e^{i\theta} = a \cdot (\cos\theta + i \cdot \sin\theta) = a \cdot \sin(\theta + \pi/2) + i \cdot a \cdot \sin\theta$$

However, a minus sign in front of our $\exp(i\theta)$ function reverses the direction of the oscillation – in space and, importantly, *in time too*. Here we can use the $\cos\theta = \cos(-\theta)$ and $\sin\theta = -\sin(-\theta)$ formulas to relate $-\exp(i\theta)$ to the complex conjugate. We write:

$$-\psi = -\exp(i\theta) = -(\cos\theta + i \cdot \sin\theta) = \cos(-\theta) + i \cdot \sin(-\theta) = \exp(-i\theta) = \psi^*$$

This should make us feel uneasy. Yes. We should think of this. We should *not* scrap one ambiguity in the description to introduce another. Things should be clean: the math has to match the physics. So… Does it? We think it does. We need to highlight a subtle point here. Time has one direction only. We cannot reverse time. We can only reverse the direction in space. We can do so by reversing the momentum of a particle. If we do so, the $\mathbf{k} = \mathbf{p}/\hbar$ in the argument of the wavefunction becomes $-\mathbf{k} = -\mathbf{p}/\hbar$. However, the energy remains what it is and, hence, nothing happens to the $\omega \cdot t = (E/\hbar) \cdot t$ term. Hence, our wavefunction becomes $\exp[i(-\mathbf{k} \cdot \boldsymbol{x} - \omega \cdot t)]$, and we can calculate the wave velocity as negative: $v = -\omega/|\mathbf{k}| = -\omega/k$. The wave effectively travels in the opposite direction (i.e. the *negative* x-direction in one-dimensional space). Hence, we can think of opposite directions in space, but we can't reverse time. Why not?

Time and well-behaved functions

The answer is related to how our mind works. Time has one direction only because – if it wouldn't – we would not be able to describe trajectories in spacetime by a well-behaved function. We really don't need to think of entropy or of other more convoluted explanations here. The diagrams below illustrate the point. The spacetime trajectory in the diagram on the right is not *kosher*, because our object travels back in time in not less than three sections of the graph. Spacetime trajectories need to be described by well-defined function: for every value of t, we should have one, and only one, value of x. The reverse is not true, of course: a particle can travel back to where it was. Hence, it is easy to see that our concept of time going in one direction, and in one direction only, implies that we should only allow well-behaved functions.

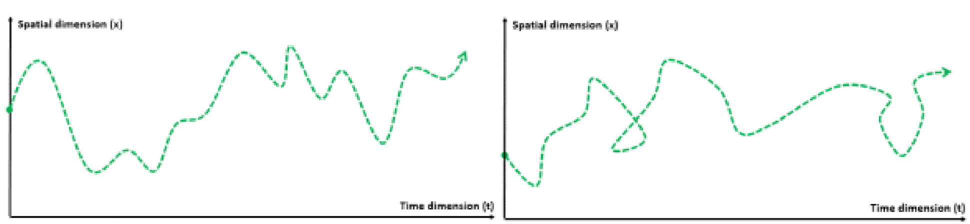


Figure 47: A well- and a not-well behaved trajectory in spacetime

It may be a self-evident point to make but it is an important one. It shows us we should not be worried: our *new* interpretation of the wavefunction – incorporating spin – is fully consistent. It rules out any ambiguity. It we would *not* accept it, then we would have two *mathematical* possibilities to describe a theoretical spin-zero particle that would travel in one direction or the other: $\psi = \exp[i(-kx-\omega t)]$ *or*, alternatively, $-\psi = \psi^* = \exp[i(kx+\omega t)]$.

An added benefit of our interpretation is that it eliminates the logic that leads to the rather uncomfortable conclusion that the wavefunction of spin-1/2 particles (read: electrons, practically speaking) has some weird 720-degree symmetry in space. This conclusion is uncomfortable because we cannot imagine such objects in space without invoking the idea of some kind of relation between the subject and the object (the reader should think of the Dirac belt trick here). It has, therefore, virtually halted all creative thinking on a physical interpretation of the wavefunction.

Interpreting state vectors and absolute squares

This may sound like Chinese to the reader, so let us proceed to something else: how should we interpret the product of the elementary function with its complex conjugate? In orthodox quantum mechanics, it is just this weird thing: some number that will be *proportional* to some *probability*. In our interpretation, this probability is proportional to energy densities – or, because of the energy-mass equivalence – to mass densities. Let us take the simplest of cases and think of the $\langle\psi|$ *state* as some very *generic* thing being represented by a *generic* complex function[140]:

$$\langle\psi| \cong a \cdot e^{i\theta}$$

The $\langle\psi|\langle\psi|^* = \langle\psi||\psi\rangle$ product then just eliminates the *oscillation*. It freezes time, we might say:

$$\langle\psi|\langle\psi|^* = \langle\psi||\psi\rangle = a \cdot e^{i\theta} \cdot a \cdot e^{-i\theta} = a^2 \cdot e^0 = a^2$$

Hence, we end up with one factor of the energy of an oscillation: its amplitude (a). Let us think about this for a brief moment. To focus our minds, let us think of a photon. The energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Hence, if we write the proportionality coefficient as k, then the energy of our photon will be equal to:

$$\mathrm{E} = k \cdot a^2 \cdot \omega^2$$

What should we use for the amplitude of the oscillation here? It turns out we get a nice result using the *wavelength*[141]:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

[140] Our critics will cry wolf and say we should be more general. They are right. However, let us make two remarks here. First, we should note that QED is a linear theory and, hence, we can effectively - and very easily – generalize anything we write to a Fourier superposition of waves. We use the ≅ symbol to indicate an *equivalence*. It's not an identity. To mathematical purists – who will continue to cry wolf no matter what we write because they won't accept the $e^{-\pi} \neq e^{-\pi}$ expression either – we will admit it is more like a symbol showing *congruence*. Second, we do get some physical laws out of physics (both classical as well as quantum-mechanical) that are likely to justify the general $a \cdot e^{i\theta}$ shape.

