Thoughts on a New Definition of Momentum That Makes Physics Simpler and More Consistent

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Abstract

We suggest that momentum should be redefined in order to help make physics more consistent and more logical. In this paper, we will propose that there is a rest-mass momentum, a kinetic momentum, and a total momentum.

Key words: momentum, kinetic momentum, rest-mass momentum.

1 Introduction

Today there is no rest-mass momentum in modern physics, which leads to unnecessary complexity and even inconsistency in the field. In modern physics, the momentum for a particle with mass is given by [5]

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

and when $v << c$, we can use the first term of a Taylor series expansion and approximate the momentum very well with $p \approx mv$.

The relativistic energy momentum relation is very important in modern physics

$$E^2 = p^2c^2 + (mc^2)^2 \tag{2}$$

To find the momentum of a photon, we can set the mass to zero in the last part of the equation above, solve with respect to momentum, and we will get

$$p = \frac{E}{c} = \frac{\hbar}{\lambda} \tag{3}$$

Relativistic momentum is given by equation 1. In modern physics, photons are always treated as something special. They are special, but do we truly need one set of momentum equations for particles with mass and one set for photons? Based on recent analysis, we will show that this is not necessary.

For photons, the standard relativistic momentum formulas do not work, so here we have defined momentum as $p = \frac{\hbar}{\lambda}$ (as derived from the relativistic momentum relation).

2 New Momentum Definition

We suggest that the total momentum is given by

$$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4}$$

and that the rest-mass momentum is given by $p_r = mc$. Then a moving particle with mass has a kinetic momentum of

$$p_k = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} - mc \tag{5}$$

and when $v << c$, this can be very well approximated by the first term of a Taylor series expansion

$$p_k \approx \frac{1}{2}\frac{mv^2}{c} \tag{6}$$

In our new momentum equation, energy is always equal to momentum times the speed of light. The relationship $E = pc$ is often used in physics, but with the old version of momentum it actually only holds for photons and not for particles like electrons. And the relativistic momentum equation for particles with mass does not hold for photons; we are operating with two different frameworks that have been merged in a rather ad-hoc way to keep the energy to line up with experiments.

Our new momentum definition leads to a new relativistic energy momentum relation of

$$E = p_k c + mc^2 \tag{7}$$

That is, we have

$$E = \left(\frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} - mc \right) c + mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{8}$$

We claim that this will also hold for photons. The key is to combine it with Haug's maximum velocity [7–10] of matter $v_{max} = c\sqrt{1 - \frac{l_p^2}{\lambda^2}}$. As discussed in previous papers, in the special case of the Planck mass particle, the maximum velocity is zero

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{l_p^2}} = 0 \tag{9}$$

This sounds absurd, but in our view it represents the collision point between two photons. This means for light there is only rest-mass momentum of the form $p = mc$, and the relativistic momentum formula and all other relativistic formulas now hold for both light and traditional matter. Modern physics often operates with two sets of rules, as a full connection made between light and matter has not been determined.

We also note that the Planck mass is observational time dependent and is approximately 10^{-51} kg in a one second observational time-window, but indeed has an enormous traditional value of approximately $10^{-8}kg$ in a one Planck time observational time window.

This leads to a new quantum probability theory that is much less mysterious than the existing quantum mechanics theory. Further, it produces one set of equations that apply equally to photons and all other matter. This stands in contrast to modern physics, which relies more on a series of mathematical tricks and complexities to compensate for the lack of a fully understood connection between photons and matter.

3 The Two Matter Waves: The de Broglie Wave and the Compton Wave

By the time of the photo-electronic work of Einstein, it was clear that light was both a particle and a wave. In 1924, Louis de Broglie [1] suggested that matter also had wave properties. He calculated the wavelength of matter from momentum and got

$$\lambda_B \approx \frac{h}{mv} \tag{10}$$

where m is the rest-mass and v is the velocity of the particle in question; this is known today as the de Broglie wavelength, or in the relativistic form

$$\lambda_B = \frac{h}{\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}} \tag{11}$$

Shortly after his conjecture, experimental research confirmed that matter did have wave-like properties and the de Broglie hypothesis was quickly accepted and incorporated. It was further developed later in quantum mechanics. We fully agree that matter has both a particle and a wave-like nature. Still, we think de Broglie made a serious mistake in how he calculated this wavelength. We also think there are errors in how it has been incorporated in modern physics. The de Broglie wavelength has a series of mystical properties; it is infinite for a particle when the velocity is zero, for example, and it is also linked to superluminal phase velocity.

