The Emperor has No Clothes

A classical interpretation of quantum mechanics

Jean Louis Van Belle

Summary : This voluminous paper organizes all of my previous papers in one volume – which might become a book if my intended co-author (Ines Urdaneta) will manage to structure, rationalize and clean up.

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I. Introduction

If you are reading this book, then you are like me. You want to *understand*. Something inside of you tells you that the idea that we will never be able to understand quantum mechanics "the way we would like to understand it" does not make any sense. The quote is from Richard Feynman – probably the most eminent of all post-World War II physicists – and, yes, this is an aggressive opening. But you are right: our mind is flexible. We can imagine weird shapes and hybrid models. Hence, we *should* be able to understand quantum physics in some kind of intuitive way.

But what is intuitive? A lot of the formulas in this book feel intuitive to me, but that is only because I have been working with them for quite a while now. They may *not* feel very intuitive to you. However, I have confidence you will also sort of *understand* what they represent – *intuitively*, that is – because all of the formulas I use represent something we can *imagine* in space and in time – and I mean 3D space here: *our* Universe. Not the Universe of strings and hidden dimensions. An *intuitive* understanding of things means an understanding in terms of their geometry and their *physicality*.

You bought the right book. No hocus-pocus here. All physicists – and popular writers on physics – will tell you it is not possible. You see, the wavefunction of a particle – say, an electron – has this weird 720-degree symmetry, which we cannot really *imagine*. Of course, we have these professors doing the *Dirac belt trick* on YouTube – and wonderful animations (Jason Hise – whom I've been in touch with – makes the best ones) but, still, these visualizations all assume some weird relation between the object and the subject. In short, we cannot *really* imagine an object with a 720-degree symmetry.

The good news is: you don't have to. The early theorists made a small mistake: they did not fully exploit the power of Euler's ubiquitous $a \cdot e^{i\theta}$ function. The mistake is illustrated below – but don't worry if this looks like you won't understand: we'll come back to it. It is a very subtle thing. Quantum physicists will tell you they don't *really* think of the elementary wavefunction as representing anything real but, in fact, they do. And they will tell you, rather reluctantly because they are not so sure about what is what, that it might represent some theoretical spin-zero particle. Now, we all know spin-zero particles do not exist. All *real* particles – electrons, photons, anything – have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. So, it's completely odd that the plus (+) or the minus (−) sign of the imaginary unit (i) in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include the spin *direction* in the mathematical description.

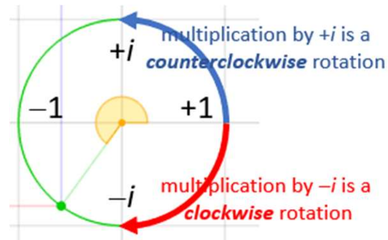


Figure 1: The meaning of +*i* and –*i*

Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta} = a \cdot e^{-i \cdot (\omega t - kx)}$ and $a \cdot e^{+i \cdot \theta} = a \cdot e^{+i \cdot (\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). We would think physicists would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention." In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*).

That leads to the *false* argument that the wavefunction of spin-½ particles have a 720-degree symmetry. Again, you should not worry if you don't get anything of what I write here – because I will come back to it – but the gist of the matter is the following: *because* they think the elementary wavefunction describes some theoretical *zero*-spin particle, physicists treat –1 as a *common phase factor*: they think we can just multiply a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with –1 and we're going to get the same *states*. We find it rather obvious that that is not necessarily the case: –1 is *not* necessarily a common phase factor. We should think of –1 as a complex number itself: the phase factor may be +π or, alternatively, –π. To put it simply, when going from +1 to –1, it matters how you get there – and vice versa – as illustrated below.

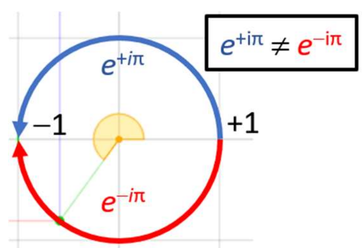


Figure 2: $e^{+i\pi} \neq e^{-i\pi}$

I know this sounds like a bad start for a book that promises to be easy – but I just thought it would be good to be upfront about why this book is very different than anything you've ever read about quantum physics. If we exploit the full descriptive power of Euler's function, then all weird symmetries disappear – and we just talk standard 360-degree symmetries in space. Also, weird mathematical conditions – such as the *Hermiticity* of quantum-mechanical operators – can easily be explained as embodying some common-sense *physical* law. In this particular case (*Hermitian* operators), we are talking *physical* reversibility: when we see something happening *at the elementary particle level*, then we need to be able to play the movie backwards. Physicists refer to it as CPT-symmetry, but that's what it is really: *physical* reversibility.

The argument above involved *geometry*, and this brings me to a second mistake of the early quantum physicists: a total neglect of what I refer to as the *form factor* in physics. Why would an electron be some perfect *sphere*, or some perfect *disk*? In fact, we will argue it is *not*. It is a regular geometric shape – the Dirac-Kerr-Newman model suggests it's an oblate spheroid – but so that's *not* a perfect sphere. Once you acknowledge that, the so-called anomalous magnetic moment is not-so-anomalous anymore.

The mistake is actually more general than what I wrote above. We are thinking of the key constants in *Nature* as some number. Most notably, we think of Planck's quantum of action ($h \approx 6.626\times10^{-34}$ N·m·s) as some (scalar) number. Why would it be? It is – obviously – some vector quantity or – let me be precise – some *matrix* quantity: h is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time (one direction only). Somehow, those dimensions disappeared in the analysis. Vector equations became *flat*: vector quantities became magnitudes. Schrödinger's equation should be rewritten as a vector or matrix equation. We do think of Planck's quantum of action as some vector. We, therefore, think that the uncertainty – or the probabilistic nature of Nature, so to speak[1] – is not in its *magnitude*: it's in its *direction*. But we are getting ahead of ourselves here – as usual. We should go step by step. Let us first acknowledge where we came from.

Before doing so, I would like to make yet another remark – one that is actually *not* so relevant for what we are going to try to do this in this book – and that is to understand the QED sector of the Standard Model geometrically – or *physically*, I should say. The *innate* nature of man to *generalize* did not contribute to greater clarity – in my humble opinion, that is. Feynman's weird *Lecture* (Volume III, Chapter 4) on the key difference between bosons and fermions does not have any practical value: it just confuses the picture.

Likewise, it makes perfect sense to me to think that each sector of the Standard Model requires its own mathematical approach. I will briefly summarize this idea in totally non-scientific language. We may say that mass comes in one 'color' only: it is just some scalar number. Hence, Einstein's geometric approach to gravity makes total sense. In contrast, the electromagnetic force is based on the idea of an electric charge, which can come in two 'colors' (+ or −), so to speak. Maxwell's equation seemed to cover it all until it was discovered the nature of Nature – sorry for the wordplay – might be discrete and probabilistic. However, that's fine. We should be able to modify the classical theory to take that into account. There is no need to invent an entirely new mathematical framework (I am talking quantum field and gauge theories here). Now, the strong force comes in three colors, and the rules for mixing them, so to speak, are very particular. It is, therefore, only natural that its analysis requires a wholly different approach. Hence, I would think the new mathematical framework should be reserved for that sector. I don't like the reference of Aitchison and Hey to gauge theories as 'the electron-figure'. The electron figure is a pretty classical idea to me. Hence, I do hope one day some alien will show us that the application of the Dyson-Feynman-Schwinger-Tomonaga 'electron-figure' to what goes on inside of the nucleus of an atom was, perhaps, not all that useful. A simple exponential series should not be explained

[1] A fair amount of so-called thought experiments in quantum mechanics – and we are not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.

by calculating a zillion integrals over 891 Feynman diagrams. It should be explained by a simple set of equations. If I have not lost you by now, please follow me to the acknowledgments section.

II. History and acknowledgments

This journey – this long search for understanding – started about thirty-five years ago. I was just a teenager then – reading popular physics books. Gribbin's *In Search of Schrödinger's Cat* is just one of the many that left me unsatisfied in my quest for knowledge.

However, my dad never pushed me and so I went the easy route: humanities, and economics – plus some philosophy and a research degree afterwards. Those rather awkward qualifications (for an author on physics, that is) have served me well – not only because I had a great career abroad, but also because I now realize that physics, as a science, is in a rather sorry state: the academic search for understanding has become a race to get the next non-sensical but conformist theory published.

Why do we want to understand? What is understanding? I am not sure, but my search was fueled by a discontent with the orthodox view that we will never be able to understand quantum mechanics "the way we would like to understand it", as Richard Feynman puts it. Talking Feynman, I must admit his meandering *Lectures* are the foundation of my current knowledge, and the reference point from where I started to think for myself. I had been studying them on and off – an original print edition that I had found in a bookshop in Old Delhi – but it was really the 2012 Higgs-Englert experiments in CERN's LHC accelerator, and the award of the Nobel prize to these two scientists, that made me accelerate my studies. It coincided with my return from Afghanistan – where I had served for five years – and, hence, I could afford to reorient myself. I had married a wonderful woman, Maria, who gave me the emotional and physical space to pursue this intellectual adventure.

I started a blog (readingfeynman.org) as I started struggling through it all – and that helped me greatly. I fondly recall that, back in 2015, Dr. Lloyd N. Trefethen from the Oxford Math Institute reacted to a post in which I had pointed out a flaw in one of Richard Feynman's arguments. It was on a topic that had nothing to do with quantum mechanics – the rather mundane topic of electromagnetic shielding, to be precise – but his acknowledgement that Feynman's argument was, effectively, flawed and that he and his colleagues had solved the issue in 2014 only (Chapman, Hewett and Trefethen, *The Mathematics of the Faraday Cage*) was an eye-opener for me. Trefethen concluded his email as follows: "Most texts on physics and electromagnetism, weirdly, don't treat shielding at all, neither correctly nor incorrectly. This seems a real oddity of history given how important shielding is to technology." This resulted in a firm determination to *not* take any formula for granted – even if they have been written by Richard Feynman! With the benefit of hindsight, I might say this episode provided me with the guts to question orthodox quantum theory.

The informed reader will now wonder: what do I mean with orthodox quantum theory? I should be precise here, and I will. It is the modern theory of quantum electrodynamics (QED) as established by Dyson, Schwinger, Feynman, Tomonaga and other post-World War II physicists. It's the explanation of the behavior of electrons and photons – and their interactions – in terms of Feynman diagrams and propagators. I instinctively felt their theory might be incomplete because it lacks a good description of what electrons and photons actually *are*. Hence, all of the weirdness of quantum mechanics is now in this weird description of the fields – as reflected in the path integral formulation of quantum mechanics.

Whatever an electron or a photon might be, we cannot *really* believe that it sort of travels along an infinite number of possible spacetime trajectories all over space *simultaneously*, can we?

I also found what Brian Hayes refers to as "the tennis match between experiment and theory" – the measurement (experiment) or calculation (theory) of the so-called anomalous magnetic moment – a rather weird business: the complexity in the mathematical framework just doesn't match the intuition that, if the theory of QED has a simple circle group structure, one should not be calculating a zillion integrals all over space over 891 4-loop Feynman diagrams to explain the magnetic moment of an electron in a Penning trap. There must be some *form factor* coming out of a decent electron model that can explain it, right?

Of course, all of the above sounds very arrogant, and it is. However, I always felt I was in good company, because I realized that not only Einstein but the whole first generation of quantum phycisists (Schrödinger, Dirac, Pauli and Heisenberg) had become skeptical about the theory they had created – if only because perturbation theory yielded those weird diverging higher-order terms. With the benefit of hindsight, we may say that the likes of Dyson, Schwinger, Feynman – the whole younger generation of mainly American scientists who dominated the *discourse* at the time – lacked a true *general*: they kept soldiering on by inventing renormalization and other mathematical techniques to ensure those weird divergences cancel out, but they had no direction.

However, I should not get ahead of myself here. This is just an introduction, after all. Before getting to the meat of the matter, I should just make some remarks and acknowledge all the people who supported me in this rather lonely search. First, whom am I writing for? I am writing for people like me: amateur physicists. *Not-so-dummies*, that is. People who don't shy away from calculations. People who understand a differential equation, some complex algebra and classical electromagnetism – all of which are, indeed, necessary, to understand anything at all in this field. I have good news for these people: I have come to the conclusion that we do *not* need to understand anything about gauges or propagators or Feynman diagrams to understand quantum electrodynamics.

Indeed, rather than "using his renormalized QED to calculate the one loop electron vertex function in an external magnetic field", Schwinger should, perhaps, have listened to Oppenheimer's predecessor on the Manhattan project, Gregory Breit, who wrote a number of letters to both fellow scientists as well as the editors of the *Physical Review* journal suggesting that the origin of the so-called discrepancy might be due to an "intrinsic magnetic moment of the electron of the order of $\alpha\mu_B$." In other words, I do *not* think Breit was acting schizophrenic when complaining about the attitude of Kusch and Lamb when they got the 1955 Nobel Prize for Physics for their work on the anomalous magnetic moment. I think he was just making a very sensible suggestion – and that is that one should probably first try investing in a good theory of the electron before embarking on mindless quantum field calculations.

My search naturally led me to the *Zitterbewegung* hypothesis. *Zitter* is German for shaking or trembling. It refers to a presumed local oscillatory motion – which I now believe to be *true*, whatever that means. Erwin Schrödinger found this *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which

> seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Dirac obviously refers to the phenomenon of *Compton* scattering of light by an electron. Indeed, as we shall see, the *Zitterbewegung* model naturally yields the Compton *radius* of an electron and – as such – effectively provides some geometric explanation of what might be happening. It took me a while to figure out that some *non*-mainstream physicists had actually continued to further explore this concept, and the writings of David Hestenes from the Arizona State University of Arizona who – back in 1990 – proposed a whole new interpretation of quantum mechanics based on the *Zitterbewegung* concept (Hestenes, 1990, *The Zitterbewegung Interpretation of Quantum Mechanics*) made me realize there was sort of a parallel universe of research out there – but it is *not* being promoted by the likes of MIT, Caltech or Harvard University – and, even more importantly, their friends who review and select articles for scientific journals.

I reached out to Hestenes, but he is 85 by now – and I don't have his private email, so I never got any reply to the one or two emails I sent him on his ASU address. In contrast, Giorgio Vassallo – one of the researchers of an Italian group centered around Francesco Celani – who followed up on the Schrödinger-Hestenes *zbw* model of an electron – politely directed me towards Dr. Alex Burinskii (I should have put a *Prof.* and/or *Dr.* title in front of every name mentioned above, because they all are professors and/or doctors in science). Both have been invaluable – not because they would want to be associated with any of our ideas – but because they gave me the benefit of the doubt in their occasional but consistent communications. Hence, I would like to thank them here for reacting and encouraging me for at least *trying* to understand.

I think Mr. Burinskii deserves a Nobel Price, but he will probably never get one – because it would question not one but *two* previously awarded Nobel Prizes (1955 and 1965). I feel validated because, in his latest communication, Dr. Burinskii wrote me to say he takes my idea of trying to corroborate his Dirac-Kerr-Newman electron model by inserting it into models that involve some kind of slow orbital motion of the electron – as it does in the Penning trap – seriously.

It is now time to start the book. However, before I do so, I should wrap up the acknowledgments section, so let us do that here. I have also been in touch with Prof. Dr. John P. Ralston, who wrote one of a very rare number of texts that, at the very least, *tries* to address some of the honest questions of amateur physicists and philosophers upfront. I was not convinced by his interpretation of quantum mechanics, but I loved the self-criticism of the profession: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." We exchanged some messages, but then concluded that our respective interpretations of the wavefunction are very different and, hence, that we should not "waste any electrons" (his expression) on trying to convince each other. In the same vein, I should mention some other seemingly random exchanges – such as those with the staff and fellow students when going through the MIT's edX course on quantum mechanics which – I admit – I did not fully complete because, while I don't mind calculations in general, I do mind *mindless* calculations.

I am also *very* grateful to my brother, Prof. Dr. Jean Paul Van Belle, for totally unrelated discussions on his key topic of research (which is information systems and artificial intelligence), which included discussions on Roger Penrose's books – mainly *The Emperor's New Mind* and *The Road to Reality*. These discussions actually provided the inspiration for the earlier draft title of this book: *The Emperor has no clothes: the sorry state of Quantum Physics*. We will go for another mountainbike or mountain-climbing adventure when this project is over.

Among other academics, I would like to single out Dr. Ines Urdaneta. Her independent research is very similar to ours. She has, therefore, provided much-needed moral support and external validation. I also warmly thank Jason Hise, whose wonderful animations of 720-degree symmetries did *not* convince me that electrons – as spin-1/2 particles – actually *have* such symmetries – but whose communications stimulated my thinking on the subject-object relation in quantum mechanics.

Finally, I would like to thank all of my friends (my university friends, in particular (loyal as ever), and I will also single out Soumaya Hasni, who has provided me with a whole new fan club here here in Brussels) and, of course, my family, for keeping me sane. I would like to thank, in particular, my children – Hannah and Vincent – and my wife, Maria, for having given me the emotional, intellectual and financial space to grow into the person I am right now.

So, now we should really start the book. Its structure is simple. In the first chapters, I'll just introduce the most basic math – Euler's function, basically – and then we'll take it from there. I will regularly refer to a series of papers I published on what I refer to as the *Los Alamos Site for Spacetime Rebels*: vixra.org. The site is managed by Phil Gibbs. I would like to acknowledge and thank him here for providing a space for independent thinkers. You can find my papers on http://vixra.org/author/jean_louis_van_belle. They are numbered, and will often refer to those papers by mentioning their number between square brackets – like this: [xx]. In fact, this very first version of this book follows the structure of paper [17]. Click on the link above, and you'll understand. 😊

Or so I hope. This brings me to the final point in my introduction. This is just the first version of this book. It is rather short – *cryptic*, I'd say. As such, you might give up after a few pages and say: this may be a *classical* interpretation of quantum physics, but it is *not* an easy one. To those, I'll say two things:

1. It may not be easy, but it is definitely *easier* than whatever else you'll read when exploring the more serious stuff.
2. To get my degree in philosophy, I had to study Wittgenstein's *Tractatus Logico-Philosophicus*. I hated that booklet – not because it is dense but because it is nonsense. Wittgenstein I wasn't even aware of the scientific revolution that was taking place while he was writing it. Still, it became a bestseller. Why? Because it was so abstruse *it made people think for themselves*.

Hence, I hope this book will do the same: it should make you think for yourself. The first version of this book is going to be dense but – hopefully – you will find it is full of sense. If so – I should find out from the number of copies sold – then I might go through the trouble of unpacking it somewhat more in the second edition. At the same time, you will also find there is a lot of overlap between the various chapters as we wanted them to be logically independent. Hence, the reader should not hesitate to skip some material here and there as there is a good chance the same idea or principle will be revisited in a subsequent chapter.

I may also switch to that weird *pluralis majestatis* which authors use: the *we*-form. It does not matter all that much: if *we* are wrong, you – the reader – will understand that *I* am wrong. 😉

III. The wavefunction and the electron

Euler's function is a wonderful mathematical object. We must assume that the reader is already familiar with it:

$$a \cdot e^{i\theta} = a \cdot \cos(\theta) + i \cdot \sin(\theta)$$

We can immediately visualize this using the *Zitterbewegung* model of an electron. We described the origin of the model (see the quote from Dirac's Nobel Prize speech) above: the illustration below represents the circular oscillatory motion of the electron (the *Zitterbewegung*) or – possibly – of any charged particle.

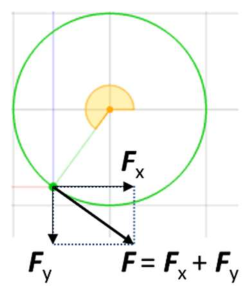


Figure 3: The *Zitterbewegung* model of an electron

It is driven by a force – which must be electromagnetic, because the force has only a charge to grab onto. We think of this charge as a pointlike object that has no rest mass. Hence, the charge spins around at the speed of light. We have a dual view of the reality of the wavefunction here. On the one hand, it will describe the physical position (i.e. the *x*- and *y*-coordinates) of the pointlike charge – the green dot in the illustration, whose motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

As such, the (elementary) wavefunction is viewed as an implicit function: it is equivalent to the $x^2 + y^2 = a^2$ equation, which describes the same circle.