[141] We use the $\mathrm{E}\lambda = \mathrm{h}c \Leftrightarrow \lambda = \mathrm{h}c/\mathrm{E}$ identity. The reader might think we should use the amplitude of the electric and magnetic field. We could – the model is consistent – but it requires some extra calculations as we then need to think of the energy as some force over a distance. We refer to our papers for more details.

However, we should immediately note that – in our interpretation(s) of the wavefunction – this assumes a circularly polarized wave. Its linear components – the sine and cosine, that is – will only pack *half* of that energy. Our electron model – *zbw* electron as well an orbital electron – is based on the same.

So, yes, now that we are here, let us quickly recap the formulas we found:

Table 7: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr orbitals)**
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I \cdot \omega = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu = \mathrm{I} \cdot \pi r_\mathrm{C}{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi r_n{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

What's Hermiticity?

We will come back to this in the next section of this book. Let us first relate the discussion to the discussion to the *Hermiticity* of (many) operators. If A is an operator[142], then it could operate on some state $|\psi\rangle$. We write this operation as:

$$A|\psi\rangle$$

Now, we can then think of some (probability) amplitude that this operation produces some other state $|\varphi\rangle$, which we would write as:

$$\langle\varphi|A|\psi\rangle$$

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

$A^\dagger$ is, of course, the conjugate transpose of A: $A^\dagger_{ij}=(A_{ji})^*$, and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix,

[142] We should use the *hat* because the symbol without the hat is reserved for the *matrix* that does the operation and, therefore, already assumes a representation, i.e. some chosen set of base states. However, let us skip the niceties here.

so that is if A† = A. Many operators are Hermitian. Why? Well… What is the meaning of ⟨φ|A|ψ⟩* = ⟨ψ|A†|φ⟩ = ⟨ψ|A|φ⟩? Well… In the ⟨φ|A|ψ⟩ we go from some state |ψ⟩ to some other state ⟨φ|. Conversely, the ⟨ψ|A|φ⟩ expression tells us we were in state |φ⟩ but now we are in the state ⟨ψ|.

So, is there some meaning to the complex conjugate of an amplitude like ⟨φ|A|ψ⟩? We say: yes, there is! Read up on time reversal and CPT symmetry! Based on the above – and your reading-up on CPT symmetry – we would think it is fair to say we should interpret the Hermiticity condition as a *physical* reversibility condition.

We are not talking mere time symmetry here: reversing a physical process is like playing a movie backwards and, hence, we are actually talking CPT symmetry here. Of course, it may be difficult to *prove* this interpretation – can one prove interpretations, really? – but, at the very least, we made a start, right? 😊

Summary: explaining QED using classical theory

The following series of diagrams summarizes most of what we covered in the previous chapters.

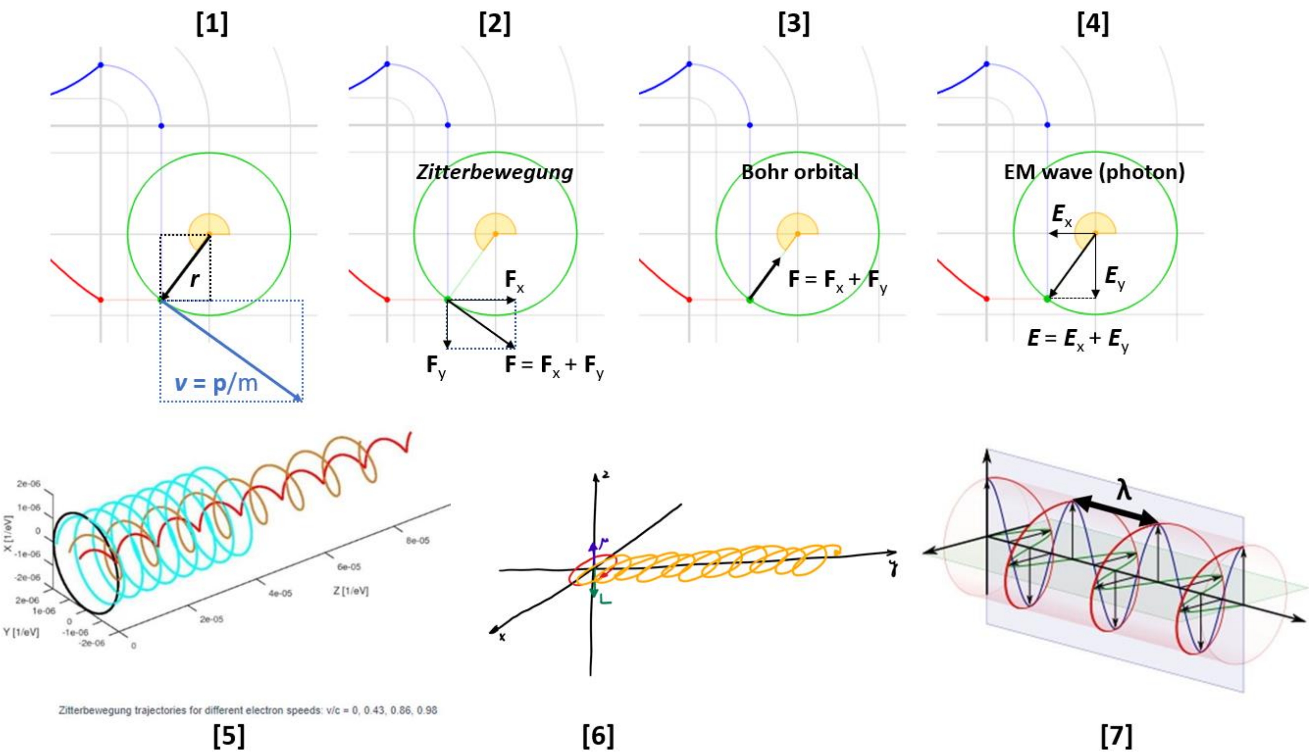


Figure 48: Physical interpretations of the wavefunction

We refer to our previous papers for a detailed discussion of each of these.[143] Here we will just sum up the basics:

[143] See our series of viXra papers (http://vixra.org/author/jean_louis_van_belle). If we would have to choose one which sort of sums most, we would select our *Layered Motions: The Meaning of the Fine-Structure Constant* (http://vixra.org/pdf/1812.0273v3.pdf).