In 1923, working at around the same time as de Broglie, Compton [2] discovered a wave related to electrons – the so-called Compton wavelength that is given by

$$\lambda_c = \frac{h}{mc} \tag{12}$$

And for a moving particle

$$\lambda_c = \frac{h}{\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}} \tag{13}$$

and when $v << c$ this can be very well approximated with the first term of a Taylor series expansion, $\lambda_c \approx \frac{h}{mc}$.

The Compton wavelength of an electron has been measured in many experiments, it is a short wavelength of about $2.4263102367 \times 10^{-12}$ m (2014 NIST CODATA). It fits perfectly with theory. No one has measured the length of the de Broglie wavelength, even though some may have claimed to do so. If one knows the Compton wavelength, however, one can easily find the mass of the electron, since the mass is related to the Compton wavelength

$$m_e = \frac{h}{\lambda_c}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c} \tag{14}$$

Which means the mass of an elementary particle can be found by measuring the Compton wavelength of the particle, as has been done experimentally with electrons, see [13]. Still, the de Broglie wavelength is a mathematical function of the physical Compton wavelength, namely

$$\lambda_B = \lambda_c \frac{c}{v} \tag{15}$$

So, we can indirectly measure the de Broglie wavelength, which is a mathematical "artifact," indirectly from the physical Compton wavelength. Further, the link between mass and Compton time frequency has recently been explored and supported by recent experimental research. Dolce and Perali [4] conclude that "*the rest-mass of a particle is associated to a rest periodicity known as Compton periodicity*".

We claim that the de Broglie wavelength is a pure mathematical "artifact." Notice also that the Compton wavelength of an electron is calculated by dividing the Planck constant by $mc = \frac{h}{\lambda}\frac{1}{c}c = \frac{h}{\lambda}$. Strangely, mc is the momentum of a photon, and not the momentum of anything with rest-mass, according to standard physics. And still the photon momentum definition is what is used to calculate a measurable wave length that is directly linked to mass of elementary particles with mass. And why should there be two different matter waves, the de Broglie and the Compton wave? Naturally no one in modern physics likes to point out that they can predict a consistent wavelength of matter with mass by dividing the Planck constant by a photon-like momentum for matter. On the other hand, if we say there exists a total momentum equal to our newly introduced momentum, namely

$$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} \tag{16}$$

then the Compton wavelength is indeed simply the Planck constant divided by the total momentum, just as the idea of de Broglie, but with a correct momentum. It is identical to the Compton formula, but here we have a simple explanation for what the parts are. In addition, our wave formula holds for photons as long as we use the maximum velocity formula for matter; it is zero for a photon. That is, the Compton wavelength of a photon is

$$\lambda = \frac{h}{p_t} = \frac{h}{\frac{mc}{\sqrt{1-\frac{0^2}{c^2}}}} = \frac{h}{mc} = \frac{h}{\frac{h}{\lambda_c}\frac{1}{c}c} = \frac{h}{p} \tag{17}$$

4 Inconsistencies and Mystical Interpretations in Modern Physics Related to Non-Optimal Momentum Definition

The relativistic energy mass relation is again given by

$$E = \sqrt{p^2c^2 + (mc^2)^2} \tag{18}$$

It is important to realize that this indirectly allows negative energy, negative mass, and negative momentum, since we must have $E = \sqrt{(\pm p)^2c^2 + (\pm mc^2)^2}$. This has been a significant challenge for modern physics, as negative energy, negative mass, and negative momentum have never been observed (except in the fantasy of a few physicists possibly). However, there is much speculation in modern physics about negative mass, negative energy, and even negative probabilities to arrive at a fully consistent theory. For example, Dirac [3] had interesting discussions concerning how negative probabilities show up in quantum mechanics:

> *Thus the two undesirable things, negative energy and negative probability, always occur together.*
> – Paul Dirac, 1942

Pauli, Feynman, [6, 12] and many others also speculated on negative probabilities. Negative probabilities actually make no logical sense at all, just as negative matter and negative energy defy common sense and logic. We would claim that this is all rooted in an incorrect definition of momentum, which is not physical, but simply a mathematical non-optimal defined "derivative." The relativistic energy mass relation is one of the cornerstones in modern quantum mechanics. A number of relativistic quantum mechanics equations, such as the Klein-Gordon equation, for example, are directly linked to the relativistic energy momentum relation. The relativistic energy momentum relation gives the correct energy, but it is unnecessarily complex, as it is rooted in an ill-specified momentum. Further, negative energy states coming out from quantum mechanics (the relativistic energy momentum equation) were interpreted by some famous physicists like Feynman as particles moving backwards in time, see for example [11].