On the other hand, the *zbw* model implies the circular motion of the pointlike charge is driven by a tangential force, which we write as:

$$\boldsymbol{F} = F_x \cdot \cos(\omega t + \pi/2) + i \cdot F_x \cdot \sin(\omega t + \pi/2) = F \cdot e^{i(\theta + \pi/2)}$$

The line of action of the force is the orbit because a force needs something to grab onto, and the only thing it can grab onto in this model is the oscillating (or rotating) charge. We think of ***F*** as a composite force: the resultant force of two perpendicular oscillations. A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree angle ensures the independence of both motions. The kinetic and potential energy of *one* harmonic oscillator add up to E = m·a^2·ω^2/2. If we have two, we can drop the ½ factor. We can then boldly equate the E = mc^2 and E = m·a^2·ω^2 formulas to get the *zbw* radius. We can think of this as follows. The *zbw* model – which is derived from Dirac's wave equation for free electrons – tells us the velocity of the pointlike charge is equal to *c*. If the *zbw* frequency is given by Planck's energy-frequency relation (ω = E/ħ), then we can combine Einstein's E = mc^2 formula with the radial velocity formula ($c = a\cdot\omega$) and, hence, we get the *zbw* radius, which is nothing but the (reduced) Compton wavelength – or the Compton *radius* of the electron:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The amount of *physical* action – which we will denote by *S* as per the usual convention – that is associated with one loop along the *zbw* circumference over its cycle time is equal to Planck's constant:

$$S = F \cdot \lambda_C \cdot \mathrm{T} = \frac{\mathrm{E}}{\lambda_C} \cdot \lambda_C \cdot \frac{1}{f_C} = \mathrm{E} \cdot \frac{h}{\mathrm{E}} = h$$

Planck's constant *h* is equal to 6.62607015×10^{-34} J·s. Hence, it is a small unit - but small and large are relative. In fact, because of the tiny time and distance scale, we have a rather enormous force here. We can calculate the force because the energy in the oscillator must be equal to the magnitude of the force times the length of the loop, we can calculate the magnitude of the force, which is – effectively – rather enormous in light of the sub-atomic scale:

$$\mathrm{E} = \mathrm{F}\lambda_C \Leftrightarrow F = \frac{\mathrm{E}}{\lambda_C} \approx \frac{8.187 \times 10^{-1}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\ \mathrm{N}$$

The associated current is equally humongous:

$$\mathrm{I} = \mathrm{q_e}f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

A household-level current at the sub-atomic scale? The result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. As the oscillator model implies the effective mass of the electron will be spread over the circular disk, we should use the 1/2 form factor for the moment of inertia (*I*). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

The vector notation for **μ** and **L** (boldface) in the equation above should make us think about the plane of oscillation. This question is related to the question of how we should analyze all of this is a moving reference frame. This is a complicated question. The Stern-Gerlach experiment suggests we may want to think of an oscillation plane that might be perpendicular to the direction of motion, as illustrated below.

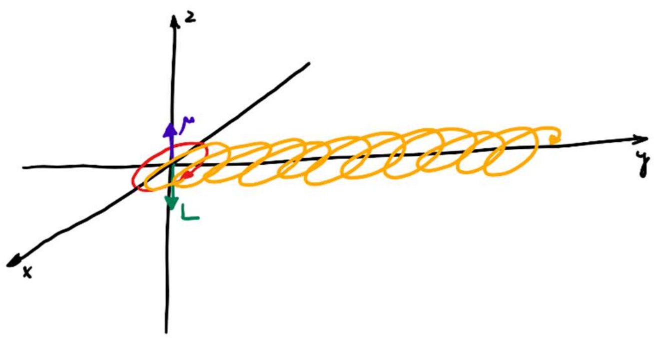


Figure 4: The *zbw* electron traveling through a Stern-Gerlach apparatus?

Of course, the Stern-Gerlach experiment assumes the application of a (non-homogenous) magnetic field. In the absence of such field, we may want to think of the plane of oscillation as something that is rotating in space itself. The idea, then, is that it sort of snaps into place when an external magnetic field is applied.

We should think some more about the nature of the force. The assumption is that the force grabs onto a pointlike charge. Hence, the force must be electromagnetic and we can write it as the product of the unit charge and the field (***E***). We write:

$$\boldsymbol{F} = q_e\boldsymbol{E}.$$

Because the force is humongous (a force of 0.0375 N is equivalent to a force that gives a mass of 37.5 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second), and the charge is tiny), we get an equally huge field strength:

$$E = \frac{F}{\mathrm{q_e}} \approx \frac{3.3743 \times 10^{-2}\ \mathrm{N}}{1.6022\ \times 10^{-1}\ \ \mathrm{C}} \approx 0.21 \times 10^{18}\ \mathrm{N/C}$$

Just as a yardstick to compare, we may note that the most powerful man-made accelerators may only reach field strengths of the order of 10^9 N/C (1 GV/m). Does this make sense? Can we calculate an energy density? Using the classical formula, we get:

$$u = \epsilon_0 E^2 \approx 8.854 \times 10^{-12} \cdot (0.21 \times 10^{18})^2 \frac{\mathrm{J}}{\mathrm{m}^3} = 0.36 \times 10^{24}\ \frac{\mathrm{J}}{\mathrm{m}^3} = 0.63 \times 10^{24}\ \frac{\mathrm{J}}{\mathrm{m}^3}$$

This amounts to about 7 kg per mm^3 (cubic *milli*meter). Is this a sensible value? Maybe. Maybe not. The rest mass of the electron is tiny, but then the *zbw* radius of an electron is also exceedingly small. It is very interesting to think about what might happen to the curvature of spacetime with such mass densities: perhaps our pointlike charge just goes round and round on a geodesic in its own (curved) space. We are not well-versed in general relativity and we can, therefore, only offer some general remarks here:

1. If we would pack all of the mass of an electron into a black hole, then the Schwarzschild formula gives us a radius that is equal to:

$$r_s = \frac{2\mathrm{Gm}}{c^2} \approx 1.35 \times 10^{-57}\mathrm{m}\ (meter)$$

This exceedingly small number has no relation whatsoever with the Compton radius. In fact, its scale has no relation with whatever distance one encounters in physics: it is *much* beyond the Planck scale, which is of the order of 10^{-35} meter and which, for reasons deep down in relativistic quantum mechanics, physicists consider to be the smallest possibly sensible distance scale.

2. We are intrigued, however, by suggestions that the Schwarzschild formula should not be used as it because an electron has angular momentum, a magnetic moment and other properties, perhaps, that do not apply when calculating, say, the Schwarzschild radius of the mass of a baseball. To be precise, we are particularly intrigued by models that suggest that, when incorporating the above-mentioned properties of an electron, the Compton radius might actually be the radius of an electron-sized black hole (Burinskii, 2008, 2016).

Let us now look at this motion in a moving reference frame. Let us consider the idea of a particle traveling in the positive *x*-direction at constant speed *v*. This idea implies a pointlike concept of position and time: we think the particle will be somewhere at some point in time. The *somewhere* in this expression does not necessarily mean that we think the particle itself will be dimensionless or pointlike. It just implies that we can associate some *center* with it. In fact, that's what we have in our *zbw* model here: we have an oscillation around some center, but the oscillation has a *physical* radius, which we referred to as the Compton radius of the electron. Of course, two extreme situations may be envisaged: $v = 0$ or $v = c$. However, let us consider the more general case. In our reference frame, we will have a position – a mathematical *point* in space, that is – which is a function of time: $x(t) = v{\cdot}t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t'. Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t', and the position and time in the two reference frames will be related as follows:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a{\cdot}e^{i\theta}$ can be re-written as follows:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\left(\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{\mathrm{E}_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}\mathrm{E}_0\left(\frac{t}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{\mathrm{E}_0}{\hbar}t'$$

We have just shown that the argument of the wavefunction is relativistically invariant (E_0 is, obviously, the rest energy and, because p' = 0 in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0t'/\hbar$ in the reference frame of the electron itself). It makes us think that of the argument of the wavefunction and – therefore – the wavefunction itself – might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution. Let us, therefore, further explore this. We have been interpreting the wavefunction as an implicit function again: for each *x*, we have a *t*, and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by $x(t) = v \cdot t$. We will get some linear motion. If we look at the ψ = $a \cdot \cos(\mathrm{p} \cdot x/\hbar - \mathrm{E} \cdot t/\hbar) + i \cdot a \cdot \sin(\mathrm{p} \cdot x/\hbar - \mathrm{E} \cdot t/\hbar)$ once more, we can write p·x/ħ as Δ and think of it as a phase factor. We will, of course, be interested to know for what *x* this phase factor Δ = p·*x*/ħ will be equal to 2π. Hence, we write:

$$\Delta = \mathrm{p} \cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi \cdot \hbar/\mathrm{p} = h/\mathrm{p} = \lambda$$

We now get a meaningful interpretation of the *de Broglie* wavelength. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.

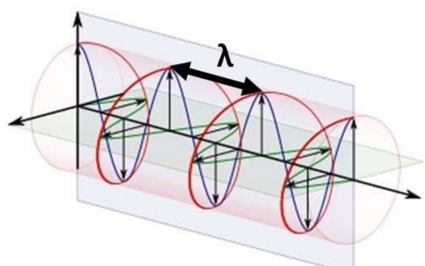


Figure 5: An interpretation of the *de Broglie* wavelength

Of course, we should probably think of the plane of oscillation as being *perpendicular* to the plane of motion – or as oscillating in space itself – but that doesn't matter. Let us explore some more. We can, obviously, re-write the argument of the wavefunction as a function of *time* only:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}vt\right) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)t = \sqrt{1-\frac{v^2}{c^2}} \cdot \frac{\mathrm{E}_0}{\hbar}t$$

We recognize the *inverse* Lorentz factor here, which goes from 1 to 0 as *v* goes from 0 to *c*, as shown below.

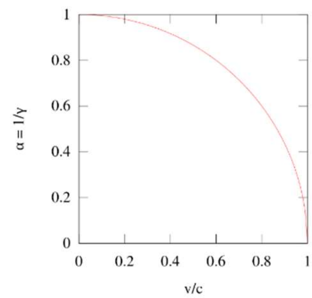


Figure 6: The inverse Lorentz factor as a function of (relative) velocity (*v*/*c*)

Note the shape of the function: it is a simple circular arc. This result should not surprise us, of course, as we also get it from the Lorentz formula:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{v^2}{c^2}t}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{1 - \frac{v^2}{c^2}} \cdot t$$

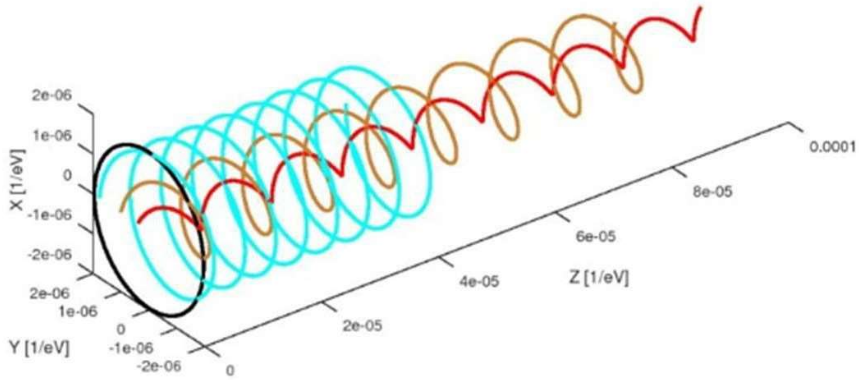
What does it all mean? We can go through a simple numerical example to think this through. Let us assume that, for example, that we are able to speed up an electron to, say, about one tenth of the speed of light. Hence, the Lorentz factor will then be equal to $\gamma = 1.005$. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0: $E_v = \gamma E_0 \approx 1.005 \cdot 0.511$ MeV ≈ 0.5135 MeV. The relativistic momentum will then be equal to $m_v v = (0.5135 \text{ eV}/c^2)\cdot(0.1\cdot c) = 5.135 \text{ eV}/c$. We get:

$$\theta = \frac{E_0}{\hbar}t' = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = 0.955\frac{E_0}{\hbar}t$$

This is interesting: we get an explanation for time dilation. A more interesting question is what happens to the *radius* of the oscillation. Does it change? It must, but how should we interpret this? In the moving reference frame, we measure higher mass and, therefore, higher energy – as it includes the kinetic energy. The $c^2 = a^2\cdot\omega^2$ identity must now be written as $c^2 = a'^2\cdot\omega'^2$. Instead of the rest mass m_0 and rest energy E_0, we must now use $m_v = \gamma m_0$ and $E_v = \gamma E_0$ in the formulas for the Compton radius and the Einstein-Planck frequency, which we just write as m and E in the formula below:

$$\mathrm{m}a'^2\omega'^2 = \mathrm{m}\frac{\hbar^2}{\mathrm{m}^2c^2}\frac{\mathrm{m}^2c^4}{\hbar^2} = \mathrm{m}c^2$$

This is easy to understand intuitively: we have the mass factor in the denominator of the formula for the Compton radius, so it must increase as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, increase with velocity. It is interesting to note that we have a simple (inverse) proportionality relation here. The idea is visualized in the illustration below (for which credit goes to the modern *zbw* theorists Celani et al.): the *radius* of the circulatory motion must effectively diminish as the electron gains speed. Once again, however, we should warn the reader that he or she should also imagine the plane of oscillation to be possibly parallel to the direction of propagation, in which case the circular motion becomes elliptical.



Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

Figure 7: The Compton radius must decrease with increasing velocity

Can the velocity go to *c*? In the limit, yes. This is very interesting, because we can see that the circumference of the oscillation becomes a wavelength in the process! This, then, links the *zbw* electron model with our photon model, which we will explain later. We first need to talk about orbital electron motion. Before we do so, we will resume the model that we have here.

We should note that the center of the *Zitterbewegung* was plain nothingness and we must, therefore, assume some two-dimensional oscillation makes the charge go round and round. The angular frequency of the *Zitterbewegung* rotation is given by the Planck-Einstein relation ($\omega = E/\hbar$) and we get the *Zitterbewegung* radius (which is just the Compton radius $a = r_C = \hbar/mc$) by equating the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equation. The energy and, therefore, the (equivalent) mass is in the oscillation and we, therefore, should associate the momentum $p = E/c$ with the electron as a whole or, if we would really like to associate it with a single mathematical *point* in space, with the center of the oscillation – as opposed to the rotating massless *charge*.

We should note that the distinction between the pointlike *charge* and the electron is subtle, perhaps, but essential. The electron is the *Zitterbewegung* as a whole: the pointlike charge has no rest mass, but the electron as a whole does. In fact, that is the whole point of our *Zitterbewegung* model: we explain the rest mass of an electron by introducing a *rest matter oscillation*. The model cannot be verified because of the extreme frequency ($f_e = \omega_e/2\pi = E/h \approx 0.123 \times 10^{-21}$ Hz) and sub-atomic scale ($a = r_C = \hbar/mc \approx 386 \times 10^{-15}$ m). It is, therefore, a *logical* model only: it gives us the right values for the angular momentum ($L = \hbar/2$), the magnetic moment ($\mu = (q_e/2m) \cdot \hbar$, and the gyromagnetic factor ($g = 2$).

This subtle combination of the idea of a pointlike charge and an oscillation is interesting because it opens the door to a plain classical explanation of interference and/or diffraction. In this regard, we would link this to more recent theory and experiments that focus on how slits or holes affect wave *shapes* as electrons – or photons – go through them. The diagram below illustrates the point that we are trying to make here.[2] We do think these are *very* promising in terms of offering some kind of classical (physical) explanation for interference and/or diffraction.

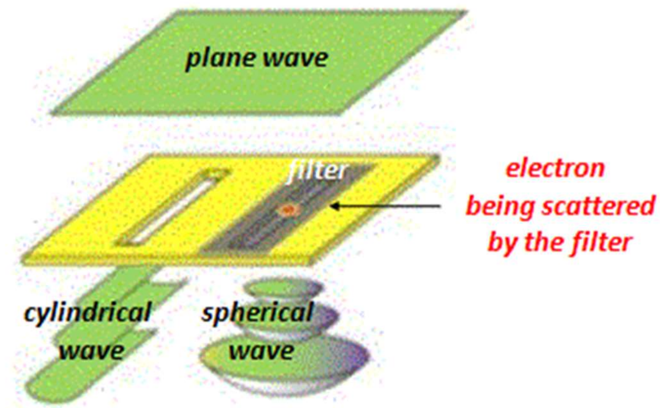


Figure 8: Physical interpretations of the electron wave

All that is left to explain – for the photon as well as the electron – is why the whole oscillation seems to stick together upon detection. We admit that's not easy to do. But – as an idea – it is definitely easier to accept this *axiom* than whatever other theory is on the market right now.

Let us go back to the idea of a two-dimensional oscillation. The $E = ma^2\omega^2 = mc^2$ is intuitive: the energy of any oscillation will be proportional to the square of (*i*) the (maximum) amplitude of the oscillation and (*ii*) the frequency of the oscillation, with the mass as the proportionality coefficient. At the same time, we should wonder: what could it possibly *mean*?

This question is difficult to answer. Is there any other idea – we mean: other than the idea of a two-dimensional oscillation – to explain the *Zitterbewegung*? We do not see any. We explored the basic ideas in our papers and, hence, we will not dwell too much on it here. We will only make one or two remarks below which may or may not help the reader to develop his or her own interpretation of what might be going on *in reality*.

The first remark is this: when everything is said and done, we should admit that the bold $c^2 = a^2\cdot\omega^2$ assumption interprets spacetime as a relativistic aether. It is a term that is, unfortunately, taboo but, fortunately, some respected academics, such as Nobel Prize Laureate Robert Laughlin, are still defending it. This interpretation is inspired by the most obvious implication of Einstein's $E = mc^2$ equation, and that is that the ratio between the energy and the mass of *any* particle is always equal to c^2:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proto}}{m_{proton}} = \frac{E_{photon}}{m_{photo}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

[2] The definition is somewhat random but we think of diffraction if there is only one slit or hole. In contrast, the idea of interference assumes two or more wave sources. The research we refer to is the work of the Italian researchers Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-doubleslit.html). The illustration was taken from the same source, but the author of this paper added the explanatory tags.

This reminds us of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ of harmonic oscillators – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom.[3] In contrast, $c^2 = E/m$ for *any* particle, *always*. This is the point: we can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light *c* emerges here as *the* defining property of spacetime. It is, in fact, tempting to think of it as some kind of *resonant frequency* but the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will now refer to as *the rest energy oscillation*: it is that what gives mass to our electron. Indeed, its rest mass is nothing but the equivalent mass of the energy in the oscillation.

We have, therefore, a rather nice explanation of Einstein's $E = m \cdot c^2$ equation.

IV. The wavefunction and the atom

The illustration below depicts the geometry of a Bohr orbital. We describe such orbital by the same mathematical object – the elementary wavefunction (Euler's function) – but we do have a different geometry here. In fact, the situation is *very* different. The Bohr model has a positively charged nucleus at its center and its electron has an effective rest mass: the radial velocity $v = a \cdot \omega$ of the electron is, therefore, some *fraction* of the speed of light ($v = \alpha \cdot c$). It also has some non-zero momentum $p = m \cdot v$ which we can relate to the electrostatic centripetal force using the simple classical formula $F = p \cdot \omega = mv^2/a$. In contrast, the model of an electron in free space is based on the presumed *Zitterbewegung*, which combines the idea of a very high-frequency circulatory motion with the idea of a pointlike *charge* which – importantly – has *no inertia* and can, therefore, move at the speed of light ($v = c$).

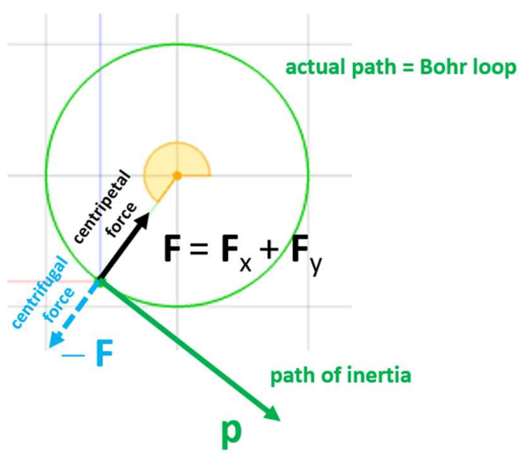


Figure 9: The position, force and momentum vector in a Bohr loop

[3] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the inertia, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as $R = \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

The formulas in the Bohr-Rutherford model are derived from the quantum-mechanical that angular momentum comes in units of $\hbar = h/2\pi$. We rephrased that rule as: physical action comes in unit of h. We also associated Planck's quantum of action with a *cycle*: one rotation will pack some energy over some time (the cycle time) or – what amounts to the same – some momentum over some distance (the circumference of the loop). We wrote:

$$S = \mathrm{h} = \mathrm{E}\cdot\mathrm{T} = \mathrm{L}\cdot 2\pi\cdot r_{\mathrm{B}}$$

Using the $v = \alpha\cdot c$ and $r_{\mathrm{C}} = \alpha\cdot r_{\mathrm{B}}$ relations[4] one can easily verify this for the momentum formulation:

$$S = \mathrm{p}\cdot 2\pi\cdot r_{\mathrm{B}} = \mathrm{m}v\cdot(r_{\mathrm{C}}/\alpha) = \mathrm{m}\alpha c\cdot\frac{2\pi\hbar}{\alpha\mathrm{mc}} = \mathrm{h}$$

We can also calculate S by calculating the force and then multiply the force with the distance and the time. The force is just the (centripetal) electrostatic force between the charge and the nucleus

$$\mathrm{F} = \frac{{\mathrm{q_e}}^2}{4\pi\varepsilon_0 {r_{\mathrm{B}}}^2} = \alpha\cdot\frac{\hbar c}{{r_{\mathrm{B}}}^2}$$

We can then recalculate S as:

$$S = \mathrm{F}\cdot r_{\mathrm{B}}\cdot\mathrm{T} = \alpha\cdot\frac{\hbar c}{{r_{\mathrm{B}}}^2}\cdot r_{\mathrm{B}}\cdot\frac{2\pi r_{\mathrm{B}}}{v} = \alpha\cdot\frac{\mathrm{h}c}{\alpha c} = \mathrm{h}$$

All is consistent. However, we should note the implied energy concept is somewhat surprising:

$$S = h = \mathrm{E}\cdot\mathrm{T} = \mathrm{E}\cdot\frac{2\pi r_{\mathrm{B}}}{v} = \mathrm{E}\cdot\frac{\frac{h}{\alpha\mathrm{mc}}}{\alpha c} \Leftrightarrow \mathrm{E} = \alpha^2\mathrm{m}c^2$$

This is *twice* the ionization energy of hydrogen ($\mathrm{Ry} = \alpha^2\mathrm{m}c^2/2$), and it is also *twice* the kinetic energy ($\hbar^2/2\mathrm{m}a^2 = \alpha^2\mathrm{m}c^2/2$). It is also just a fraction ($\alpha^2 \approx 0.00005325$) of the rest energy of the electron.[5] This somewhat odd result can be explained if we would actually be thinking of a two-dimensional oscillation here. In that case, we would effectively write the force as $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ (as suggested in the illustration above) in a moment) and, hence, we should therefore *add* the kinetic and potential energy of *two* oscillators.