1. We had a *Zitterbewegung* model, in which the elementary wavefunction represents a pointlike *charge* with zero rest mass and which, therefore, moves at the speed of light. This model explains Einstein's energy-mass equivalence relation in terms of a two-dimensional oscillation. The radius of the oscillation is the Compton radius of the electron.

2. The *Zitterbewegung* electron – which combines the idea of a pointlike charge and Wheeler's idea of mass without mass[144] – can then be inserted into Bohr's quantum-mechanical model of an atom, which can also be represented using the elementary wavefunction. We have a different force configuration here (because of the positively charged nucleus, we have a centripetal force now – as opposed to the tangential *zbw* force) but Euler's $a \cdot e^{\pm i\theta}$ function still represents an actual *position vector* of an electron which – because it acquired a rest mass from its *Zitterbewegung* – now moves at velocity $v = (\alpha/n) \cdot c$.[145] This should suffice to explain diagram 1, 2 and 3 below.

3. Diagram 4 represents the idea of a photon that we get out of the Bohr model. We referred to it as the one-cycle photon model. The idea is the following. The Bohr orbitals are separated by a amount of (physical) action that is equal to h. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. Our photon will have to pack that, somehow. It will also have to pack the related energy, which is given by the *difference* of the energies of the two orbitals. This gives us not only the Rydberg formula – Bohr sort of *explained* that formula in 1913 already, but not like we do here – but also a delightfully simple model of a photon and an intuitive interpretation of the Planck-Einstein relation ($f = 1/T = E/h$) for a photon. Indeed, we can do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the $E = mc^2$ mass-energy equivalence relation can be written as $p = mc = E/c$ for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

We calculated the related force and field strength in our paper[146] so we won't repeat ourselves here. We would just like to point out something interesting – using diagram 5 above. Diagram 5 was copied from one of the many papers of Celani, Vassallo and Di Tommaso on the *Zitterbewegung* model, but we can use it to illustrate how and why we can associate a *radius* with the wavelength of a photon. Indeed, the diagram shows that, as an electron starts moving along some trajectory at a relativistic velocity – a velocity that becomes a more substantial fraction of c, that is – then the radius of the *Zitterbewegung*

[144] The mass of the electron is the equivalent mass of the energy in the oscillation.
[145] The n is the number of the Bohr orbital (n = 1, 2, 3…). The α and c are the fine-structure constant and the speed of light. This formula comes out naturally of the Bohr model. See the referenced papers.
[146] See the above-mentioned paper: Jean Louis Van Belle, *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

oscillation becomes smaller and smaller. In the limit ($v \to c$), it becomes zero ($r \to 0$), and the *circumference* of the oscillation becomes a simple (linear) wavelength in the process (this is illustrated in diagram 5 and 7, which provides a geometric interpretation of the *de Broglie* wavelength). Now, if we write this wavelength as λ_C (this is, of course, the Compton *wavelength*), then we get the usual relationship between a radius and a wavelength: $r_C = \lambda_C/2\pi$. This, then, provides an intuitive interpretation of the Eλ = *hc* equation for the photon and – more importantly – an intuitive explanation of the 2π factor in the formula for the fine-structure constant as a coupling constant. We write:

$$\alpha = \frac{2\pi \cdot \mathrm{q_e}^2}{h \cdot c} = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F_\gamma} \cdot r_\gamma \cdot r_\gamma} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F_\gamma} \cdot r_\gamma^2} = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E_\gamma} \cdot r_\gamma}$$

Needless to say, $\mathrm{E_B}$, $\mathrm{F_B}$, r_B and $\mathrm{E_\gamma}$, $\mathrm{F_\gamma}$, r_γ are the energies, forces and radii that are associated with the Bohr orbitals and our one-cycle photon.[147]

Finally – but this is a much finer and more philosophical point – diagram 5 gives us an intuitive geometric interpretation of one of the many ways in which Planck's quantum of action may express itself: the quantization of space. Indeed, at $v = 0$ (diagram 2), we have perfectly circular motion of a pointlike charge moving at the velocity of light, and we may associate Planck's quantum of action with the surface area of the circle. However, at $v = c$, the motion is purely linear – but we still think of the rotating field vector at the core (diagram 4). Planck's quantum of action now expresses itself space as a linear distance: the wavelength of the photon. We like to express this dual view as follows:

$$zbw\ electron: \mathrm{S} = h = \mathrm{p}_{Compton} \cdot \lambda_{Compton} = \mathrm{m_e} c \lambda_\mathrm{C} = \mathrm{m_e} c \cdot 2\pi r_\mathrm{C} = \mathrm{m_e} c \frac{h}{\mathrm{m_e} c} = h$$

$$photon: \mathrm{S} = h = \mathrm{p}_{photon} \cdot \lambda_{photon} = \frac{\mathrm{E_\gamma}}{c} \lambda_\gamma = \mathrm{m_\gamma} c \lambda_\gamma = \mathrm{m_\gamma} c \cdot 2\pi r_\gamma = \mathrm{m_\gamma} c \frac{hc}{\mathrm{E_\gamma}} = h$$

To be fully complete, we can add the same equation for the Bohr orbitals:

$$n^{th}\ Bohr\ orbital: \mathrm{S} = n \cdot h = \mathrm{p}_n \cdot \lambda_n = \mathrm{m}_e v_n \lambda_n = \mathrm{m}_e \frac{\alpha c}{n} 2\pi \frac{n^2 \hbar}{\alpha \mathrm{m}_e c} = n \cdot h$$

We like these expressions because – in our humble view – there is no better way to express the idea that we should associate Planck's quantum of action (or any multiple of it) with the idea of a *cycle* in Nature.[148]

We can imagine the reader is, by now, quite tired of these gymnastics. He or she should ask: what does it all mean? We would like to refer to some history here. Prof. Dr. Alexander Burinskii – the author of the Dirac-Kerr-Newman electron model – told us he had started to further elaborate the *Zitterbewegung* model in the year the author of this book was born – that is in 1969. He published an article on this in the *Journal of Experimental and Theoretical Physics* (JETP)[149]. However, he told us he had always been

[147] These formulas may appear as mind-boggling to the reader. If so, we advise the reader to first look at our papers, whose pace is much more gradual.
[148] Our model also offers a much more comprehensive understanding of the fine-structure constant as a scaling constant. See: Jean Louis Van Belle, *Layered Motions : The Meaning of the Fine-Structure Constant*, http://vixra.org/abs/1812.0273.
[149] Burinskii, A.Y., *Microgeons with spin*, Sov. Phys. JETP 39 (1974) 193. One should note that Prof. dr. Burinskii refers to the *zbw* charge as an 'electron photon' or the 'electron EM wave'. However, its function in the model is basically the same. Prof. dr.

puzzled about this one question: what keeps the pointlike charge in the *zbw* electron in its circular orbit? He, therefore, moved to exploring Kerr-Newman geometries – which has resulted in his Dirac-Kerr-Newman model of an electron.[150]

While the Dirac-Kerr-Newman model is a much more advanced model – it accommodates the theory of the supersymmetric Higgs field and string theory – we understand it does reduce to its classical limit, which is the *Zitterbewegung* model, if one limits the assumptions to general relativity and classical electromagnetism only. In our modest view, this validates our model. There is no mystery on the *zbw* force, we think: it is just the classical Lorentz force $\mathbf{F} = q\mathbf{E} + q\boldsymbol{v}\times\mathbf{B}$. We, therefore, think that the *zbw* force results from the very same electric and magnetic field oscillation that makes up the photon. It is just the way that Planck's quantum of action expresses itself in space that is different here: we just get a different *form factor*, so to speak, when we look at the pointlike *zbw* charge. This, then, should solve Mr. Burinskii's puzzle – in our humble view, that is.

Finally, the attentive reader will have noticed that we did *not* discuss diagram 6. We inserted this diagram because when we considered the various *degrees of freedom* in interpreting Euler's wavefunction, we thought we should, perhaps, not necessarily assume that the plane of the circulatory motion – the *zbw* motion of the pointlike charge in the diagram – is perpendicular to the direction of propagation. In fact, the Stern-Gerlach experiment tells us the magnetic moment is literally *up* or *down*, which assumes the plane of the electric current should be parallel to the direction of motion. We like this alternative picture of the *zbw* electron because – intuitively – we feel it might provide us with some kind of *physical* explanation of relativistic length contraction: as velocities increase, the radius of the circular motion becomes smaller which, in this model, may be interpreted as a contraction of the size of the *zbw* electron.[151]

Burinskii also told us that he was told *not* to refer to the *Zitterbewegung* model at the time, because it was seen as a classical model and, therefore, not in tune with the modern ideas of quantum mechanics.

[150] See the references above.

[151] This is just a random thought at the moment. It needs further exploration.

XIV. The interference of a photon with itself

Can we explain quantum-mechanical interference, i.e. the interference of a photon with itself in, say, a Mach-Zehnder interferometer? We think we can. We think of a photon as the sum of two linearly polarized waves. We write:

$$cos\theta + i \cdot sin\theta = e^{i \cdot \theta} \text{ (RHC)}$$

$$cos(-\theta) + i \cdot sin(-\theta) = cos\theta - i \cdot sin\theta = e^{-i \cdot \theta} \text{ (LHC)}$$

We, therefore, have an alternative theory of what happens in the Mach-Zehnder interferometer:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

The idea of a photon

Our analysis of Feynman's argument on the 720-degree of spin-1/2 particles should *not* be construed as a criticism of Feynman: it's not *his* argument – it's just orthodox QM. In general, we think Feynman's *Lectures* are still the best lectures on physics one can possibly get – if only because they make one *think* about what one is taught. We, therefore, borrow with very much pleasure two diagrams of his *Lectures* to complete the classical picture of a photon.

The first diagram (Feynman, I-34-9) brings in the oft-neglected magnetic field.[152] Feynman uses it to explain what he refers to as the 'pushing momentum' of light – which is more commonly referred to as radiation or *light pressure*. It is a bit of a strange term, because we are talking a *force*, really.

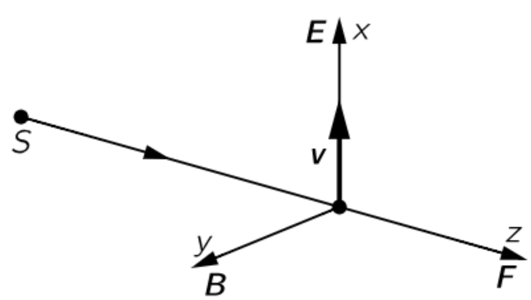


Figure 49: Feynman's explanation of the momentum of light

The basic idea is illustrated in another diagram, which is – unfortunately – separated from the diagram above by a full volume of lectures.[153] An electromagnetic wave – we take it to be a photon – will *drive* an electron, as shown below (Feynman, III-17-4). Hence, the magnetic force comes into play – as there is a

[152] Oft-neglected in the context of a *photon* model, that is.

[153] The first illustration comes from Feynman's volume on classical mechanics (Volume I), while the second comes from his lectures on quantum mechanics (Volume III). The volume in-between (Volume II) is on (classical) electromagnetism.

charge and a velocity to play with now. 😊 The magnetic force – which is just denoted as ***F*** in the diagram above – will be equal to ***F*** = q***v***×**B**.