Let us explain and generalize these results for all electron orbitals. In other words, let us explain it in terms of the Bohr atom. The quantum of action effectively underpins the Rutherford-Bohr model of an atom. This 105-year old model[6] was designed to explain the wavelength of a photon that is emitted or absorbed by a hydrogen atom – a one-electron atom, basically – and does a superb job of it. The idea is

[4] These relations come out of the model. They are, therefore, not some new hypothesis. The α in the formula is the fine-structure constant. It pops up in (almost) all of the equations we get. As such, it does appear as some magical dimensionless number that relates almost all (physical) dimensions of the electron (radii, circumferences, energies, momenta, etcetera).

[5] The reader can check the conversion of the Rydberg energy in terms of the fine-structure constant and the rest mass (or rest energy) of the electron.

[6] Around 1911, Rutherford had concluded that the nucleus had to be very small. Hence, Thomson's model – which assumed that electrons were held in place because they were, somehow, embedded in a uniform sphere of positive charge – was summarily dismissed. Bohr immediately used the Rutherford hypothesis to explain the emission spectrum of hydrogen atoms, which further confirmed Rutherford's conjecture, and Niels and Rutherford jointly presented the model in 1913. As Rydberg had published his formula in 1888, we have a gap of about 25 years between experiment and theory here.

that the energy of such photon is equal to the *difference* in energy between the various orbitals. The energy of these orbitals is usually expressed in terms of the energy of the *first* Bohr orbital, which is usually referred to as the *ground state* of (the electron in) the hydrogen atom. The *Rydberg* energy E_R is just the combined kinetic and potential energy of the electron in the first Bohr orbital and it can be expressed in terms of the fine-structure constant (α) and the rest energy ($E_0 = mc^2$) of the electron[7]:

$$\mathrm{E}_R = \frac{\alpha^2 \mathrm{m}c^2}{2} = \frac{1}{2}\left(\frac{\mathrm{q_e}^2}{2\varepsilon_0 \mathrm{h}c}\right)^2 \mathrm{m}c^2 = \frac{\mathrm{q_e}^4 \mathrm{m}}{8\varepsilon_0^2 \mathrm{h}^2} \approx 13.6\ \mathrm{eV}$$

To be precise, the *difference* in energy between the various orbitals should be equal to:

$$\Delta\mathrm{E} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot \mathrm{E}_R$$

The Rydberg formula then becomes self-evident. The idea of the wavelength of a wave (λ), its velocity of propagation (c) and its frequency (f)[8] are related through the $\lambda = c/f$ relation, and the Planck-Einstein relation ($E = h \cdot f$) tells us the energy and the wavelength of a photon are related through the frequency:

$$\lambda = \frac{c}{f} = \frac{hc}{\mathrm{E}}$$

Hence, we can now write the Rydberg formula by combining the above:

$$\frac{1}{\lambda} = \frac{\mathrm{E}}{\mathrm{h}c} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot \frac{\mathrm{E}_R}{\mathrm{h}c} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot \frac{\alpha^2 \mathrm{m}c^2}{2\mathrm{h}c}$$

The Rydberg formula uses the fine-structure constant, but actually describes the so-called *gross* structure of the hydrogen spectrum only (illustrated below). Indeed, when the spectral lines are examined at *very* high resolution, the spectral lines are split into finer lines. This is due to the intrinsic spin of the electron. This *intrinsic* spin of the electron is to be distinguished from its orbital motion. It shows we should not be thinking of the electron as a pointlike (infinitesimally small) particle: it has a radius.[9] Hence, we speak of *spin* angular momentum versus *orbital* angular momentum. However, as we will explain, there is some *coupling* between the two motions. We will come back to this later.

[7] We should write m_0 instead of m – *everywhere*. But we are using non-relativistic formulas for the velocity and kinetic energy everywhere. Hence, we dropped the subscript.

[8] Our paper relates mathematical and physical concepts. Hence, we prefer to think of a wavelength as a mathematical *idea* right now, as opposed to some (physical) *reality*. Our ontological viewpoint is very simple: language describes reality. Hence, math describes physics. There is an intimate relation between both but – at the same time – we should not confuse the two.

[9] We argue its radius is the Compton radius. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

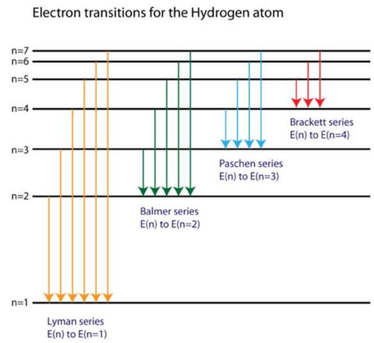


Figure 10: The gross structure of the hydrogen spectrum

The Copenhagen interpretation of quantum mechanics – which, privately, we have started to think of as the *Heisenberg Diktatur*[10] – dismisses Bohr's model. However, it is actually a proper quantum-mechanical explanation and Schrödinger's equation does not seem to add much in terms of a scientific explanation for the atomic electron orbitals. Feynman (*Lectures*, III-2-4) derives it from the momentum-space expression of the Uncertainty Principle which we may loosely state as follows: the product of the uncertainty in the momentum (Δp) and the uncertainty in the position (Δx) has an order of magnitude that is equal to Planck's quantum (*h*). His equation is the following:

$$\mathrm{p} \cdot a \approx \hbar \Leftrightarrow \mathrm{p} \approx \hbar / a$$

This allows him to write the kinetic energy of the electron as $mv^2/2 = p^2/2m = \hbar^2/2ma^2$. The potential energy is just the electrostatic energy $-e^2/a$.[11] The idea is then that the configuration must minimize the total energy $E = \hbar^2/2ma^2 - e^2/a$. The variable is the radius *a* and, hence, we get *a* by calculating the dE/da derivative and equating it to zero. We thus get the correct Bohr radius:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{mq_e}^2} = \frac{1}{\alpha} \cdot r_{\mathrm{Compton}} \approx 53 \times 10^{-12}\ \mathrm{m}$$

We find it useful to write the Bohr radius as the Compton radius divided by the fine-structure constant:

$$r_B = r_C/\alpha = \hbar/\alpha mc \approx (386/0.0073)\times 10^{-15}\ \mathrm{m} \approx 53\times 10^{-9}\ \mathrm{m}.$$

We can now calculate the Rydberg energy – which is the *ionization* energy of hydrogen – by using the Bohr radius to calculate the energy $E = \hbar^2/2ma^2 - e^2/a$:

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[10] No one should take offense here. It is an opinion which is rooted in our experience trying to submit articles to scientific journals as well as interactions with academics. In fact, we should tone down and *not* specifically associate the Copenhagen interpretation with Heisenberg and other founding fathers of the quantum-mechanical framework, as they were part of the group of 'founding fathers' who actually became quite skeptical about the theory they had created because of the divergences in perturbative quantum electrodynamics (QED). Todorov (2018) specifically Heisenberg, Dirac, and Pauli in this regard, and mentions that QED, as a theory, only survived because of the efforts of the second generation of quantum physicists (Feynman, Schwinger, Dyson, etcetera). See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

[11] The e^2 in this formula is the squared charge of an electron (q_e^2) divided by the electric constant ($4\pi\varepsilon_0$). The formula assumes the potential is zero when the distance between the positively charged nucleus and the electron is infinite, which explains the minus sign. We also get the minus sign, of course, by noting the two charges (electron and nucleus) have equal magnitude but opposite sign. One should note that the formulas are non-relativistic. This is justified by the fact that the velocities in this model are non-relativistic (the electron velocity in the Bohr orbital is given by $v_e = \alpha \cdot c \approx 0.0073 \cdot c$. This is an enormous speed but still less than 1% of the speed of light.

$$E_R = \frac{1}{2}\frac{\hbar^2}{m}\frac{m^2e^4}{\hbar^4} - e^2\frac{me^2}{\hbar^2} = -\frac{1}{2}\frac{me^4}{\hbar^2} \approx -13.6\text{ eV}$$

This amount equals the kinetic energy ($\hbar^2/2ma^2 = \alpha^2mc^2/2$). The electrostatic energy itself is *twice* that value ($-e^2/r_{Bohr} = -\alpha^2mc^2$).

Feynman's Uncertainty Principe is suspiciously certain. He basically equates the uncertainty in the momentum as the momentum itself (Δp = p) and the uncertainty in the position as a precise radius. We offer an alternative interpretation. If Planck's constant is, effectively, a physical constant ($h \approx 6.626\times10^{-34}$ N·m·s), then we should interpret it as such. If physical action – some force over some distance over some time – comes in units of h, then the relevant distance here is the loop, so that is $2\pi\cdot r_{Bohr}$. We would, therefore, like to re-write Feynman's $p\cdot a \approx \hbar$ assumption as:

$$S = h = p\cdot 2\pi\cdot r_{Bohr} = p\cdot\lambda$$

The λ is, of course, the circumference of the loop. The equation resembles the *de Broglie* equation $\lambda = h/p$. How should we interpret this? We can associate Planck's quantum of action with a *cycle*: let us refer to it as a Bohr loop and, yes, we think of it as a circular orbit. As such, we can write h either as the energy times the cycle time or, else, as the (linear) momentum times the loop: $h = p\cdot 2\pi\cdot r_B$. The latter expression not only reflects the second *de Broglie* relation but also the quantum-mechanical rule that angular momentum should come in units of $\hbar = h/2\pi$. Indeed, the angular momentum can always be written in terms of the tangential velocity, the radius and the mass. As such, the two formulas below amount to the same:

$$L = m\cdot v\cdot r_B = p\cdot r_B = \hbar \Leftrightarrow S = p\cdot 2\pi\cdot r_B = p\cdot\lambda_B = h$$

Let us continue our calculations. We get the velocity out of the expression for the kinetic energy:

$$\text{K.E.} = \frac{mv^2}{2} = \frac{\alpha^2mc^2}{2} \Leftrightarrow v = \alpha\cdot c \approx 0.0073\cdot c$$

Of course, we should also be able to express the velocity as the product of the radius and an angular frequency, which we can do as follows:

$$v = \alpha\cdot c = r_B\cdot\omega_B = \frac{\hbar}{\alpha mc}\cdot\frac{\alpha^2mc^2}{\hbar} = \alpha\cdot c \Leftrightarrow \omega_B = \frac{\alpha^2mc^2}{\hbar}$$

We then calculate the cycle time T as $T = 1/f_B = 2\pi/\omega_B$. Interestingly, the formula for f_B (or, thinking in terms of *angular* frequencies, for ω_B) reflects the first *de Broglie* relation: $f_B = E/h = \alpha^2mc^2/h$. However, we should note that α^2mc^2 is *twice* the Rydberg energy – and, unlike some physicists, we do care about a 1/2 or π factor in our model of a Bohr electron. Hence, we should have a look at this energy concept. We will do so later. Let us – just for now – roll for a moment with this $E = \alpha^2mc^2$ energy concept. It is, obviously, the energy that is associated with the loop. We wrote the quantum of action as the product of the (linear) momentum and the distance along the loop: $h = p\cdot\lambda_B = p\cdot 2\pi\cdot\lambda_B$. Likewise, we can write:

$$h = E\cdot T = \alpha^2mc^2\cdot\frac{2\pi\cdot r_B}{v} = \alpha^2mc^2\cdot\frac{2\pi\cdot r_C}{\alpha\cdot c\cdot\alpha} = mc^2\frac{2\pi\cdot\hbar}{c\cdot m\cdot c} = h$$

Let us now generalize our formulas for all of the Bohr orbitals:

Table 1: Generalized formulas for the Bohr orbitals

Orbital electron (Bohr orbitals)
$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v_n = \frac{1}{n}\alpha c$
$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu_n = \mathrm{I} \cdot \pi r_n{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

The reader can easily verify these formulas – by googling them, doing the calculations himself or, preferably, just doing some substitutions here and there. Let us substitute the equation for ω_n in the L_n formula, for example:

$$\mathrm{L}_n = I \cdot \omega_n = \mathrm{m} \cdot r_n{}^2 \cdot \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \mathrm{m} \cdot \frac{n^4}{\alpha^2}\frac{\hbar^2}{\mathrm{m}^2c^2} \cdot \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = n\hbar$$

The reader should note that these formulas are *not* so obvious as they seem. The table below shows what happens with radii, velocities, frequencies and cycle times as we move out. The velocities go down, all the way to zero for $n \to \infty$, and the corresponding cycle times increases as the *cube* of *n*. Using totally non-scientific language, we might say the numbers suggest the electron starts to lose interest in the nucleus so as to get ready to just wander about as a free electron.

Table 2: Functional behavior of radius, velocity and frequency of the Bohr orbitals

n	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***
$r_n \propto n^2$	1	4	9	16	25	36	49	64	81
$v_n \propto 1/n$	1	0.500	0.333	0.250	0.200	0.167	0.143	0.125	0.111
$\omega_n \propto 1/n^3$	1	0.125	0.037	0.016	0.008	0.005	0.003	0.002	0.001
$T_n \propto n^3$	1	8	27	64	125	216	343	512	729

The important thing is the energy formula, of course, because it should explain the Rydberg formula, and it does:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{n_2{}^2}\mathrm{E}_R + \frac{1}{n_1{}^2}\mathrm{E}_R = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right) \cdot \mathrm{E}_R = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right) \cdot \frac{\alpha^2\mathrm{m}c^2}{2}$$

Let us know look at the energies once again and try to connect this model with the idea of a photon.

V. The wavefunction and the photon

The Bohr orbitals are separated by a amount of *action* that is equal to h. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. Our photon will have to pack that, somehow. It will also have to pack the related energy, which is given by the Rydberg formula (see above). To focus our thinking, let us consider the transition from the second to the first level, for which the $1/1^2 - 1/2^2$ is equal 0.75. Hence, the photon energy should be equal to $(0.75)\cdot E_R \approx 10.2$ eV.[12] Now, if the total action is equal to h, then the cycle time T can be calculated as:

$$\mathrm{E}\cdot\mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135\times 10^{-15}\,\mathrm{eV\cdot s}}{10.2\ \mathrm{eV}} \approx 0.4\times 10^{-15}\ \mathrm{s}$$

This corresponds to a wave train with a length of $(3\times10^8$ m/s$)\cdot(0.4\times10^{-15}$ s$) = 122$ nm. That is the size of a large molecule and it is, therefore, much more reasonable than the length of the wave trains we get when thinking of transients using the supposed Q of an atomic oscillator.[13] In fact, this length is *exactly* equal to the wavelength $\lambda = c/f = c\cdot\mathrm{T} = hc/\mathrm{E}$.

What picture of the photon are we getting here? Because of the angular momentum, we will probably want to think of it as a circularly polarized wave, which we may represent by the elementary wavefunction, as shown below.[14] We will call this interpretation of the wavefunction the *one-cycle photon*: the wavefunction represents the rotating field vector itself or, remembering the $F = q_eE$ equation, the force field.

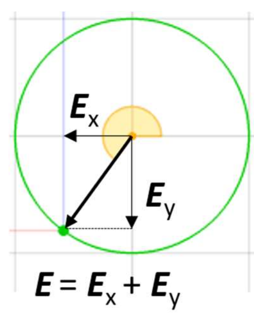

Figure 11: The one-cycle photon

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[12] This is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. The ozone layer of our atmosphere blocks most of it.

[13] In one of his famous *Lectures* (I-32-3), Feynman thinks about a sodium atom, which emits and absorbs sodium light, of course. Based on various assumptions – assumption that make sense in the context of the blackbody radiation model but *not* in the context of the Bohr model – he gets a Q of about 5×10^7. Now, the frequency of sodium light is about 500 THz (500×10^{12} oscillations per second). Hence, the *decay time* of the radiation is of the order of 10^{-8} seconds. So that means that, after 5×10^7 oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. That seems to be very short, but it still makes for 5 million oscillations and, because the wavelength of sodium light is about 600 nm (600×10^{-9} meter), we get a wave train with a considerable length: $(5\times10^6)\cdot(600\times10^{-9}$ meter$) = 3$ meter. *Surely you're joking, Mr. Feynman!* A photon with a length of 3 meter – or longer? While one might argue that relativity theory saves us here (relativistic length contraction should cause this length to reduce to zero as the wave train zips by at the speed of light), this just doesn't feel right – especially when one takes a closer look at the assumptions behind.

[14] Note that the wave could be either left- or right-handed.

It is a delightfully simple model: the photon is just one single cycle traveling through space and time, which packs one unit of angular momentum ($\hbar$) or – which amounts to the same, one unit of physical action (h). This gives us an equally delightful interpretation of the Planck-Einstein relation (f = 1/T = E/h) and we can, of course, do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the E = mc^2 mass-energy equivalence relation can be written as p = mc = E/c for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

Let us now try something more adventurous: let us try to calculate the strength of the electric field. How can we do that? Energy is some force over a distance. What distance should we use? We could think of the wavelength, of course.

However, the formulas above imply the following equation: E·λ = h·c. This suggest we should, perhaps, associate some *radius* with the wavelength of our photon. We write:

$$\mathrm{E} \cdot \frac{\lambda}{2\pi} = \mathrm{E} \cdot r = \hbar \cdot c \Leftrightarrow r = \frac{\lambda}{2\pi} = \frac{\hbar \cdot c}{\mathrm{E}}$$

A strange formula? The reader can check the physical dimensions. They all work out: we do get a distance – something that is expressed in *meter*. But why the 2π factor? We do not want to confuse the reader too much but let us quickly re-insert the graph on the presumed *Zitterbewegung* of a free electron – which is interpreted as an oscillation of a pointlike *charge* (with *zero* rest mass) moving about a center at the speed of light. Now, as the electron starts moving along some trajectory at a relativistic velocity (i.e. a velocity that is a *substantial* fraction of c), the *radius* of the oscillation will have to diminish – because the tangential velocity remains what it is: c. The geometry of the situation (see below) shows the circumference becomes a wavelength in this process.