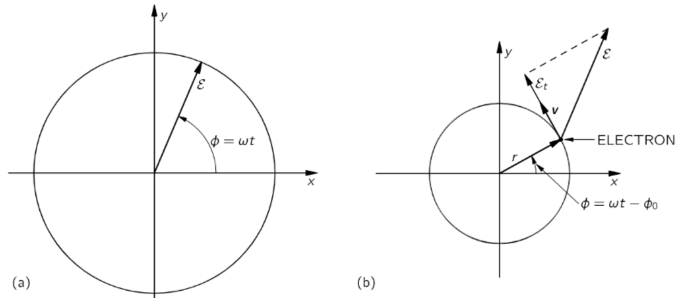


Figure 50: How the electric field of a photon might drive an orbital electron

Feynman then goes off on a bit of a tangent – analyzing the *average* force over time, which makes sense when one continues to take a classical view of an atom (or a Bohr (electron) orbital, practically speaking), and which gives some kind of meaning to the momentum of light.[154] The point is: his analysis fails to bridge classical mechanics with quantum mechanics because he fails to interpret Planck's quantum of action as a quantum: we're not only transferring energy here. We're also transferring angular momentum. In short: photon absorption and emission should respect *the integrity of a cycle*. What is this rule? Some new random interpretation of quantum mechanics? Yes. That is the one we offer here.

What happens when an electron jumps several Bohr orbitals? The angular momentum between the orbitals will then differ by several units of ħ. What happens to the photon picture in that case? It will pack the energy difference, but it will also pack several units of *ħ* (angular momentum) or – what amounts to the same – several units of *h* (physical action). In our humble opinion, we should still think of the photon a one-cycle oscillation. Hence, we do *not* think its energy will be spread over several cycles.[155] The two equations below need to make sense for *all* transitions[156]:

$$photon\text{: } \mathrm{S} = h = \mathrm{p}_\gamma \cdot \lambda_\gamma = \frac{\mathrm{E}_\gamma}{c}\lambda_\gamma = \frac{\mathrm{E}_\gamma}{f_\gamma} = \mathrm{E}_\gamma \cdot \mathrm{T}_\gamma$$

$$electron\ transition\text{: } \mathrm{S} = n \cdot h = \mathrm{p}_n \cdot \lambda_n = \mathrm{m}_e v_n \lambda_n = \mathrm{E}_n \cdot \mathrm{T}_n$$

[154] Mr. Feynman gets some kind of explanation for the p = E/*c* relation out of his analysis.

[155] When discussing the Mach-Zehnder experiment in the next version of this book, we may bring a subtle but essential *nuance* to this point of view.

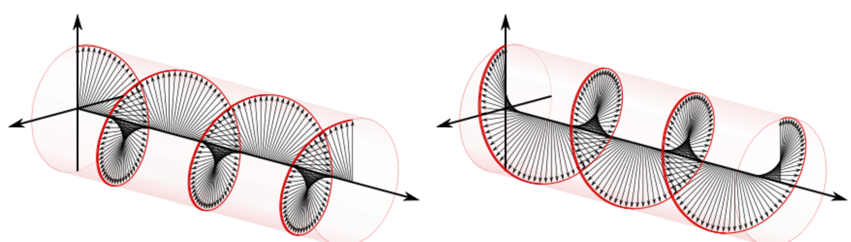
[156] The use of the same integer *n* for the *difference* in energy between Bohr orbitals might be confusing but we did not want to use another symbol – such as *m*, for example – because *m* would make one think of the fine-structure transitions (which we haven't discussed at all – not in this book, and not in any of ours papers) and – more importantly – because we want to encourage the reader to think these things through for him- or herself. Symbols acquire meaning from the context in which they are used. We are tempted to go off on a tangent on Wittgenstein but we should restrain ourselves here. There is too much philosophy in this book already. We advise the reader to critically cross-check the formula for electron transitions with what we wrote in previous papers. We warmly welcome comments. Our email is mentioned on the first page.

The formulas above express the two most common expressions of what we referred to as the *Certainty Principle*. Pun intended.[157] We will leave it as an exercise for the reader to re-write these formulas in terms of a product of *force*, *distance*, and *time*.

So, what about Uncertainty, then? Nothing – absolutely *nothing* – of what we wrote above involves any uncertainty. It must be there somewhere, right? We would like to offer the following reflection. We have a few footnotes in previous papers, in which we suggest that Planck's quantum of action should be interpreted as a *vector*. The uncertainty – or the probabilistic nature of Nature, so to speak[158] – might, therefore, not be in its *magnitude*. We feel the uncertainty is in its *direction*. This may seem to be restrictive. However, because *h* is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time, we think we have the mathematical framework comes with sufficient degrees of freedom to describe any situation. Quantum-mechanical equations – such as Schrödinger's equation – should probably be written as vector equations.[159]

The photons above make for a circularly polarized beam. The spin direction may be left-handed or right-handed, as shown below.

Figure 51: Left- and right-handed polarization[160]



We can think of these photons as the sum of two linearly polarized waves. We write:

$$cos\theta + i\cdot sin\theta = e^{i\cdot\theta} \text{ (RHC)}$$

$$cos(-\theta) + i\cdot sin(-\theta) = cos\theta - i\cdot sin\theta = e^{-i\cdot\theta} \text{ (LHC)}$$

Huh? *What is the geometry here?* It is quite simple. Let us spell it out so we have no issues of interpretation. If ***x*** is the direction of propagation of the wave, then the ***z***-direction will be pointing upwards, and we get the ***y***-direction from the righthand rule for a Cartesian reference frame.[161] We may now think of the oscillation along the ***y***-axis as the cosine, and the oscillation along the ***z***-axis as the sine.

[157] As we argued in previous papers, Planck's quantum of action should probably be interpreted as a *vector*. The uncertainty might not be in its *magnitude*. We feel the uncertainty is in its *direction*. Because *h* is the product of a force, a distance and time, we have a lot of dimensions to consider.

[158] A fair amount of so-called thought experiments in quantum mechanics – and I am not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.

[159] We made a start with this in a previous paper: Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, http://vixra.org/pdf/1812.0202v1.pdf.

[160] Credit: https://commons.wikimedia.org/wiki/User:Dave3457.