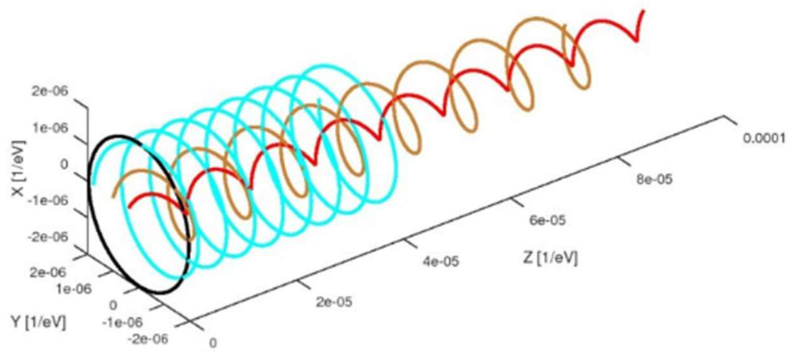

Figure 12: The Compton radius must decrease with increasing velocity

We have probably confused the reader now, but he or she should just hang on for a while. Let us just jot down the following expression and then we can think about it:

$$\mathrm{E}_\gamma = \mathrm{F}_\gamma \cdot r_\gamma = \mathrm{F}_\gamma \cdot \frac{\lambda_\gamma}{2\pi}$$

We use the γ subscript to denote we're talking the energy, force and radius in the context of a photon because – in order to justify the formula above – we will remind ourselves of one of the many meanings of the fine-structure constant here: as a coupling constant, it is defined as the ratio between (1) $k \cdot q_e^2$ and (2) $E \cdot \lambda$. We can interpret this as follows:

1. The $k \cdot q_e^2$ in this ratio is just the product of the electric potential between two elementary charges (we should think of the proton and the electron in our hydrogen atom here) and the distance between them:

$$\mathrm{U}(r) = \frac{k \cdot \mathrm{q_e}^2}{r} = \frac{\mathrm{q_e}^2}{4\pi\varepsilon_0 r} \Leftrightarrow k \cdot \mathrm{q_e}^2 = \mathrm{U}(r) \cdot r$$

2. The fine-structure constant can then effectively be written as:

$$\alpha = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{U}(r) \cdot r}{\mathrm{E}_{photon} \cdot r_{photon}}$$

We can also write this in terms of forces times the *squared* distance:

$$\alpha = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}{}^2}{\mathrm{F}_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}{}^2}{\mathrm{F}_\gamma \cdot r_\gamma{}^2} = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma}$$

This doesn't look too bad. We use B as a subscript in the denominator to remind ourselves we are talking the Bohr energies and radii. Let us write it all out – using the generalized formulas (n = 1, 2,…) above – to demonstrate the consistency of this formula:

$$\alpha = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 \mathrm{m}c^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}}{\mathrm{E}_\gamma \cdot \frac{\hbar \cdot c}{\mathrm{E}_\gamma}} = \alpha$$

Onwards! We think the following formula for the force may make sense now:

$$\mathrm{F}_\gamma = \frac{\mathrm{E}_\gamma}{r_\gamma} = \frac{2\pi \cdot \mathrm{E}_\gamma}{\lambda_\gamma} = \frac{2\pi \cdot h \cdot f_\gamma}{\lambda_\gamma} = \frac{2\pi \cdot h \cdot c}{{\lambda_\gamma}^2}$$

The electric field (E) is the force per unit charge which, we should remind the reader, is the *coulomb* – *not* the electron charge. Dropping the subscript, we get a delightfully simple formula for the strength of the electric field vector for a photon[15]:

$$E = \frac{\frac{2\pi hc}{\lambda^2}}{1} = \frac{2\pi hc}{\lambda^2} = \frac{2\pi \mathrm{E}}{\lambda}\left(\frac{N}{C}\right)$$

Let us calculate its value for our 10.2 eV photon. We should, of course, express the photon energy in SI units here:

$$E \approx \frac{2\pi \cdot 1.634 \times 10^{-18}\, J}{122 \times 10^{-9}\, m \cdot C} \approx 84 \times 10^{-1}\ \frac{\mathrm{N}}{\mathrm{C}}$$

This seems pretty reasonable![16] Let us make a final check on the logical consistency of this model. The energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Do we get any meaningful result when we apply that principle here? If we write the proportionality coefficient as k, we could write something like this:

$$\mathrm{E} = k \cdot a^2 \cdot \omega^2$$

It would be wonderful if this would give some meaningful result – and even more so if we could interpret the proportionality coefficient k as the mass m. Why? Because we have used the $\mathrm{E} = \mathrm{m} \cdot a^2 \cdot \omega^2$ equation before: it gave us this wonderful interpretation of the *Zitterbewegung* as what we referred to as the *rest matter oscillation*. We will show, in the next section, that the idea of a two-dimensional oscillation can also be applied to the Rutherford-Bohr model. Hence, can we repeat the trick here? We can, but the amplitude of the oscillation here is the *wavelength*. We can then write:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

Sometimes physics can be *just nice*. I think we have a pretty good photon model here.

Before we move on, we need to answer an obvious question: what happens when an electron jumps several Bohr orbitals? The angular momentum between the orbitals will then differ by *several* units of $\hbar$. What happens to the photon picture in that case? It will pack the energy difference, but should it also pack several units of $\hbar$. In other words, should we still think of the photon as a one-cycle oscillation, or will the energy be spread over several cycles?

[15] The E and E symbols should not be confused. E is the magnitude of the electric field vector and E is the energy of the photon. We hope the italics (E) – and the context of the formula, of course ! – will be sufficient to distinguish the electric field vector (E) from the energy (E).

[16] We got a rather non-sensical value in another paper (http://vixra.org/abs/1812.0028) but that's because we used the electron charge instead of the unit charge to calculate the field.

We will let the reader think about this, but our intuitive answer is: the photon is a spin-one particle and, hence, its energy should, therefore, be packed in one cycle only. This is also necessary for the consistency of the interpretation here: when everything is said and done, we do interpret the wavelength as a *physical* distance. To put it differently, the equation below needs to make sense:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

VI. The two-dimensional oscillator

Let us summarize what we have presented so far. We explained the rest mass of the electron in terms of its *Zitterbewegung*. This interpretation of an electron, which goes back to Schrödinger and Dirac[17], combines the idea of motion with the idea of a pointlike *charge*, which has no inertia and can, therefore, move at the speed of light. The illustration below described the presumed circular oscillatory motion of the charge (the *Zitterbewegung*). We got wonderful results. The most spectacular result is the explanation for the *rest* mass of an electron: it is the equivalent mass of what we referred to as the *rest matter oscillation*.

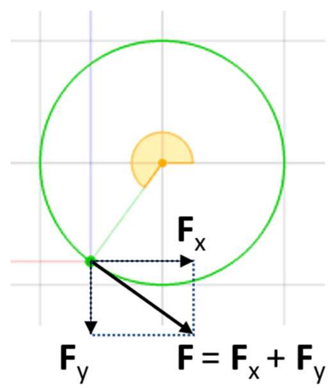

Figure 13: The *Zitterbewegung* model of an electron

The table summarizes the properties – angular momentum, magnetic moment, g-factor, etc. – we calculated:

[17] Erwin Schrödinger derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Table 3: The properties of the free electron (spin-only)

Spin-only electron (*Zitterbewegung*)
$\mathrm{S} = \mathrm{h}$
$\mathrm{E} = \mathrm{m}c^2$
$r = r_\mathrm{C} = \dfrac{\hbar}{\mathrm{m}c}$
$v = c$
$\mathrm{L} = I \cdot \omega = \dfrac{\hbar}{2}$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \dfrac{\mathrm{q_e}}{2\mathrm{m}}\hbar$
$\mathrm{g} = \dfrac{2\mathrm{m}}{\mathrm{q_e}}\dfrac{\mu}{\mathrm{L}} = 2$

The reader should keep his wits about him[18] here: the *Zitterbewegung* model should not be confused with our Bohr atom. We do not have any centripetal force here. There is no nucleus or other charge at the center of the *Zitterbewegung*. Instead of a tangential momentum vector, we have a tangential *force* vector (**F**), which we thought of as being the resultant force of two perpendicular oscillations.[19] This led us to boldly equate the E = mc^2, E = m·a^2·ω^2 and E = $\hbar$·ω equations – which gave us all the results we wanted. The *zbw* model – which, as we have mentioned in the footnote above, is inspired by the solution(s) for Dirac's wave equation for free electrons – tells us the velocity of the pointlike *charge* is equal to *c*. Hence, *if* the *zbw* frequency would be given by Planck's energy-frequency relation (ω = E/ħ), *then* we can easily combine Einstein's E = mc^2 formula with the radial velocity formula ($c = a \cdot \omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$r_{\mathrm{Compton}} = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_\mathrm{e}}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

The calculations relate the Bohr radius to the Compton radius through the fine-structure constant:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{m}{\mathrm{q_e}}^2} = \frac{1}{\alpha} \cdot r_{\mathrm{Compton}} = \frac{\hbar}{\alpha\mathrm{m}c} \approx 53 \times 10^{-12}\ \mathrm{m}$$

The fine-structure constant also relates the respective velocities, frequencies and energies of the two oscillations. We wrote:

$$v = \alpha \cdot c = r_\mathrm{B} \cdot \omega_\mathrm{B} = \frac{\hbar}{\alpha\mathrm{m}c} \cdot \frac{\alpha^2\mathrm{m}c^2}{\hbar} = \alpha \cdot c \Leftrightarrow \omega_\mathrm{B} = \frac{\alpha^2\mathrm{m}c^2}{\hbar}$$

As we mentioned before, the formula for the frequency of the motion of the electron in the Bohr orbitals reflects the first *de Broglie* relation: f_B = E/h = α^2mc^2/h. Needless to say, the cycle time T is given

[18] The him could be a her, of course.

[19] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

as a function of the Bohr loop frequency by $T = 1/f_B = 2\pi/\omega_B$. [In this section, we will just use the formulas for the first Bohr orbital ($n = 1$). It is easy generalize for $n = 2, 3, 4$, etc.] However, we noted that the α^2mc^2 is *twice* the Rydberg energy – and, unlike some physicists, we do care about a 1/2 or π factor in our model of a Bohr electron. Hence, we should have a look at this energy concept.

The $E = \alpha^2mc^2$ energy concept is the energy that is associated with the loop. It is *twice* the kinetic energy, but it is a different energy concept altogether. In line with our interpretation of the elementary wavefunction in the context of our one-cycle photon and our free (spin-only) electron, we are thinking of the orbital motion as being driven by a two-dimensional oscillation, as illustrated below.

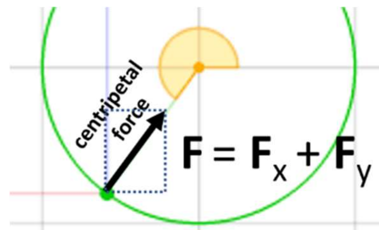

Figure 14: The oscillator model of the Bohr orbital

We look at the centripetal force as a *resultant* force here – a vector sum of two perpendicular components: $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Needless to say, the **boldface** here indicates *vectors*: these force *components* have a magnitude as well as a direction. We can now develop yet another interpretation of the elementary wavefunction and think of a dual view of what is going on. Let us start with the description of the physical position (i.e. the *x*- and *y*-coordinates) of the electron. This is the green dot in the illustration of Euler's function above. Its motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot cos(\theta) + i \cdot a \cdot sin(\theta) = (x, y)$$

We can now think of this motion being driven by two perpendicular oscillations. These oscillations are associated with a kinetic and a potential energy. We illustrate this below for *one* oscillator only.

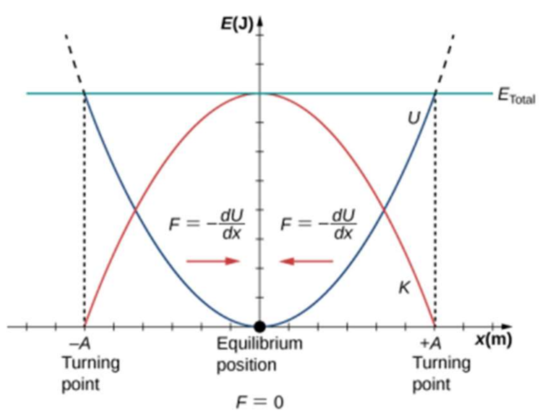

Figure 15: Kinetic (K) and potential energy (U) of an oscillator

Now, if the amplitude of the oscillation is equal to a, then we know that the sum of the kinetic and potential energy of the oscillator will be equal to $(1/2) \cdot m \cdot a^2 \cdot \omega^2$. In this case (the Bohr orbital), we have two oscillators, and we can add their kinetic and potential energies because of the 90-degree phase difference. Indeed, it is easy to see that the *total* kinetic energy – added over the *two* oscillators – will effectively be constant *over the cycle* and will be equal to:

$$\text{K.E.} = \frac{1}{2}\text{m}\cdot{r_\text{B}}^2\cdot\omega^2 = \frac{1}{2}\text{m}\cdot v^2 = \frac{1}{2}\alpha^2\cdot\text{m}\cdot c^2$$

The potential energy will be equal to the kinetic energy and we, therefore, get the desired result: the *total* energy of the loop is equal to E = $\alpha^2 mc^2$. We can now re-write the quantum of action as the product of the energy and the cycle time:

$$h = \text{E}\cdot\text{T} = \alpha^2\text{m}c^2\cdot\frac{2\pi\cdot r_\text{B}}{v} = \alpha^2\text{m}c^2\cdot\frac{2\pi\cdot r_\text{C}}{\alpha\cdot c\cdot\alpha} = \text{m}c^2\frac{2\pi\cdot\hbar}{c\cdot\text{m}\cdot c} = h$$

Of course, we can also write it as the product of the (linear) momentum and the distance along the loop:

$$h = \text{p}\cdot\lambda_\text{B} = \text{m}\cdot v\cdot 2\pi\cdot r_\text{B} = \text{m}\cdot\alpha\cdot c\cdot 2\pi\cdot\frac{\hbar}{\alpha\text{m}c} = h$$

All makes sense. Now, we said we have a *dual* view of the meaning of the wavefunction here. What is the dual view? It is that of the force vector: we will want to write the energy as the product of a force over a distance. Hence, what is the force and what is the distance here? The Bohr model implies the circular motion of the *electron* is driven by (1) its inertia[20] and (2) a centripetal force (because of the presence of a nucleus with the opposite charge). The geometry of the situation shows we can write $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ as:

$$\boldsymbol{F} = -F\cdot\cos(\omega t) - i\cdot F\cdot\sin(\omega t) = -F\cdot e^{i\theta}$$

The nature of this force is electric, of course. Hence, we should write in in terms of the electric field vector E[21]: $\boldsymbol{F} = q_e\boldsymbol{E}$. The electric field is, of course, the force on the unit charge which, in this case, is a force between q_e (the electron) and $-q_e$ (the proton or hydrogen nucleus).[22] Let us calculate the magnitude of the force by using the fine-structure constant to check the consistency of the model:

$$F = \text{q}_\text{e}E = \frac{{\text{q}_\text{e}}^2}{4\pi\varepsilon_0{r_\text{B}}^2} = \frac{\alpha\hbar c}{{r_\text{B}}^2} = \frac{\hbar v}{{r_\text{B}}^2} = \frac{\hbar r_\text{B}\omega_\text{B}}{{r_\text{B}}^2} = \frac{\hbar}{r_\text{B}}\frac{\alpha^2\text{m}c^2}{\hbar} = \frac{\text{E}}{r_\text{B}}$$

This $F = q_eE = \text{E}/r_B$ is confusing (E is the electric field, but E is the *energy*) but very interesting because it allows us to write the quantum of action in its usual dimensions – which is the product of a force, a distance (the radius of the oscillation, in this particular case), and a time:

---

[20] Note the difference with the *Zitterbewegung* model, which assumes a pointlike *charge* with *no inertia to motion*. Its orbital velocity is, therefore, effectively equal to the speed of light (c). This is *very* different from the Bohr model, in which the electron moves at a non-relativistic speed $v = \alpha c$ with $\alpha \approx 0.0073$. However, the two models are obviously complementary: the *Zitterbewegung* model – Dirac's electron, we might say – effectively explains the (rest) mass of the Bohr electron.

[21] Symbols may be confusing. We use E for the energy, but E for the electric field vector. Likewise, I is a moment of inertia, and I is an electric current. The context is usually clear enough to make out what is what.

[22] The concepts of potential, potential energy and the electric field can be quite confusing. The potential and the potential energy of a charge in a field vary with $1/r$. The electric field is the electric force – generally defined as the *Lorentz* force $\boldsymbol{F} = q\boldsymbol{E} + q(\boldsymbol{v}\times\boldsymbol{B})$ – on the unit charge. Hence, the $\boldsymbol{F} = q\boldsymbol{E}$ formula here is nothing but the $\boldsymbol{E} = \boldsymbol{F}/q$ formula. The electric field varies with $1/r^2$ and is, therefore, associated with the inverse-*square* law. It is also quite confusing that q_e is actually the (supposedly negative) electron charge and that we have to, therefore, use a minus sign for the charge of the (supposedly positive) proton charge – but then the signs always work out, of course.

$$h = F \cdot r \cdot \mathrm{T} = \frac{\mathrm{E}}{r_{\mathrm{B}}} \cdot r_{\mathrm{B}} \cdot \frac{1}{f} = \frac{\mathrm{E}}{r_{\mathrm{B}}} \cdot r_{\mathrm{B}} \cdot \frac{h}{\mathrm{E}} = h$$

Hence, we have a bunch of equivalent expressions for Planck's quantum of action – all of which help us to understand the *complementarity* of the various viewpoints:

$$h = \mathrm{p} \cdot 2\pi \cdot r = \mathrm{p} \cdot \lambda$$

$$h = \mathrm{E} \cdot \mathrm{T} = \mathrm{E}/f$$

$$h = r \cdot \mathrm{T} \cdot F = r \cdot \mathrm{T} \cdot \mathrm{q_e} E = r \cdot \mathrm{T} \cdot \mathrm{E}/r = \mathrm{E} \cdot \mathrm{T}$$

We could also combine these formulas with the classical formulas for a centripetal force – think of the $F = \mathrm{m} \cdot r \cdot \omega^2$ and $\mathrm{F} = \mathrm{m} \cdot v^2/r = \mathrm{p} \cdot v/r$ formulas here – but we will let the reader play with that.

The point is: there is an *energy* in this oscillation, and the energy makes sense if we think of it as a two-dimensional oscillation. We can write this two-dimensional oscillation – using Euler's formula - in various but complementary ways. We can use the position vector, the force vector, or the electric field vector:

$$\boldsymbol{F} = -F_x \cdot \cos(\omega t) - i \cdot F_x \cdot \sin(\omega t) = -F \cdot e^{i\theta}$$

$$\boldsymbol{E} = -(E/\mathrm{q_e}) \cdot \cos(\omega t) - i \cdot (E/\mathrm{q_e}) \cdot \sin(\omega t) = -F \cdot e^{i\theta}$$

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot cos(\theta) + i \cdot a \cdot sin(\theta) = (x, y)$$

The various viewpoints of the oscillation are complementary. They pack the same energy ($\mathrm{E} = \alpha^2 \mathrm{m} c^2$), and they pack one unit of physical action (h). We will leave it to the reader to generalize for the n = 2, 3, etc. orbitals. It is an easy exercise: the energy for the higher loops is equal to $\mathrm{E}_n = \alpha^2 \mathrm{m} c^2/n^2$ and the associated action is equal to $\mathrm{S} = n \cdot h$. One obvious way to relate both is through the frequency of the loop. We write:

$$f_n = \frac{\mathrm{E}_n}{\mathrm{S}_n} = \frac{\frac{1}{n^2}\alpha^2 \mathrm{m} c^2}{nh} = \frac{\alpha^2}{n^3 h} \mathrm{m} c^2$$

VII. The fine-structure constant as a scaling constant

The fine-structure constant pops up as a dimensional scaling constant in the calculations above. It relates the Bohr radius to the Compton radius, for example:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0 \hbar^2}{\mathrm{m}{\mathrm{q_e}}^2} = \frac{1}{\alpha} \cdot r_{\mathrm{Compton}} = \frac{\hbar}{\alpha \mathrm{m} c} \approx 53 \times 10^{-12} \mathrm{~m}$$

But it also relates the respective velocities, frequencies and energies of the two oscillations. We may summarize these relations in the following equations:

$$v = \alpha \cdot c = r_{\mathrm{B}} \cdot \omega_{\mathrm{B}} = \frac{\hbar}{\alpha \mathrm{m} c} \cdot \frac{\alpha^2 \mathrm{m} c^2}{\hbar} = \alpha \cdot c$$

But this is not the only meaning of the fine-structure constant. We know it pops up in many other formulas as well. To name just a few:

1. It is the mysterious quantum-mechanical *coupling* constant.

2. It explains the so-called anomalous magnetic moment – which, as we will explain in a moment, might *not* be anomalous at all!
3. Last but not least, it explains the *fine structure* of the hydrogen spectrum – which is where it got its name from, of course!

Can we make some more sense of this as a result of the interpretations we have offered above? Let us start with the coupling constant because there is a lot of nonsensical writing on that.[23] We basically showed that, as a coupling constant, the fine-structure continues to act as a dimensional scaling constant. We wrote:

$$\alpha = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F}_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F}_\gamma \cdot r_\gamma^2} = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma}$$

We use B as a subscript in the denominator to remind ourselves we are talking the Bohr energies and radii. Let us use the generalized formulas (*n* = 1, 2,…) for the Bohr orbitals once again and write it all out:

$$\alpha = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 \mathrm{m}c^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}}{\mathrm{E}_\gamma \cdot \frac{\hbar \cdot c}{\mathrm{E}_\gamma}} = \alpha$$

While the formula is obvious, its interpretation is not necessarily as obvious: what *is* this product of an energy and a radius? How should we interpret this? The physical dimension of this product (in the denominator and the numerator, of course) is J·m = N·m·m = N·m^2. We get the same physical dimension if we multiply action or angular momentum with a velocity, so let us try this to check if it makes us any wiser:

$$\alpha = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E}_\gamma \cdot r_\gamma} = \frac{\mathrm{L}_n \cdot v_n}{\mathrm{L}_\gamma \cdot v_\gamma} = \frac{n\hbar \cdot \frac{1}{n}\alpha c}{\hbar \cdot c} = \alpha = \frac{\mathrm{S}_n \cdot v_n}{\mathrm{S}_\gamma \cdot v_\gamma} = \frac{nh \cdot \frac{1}{n}\alpha c}{h \cdot c} = \alpha$$

The formulas show we should, most probably, just think of them as yet another expression of the idea of a scaling constant.