[161] Note the reference frame in the illustrations of the LHC and RHC wave – which we took from Wikipedia – is left-handed. Our argument will use a regular right-handed reference frame.

If we then think of the imaginary unit *i* as a 90-degree counterclockwise rotation in the ***yz***-plane (and remembering the convention that angles (including the phase angle θ) are measured counterclockwise), then the right- and left-handed waves can effectively be represented by the wavefunctions above.

The point here is that easy visualizations like this strongly encourage us to think of a geometric representation of the wavefunction—if only because, conversely, one may also adopt the convention that the imaginary unit should be interpreted as a unit vector pointing in a direction that is perpendicular to the direction of propagation of the wave and one may then write the magnetic field vector as **B** = –*i*·**E**/*c*.[162] The minus sign in the **B** = –*i*·**E**/*c*. It is there because of consistency: we must combine a classical *physical* right-hand rule for **E** and **B** here as well as the *mathematical* convention that multiplication with the imaginary unit amounts to a *counter*clockwise rotation by 90 degrees. This allows us to re-write Maxwell's equations using complex numbers. We have done that in other papers, so if the reader is interested he can check there.[163] The point to note is that, while we will often sort of forget to show the magnetic field vector, the reader should always think of it – because it is an integral part of the electromagnetic wave: when we think of **E**, we should also think of **B**. Both oscillations carry energy.

The mention of energy brings me to another important point. As mentioned above, we think of a circularly polarized beam – and a photon – as a superposition of two linear waves. Now, these two linearly polarized waves will each pack *half* of the energy of the combined wave. It is a very important point to make because any classical explanation of interference – like the one we will offer in the next section – will need to respect the energy conservation law. Note that, while each wave packs *half* of the energy of the combined wave, their (maximum) amplitude is the same: there is no change there. Let us briefly elaborate this point. The energy of any oscillation will always be proportional to (1) its amplitude (*a*) and (2) its frequency (*f*). Hence, if we write the proportionality coefficient as *k*, then the energy of our photon will be equal to:

$$\mathrm{E} = k \cdot a^2 \cdot \omega^2$$

What should we use for the amplitude of the oscillation here? It turns out we get a nice result using the *wavelength*[164]:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

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[162] As usual, we use **boldface** letters to represent geometric vectors – the electric (**E**) and magnetic field vectors (**B**), in this case. There is a risk of confusion between the energy E and the electric field E because we use the same symbols, but the context should make clear what is what.

[163] See, for example, Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, http://vixra.org/pdf/1812.0202v1.pdf.

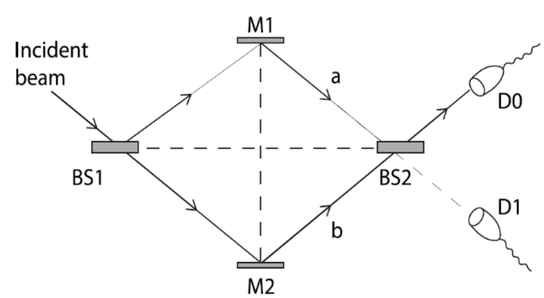
[164] We use the Eλ = h*c* ⇔ λ = h*c*/E identity. The reader might think we should use the amplitude of the electric and magnetic field. We could – the model is consistent – but it requires some extra calculations as we then need to think of the energy as some force over a distance. We refer to our papers for more details.

However, we should note this assumes a circularly polarized wave. Its linear components – the sine and cosine, that is – will only pack *half* of that energy. We can now offer the following classical explanation of the Mach-Zehnder experiment *for one photon only*.[165]

A classical explanation for the one-photon Mach-Zehnder experiment

We offered a geometric interpretation of the wavefunction. When analyzing interference in quantum mechanics, the wavefunction concept gives way to the concept of a probability amplitude which we associate with a possible *path* rather than a particle. The math looks somewhat similar but models very different ideas and concepts. Before the photon enters the beam splitter, we have one wavefunction: the photon. When it goes through, we have two probability amplitudes that – somehow – recombine and interfere with each other. What we want to do here is to explain this classically. So let us look at the Mach-Zehnder interferometer once again. We have two beam splitters (BS1 and BS2) and two perfect mirrors (M1 and M2). An incident beam coming from the left is split at BS1 and recombines at BS2, which sends two outgoing beams to the photon detectors D0 and D1. More importantly, the interferometer can be set up to produce a precise interference effect which ensures all the light goes into D0, as shown below. Alternatively, the setup may be altered to ensure all the light goes into D1.

Figure 52: The Mach-Zehnder interferometer[166]



What is the classical explanation? The classical explanation is something like this: the first beam splitter (BS1) splits the beam into *two* beams. These two beams arrive in phase or, alternatively, out of phase and we, therefore, have constructive or destructive interference that recombines the original beam and makes it go towards D0 or, alternatively, towards D1. When we analyze this in terms of a single photon, this classical picture becomes quite complicated – but we argue there is such classical picture. Our alternative theory of what happens in the Mach-Zehnder interferometer is the following:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

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165 We have written about this topic before (see: Jean Louis Van Belle, *Linear and circular polarization states in the Mach-Zehnder interference experiment*, 5 November 2018, http://vixra.org/pdf/1811.0056v1.pdf). Hence, we will only offer a summary of what we wrote there.

166 Source of the illustration: MIT edX Course 8.04.1*x* (Quantum Physics), Lecture Notes, Chapter 1, Section 4 (*Quantum Superpositions*).

We will detail this in the next section, because what happens in a Mach-Zehnder interferometer is not all that straightforward. We should note, for example, that there are phase shifts along both paths: classical physics tells us that, on transmission, a wave does not pick up any phase shift, but it does so on reflection. To be precise, it will pick up a phase shift of π on reflection. We will refer to the standard textbook explanations of these subtleties and just integrate them in our more detailed explanation in the next section.[167] Before we do so, we will show the assumption that the two linear waves are orthogonal to each other is quite crucial. If they weren't, we would be in trouble with the energy conservation law. Let us show that before we proceed.