Let us think of the fine-structure constant in yet one more way. We know the Compton and Bohr radius are related through the fine-structure constant. We used this formula many times already:

$$r_\mathrm{C} = \alpha \cdot r_\mathrm{B}$$

Let us write this out:

$$r_\mathrm{C} = \frac{\hbar c}{\mathrm{m}c^2} = \frac{\hbar c}{\mathrm{E_e}}$$

[23] Feynman's *QED: The Strange Theory of Light and Matter* (1985) refers to its (negative) *square root* as the coupling constant, and states that is "the amplitude for a real electron to emit or absorb a real photon." We take it to be just one example of an ambiguous remark by a famous physicist that is being explained by an amateur physicist. The book was not written by Richard Feynman: it is a transcription of a short series of lectures by Feynman for a popular audience. We are not impressed by the transcription.

$$r_{\mathrm{B}} = \frac{\hbar c}{\alpha \mathrm{m} c^2} = \frac{\hbar c}{\mathrm{E_B}}$$

The E_e is just the (rest) energy of the electron, and E_B is the energy in the (first) Bohr orbital. Hence, we can also write the fine-structure constant as the ratio between these two energies:

$$\alpha = \frac{r_{\mathrm{C}}}{r_{\mathrm{B}}} = \frac{\frac{\hbar c}{\mathrm{E_e}}}{\frac{\hbar c}{\mathrm{E_B}}} = \frac{\mathrm{E_B}}{\mathrm{E_e}}$$

Because $r_n = n^2 r_{\mathrm{B}}$ and $E_n = E_B/n^2$, we know that $r_n = \frac{n^2 \hbar c}{\alpha \mathrm{m} c^2} = \frac{\hbar c}{\mathrm{E}_n}$ and, hence, we can easily generalize for the n = 2, 3,… orbitals:

$$\alpha = \frac{r_{\mathrm{C}}}{r_n} = \frac{\frac{\hbar c}{\mathrm{E_e}}}{\frac{\hbar c}{\mathrm{E}_n}} = \frac{\mathrm{E}_n}{\mathrm{E_e}}$$

The explorations above - and the interpretation of the fine-structure constant as a scaling constant – raise an interesting question. We know there is also the idea of a classical electron radius, which is related to the Compton radius in the same way as the Compton radius to the Bohr radius:

$$r_{\mathrm{e}} = \alpha \cdot r_{\mathrm{C}} = \alpha^2 \cdot r_{\mathrm{B}}$$

We have already explained the second identity ($\alpha r_{\mathrm{C}} = \alpha^2 r_{\mathrm{B}} \Leftrightarrow r_{\mathrm{C}} = \alpha r_{\mathrm{B}}$) but what about $r_{\mathrm{e}} = \alpha r_{\mathrm{C}}$? Let us think about that in a separate section

VIII. The fine-structure constant and the classical electron radius

Let us write all out and see if there is something triggering some idea:

$$r_{\mathrm{e}} = \frac{\mathrm{e}^2}{\mathrm{m} c^2} = \alpha \frac{\hbar c}{\mathrm{m} c^2}$$

We, once again, have two energies in the numerator – but they are the same ! Hence, when writing the fine-structure constant as the ratio between the two radii, we get:

$$\alpha = \frac{r_e}{r_{\mathrm{C}}} = \frac{\frac{\mathrm{e}^2}{\mathrm{m} c^2}}{\frac{\hbar c}{\mathrm{m} c^2}} = \frac{\mathrm{e}^2}{\hbar c} = k \mathrm{q_e}^2 = \frac{1}{4\pi\varepsilon_0} \frac{\mathrm{q_e}^2}{\hbar c}$$

We just get the usual formula for the fine-structure constant here. What does it mean in terms of interpretation? Here we should probably try to think of the meaning of e^2. There is something interesting here: the elementary charge e^2 has the same physical dimension – the *joule-meter* (J·m) – as the h·c = E·λ product:

$$[\mathrm{e}^2] = \left[\frac{1}{4\pi\varepsilon_0} \frac{\mathrm{q_e}^2}{\hbar c}\right] = \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{C}^2} \cdot \mathrm{C}^2 = \mathrm{N} \cdot \mathrm{m}^2 = \mathrm{J} \cdot \mathrm{m}$$

What was that h·c = E·λ product again? We got it in the context of our photon model. To be precise, we got it by applying the *second* de Broglie equation to a photon:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

In fact, it appears we may apply this relation to any particle that is traveling at the speed of light. *Huh?* What other particle do we have? Our pointlike *charge* in the *Zitterbewegung* model of an electron: this charge has, effectively, no rest mass and, therefore, does make us think of a photon. But we should be precise here: it is the *square* of the elementary charge that that *joule-meter* dimension. We write:

$$[\mathrm{e}^2] = [\mathrm{E}] \cdot [\lambda] = [h] \cdot [c]$$

This is strange: what energy and what wavelength would we associate with this pointlike charge. I am not sure – but if we try the energy and the circumference of the loop of the *Zitterbewegung,* we get a sensible relation on the right-hand side:

$$\mathrm{E} \cdot \lambda = \mathrm{m}c^2 \cdot \frac{\hbar}{\mathrm{m}c} = h \cdot c$$

Obvious, you'll say. But, *no*, this is not obvious: we are not talking the energy and the mass of a photon here but the energy and the mass of… Well… Our pointlike charge in its *Zitter* motion.

And what about the suggestion we should be able to write something like e^2 = E·λ? Well… We can start by re-writing the formula for the classical electron radius so it gives us a product of an energy and a distance:

$$\mathrm{e}^2 = r_\mathrm{e}\mathrm{m}c^2 = r_e\mathrm{E}$$

Does this make sense? Yes, it does. It gives us the formula for the fine-structure constant once again:

$$\mathrm{e}^2 = r_\mathrm{e}\mathrm{m}c^2 = \alpha r_\mathrm{C}\mathrm{E} = \alpha\frac{\hbar c}{\mathrm{m}c^2}\mathrm{E} = \alpha\frac{\hbar c}{\mathrm{m}c^2}\mathrm{E} = \alpha\hbar c \Leftrightarrow \alpha = \frac{\mathrm{e}^2}{\hbar c}$$

By now, the reader is probably tired of these gymnastics and, hence, we will stop here. What was the use? Interpretation. The formulas are not presenting anything new: we have just been substituting and re-arranging equations but we have, hopefully, succeeded in presenting a coherent picture while doing so.

IX. The fine-structure constant and the anomalous magnetic moment

Introduction

Let us briefly remind the reader of the context. We recently suggested[24] that it might be possible to explain the anomalous magnetic moment based on some *form factor* that would come out of a classical electron model. While we initially thought about these things from a learning perspective only – we just

[24] Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018, http://vixra.org/pdf/1812.0233v3.pdf. The paper should, preferably, be read in conjunction with a more recent paper on how the fine-structure constant relates the various layers in the motion of an electron: *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

wanted to possibly identify a better *didactic* approach to teaching quantum mechanics – the idea seems to have taken some life on its own now.[25]

What is a 'classical' electron model? We use this term to refer to any theory of an electron that does *not* invoke perturbation theory. We do not like perturbation theory because of the very same reason that made the founding fathers (Heisenberg, Dirac, Pauli, ...) skeptical about the theory they had created.[26] Interestingly, Ivan Todorov – whose paper notes the above – also speaks of the theoretical value of the spin angular momentum (g_{spin} = 2) as a "dogma" and mentions two letters of Gregory Breit to Isaac Rabi, which may be interpreted as Breit defending the idea that an intrinsic magnetic moment "of the order of $\alpha\mu_B$" may not be anomalous at all.[27] Needless to say, the issue is quite controversial because a classical explanation of the anomalous magnetic moment would question some of the rationale behind the award of two Nobel Prizes for physics.

Let us be precise here. Polykarp Kusch got (half of) the 1955 Nobel Prize "for his precision determination of the magnetic moment of the electron."[28] As such, we should *not* associate him with the theory behind. Having said that, the measurement obviously corroborated the new theories of what Todorov refers to as "the younger generation" of physicists – in particular Richard Feynman, Julian Schwinger and Shinichiro Tomonaga, who got their 1965 Nobel Prize for "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles" – for the *theory*, that is.

What Brian Hayes refers to as "the tennis match between experiment and theory" [29] seems to be a game without end. The question is: is there another game in town? We think there might be one.

The new quantum physics

We will *not* explain perturbation theory here.[30] We only want to give a quick overview of its results in the context of the *theoretical* explanation of the anomalous magnetic moment. Indeed, we described the methodology of its *measurement* in the above-mentioned paper and, hence, we will not repeat ourselves here. In fact, we suggest the reader directly consults the 2009 article of the Harvard University group that does these experiments.[31] We will just note that the confusion starts with the definition of

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[25] Our physics blog attracts a fair amount of comments from fellow amateur physicists. These remarks are encouraging but do not add any credibility to the model (on the contrary, we'd say). However, we also had discussions with some researchers on Kerr-Newman and *Zitterbewegung* models. While we speak a very different language, these discussions suggest the key ideas might make some sense.

[26] See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

[27] For a more detailed account of the substance of these conversations, see: Silvan S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga* , p. 222-223.

[28] See: https://www.nobelprize.org/prizes/physics/1955/summary/ and https://www.nobelprize.org/prizes/physics/1965/summary/.

[29] See: Brian Hayes, *Computing Science: g-ology*, in: *American Scientist*, Vol. 92, No. 3, May-June 2004, pages 212-216. The subtitle says it all: it is an article 'on the long campaign to refine measurements and theoretical calculations of a physical constant called the *g* factor of the electron.' https://pdfs.semanticscholar.org/4c12/50f66fc1fb799610d58f25b9c1e1c2d9854c.pdf.

[30] The interested reader may consult any standard textbook on that. See, for example, Jon Mathews and R.L. Walker, *Mathematical Methods of Physics*, 1970.

[31] D. Hanneke, S. Fogwell, N. Guise, J. Dorr and G. Gabrielse, *More Accurate Measurement of the Electron Magnetic Moment and the Fine-Structure Constant*, in: Proceedings of the XXI International Conference on Atomic Physics, 2009. We prefer this article over the original 2006 article (G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *New Determination of the*

the anomalous magnetic moment. It is actually *not* a magnetic moment but a *gyromagnetic ratio* (i.e. a *ratio* between a magnetic moment and an angular momentum) and it's defined as:

$$a_e = \frac{g}{2} - 1$$

The 2009 article states that the *measured* value of *g* is equal to 2.00231930436146(56). The 56 (between brackets) is the (un)certainty: it is equal to 0.00000000000056, i.e. 56 *parts per trillion* (ppt) and it is measured as a standard deviation.[32] Hence, a_e is equal to 0.00115965218073(28).

The so-called anomaly is the difference with the theoretical value for the *spin* angular momentum which came out of Dirac's equation for the free electron, which is equal to 2. The confusion starts here because there is no obvious explanation of why one would use the (theoretical) *g*-factor for the intrinsic spin of an electron (*g* = 2). The electron in the Penning trap that is used in these experiments is *not* a spin-only electron. It follows an orbital motion – that is one of the three or four layers in its motion, at least – and, hence, if some theoretical value for the *g*-factor has to be used here, then one should also consider the *g*-factor that is associated with the orbital motion of an electron, which is that of the Bohr orbitals (*g* = 1). In any case, one would expect to see a *classical* coupling between (1) the precession, (2) the orbital angular momentum and (3) the spin angular momentum, and the situation is further complicated because of the electric fields in the Penning trap, which add another layer of motion. We illustrate the complexity of the situation below[33].

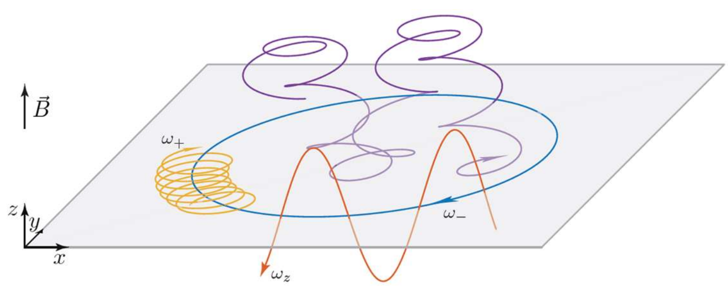

Figure 16: The three principal motions and frequencies in a Penning trap

The point we are trying to make is the following: the theoretical value for a_e (*zero*) would seem to need a better explanation. However, let us roll for a moment with the idea that – through the magic of classical coupling – that its theoretical value should be zero and that we, therefore, do have some anomaly here of the measured order of magnitude, i.e. a_e = 0.00115965218073(28). How is it being explained? The new quantum physicists write it as (the sum of) a series of first-, second-, third-,…, n^{th}-order *corrections*:

Fine Structure Constant from the Electron g Value and QED, Phys. Rev. Lett. 97, 030802, 2006) because it can be freely consulted online: http://gabrielse.physics.harvard.edu/gabrielse/papers/2009/PushingTheFrontiersOfAtomicPhysics.pdf.

[32] To be precise, the article gives the *measured* value for *g*/2, which is equal to 1.00115965218073(28).

[33] We took this illustration from an excellent article on the complexities of a Penning trap: *Cylotron frequency in a Penning trap*, Blaum Group, 28 September 2015, https://www.physi.uni-heidelberg.de/Einrichtungen/FP/anleitungen/F47.pdf. The motions are complicated because the Penning trap traps the electron using both electric as well as magnetic fields (the electric field is *not* shown in the illustration, but it is there). One should note the illustration does *not* show the intrinsic spin of the electron, which we should also consider. See our above-mentioned paper for a more detailed description of the various layers of motion.

$$a_e = \sum_n a_n \left(\frac{\alpha}{\pi}\right)^n$$

The first coefficient (a_1) is equal to 1/2 and the associated first-order correction is, therefore, equal to:

$$\alpha/2\pi \approx 0.00116141$$

Using "his renormalized QED theory", Julian Schwinger had already obtained this value back in 1947. He got it from calculating the "one loop electron vertex function in an external magnetic field." I am just quoting here from the above-mentioned article (Todorov, 2018). Julian Schwinger is, of course, one of the most prominent representatives of the second generation of quantum physicists, and he has this number on this tombstone. Hence, we surely do not want to question the depth of his understanding of this phenomenon. However, the difference that needs to be explained by the 2nd, 3rd, etc. corrections is only 0.15%, and Todorov's work shows all of these corrections can be written in terms of a sort of exponential series of α/2π and a *phi*-function ϕ(*n*) which had intrigued Euler for all of his life. We copy the formula for (the sum of) the first-, second- and third-order term of the theoretical value of a_e as calculated in 1995-1996 (*th* : 1996).[34]

$$\begin{aligned} a_e(th:1996) &= \frac{1}{2}\frac{\alpha}{\pi} + \left[\phi(3) - 6\,\phi(1)\,\phi(2) + \phi(2) + \frac{197}{2^4\,3^2}\right]\left(\frac{\alpha}{\pi}\right)^2 \\ &+ \left[\frac{2}{3^2}\left(83\,\phi(2)\,\phi(3) - 43\,\phi(5)\right) - \frac{50}{3}\,\phi(1,3) + \frac{13}{5}\,\phi(2)^2\right. \qquad (19) \\ &+ \left.\frac{278}{3}\left(\frac{\phi(3)}{3^2} - 12\,\phi(1)\,\phi(2)\right) + \frac{34202}{3^3\,5}\,\phi(2) + \frac{28259}{2^5\,3^4}\right]\left(\frac{\alpha}{\pi}\right)^3 + \ldots \\ &= 1.159652201(27) \times 10^{-3} \end{aligned}$$

We also quote Todorov's succinct summary of how this result was obtained: "Toichiro Kinoshita of Cornell University evaluated the 72 [third-order loop Feynman] diagrams numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. A year later, the last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna."

Apparently, the calculations are even more detailed now: the mentioned Laporta claims to have calculated 891 *four*-loop contributions to the anomalous magnetic moment.[35] One gets an uncanny feeling here: if one has to calculate a zillion integrals all over space using 72 third-order diagrams to calculate the 12th digit in the anomalous magnetic moment, or 891 fourth-order diagrams to get the next level of precision, then there might something wrong with the theory. Is there an alternative? We think there is, and the idea is surprisingly simple.

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[34] It is worth quoting Todorov's succinct summary of how this result was obtained: Toichiro Kinoshita of Cornell University evaluated the 72 [Feynman] diagrams [corresponding to the third-order loop] numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. later the last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna.

[35] See: Stefano Laporta, *High-precision calculation of the 4-loop contribution to the electron g-2 in QED*, as reported in: https://www.sciencedirect.com/science/article/pii/S0370269317305324.

Classical electron models

Mr. Burinskii would probably *not* wish to describe his Dirac-Kerr-Newman model of an electron as a classical electron model – and neither would he want to be considered as a classical physicist[36] – but that is what it is for us: a charge with a geometry in three-dimensional space. To be precise, it is a disk-like structure, and its *form factor* – read: the ratio between the radius and thickness of the disk – depends on various assumptions (as illustrated below) but reduces to the ratio between the Compton and Thomson radius of an electron when assuming classical (non-perturbative) theory applies. We quote from Mr. Burinskii's 2016 paper: "It turns out that the flat Compton zone free from gravity may be achieved without modification of the Einstein-Maxwell equations."

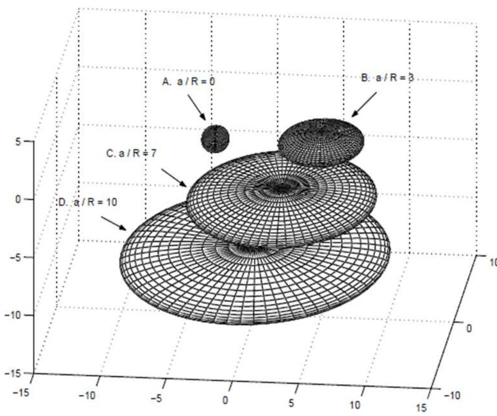

Figure 17: Alexander Burinskii's electron model

Hence, it would seem we get the fine-structure constant as the ratio of the Compton radius – i.e. the radius of the disk *R* – and the classical electron radius – i.e. the thickness of the disk *r* – out of a smart model based on Maxwell's and Einstein's equations, i.e. classical electromagnetism and general relativity theory:

$$\alpha = \frac{r}{R} = \frac{r_e}{r_C} = \frac{\mathrm{e}^2/\mathrm{m}c^2}{\hbar c/\mathrm{m}c^2} = \frac{\mathrm{e}^2}{\hbar c}$$

There is no need for smart quantum mechanics here! These results, therefore, confirm the intuitive but, admittedly, rather primitive *Zitterbewegung* model we introduced in our own papers. To illustrate the point, we would like to summarize one of the many possible interpretations of the fine-structure constant as a dimensional scaling constant here.[37]

First, we need to think about the meaning of e^2. There is something interesting here: the elementary charge e^2 has the same physical dimension – the *joule-meter* (J·m) – as the *hc* = Eλ product.

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[36] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008, https://arxiv.org/abs/hep-th/0507109. A more recent article of Mr. Burinskii (*New Path to Unification of Gravity with Particle Physics*, 2016, https://arxiv.org/abs/1701.01025, relates the model to more recent theories – most notably the "supersymmetric Higgs field" and the "Nielsen-Olesen model of dual string based on the Landau-Ginzburg (LG) field model." We admit we do not understand a word of this. As for Mr. Burinskii's general expertise (which is *quantum* physics, mainly), see his profile: https://www.researchgate.net/profile/Alexander_Burinskii.

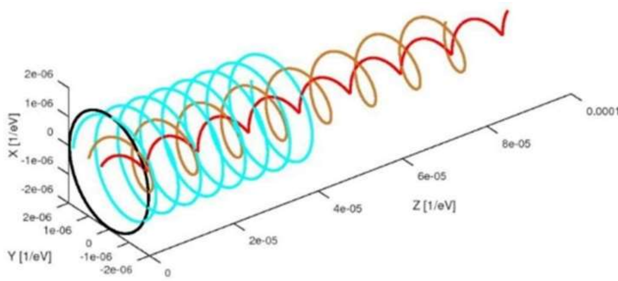
[37] See the above-mentioned paper: Jean Louis Van Belle, *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

$$[e^2] = \left[\frac{q_e{}^2}{4\pi\epsilon_0}\right] = \frac{Nm^2C^2}{C^2} = Nm^2 = Jm$$

Now, what was that *hc* = Eλ product again? We get it in the context of the description of a photon. To be precise, we get it by applying one of the two *de Broglie* equations to a photon:

$$h = p\lambda = \frac{E}{c}\lambda \Longleftrightarrow \lambda = \frac{hc}{E}$$

The energy (E) and wavelength (λ) are, of course, the energy and the wavelength of our photon. However, it turns out it makes sense to apply these equations to any particle that moves at the speed of light. The reader will wonder: what other particle? Our electron has a rest mass, right? It does, but our *Zitterbewegung* model assumes this rest mass is the equivalent mass of the *rest matter oscillation*. This rest matter oscillation is a two-dimensional oscillation: a local circulatory motion, in fact. It is illustrated below.



Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

Figure 18: The *Zitterbewegung* model of an electron

The illustration above does not only show the *Zitterbewegung* itself but also another aspect of the theory. As the electron starts moving along some trajectory at a relativistic velocity (i.e. a velocity that is a substantial fraction of *c*), then the radius of the oscillation will have to diminish. Why? Because the tangential velocity remains what it is: *c*. Hence, the geometry of the situation shows that the radius of the oscillation becomes a wavelength in the process.[38] As Dirac noted in his Nobel Prize speech[39], the idea of the Zitterbewegung is very intuitive – and, therefore, very attractive – because it seems to give us a geometric (or, we might say, physical) explanation of the (reduced) Compton wavelength as the Compton scattering radius of an electron (a = ħ/mc).15 However, if we think of an actual physical interpretation, then it is quite obvious that the suggested plane of circulatory motion is not consistent with the measured direction of the magnetic moment – which, as the Stern-Gerlach experiment has shown us, is either up or down. Hence, we may want to think the plane of oscillation might be parallel to the direction of propagation, as drawn below.

[38] We refer to the mentioned paper for a more elaborate *exposé* of the geometry.
[39] Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf.

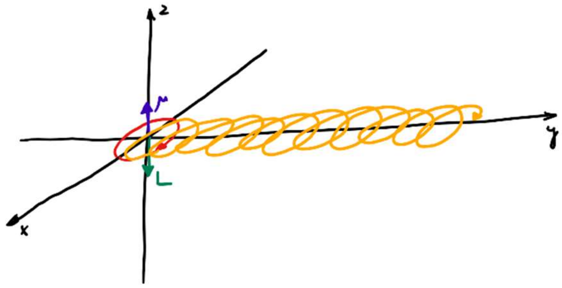


Figure 19: An alternative orientation of the *zbw* plane of rotation

We like the alternative picture of the *zbw* electron above not only because it is more consistent with the idea of the up-or-down orientation of the magnetic moment (cf. the Stern-Gerlach experiment) but also because it might provide us with a *physical* explanation of relativistic length contraction: as velocities increase, the radius of the circular motion becomes smaller (as illustrated above) which, in this model, may be interpreted as a contraction of the size of the *zbw* electron.

However, these remarks are not the point here. Let us return to our discussion of the anomalous magnetic moment.

How to test the classical electron models

Mr. Burinskii's model is very flexible. If one limits the assumptions - combining gravity and electromagnetism, we get the *Zitterbewegung* electron – a simple disk-like structure whose *form factor* is given by the fine-structure constant:

$$\alpha = \frac{r}{R} = \frac{r_e}{r_C} = \frac{\mathrm{e}^2/\mathrm{m}c^2}{\hbar c/\mathrm{m}c^2} = \frac{\mathrm{e}^2}{\hbar c}$$

When calculating the angular momentum, this form factor translates into a simple ½ factor when calculating the moment of inertia. We write $I = \mathrm{m}r^2/2$ – as opposed to the $I = \mathrm{m}r^2$ formula we would use for a pure orbital moment. This effectively gives us Dirac's theoretical value for the gyromagnetic ratio (*g*-factor) of the *spin-only* electron: g = 2. The table below summarizes the difference between the spin and orbital angular momentum.

Table 4: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr orbitals)**
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I \cdot \omega = \mathrm{m}\frac{\hbar^2}{\mathrm{m}^2c^2}\frac{\mathrm{E}}{\hbar} = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

As we mentioned in our paper[40], we will have a classical coupling between the two moments because of the Larmor precession of the electron in the Penning trap, as illustrated below. The *effective* current and the *effective* radius of the orbital motion will, therefore, *not* be equal to the values one would get from using the formulas in the right-hand column of the table above.[41]

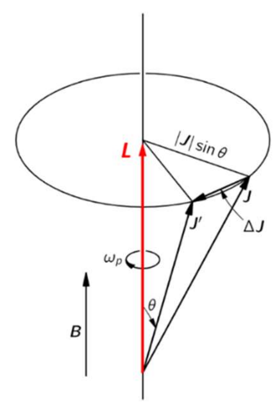


Figure 20: The precession of an orbital electron

Now, this classical coupling may or may not explain the bulk of what is actually being measured in these famous experiments measuring the (anomalous or not) magnetic moment of an electron in a Penning

[40] Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018, http://vixra.org/pdf/1812.0233v3.pdf.
[41] Note that the formulas in the right column are the formulas for the properties of the Bohr orbitals. These *resemble* the cyclotron orbitals – to some extent – but one should not confuse them: the cyclotron orbitals have no nucleus at their center. In fact, the oft-quoted description of the electron in the Penning trap as an artificial atom is quite confusing and, therefore, not very useful: the radius and kinetic energy of the electron in a magnetron is of an entirely different order of magnitude! However, we would expect the formulas to be similar.

trap. However, we would suspect there will, effectively, be a small anomaly left – which is only natural because all of the formulas above assume the electron is a perfect disk (when calculating the values for the spin-only moment), or a perfect sphere (when calculating the values for the orbital moment). However, the Dirac-Kerr-Newman model of an electron tells us that is, perhaps, *not* the case. Let us copy the illustration again.

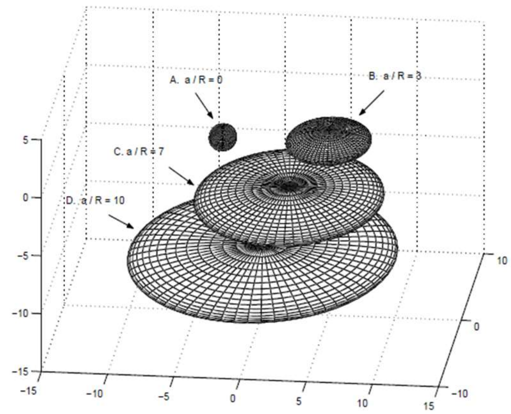


Figure 21: Burinskii's electron model

Despite all of the complexities of Mr. Burinskii's model, the shape of the electron can be characterized by a simple a/R ratio. Somewhat confusingly, the R in this formula is actually the surface area. Hence, if we re-use the r symbol for the radius of the disk, then R will be – roughly – equal to πr^2. The a is the ratio between the angular momentum (J) and the electron mass. Hence, the a/R ratio can be written as:

$$\frac{a}{R} = \frac{J}{\mathrm{m}}\frac{1}{\pi r^2}$$

We have not only the angular momentum here, but also the surface area here (πr^2) which co-determines the magnetic moment of the loop of current (I).[42] In short, all of the variables that could, potentially, explain the anomalous magnetic moment in terms of a form factor are there. Hence, the next logical step would be to validate this classical electron model by inserting it into some other model. Indeed, as Dirac noted, "the very-high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us", as a result of which "the velocity of the electron at any time equals the velocity of light" is a "prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small."[43]

[42] The symbols in the table may be somewhat confusing: I (*italicized*) is a moment of inertia, but I (non-italicized) is a current. We did not want to use new symbols because the context of the formula makes clear what it what.

[43] Erwin Schrödinger had, effectively, already derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth to now quote all of Dirac's summary of Schrödinger's discovery in his 1933 Nobel Prize speech: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one

However, we can, of course, *insert* this *Zitterbewegung* model – or, preferably, the more flexible model of Mr. Burinskii – into models that do *not* involve micro-motion at the speed of light. What models? Models involving the slow motion of an electron around a nucleus (atomic orbitals) or – in this particular case – the motion of an electron in a Penning trap.

Theoretical implications

The reader may wonder: what's the use if there is already a satisfactory theory (perturbative theory)? The answer to this question is quite obvious. First, a classical theory would be simpler, and Occam's Razor Principle, therefore, tells us we should consider it. More generally, all physicists would agree the King of Science should respect Boltzmann's adage: "Bring forth the truth. Write it so it's clear. Defend it to your last breath." Indeed, even if the results would only remotely explain the anomaly, we would still have achieved two very significant scientific breakthroughs. First, it would show that these seemingly irrelevant micro-models can be validated externally. More importantly, it would prove that an alternative (classical) explanation of the anomalous magnetic moment would be possible.

One may, of course, wonder, further down the line, if an *augmented* classical explanation of QED would upset the theoretical approach in other sectors of the Standard Model. Indeed, as Aitchison and Hey write, the new quantum electrodynamical theory (QED) provided physicists with a model – they refer to it as the 'electron-figure' but what we are talking about are gauge theories, really[44] – to analyze the forces in the nucleus – i.e. the strong and weak force. We do not think so, because these forces are *non*-linear and are also quite different in their *nature* in other respects.

Using totally non-scientific language, we may say that mass comes in one 'color' only: it is just some scalar number. Hence, Einstein's geometric approach to it makes total sense. In contrast, the electromagnetic force is based on the idea of an electric charge, which can come in two 'colors' (+ or −), so to speak. Maxwell's equation seemed to cover it all until it was discovered the nature of Nature – sorry for the wordplay – might be discrete and probabilistic.[45] Now, the strong force comes in three colors, and the rules for mixing them, so to speak, are *very* particular. It is, therefore, only natural that its analysis requires a wholly different approach. In fact, who knows? Perhaps one day some alien will show us that the application of the 'electron-figure' to these sectors was actually *not* so useful. Don't get us wrong: we think these models are all very solid, but history has shown us that one can never exclude a scientific revolution!

We will send this paper to Mr. Burinskii. If he – or others – would take up this suggestion and show that it can be done, Mr. Burinskii should probably be considered for the next Nobel Prize. As for us – amateur physicists – we would be happy to document the story. 😊

must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

[44] Ian J.R. Aitchison and Anthony J.G. Hey, *Gauge Theories in Particle Physics*, 4th edition, Volume I, p. 3.

[45] In the above-mentioned paper, we note it helps a lot to think of Planck's quantum of action as a *vector* quantity: the uncertainty may then be related to its *direction*, rather than its magnitude. We also note the theoretical framework might benefit from using the ± sign in the argument of the wavefunction to associate the wavefunction with a *non-zero* spin particle. We argue that the weird 720-degree symmetries which discouraged research into geometric (or *physical*) interpretations of the wavefunction might then disappear. See: Jean Louis Van Belle, Euler's Wavefunction: The Double Life of −1, 30 October 2018, http://vixra.org/pdf/1810.0339v2.pdf.

X. The fine-structure constant and the fine structure

We should now explain the final and last meaning of the fine-structure constant – the one that gave it its name! Why is that the fine-structure constant explains the fine structure of the hydrogen spectrum? However, because this is actually a topic that is well covered in standard physics textbooks – we will, effectively, refer the reader to such physics textbooks. He or she should, by now, be able to apply the knowledge gained here to *translate* the quantum-mechanical explanation into something that is *not*-so-mysterious as physicists and popular writers want us to believe.

Let us – after all this – offer some more fundamental reflections on the meaning of the wavefunction.

XI. The meaning of the wavefunction

Thomas Aquinas starts his *de Ente et Essentia* (on Being and Essence) quoting Aristotle: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions. This philosophical warning – combined with Occam's quest for mathematical parsimony – made us think about the mathematical framework of quantum mechanics: its rules explain reality, but no one understands them. Perhaps some small mistake has been made – early on – in the *interpretation* of the math. This has been a long quest – with little support along the way (see the acknowledgments above) – but we think we have found the small mistake – and we do believe it has led to some substantial misunderstandings – or, at the very least, serious *ambiguities* in the description.

We think that the power of Euler's function – as a mathematical description of what we believe to be a real particle – has not been fully exploited. We, therefore, have a redundancy in the description. The fallacy is illustrated below. When we combine −1 with an amplitude, we should not think of it as a scalar: we should think of −1 as a complex number itself. Hence, when we are multiplying a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with −1, we are *not* necessarily multiplying them with the same thing: −1 is *not* necessarily a common phase factor. The phase factor may be +π or, alternatively, −π. To put it simply, when going from +1 to −1, it matters how you get there – and vice versa.

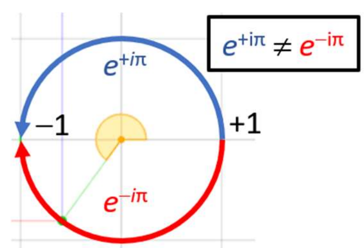


Figure 22: $e^{+i\pi} \neq e^{-i\pi}$

Let us elaborate this. Quantum physicists don't think of the elementary wavefunction as representing anything real but – if they do – they would reluctantly say it might represent some theoretical spin-zero particle. Now, we all know spin-zero particles do not exist. All *real* particles have spin – electrons, photons, anything – and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. Hence, it is rather odd that the plus/minus sign of the imaginary unit in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include spin in the mathematical description. Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta}$ =

$a \cdot e^{-i\cdot(\omega t - kx)}$ and $a \cdot e^{+i\cdot\theta} = a \cdot e^{+i\cdot(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). We would think physicists would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention." In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*).

Historical experience tells us theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things – think, for example, of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? Occam's Razor principle tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations, and the one obvious characteristic that would distinguish the two physical situations is the spin direction. Hence, we do *not* agree with the mainstream view that the choice is a matter of convention. Instead, we dare to suggest that the two mathematical possibilities represent identical particles with opposite spin. Combining this with the two possible directions of propagation (which are given by the +– or ++ signs in front of ω and k), we get the following table:

Table 5: Occam's Razor: mathematical possibilities versus physical realities

Spin and direction of travel	**Spin up (e.g. $J = +\hbar/2$)**	**Spin down (e.g. $J = -\hbar/2$)**
Positive *x*-direction	$\psi = a \cdot e^{-i\cdot(\omega t - kx)}$	$\psi^* = a \cdot e^{+i\cdot(\omega t - kx)}$
Negative *x*-direction	$\chi = a \cdot e^{-i\cdot(\omega t + kx)}$	$\chi^* = a \cdot e^{-i\cdot(\omega t + kx)}$

Let us think this through. Physicists tell us that wavefunctions of spin-1/2 particles (which is what we are thinking of here) have a weird 720° symmetry, but that this weird symmetry is not there for spin-1 particles. Hence, intuition tells us that it should disappear when we would use the two mathematical possibilities for describing the wavefunction of a particle to distinguish between two particles that are identical but have opposite spin. If our intuition is correct (we do not have a formal proof of this – but we do have a *heuristic disproof* (see: Euler's wavefunction, the double life of –1: http://vixra.org/abs/1810.0339), then the most important objection to a physical interpretation of the wavefunction would no longer be valid and, in our humble view, it would trigger a whole new wave (pun intended) of geometric (read: physical) interpretations of the wavefunction.

For starters, it would get rid of the desiccated idea that the complex conjugate of the (elementary) $\psi = \exp(i\theta) = \exp[i(kx-\omega t)]$ function – so that is $\psi^* = \exp(-i\theta) = \exp[i(\omega t - kt)]$ – is just another *mathematical* possibility to describe reality. In other words, it would get rid of the idea that it is just some *convention.* We insist on this point. Why? We readily acknowledge conventions are essential in any (mathematical) description of (physical) reality, so why don't we like *this* convention? It's Occam. Occam tells us the degrees of freedom in the mathematical description (and we are talking just some plain number here, like 3 or 5 or whatever) should match the degrees of freedom in our measurement of whatever we think reality might be. The idea of just settling on a mathematical convention in this particular context (a mathematical object describing a physical reality) is, for us, plain anathema.

Let us mention some (possible) implications so as to illustrate the point.

The idea of associating the complex conjugate of a wavefunction with a particle that's identical except for its (opposite) spin might be outlandish, which is why we should first try to connect with a much simpler idea – which might or might not be more palatable: the complex conjugate of a wavefunction obviously reverses the trajectory of the particle in space and in time: $\boldsymbol{x}$ becomes $-\boldsymbol{x}$ and t becomes $-t$.

What? Yes. A true physical interpretation will present the real and imaginary part of the elementary wavefunction $a \cdot e^{i\theta}$ as *real* field vectors driven by the same function but with a phase difference of 90 degrees:

$$a \cdot e^{i\theta} = a \cdot (cos\theta + i \cdot sin\theta) = a \cdot sin(\theta + \pi/2) + i \cdot a \cdot sin\theta$$

However, a minus sign in front of our $\exp(i\theta)$ function reverses the direction of the oscillation – in space and, importantly, *in time too*. Here we can use the $cos\theta = cos(-\theta)$ and $sin\theta = -sin(-\theta)$ formulas to relate $-\exp(i\theta)$ to the complex conjugate. We write:

$$-\psi = -\exp(i\theta) = -(cos\theta + i \cdot sin\theta) = cos(-\theta) + i \cdot sin(-\theta) = \exp(-i\theta) = \psi^*$$

This should make us feel uneasy. Yes. We should think of this. We should *not* scrap one ambiguity in the description to introduce another. Things should be clean: the math has to match the physics. So… Does it? We think it does. We need to highlight a subtle point here. Time has one direction only. We cannot reverse time. We can only reverse the direction in space. We can do so by reversing the momentum of a particle. If we do so, the $\mathbf{k} = \mathbf{p}/\hbar$ in the argument of the wavefunction becomes $-\mathbf{k} = -\mathbf{p}/\hbar$. However, the energy remains what it is and, hence, nothing happens to the $\omega \cdot t = (E/\hbar) \cdot t$ term. Hence, our wavefunction becomes $\exp[i(-\mathbf{k} \cdot \boldsymbol{x} - \omega \cdot t)]$, and we can calculate the wave velocity as negative: $v = -\omega/|\mathbf{k}| = -\omega/k$. The wave effectively travels in the opposite direction (i.e. the *negative* x-direction in one-dimensional space). Hence, we can think of opposite directions in space, but we can't reverse time. Why not?

The answer is related to how our mind works. Time has one direction only because – if it wouldn't – we would not be able to describe trajectories in spacetime by a well-behaved function. We really don't need to think of entropy or of other more convoluted explanations here. The diagrams below illustrate the point. The spacetime trajectory in the diagram on the right is not *kosher*, because our object travels back in time in not less than three sections of the graph. Spacetime trajectories need to be described by well-defined function: for every value of t, we should have one, and only one, value of x. The reverse is not true, of course: a particle can travel back to where it was. Hence, it is easy to see that our concept of time going in one direction, and in one direction only, implies that we should only allow well-behaved functions.

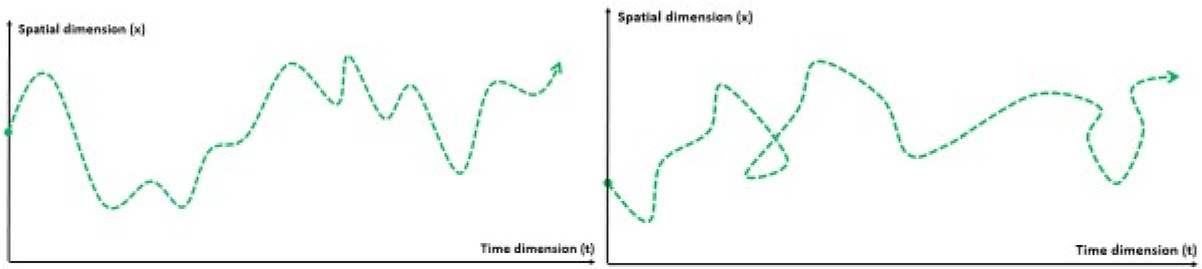


Figure 23: A well- and a not-well behaved trajectory in spacetime

It may be a self-evident point to make but it is an important one. It shows us we should not be worried: our *new* interpretation of the wavefunction – incorporating spin – is fully consistent. It rules out any ambiguity. It we would *not* accept it, then we would have two *mathematical* possibilities to describe a theoretical spin-zero particle that would travel in one direction or the other: $\psi = \exp[i(-kx-\omega t)]$ *or*, alternatively, $-\psi = \psi^* = \exp[i(kx+\omega t)]$.

An added benefit of our interpretation is that it eliminates the logic that leads to the rather uncomfortable conclusion that the wavefunction of spin-1/2 particles (read: electrons, practically speaking) has some weird 720-degree symmetry in space. This conclusion is uncomfortable because we cannot imagine such objects in space without invoking the idea of some kind of relation between the subject and the object (the reader should think of the Dirac belt trick here). It has, therefore, virtually halted all creative thinking on a physical interpretation of the wavefunction.