Suppose the beams would be polarized along the same direction. If *x* is the direction of propagation of the wave, then it may be the *y*- or *z*-direction of anything in-between. The magnitude of the electric field vector will then be given by a sinusoid. Now, we assume we have *two* linearly polarized beams, of course, which we will refer to as beam *a* and *b* respectively. These waves are likely to arrive with a phase difference – unless the apparatus has been set up to ensure the distances along both paths are exactly the same. Hence, the general case is that we would describe *a* by $cos(\omega \cdot t - k \cdot x) = cos(\theta)$ and *b* by $cos(\theta + \Delta)$ respectively. In the classical analysis, the difference in phase (Δ) will be there because of a difference of the path lengths[168] and the recombined wavefunction will be equal to the same cosine function, but with argument $\theta + \Delta/2$, multiplied by an *envelope* equal to $2 \cdot \cos(\Delta/2)$. We write[169]:

$$\cos(\theta) + \cos(\theta + \Delta) = 2 \cdot \cos(\theta + \Delta/2) \cdot \cos(\Delta/2)$$

We always get a recombined beam with the same frequency, but when the phase difference between the two incoming beams is small, its amplitude is going to be much larger. To be precise, it is going to be twice the amplitude of the incoming beams for $\Delta = 0$. In contrast, if the two beams are out of phase, the amplitude is going to be much smaller, and it's going to be zero if the two waves are 180 degrees out of phase ($\Delta = \pi$), as shown below. That does not make sense because twice the amplitude means *four* times the energy, and zero amplitude means zero energy. The energy conservation law is being violated: photons are being multiplied or, conversely, are being destroyed.

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[167] For a good *classical* explanation of the Mach-Zehnder interferometer, see: K.P. Zetie, S.F. Adams and R.M. Tocknell, January 2000, *How does a Mach–Zehnder interferometer work?* (https://www.cs.princeton.edu/courses/archive/fall06/cos576/papers/zetie_et_al_mach_zehnder00.pdf, accessed on 5 November 2018).
For a good quantum-mechanical explanation (interference of single photons), see – for example – the Mach-Zehnder tutorial from the PhysPort website (https://www.physport.org/curricula/QuILTs/, accessed on 5 November 2018).
[168] Feynman's path integral approach to quantum mechanics allows photons (or probability amplitudes, we should say) to travel somewhat slower or faster than *c*, but that should not bother us here.
[169] We are just applying the formula for the sum of two cosines here. If we would add sines, we would get $\sin(\theta) + \sin(\theta + \Delta) = 2 \cdot \sin(\theta + \Delta/2) \cdot \cos(\Delta/2)$. Hence, we get the same envelope: $2 \cdot \cos(\Delta/2)$.

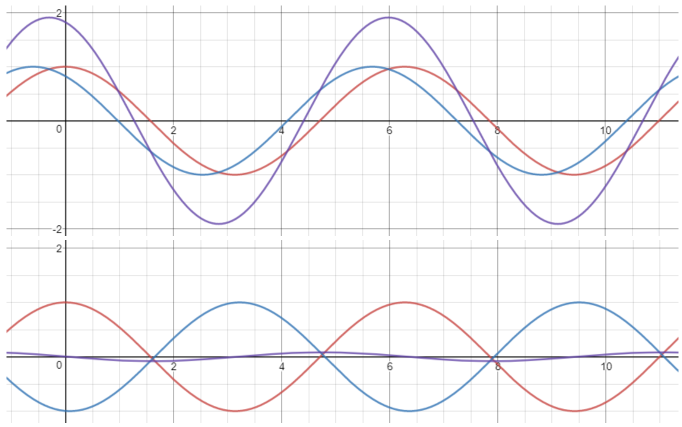

Figure 53: Constructive and destructive interference for linearly polarized beams

Let us be explicit about the energy calculation. We assumed that, when the incoming beam splits up at BS1, that the energy of the *a* and *b* beam will be split in half too. We know the energy is given by (or, to be precise, proportional to) the square of the amplitude (let us denote this amplitude by *A*).[170] Hence, if we want the energy of the two individual beams to add up to $A^2 = 1^2 = 1$, then the (maximum) amplitude of the *a* and *b* beams must be 1/√2 of the amplitude of the original beam, and our formula becomes:

$$(1/\sqrt{2})\cdot\cos(\theta) + (1/\sqrt{2})\cdot\cos(\theta + \Delta) = (2/\sqrt{2})\cdot\cos(\theta + \Delta/2)\cdot\cos(\Delta/2)$$

This reduces to $(2/\sqrt{2})\cdot\cos(\theta)$ for $\Delta = 0$. Hence, we still get *twice* the energy – $(2/\sqrt{2})^2$ equals 2 – when the beams are in phase and zero energy when the two beams are 180 degrees out of phase. This doesn't make sense.

Of course, the mistake in the argument is obvious. This is why our assumption that the two linear waves are orthogonal to each other comes in: we cannot just add the amplitudes of the *a* and *b* beams because they have different directions. If the *a* and *b* beams – after being split from the original beam – are linearly polarized, then the angle between the axes of polarization should be equal to 90 degrees to ensure that the two oscillations are independent. We can then add them like we would add the two parts of a complex number. Remembering the geometric interpretation of the imaginary unit as a counterclockwise rotation, we can then write the sum of our a and b beams as:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \Delta) = (1/\sqrt{2})\cdot[\cos(\theta) + i\cdot\cos(\theta + \Delta)]$$

What can we do with this? Not all that much, except noting that we can write the $cos(\theta + \Delta)$ as a sine for $\Delta = \pm\,\pi/2$. To be precise, we get:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \pi/2) = (1/\sqrt{2})\cdot(\cos\theta - i\cdot\sin\theta) = (1/\sqrt{2})\cdot e^{-i\cdot\theta}$$

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta - \pi/2) = (1/\sqrt{2})\cdot(\cos\theta + i\cdot\cos\theta) = (1/\sqrt{2})\cdot e^{i\cdot\theta}$$

This gives us the classical explanation we were looking for:

1. The incoming photon is circularly polarized (left- or right-handed).

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[170] If we would reason in terms of average energies, we would have to apply a 1/2 factor because the average of the $\sin^2\theta$ and $\cos^2\theta$ over a cycle is equal to 1/2.