This may sound like Chinese to the reader, so let us proceed to something else: how should we interpret the product of the elementary function with its complex conjugate? In orthodox quantum mechanics, it is just this weird thing: some number that will be *proportional* to some *probability*. In our interpretation, this probability is proportional to energy densities – or, because of the energy-mass equivalence – to mass densities. Let us take the simplest of cases and think of the $\langle\psi|$ *state* as some very *generic* thing being represented by a *generic* complex function[46]:

$$\langle\psi| \cong a\cdot e^{i\theta}$$

The $\langle\psi|\langle\psi|^* = \langle\psi||\psi\rangle$ product then just eliminates the *oscillation*. It freezes time, we might say:

$$\langle\psi|\langle\psi|^* = \langle\psi||\psi\rangle = a\cdot e^{i\theta}\cdot a\cdot e^{-i\theta} = a^2\cdot e^0 = a^2$$

Hence, we end up with one factor of the energy of an oscillation: its amplitude (a). Let us think about this for a brief moment. To focus our minds, let us think of a photon. The energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Hence, if we write the proportionality coefficient as k, then the energy of our photon will be equal to:

$$\mathrm{E} = k\cdot a^2\cdot\omega^2$$

[46] Our critics will cry wolf and say we should be more general. They are right. However, let us make two remarks here. First, we should note that QED is a linear theory and, hence, we can effectively - and very easily – generalize anything we write to a Fourier superposition of waves. We use the ≅ symbol to indicate an *equivalence*. It's not an identity. To mathematical purists – who will continue to cry wolf no matter what we write because they won't accept the $e^{-\pi} \neq e^{-\pi}$ expression either – we will admit it is more like a symbol showing *congruence*. Second, we do get some physical laws out of physics (both classical as well as quantum-mechanical) that are likely to justify the general $a\cdot e^{i\theta}$ shape.

What should we use for the amplitude of the oscillation here? It turns out we get a nice result using the *wavelength*[47]:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

However, we should immediately note that – in our interpretation(s) of the wavefunction – this assumes a circularly polarized wave. Its linear components – the sine and cosine, that is – will only pack *half* of that energy. Our electron model – *zbw* electron as well an orbital electron – is based on the same. Now that we are here, we will quickly write down the formulas we found:

Table 6: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr orbitals)**
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h} \text{ for } n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c\cdot\frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I\cdot\omega = \frac{\hbar}{2}$	$\mathrm{L}_n = I\cdot\omega_n = n\hbar$
$\mu = \mathrm{I}\cdot\pi r_\mathrm{C}{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I}\cdot\pi r_n{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

We will come back to this in the next section of our paper. Let us first relate the discussion to the discussion to the *Hermiticity* of (many) operators. If A is an operator[48], then it could operate on some state |ψ⟩. We write this operation as:

$$\mathrm{A}|\psi\rangle$$

Now, we can then think of some (probability) amplitude that this operation produces some other state |φ⟩, which we would write as:

$$\langle\varphi|\mathrm{A}|\psi\rangle$$

---

[47] We use the Eλ = h*c* ⇔ λ = h*c*/E identity. The reader might think we should use the amplitude of the electric and magnetic field. We could – the model is consistent – but it requires some extra calculations as we then need to think of the energy as some force over a distance. We refer to our papers for more details.

[48] We should use the *hat* because the symbol without the hat is reserved for the *matrix* that does the operation and, therefore, A already assumes a representation, i.e. some chosen set of base states. However, let us skip the niceties here.

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

$A^\dagger$ is, of course, the conjugate transpose of A: $A^\dagger_{ij}=(A_{ji})^*$, and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^\dagger = A$. Many operators are Hermitian. Why? Well… What is the meaning of $\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle = \langle\psi|A|\varphi\rangle$? Well… In the $\langle\varphi|A|\psi\rangle$ we go from some state $|\psi\rangle$ to some other state $\langle\varphi|$. Conversely, the $\langle\psi|A|\varphi\rangle$ expression tells us we were in state $|\varphi\rangle$ but now we are in the state $\langle\psi|$.

So, is there some meaning to the complex conjugate of an amplitude like $\langle\varphi|A|\psi\rangle$? We say: yes, there is! Read up on time reversal and CPT symmetry! Based on the above – and your reading-up on CPT symmetry – we would think it is fair to say we should interpret the Hermiticity condition as a *physical* reversibility condition.

We are not talking mere time symmetry here: reversing a physical process is like playing a movie backwards and, hence, we are actually talking CPT symmetry here. Of course, it may be difficult to *prove* this interpretation – can one prove interpretations, really? – but, at the very least, we made a start, right? 😊

Explaining QED using classical theory

The following series of diagrams summarizes some of what we covered in the previous chapters.

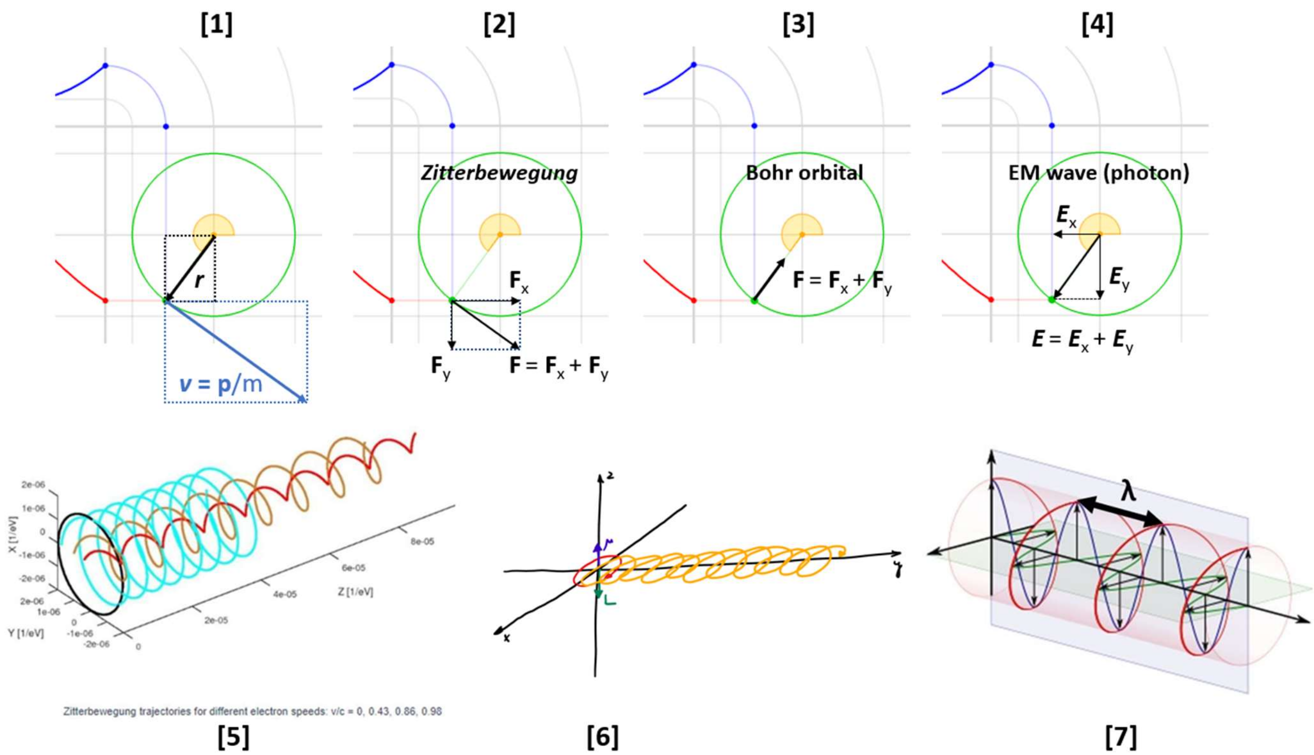


Figure 24: Physical interpretations of the wavefunction

We refer to our previous papers for a detailed discussion of each of these.[49] Here we will just sum up the basics.

1. We had a *Zitterbewegung* model, in which the elementary wavefunction represents a pointlike *charge* with zero rest mass and which, therefore, moves at the speed of light. This model explains Einstein's energy-mass equivalence relation in terms of a two-dimensional oscillation. The radius of the oscillation is the Compton radius of the electron.

2. The *Zitterbewegung* electron – which combines the idea of a pointlike charge and Wheeler's idea of mass without mass[50] – can then be inserted into Bohr's quantum-mechanical model of an atom, which can also be represented using the elementary wavefunction. We have a different force configuration here (because of the positively charged nucleus, we have a centripetal force now – as opposed to the tangential *zbw* force) but Euler's $a \cdot e^{\pm i\theta}$ function still represents an actual *position vector* of an electron which – because it acquired a rest mass from its *Zitterbewegung* – now moves at velocity $v = (\alpha/n) \cdot c$.[51] This should suffice to explain diagram 1, 2 and 3 below.

3. Diagram 4 represents the idea of a photon that we get out of the Bohr model. We referred to it as the one-cycle photon model. The idea is the following. The Bohr orbitals are separated by a amount of (physical) action that is equal to h. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. Our photon will have to pack that, somehow. It will also have to pack the related energy, which is given by the *difference* of the energies of the two orbitals. This gives us not only the Rydberg formula – Bohr sort of *explained* that formula in 1913 already, but not like we do here – but also a delightfully simple model of a photon and an intuitive interpretation of the Planck-Einstein relation ($f = 1/\mathrm{T} = \mathrm{E}/h$) for a photon. Indeed, we can do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the $\mathrm{E} = \mathrm{m}c^2$ mass-energy equivalence relation can be written as $\mathrm{p} = \mathrm{m}c = \mathrm{E}/c$ for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

[49] See our series of viXra papers (http://vixra.org/author/jean_louis_van_belle). If we would have to choose one which sort of sums most, we would select our *Layered Motions: The Meaning of the Fine-Structure Constant* (http://vixra.org/pdf/1812.0273v3.pdf).
[50] The mass of the electron is the equivalent mass of the energy in the oscillation.
[51] The n is the number of the Bohr orbital (n = 1, 2, 3…). The α and c are the fine-structure constant and the speed of light. This formula comes out naturally of the Bohr model. See the referenced papers.

We calculated the related force and field strength in our paper[52] so we won't repeat ourselves here. We would just like to point out something interesting – using diagram 5 above. Diagram 5 was copied from one of the many papers of Celani, Vassallo and Di Tommaso on the *Zitterbewegung* model, but we can use it to illustrate how and why we can associate a *radius* with the wavelength of a photon. Indeed, the diagram shows that, as an electron starts moving along some trajectory at a relativistic velocity – a velocity that becomes a more substantial fraction of *c*, that is – then the radius of the *Zitterbewegung* oscillation becomes smaller and smaller. In the limit ($v \rightarrow c$), it becomes zero ($r \rightarrow 0$), and the *circumference* of the oscillation becomes a simple (linear) wavelength in the process (this is illustrated in diagram 5 and 7, which provides a geometric interpretation of the *de Broglie* wavelength). Now, if we write this wavelength as λ_C (this is, of course, the Compton *wavelength*), then we get the usual relationship between a radius and a wavelength: $r_C = \lambda_C/2\pi$. This, then, provides an intuitive interpretation of the $E\lambda = hc$ equation for the photon and – more importantly – an intuitive explanation of the 2π factor in the formula for the fine-structure constant as a coupling constant. We write:

$$\alpha = \frac{2\pi \cdot q_e^2}{h \cdot c} = \frac{k \cdot q_e^2}{\hbar \cdot c} = \frac{F_B \cdot r_B^2}{F_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{F_B \cdot r_B^2}{F_\gamma \cdot r_\gamma^2} = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma}$$

Needless to say, E_B, F_B, r_B and E_γ, F_γ, r_γ are the energies, forces and radii that are associated with the Bohr orbitals and our one-cycle photon.[53]

Finally – but this is a much finer and more philosophical point – diagram 5 gives us an intuitive geometric interpretation of one of the many ways in which Planck's quantum of action may express itself: the quantization of space. Indeed, at $v = 0$ (diagram 2), we have perfectly circular motion of a pointlike charge moving at the velocity of light, and we may associate Planck's quantum of action with the surface area of the circle. However, at $v = c$, the motion is purely linear – but we still think of the rotating field vector at the core (diagram 4). Planck's quantum of action now expresses itself space as a linear distance: the wavelength of the photon. We like to express this dual view as follows:

$$zbw\ electron\text{: } S = h = p_{Compton} \cdot \lambda_{Compton} = m_e c \lambda_C = m_e c \cdot 2\pi r_C = m_e c \frac{h}{m_e c} = h$$

$$photon\text{: } S = h = p_{photon} \cdot \lambda_{photon} = \frac{E_\gamma}{c} \lambda_\gamma = m_\gamma c \lambda_\gamma = m_\gamma c \cdot 2\pi r_\gamma = m_\gamma c \frac{hc}{E_\gamma} = h$$

To be fully complete, we can add the same equation for the Bohr orbitals:

$$n^{th}\ Bohr\ orbital\text{: } S = n \cdot h = p_n \cdot \lambda_n = m_e v_n \lambda_n = m_e \frac{\alpha c}{n} 2\pi \frac{n^2 \hbar}{\alpha m_e c} = n \cdot h$$

We like these expressions because – in our humble view – there is no better way to express the idea that we should associate Planck's quantum of action (or any multiple of it) with the idea of a *cycle* in Nature.[54]

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[52] See the above-mentioned paper: Jean Louis Van Belle, *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

[53] These formulas may appear as mind-boggling to the reader. If so, we advise the reader to first look at our other papers, whose pace is much more gradual.

[54] Our model also offers a much more comprehensive understanding of the fine-structure constant as a scaling constant. See: Jean Louis Van Belle, *Layered Motions : The Meaning of the Fine-Structure Constant*, http://vixra.org/abs/1812.0273.

We can imagine the reader is, by now, quite tired of these gymnastics. He or she should ask: what does it all mean? We would like to refer to some history here. Prof. Dr. Alexander Burinskii – the author of the Dirac-Kerr-Newman electron model – told us he had started to further elaborate the *Zitterbewegung* model in the year the author of this paper was born – that is in 1969. He published an article on this in the *Journal of Experimental and Theoretical Physics* (JETP)[55]. However, he told us he had always been puzzled about this one question: what keeps the pointlike charge in the *zbw* electron in its circular orbit? He, therefore, moved to exploring Kerr-Newman geometries – which has resulted in his Dirac-Kerr-Newman model of an electron.[56]

While the Dirac-Kerr-Newman model is a much more advanced model – it accommodates the theory of the supersymmetric Higgs field and string theory – we understand it does reduce to its classical limit, which is the *Zitterbewegung* model, if one limits the assumptions to general relativity and classical electromagnetism only. In our modest view, this validates our model. There is no mystery on the *zbw* force, we think: it is just the classical Lorentz force $\mathbf{F} = q\mathbf{E} + q\boldsymbol{v}\times\mathbf{B}$. We, therefore, think that the *zbw* force results from the very same electric and magnetic field oscillation that makes up the photon. It is just the way that Planck's quantum of action expresses itself in space that is different here: we just get a different *form factor*, so to speak, when we look at the pointlike *zbw* charge. This, then, should solve Mr. Burinskii's puzzle – in our humble view, that is.

Finally, the attentive reader will have noticed that we did *not* discuss diagram 6. We inserted this diagram because when we considered the various *degrees of freedom* in interpreting Euler's wavefunction, we thought we should, perhaps, not necessarily assume that the plane of the circulatory motion – the *zbw* motion of the pointlike charge in the diagram – is perpendicular to the direction of propagation. In fact, the Stern-Gerlach experiment tells us the magnetic moment is literally *up* or *down*, which assumes the plane of the electric current should be parallel to the direction of motion. We like this alternative picture of the *zbw* electron because – intuitively – we feel it might provide us with some kind of *physical* explanation of relativistic length contraction: as velocities increase, the radius of the circular motion becomes smaller which, in this model, may be interpreted as a contraction of the size of the *zbw* electron.[57]

XII. The interference of a photon with itself

Can we explain quantum-mechanical interference, i.e. the interference of a photon with itself in, say, a Mach-Zehnder interferometer? We think we can. We think of a photon as the sum of two linearly polarized waves. We write:

$$cos\theta + i\cdot sin\theta = e^{i\cdot\theta} \text{ (RHC)}$$

$$cos(-\theta) + i\cdot sin(-\theta) = cos\theta - i\cdot sin\theta = e^{-i\cdot\theta} \text{ (LHC)}$$

We, therefore, have an alternative theory of what happens in the Mach-Zehnder interferometer:

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[55] Burinskii, A.Y., *Microgeons with spin*, Sov. Phys. JETP 39 (1974) 193. One should note that Prof. dr. Burinskii refers to the *zbw* charge as an 'electron photon' or the 'electron EM wave'. However, its function in the model is basically the same. Prof. dr. Burinskii also told us that he was told *not* to refer to the *Zitterbewegung* model at the time, because it was seen as a classical model and, therefore, not in tune with the modern ideas of quantum mechanics.

[56] See the references above.

[57] This is just a random thought at the moment. It needs further exploration.

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

The idea of a photon

Our analysis of Feynman's argument on the 720-degree of spin-1/2 particles should *not* be construed as a criticism of Feynman: it's not *his* argument – it's just orthodox QM. In general, we think Feynman's *Lectures* are still the best lectures on physics one can possibly get – if only because they make one *think* about what one is taught. We, therefore, borrow with very much pleasure two diagrams of his *Lectures* to complete the classical picture of a photon.

The first diagram (Feynman, I-34-9) brings in the oft-neglected magnetic field.[58] Feynman uses it to explain what he refers to as the 'pushing momentum' of light – which is more commonly referred to as radiation or *light pressure*. It is a bit of a strange term, because we are talking a *force*, really.

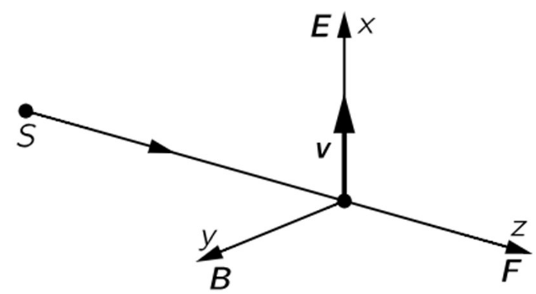


Figure 25: Feynman's explanation of the momentum of light

The basic idea is illustrated in another diagram, which is – unfortunately – separated from the diagram above by a full volume of lectures.[59] An electromagnetic wave – we take it to be a photon – will *drive* an electron, as shown below (Feynman, III-17-4). Hence, the magnetic force comes into play – as there is a charge and a velocity to play with now. 😊 The magnetic force – which is just denoted as ***F*** in the diagram above – will be equal to ***F*** = q***v***×**B**.

[58] Oft-neglected in the context of a *photon* model, that is.
[59] The first illustration comes from Feynman's volume on classical mechanics (Volume I), while the second comes from his lectures on quantum mechanics (Volume III). The volume in-between (Volume II) is on (classical) electromagnetism.

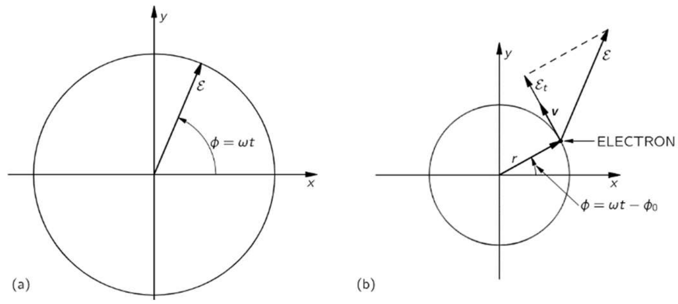


Figure 26: How the electric field of a photon might drive an orbital electron

Feynman then goes off on a bit of a tangent – analyzing the *average* force over time, which makes sense when one continues to take a classical view of an atom (or a Bohr (electron) orbital, practically speaking), and which gives some kind of meaning to the momentum of light.[60] The point is: his analysis fails to bridge classical mechanics with quantum mechanics because he fails to interpret Planck's quantum of action as a quantum: we're not only transferring energy here. We're also transferring angular momentum. In short: photon absorption and emission should respect *the integrity of a cycle*. What is this rule? Some new random interpretation of quantum mechanics? Yes. That is the one we offer here.

What happens when an electron jumps several Bohr orbitals? The angular momentum between the orbitals will then differ by several units of ħ. What happens to the photon picture in that case? It will pack the energy difference, but it will also pack several units of *ħ* (angular momentum) or – what amounts to the same – several units of *h* (physical action). In our humble opinion, we should still think of the photon a one-cycle oscillation. Hence, we do *not* think its energy will be spread over several cycles.[61] The two equations below need to make sense for *all* transitions[62]:

$$photon\!: \mathrm{S} = h = \mathrm{p}_\gamma \cdot \lambda_\gamma = \frac{\mathrm{E}_\gamma}{c}\lambda_\gamma = \frac{\mathrm{E}_\gamma}{f_\gamma} = \mathrm{E}_\gamma \cdot \mathrm{T}_\gamma$$

$$electron\ transition\!: \mathrm{S} = n \cdot h = \mathrm{p}_n \cdot \lambda_n = \mathrm{m}_e v_n \lambda_n = \mathrm{E}_n \cdot \mathrm{T}_n$$

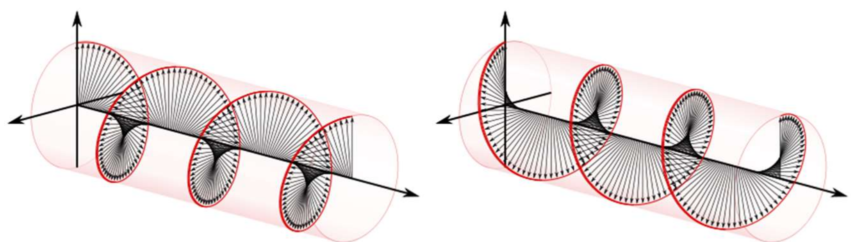
The formulas above express the two most common expressions of what we referred to as the *Certainty Principle*. Pun intended.[63] We will leave it as an exercise for the reader to re-write these formulas in terms of a product of *force*, *distance*, and *time*.

[60] Mr. Feynman gets some kind of explanation for the p = E/*c* relation out of his analysis.
[61] When discussing the Mach-Zehnder experiment in the next version of our paper, we will bring a subtle but essential *nuance* to this point of view.
[62] The use of the same integer *n* for the *difference* in energy between Bohr orbitals might be confusing but we did not want to use another symbol – such as *m*, for example – because *m* would make one think of the fine-structure transitions (which we haven't discussed at all – not in this paper, not in previous one) and – more importantly – because we want to encourage the reader to think these things through for him- or herself. Symbols acquire meaning from the context in which they are used. We are tempted to go off on a tangent on Wittgenstein but we should restrain ourselves here. There is too much philosophy in this paper already. We advise the reader to critically cross-check the formula for electron transitions with what we wrote in previous papers. We warmly welcome comments.
[63] As we argued in previous papers, Planck's quantum of action should probably be interpreted as a *vector*. The uncertainty might not be in its *magnitude*. We feel the uncertainty is in its *direction*. Because *h* is the product of a force, a distance and time, we have a lot of dimensions to consider.

So, what about Uncertainty, then? Nothing – absolutely *nothing* – of what we wrote above involves any uncertainty. It must be there somewhere, right? We would like to offer the following reflection. We have a few footnotes in previous papers, in which we suggest that Planck's quantum of action should be interpreted as a *vector*. The uncertainty – or the probabilistic nature of Nature, so to speak[64] – might, therefore, not be in its *magnitude*. We feel the uncertainty is in its *direction*. This may seem to be restrictive. However, because *h* is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time, we think we have the mathematical framework comes with sufficient degrees of freedom to describe any situation. Quantum-mechanical equations – such as Schrödinger's equation – should probably be written as vector equations.[65]

The photons above make for a circularly polarized beam. The spin direction may be left-handed or right-handed, as shown below.

Figure 27: Left- and right-handed polarization[66]



We can think of these photons as the sum of two linearly polarized waves. We write:

$$cos\theta + i\cdot sin\theta = e^{i\cdot\theta} \text{ (RHC)}$$

$$cos(-\theta) + i\cdot sin(-\theta) = cos\theta - i\cdot sin\theta = e^{-i\cdot\theta} \text{ (LHC)}$$

Huh? *What is the geometry here?* It is quite simple. Let us spell it out so we have no issues of interpretation in the next section(s) of this paper. If ***x*** is the direction of propagation of the wave, then the ***z***-direction will be pointing upwards, and we get the ***y***-direction from the righthand rule for a Cartesian reference frame.[67] We may now think of the oscillation along the ***y***-axis as the cosine, and the oscillation along the ***z***-axis as the sine. If we then think of the imaginary unit *i* as a 90-degree counterclockwise rotation in the ***yz***-plane (and remembering the convention that angles (including the phase angle θ) are measured counterclockwise), then the right- and left-handed waves can effectively be represented by the wavefunctions above.

The point here is that easy visualizations like this strongly encourage us to think of a geometric representation of the wavefunction—if only because, conversely, one may also adopt the convention

[64] A fair amount of so-called thought experiments in quantum mechanics – and I am not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.

[65] We made a start with this in a previous paper: Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, http://vixra.org/pdf/1812.0202v1.pdf.

[66] Credit: https://commons.wikimedia.org/wiki/User:Dave3457.

[67] Note the reference frame in the illustrations of the LHC and RHC wave – which we took from Wikipedia – is left-handed. Our argument will use a regular right-handed reference frame.

that the imaginary unit should be interpreted as a unit vector pointing in a direction that is perpendicular to the direction of propagation of the wave and one may then write the magnetic field vector as $\mathbf{B} = -i \cdot \mathbf{E}/c$.[68] The minus sign in the $\mathbf{B} = -i \cdot \mathbf{E}/c$. It is there because of consistency: we must combine a classical *physical* right-hand rule for **E** and **B** here as well as the *mathematical* convention that multiplication with the imaginary unit amounts to a *counter*clockwise rotation by 90 degrees. This allows us to re-write Maxwell's equations using complex numbers. We have done that in other papers, so if the reader is interested he can check there. [69] The point to note is that, while we will often sort of forget to show the magnetic field vector, the reader should always think of it – because it is an integral part of the electromagnetic wave: when we think of **E**, we should also think of **B**. Both oscillations carry energy.

The mention of energy brings me to another important point. As mentioned above, we think of a circularly polarized beam – and a photon – as a superposition of two linear waves. Now, these two linearly polarized waves will each pack *half* of the energy of the combined wave. It is a very important point to make because any classical explanation of interference – like the one we will offer in the next section – will need to respect the energy conservation law. Note that, while each wave packs *half* of the energy of the combined wave, their (maximum) amplitude is the same: there is no change there. Let us briefly elaborate this point. The energy of any oscillation will always be proportional to (1) its amplitude (*a*) and (2) its frequency (*f*). Hence, if we write the proportionality coefficient as *k*, then the energy of our photon will be equal to:

$$\mathrm{E} = k \cdot a^2 \cdot \omega^2$$

What should we use for the amplitude of the oscillation here? It turns out we get a nice result using the *wavelength*[70]:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

However, we should note this assumes a circularly polarized wave. Its linear components – the sine and cosine, that is – will only pack *half* of that energy. We can now offer the following classical explanation of the Mach-Zehnder experiment *for one photon only*.[71]

A classical explanation for the one-photon Mach-Zehnder experiment

We offered a geometric interpretation of the wavefunction. When analyzing interference in quantum mechanics, the wavefunction concept gives way to the concept of a probability amplitude which we associate with a possible *path* rather than a particle. The math looks somewhat similar but models very different ideas and concepts. Before the photon enters the beam splitter, we have one wavefunction:

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[68] As usual, we use **boldface** letters to represent geometric vectors – the electric (**E**) and magnetic field vectors (**B**), in this case. There is a risk of confusion between the energy E and the electric field E because we use the same symbols, but the context should make clear what is what.

[69] See, for example, Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, http://vixra.org/pdf/1812.0202v1.pdf.

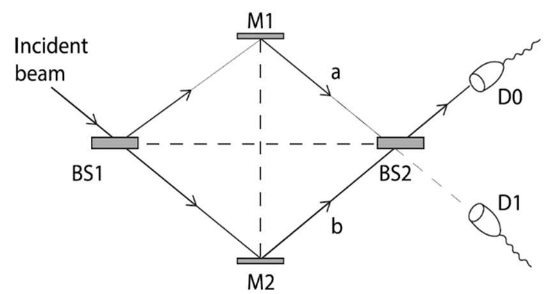
[70] We use the $\mathrm{E}\lambda = hc \Leftrightarrow \lambda = hc/\mathrm{E}$ identity. The reader might think we should use the amplitude of the electric and magnetic field. We could – the model is consistent – but it requires some extra calculations as we then need to think of the energy as some force over a distance. We refer to our papers for more details.

[71] We have written about this topic before (see: Jean Louis Van Belle, *Linear and circular polarization states in the Mach-Zehnder interference experiment*, 5 November 2018, http://vixra.org/pdf/1811.0056v1.pdf). Hence, we will only offer a summary of what we wrote there.

the photon. When it goes through, we have two probability amplitudes that – somehow – recombine and interfere with each other. What we want to do here is to explain this classically.

Let us look at the Mach-Zehnder interferometer once again. We have two beam splitters (BS1 and BS2) and two perfect mirrors (M1 and M2). An incident beam coming from the left is split at BS1 and recombines at BS2, which sends two outgoing beams to the photon detectors D0 and D1. More importantly, the interferometer can be set up to produce a precise interference effect which ensures all the light goes into D0, as shown below. Alternatively, the setup may be altered to ensure all the light goes into D1.

Figure 28: The Mach-Zehnder interferometer[72]



What is the classical explanation? The classical explanation is something like this: the first beam splitter (BS1) splits the beam into *two* beams. These two beams arrive in phase or, alternatively, out of phase and we, therefore, have constructive or destructive interference that recombines the original beam and makes it go towards D0 or, alternatively, towards D1.

When we analyze this in terms of a single photon, this classical picture becomes quite complicated – but we argue there is such classical picture. Our alternative theory of what happens in the Mach-Zehnder interferometer is the following:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

We will detail this in the next section, because what happens in a Mach-Zehnder interferometer is not all that straightforward. We should note, for example, that there are phase shifts along both paths: classical physics tells us that, on transmission, a wave does not pick up any phase shift, but it does so on reflection. To be precise, it will pick up a phase shift of π on reflection. We will refer to the standard textbook explanations of these subtleties and just integrate them in our more detailed explanation in the next section.[73] Before we do so, we will show the assumption that the two linear waves are

[72] Source of the illustration: MIT edX Course 8.04.1*x* (Quantum Physics), Lecture Notes, Chapter 1, Section 4 (*Quantum Superpositions*).
[73] For a good *classical* explanation of the Mach-Zehnder interferometer, see: K.P. Zetie, S.F. Adams and R.M. Tocknell, January 2000, *How does a Mach–Zehnder interferometer work?*

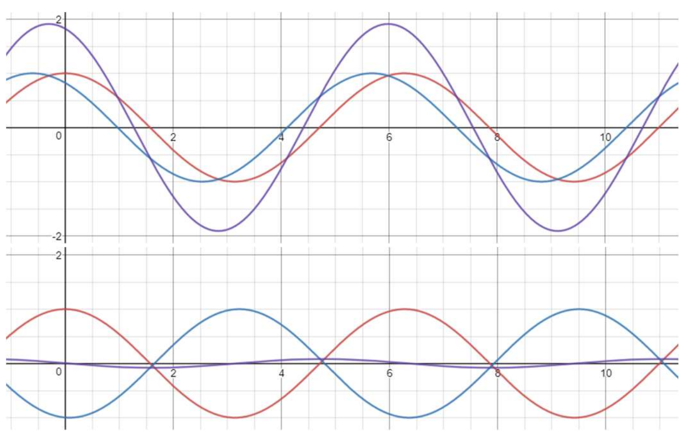
orthogonal to each other is quite crucial. If they weren't, we would be in trouble with the energy conservation law. Let us show that before we proceed.

Suppose the beams would be polarized along the same direction. If *x* is the direction of propagation of the wave, then it may be the *y*- or *z*-direction of anything in-between. The magnitude of the electric field vector will then be given by a sinusoid. Now, we assume we have *two* linearly polarized beams, of course, which we will refer to as beam *a* and *b* respectively. These waves are likely to arrive with a phase difference – unless the apparatus has been set up to ensure the distances along both paths are exactly the same. Hence, the general case is that we would describe *a* by $cos(\omega \cdot t - k \cdot x) = cos(\theta)$ and *b* by $cos(\theta + \Delta)$ respectively. In the classical analysis, the difference in phase (Δ) will be there because of a difference of the path lengths[74] and the recombined wavefunction will be equal to the same cosine function, but with argument $\theta + \Delta/2$, multiplied by an *envelope* equal to $2 \cdot \cos(\Delta/2)$. We write[75]:

$$\cos(\theta) + \cos(\theta + \Delta) = 2 \cdot \cos(\theta + \Delta/2) \cdot \cos(\Delta/2)$$

We always get a recombined beam with the same frequency, but when the phase difference between the two incoming beams is small, its amplitude is going to be much larger. To be precise, it is going to be twice the amplitude of the incoming beams for $\Delta = 0$. In contrast, if the two beams are out of phase, the amplitude is going to be much smaller, and it's going to be zero if the two waves are 180 degrees out of phase ($\Delta = \pi$), as shown below. That does not make sense because twice the amplitude means *four* times the energy, and zero amplitude means zero energy. The energy conservation law is being violated: photons are being multiplied or, conversely, are being destroyed.

Figure 29: Constructive and destructive interference for linearly polarized beams



Let us be explicit about the energy calculation. We assumed that, when the incoming beam splits up at BS1, that the energy of the *a* and *b* beam will be split in half too. We know the energy is given by (or, to

(https://www.cs.princeton.edu/courses/archive/fall06/cos576/papers/zetie_et_al_mach_zehnder00.pdf, accessed on 5 November 2018).
For a good quantum-mechanical explanation (interference of single photons), see – for example – the Mach-Zehnder tutorial from the PhysPort website (https://www.physport.org/curricula/QuILTs/, accessed on 5 November 2018).

[74] Feynman's path integral approach to quantum mechanics allows photons (or probability amplitudes, we should say) to travel somewhat slower or faster than *c*, but that should not bother us here.

[75] We are just applying the formula for the sum of two cosines here. If we would add sines, we would get $\sin(\theta) + \sin(\theta + \Delta) = 2 \cdot \sin(\theta + \Delta/2) \cdot \cos(\Delta/2)$. Hence, we get the same envelope: $2 \cdot \cos(\Delta/2)$.

be precise, proportional to) the square of the amplitude (let us denote this amplitude by *A*).[76] Hence, if we want the energy of the two individual beams to add up to $A^2 = 1^2 = 1$, then the (maximum) amplitude of the *a* and *b* beams must be 1/√2 of the amplitude of the original beam, and our formula becomes:

$$(1/\sqrt{2})\cdot\cos(\theta) + (1/\sqrt{2})\cdot\cos(\theta + \Delta) = (2/\sqrt{2})\cdot\cos(\theta + \Delta/2)\cdot\cos(\Delta/2)$$

This reduces to $(2/\sqrt{2})\cdot\cos(\theta)$ for $\Delta = 0$. Hence, we still get *twice* the energy – $(2/\sqrt{2})^2$ equals 2 – when the beams are in phase and zero energy when the two beams are 180 degrees out of phase. This doesn't make sense.

Of course, the mistake in the argument is obvious. This is why our assumption that the two linear waves are orthogonal to each other comes in: we cannot just add the amplitudes of the *a* and *b* beams because they have different directions. If the *a* and *b* beams – after being split from the original beam – are linearly polarized, then the angle between the axes of polarization should be equal to 90 degrees to ensure that the two oscillations are independent. We can then add them like we would add the two parts of a complex number. Remembering the geometric interpretation of the imaginary unit as a counterclockwise rotation, we can then write the sum of our a and b beams as:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \Delta) = (1/\sqrt{2})\cdot[\cos(\theta) + i\cdot\cos(\theta + \Delta)]$$

What can we do with this? Not all that much, except noting that we can write the $\cos(\theta + \Delta)$ as a sine for $\Delta = \pm\,\pi/2$. To be precise, we get:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \pi/2) = (1/\sqrt{2})\cdot(\cos\theta - i\cdot\sin\theta) = (1/\sqrt{2})\cdot e^{-i\cdot\theta}$$

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta - \pi/2) = (1/\sqrt{2})\cdot(\cos\theta + i\cdot\cos\theta) = (1/\sqrt{2})\cdot e^{i\cdot\theta}$$

This gives us the classical explanation we were looking for:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

What about the 1/√2 factor? If the $e^{-i\cdot\theta}$ and $e^{i\cdot\theta}$ wavefunctions can, effectively, be interpreted geometrically as a *physical* oscillation in *two* dimensions – which is, effectively, our interpretation of the wavefunction[77] – then then each of the two (independent) oscillations will pack one half of the energy of the wave. Hence, if such *circularly* polarized wave splits into two *linearly* polarized waves, then the two linearly polarized waves will effectively, pack half of the energy without any need for us to think their (maximum) amplitude should be adjusted. If we now think of the ***x***-direction as the direction of the incident beam in the Mach-Zehnder experiment, and we would want to also think of rotations in the ***xz***-plane, then we need to need to introduce some new convention here. Let us introduce *another* imaginary unit, which we'll denote by *j*, and which will represent a 90-degree counterclockwise rotation

[76] If we would reason in terms of average energies, we would have to apply a 1/2 factor because the average of the $\sin^2\theta$ and $\cos^2\theta$ over a cycle is equal to 1/2.
[77] We can assign the physical dimension of the electric field (force per unit charge, N/C) to the two perpendicular oscillations.

in the ***xz***-plane.[78] We then get the following *classical* explanation for the results of the one-photon Mach-Zehnder experiment:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = \cos\theta + i\cdot\sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta} = \cos\theta - i\cdot\sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $-j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.

Of course, we may also set up the apparatus with different path lengths, in which case the two linearly polarized beams will be out of phase when arriving at BS1. Let us assume the phase shift is equal to $\Delta = 180° = \pi$. This amounts to putting a minus sign in front of *either* the sine *or* the cosine function. Why? Because of the $\cos(\theta \pm \pi) = -\cos\theta$ and $\sin(\theta \pm \pi) = -\sin\theta$ identities. Let us assume the distance along the upper path is longer and, hence, that the phase shift affects the sine function.[79] In that case, the sequence of events might be like this:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = \cos\theta + i\cdot\sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $j\cdot\sin(\theta + \pi) = -j\cdot\sin\theta$.	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.

[78] This convention may make the reader think of the quaternion theory but we are thinking more of simple Euler angles here: *i* is a (counterclockwise) rotation around the *x*-axis, and *j* is a rotation around the *y*-axis.
[79] The reader can easily work out the math for the opposite case (longer length of the lower path).

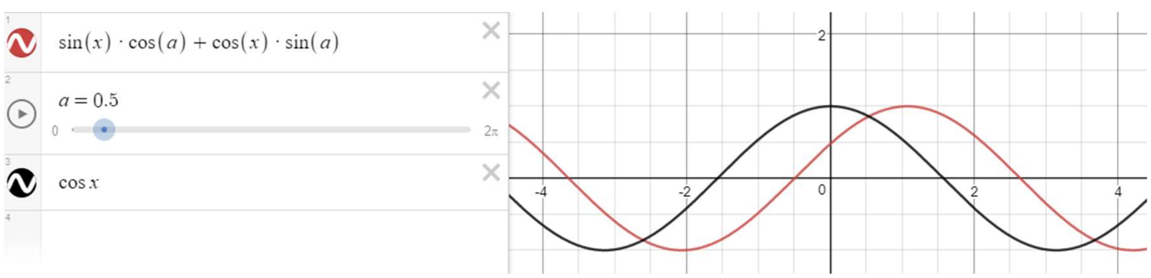
	Lower beam (horizontal oscillation) = $\cos\theta$	affected and is still represented by $\cos\theta$	The lower beam is still represented by $\cos\theta$	
LHC	Photon ($e^{-j\cdot\theta} = \cos\theta - j\cdot\sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $-j\cdot\sin(\theta + \pi) = +j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.

What happens when the difference between the phases of the two beams is not equal to 0 or 180 degrees? What if it is some random value in-between? Do we get an elliptically polarized wave or some other nice result? Denoting the phase shift as Δ, we can write:

$$\cos\theta + j\cdot\sin(\theta + \Delta) = \cos\theta + j\cdot(\sin\theta\cdot\cos\Delta + \cos\theta\cdot\sin\Delta)$$

However, this is also just a circularly polarized wave, but with a random phase shift between the horizontal and vertical component of the wave, as shown below. Of course, for the special values $\Delta = 0$ and $\Delta = \pi$, we get $\cos\theta + j\cdot\sin\theta$ and $\cos\theta - j\cdot\sin\theta$ once more.

Figure 30: Random phase shift between two waves



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Mystery solved? Maybe. Maybe not. We just wanted to show that one should try to go everywhere. 😉

XIII. Conclusions

We presented a lot of material. How can one sum it all up? We would probably just want to say this: it is about time physicists consider the *form factor* in their analysis. It somehow disappeared. Vector equations became *flat*: vector quantities became magnitudes. Schrödinger's equation should be rewritten as a vector equation.

What about uncertainty? Nothing – absolutely *nothing* – of what we wrote above involves any uncertainty. It must be there somewhere, right? We would like to offer the following reflection. We have a few footnotes in previous papers, in which we suggest that Planck's quantum of action should be interpreted as a *vector*. The uncertainty – or the probabilistic nature of Nature, so to speak[80] – might, therefore, not be in its *magnitude*. We feel the uncertainty is in its *direction*. This may seem to be restrictive. However, because *h* is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time, we think the mathematical framework comes with sufficient degrees of freedom to describe any situation.

Jean Louis Van Belle, 8 January 2018

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[80] A fair amount of so-called thought experiments in quantum mechanics – and we are not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.