2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

What about the 1/√2 factor? If the $e^{-i\cdot\theta}$ and $e^{i\cdot\theta}$ wavefunctions can, effectively, be interpreted geometrically as a *physical* oscillation in *two* dimensions – which is, effectively, our interpretation of the wavefunction[171] – then then each of the two (independent) oscillations will pack one half of the energy of the wave. Hence, if such *circularly* polarized wave splits into two *linearly* polarized waves, then the two linearly polarized waves will effectively, pack half of the energy without any need for us to think their (maximum) amplitude should be adjusted. If we now think of the ***x***-direction as the direction of the incident beam in the Mach-Zehnder experiment, and we would want to also think of rotations in the ***xz***-plane, then we need to need to introduce some new convention here. Let us introduce *another* imaginary unit, which we'll denote by *j*, and which will represent a 90-degree counterclockwise rotation in the ***xz***-plane.[172] We then get the following *classical* explanation for the results of the one-photon Mach-Zehnder experiment:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = cos\theta + i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta} = cos\theta - i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $-j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.

[171] We can assign the physical dimension of the electric field (force per unit charge, N/C) to the two perpendicular oscillations.
[172] This convention may make the reader think of the quaternion theory but we are thinking more of simple Euler angles here: *i* is a (counterclockwise) rotation around the *x*-axis, and *j* is a rotation around the *y*-axis.

Of course, we may also set up the apparatus with different path lengths, in which case the two linearly polarized beams will be out of phase when arriving at BS1. Let us assume the phase shift is equal to Δ = 180° = π. This amounts to putting a minus sign in front of *either* the sine *or* the cosine function. Why? Because of the $cos(\theta \pm \pi) = -cos\theta$ and $sin(\theta \pm \pi) = -sin\theta$ identities. Let us assume the distance along the upper path is longer and, hence, that the phase shift affects the sine function.[173] In that case, the sequence of events might be like this:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = cos\theta + i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $j\cdot\sin(\theta + \pi) = -j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta} = cos\theta - i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $-j\cdot\sin(\theta + \pi) = +j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.

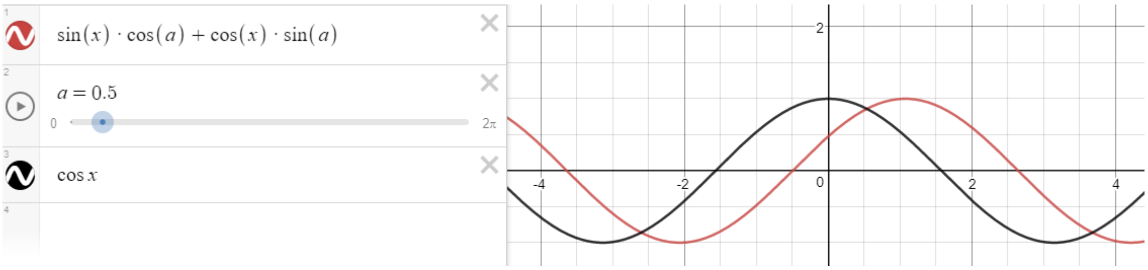
What happens when the difference between the phases of the two beams is not equal to 0 or 180 degrees? What if it is some random value in-between? Do we get an elliptically polarized wave or some other nice result? Denoting the phase shift as Δ, we can write:

$$cos\theta + j\cdot sin(\theta + \Delta) = cos\theta + j\cdot(sin\theta\cdot cos\Delta + cos\theta\cdot sin\Delta)$$

However, this is also just a circularly polarized wave, but with a random phase shift between the horizontal and vertical component of the wave, as shown below. Of course, for the special values $\Delta = 0$ and $\Delta = \pi$, we get $cos\theta + j\cdot sin\theta$ and $cos\theta - j\cdot sin\theta$ once more.

[173] The reader can easily work out the math for the opposite case (longer length of the lower path).

Figure 54: Random phase shift between two waves



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Mystery solved? Maybe. Maybe not. We just wanted to show that Bell's No-Go Theorem should not prevent you from trying to go *every*where. 😊

XV. The way forward

We presented a lot of material in this book. How can one sum it all up? Any conclusions? We would probably just want to say this: it is about time physicists consider the *form factor* in their analysis. It somehow disappeared. Vector equations became *flat*: vector quantities became magnitudes. Schrödinger's equation should be rewritten as a vector equation. We've made a start with this[174], but there is a lot more work to be done.

What about uncertainty? Nothing – absolutely *nothing* – of what we wrote above involves any uncertainty. It must be there somewhere, right? We would like to offer the following reflection. We have a few footnotes in previous papers, in which we suggest that Planck's quantum of action should be interpreted as a *vector*. The uncertainty – or the probabilistic nature of Nature, so to speak[175] – might, therefore, not be in its *magnitude*. We feel the uncertainty is in its *direction*. This may seem to be restrictive. However, because *h* is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time, we think the mathematical framework comes with sufficient degrees of freedom to describe any situation.

While what we've done so far, shows that might be the case, there is sure a lot of nitty-gritty that we haven't answered yet. For example, our description of how interference – or diffraction – actually works, both for electrons as well as for photons, was *very* rudimentary. Physicists will – and should – only accept a model if it explains stuff *exactly*. So we need to translate our models into models that have a lot more detail – so we can see if, for example, we can get the Fresnel equations out of them.

That's a long road ahead, and I am not sure if I have enough intellectual baggage to travel that. 😉

Jean Louis Van Belle, February 2019

[174] See: *A Geometric Interpretation of Schrödinger's Equation*, http://vixra.org/abs/1812.0202.

[175] A fair amount of so-called thought experiments in quantum mechanics – and we are not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.