Here we will outline a series of partial inconsistencies related to their choice of non-optimal momentum definition

- Modern physics does not have rest-mass momentum, but does have rest-mass energy, kinetic energy, and total energy. The lack of rest-mass momentum, we will claim, is inconsistent.
- Modern physics uses different formulas for momentum for photons and for matter with rest-mass. For matter, we have $p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$, while for photons, we have $p = \frac{h}{\lambda}$, For photon, we also have $E = pc$. In a series of papers, it seems to be used incorrectly: for matter with rest-mass, one uses $E = pc$ to go from momentum to energy, but this is inconsistent with the relativistic momentum of matter in modern physics. And $E = pc = mvc$ is not energy.
- When using standard momentum to calculate a matter wave, we get the de Broglie wavelength. Contrary to what modern physics claims, this wave has never been observed. The wave nature of matter has been detected, and a wave related to matter has been measured very accurately. However, this is the Compton wavelength and not the de Broglie wavelength. Still, one could claim the de Broglie wavelength exists, as it is indirectly a mathematical derivative of the Compton wavelength, namely $\lambda_B = \lambda_c \frac{c}{v}$. The de Broglie wave makes no sense for a rest-mass, as it is then infinite. The idea that an electron at rest should be everywhere in the universe, for example, simply makes no logical sense. And yet there are a series of different interpretations on this, even inside the standard paradigm.
- The relativistic energy momentum relation is unnecessarily complex, and, we would say, even mystical as an approach to scientific phenomenon. What is energy and momentum squared? There are no such things physically. The standard relativistic energy momentum relation is unnecessarily complex simply because the momentum is ill-specified in the first place, so to get the math to fit observations (energy) one needs an unnecessarily complex formula $E = p^2c^2 + (mc^2)$. This also means the momentum of a particle with mass is an unnecessarily complex function of energy, namely $p = \frac{\sqrt{E^2 - mc^2}}{c}$. At the same time, for a photon it is simply $p = E/c$ (simply by putting $m = 0$ in the relativistic energy momentum formula). By using our redefined momentum, we get a much simpler and logical relativistic energy momentum relation, namely $E_t = p_k c + mc^2$. This also means momentum is always simply the energy divided by the speed of light, which removes challenges with such things as negative energies and negative probabilities. Be aware they give the same energy.
- The relativistic energy momentum relation leads to possibility of negative momentum, negative energy, negative mass and a series of famous physicsts have even speculated on negative probabilities. This has led to a considerable amount of wild speculation in modern physics that will all disappear with a sound momentum definition.
- Standard physics momentum has led to non-physical wavelength (the de Broglie wavelength) and impossible mathematical artifacts, such as superluminal and even infinite phase velocity of $\frac{c^2}{v}$. These are mathematical derivatives that are linked to real properties, but mostly they only add complexity when what they represent is not fully understood.
- The standard momentum also leads to a series of other inconsistencies in modern physics that we will discuss in more detail in the next version of this paper.

Table 1 summarizes how our newly defined momentum brings logic and simplicity back into physics.

Entity	**Standard physics**	**New theory**
Total momentum mass	$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$	$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}$
Kinetic momentum	$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$	$p_k = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} - mc$
Kinetic momentum $v << c$	$p \approx mv$	$p_k \approx \frac{1}{2} m \frac{v^2}{c}$
Rest-mass momentum	None	$p_r = mc$
Momentum photon	$p = \frac{h}{\lambda} = mc$	$p_t = \frac{mc}{\sqrt{1-\frac{v^2}{c^2}}} = mc$ since $v = 0$
From momentum to energy	For photons multiply by c, or else complicated	Just multiply by c for photons and standard mass.
From energy to momentum	For photons divide by c, or else complicated	Just divide by c for photons and standard mass.
Matter wave-1 de Broglie	$\lambda_B = \frac{h}{\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}}$ Never observed!	de Broglie is mathematical construct
Matter wave-2 Compton wave The new momentum used	$\lambda_c = \frac{h}{\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}}$ Observed. Not understood	$\lambda_c = \frac{h}{\frac{mc}{\sqrt{1-\frac{v^2}{c^2}}}}$ The only matter wave. Understood
Mass from Compton	$m = \frac{h}{\lambda_c}\frac{1}{c}$	$m = \frac{h}{\lambda_c}\frac{1}{c}$
Mass from de Broglie	$m = \frac{h}{\lambda_B}\frac{1}{v}$ Impossible for rest-mass	$m = \frac{h}{\lambda_B}\frac{1}{v}$ Impossible for rest-mass (artifact)
de Broglie from Compton	$\lambda_B = \lambda_c \frac{c}{v}$	$\lambda_B = \lambda_c \frac{c}{v}$
Compton from de Broglie	$\lambda_c = \lambda_B \frac{v}{c}$	$\lambda_c = \lambda_B \frac{v}{c}$
Phase velocity	$v_p = \frac{E}{p} = \frac{c^2}{v}$ Not understood	$v_p = \frac{E}{p} = c$ Understood
Energy momentum relation	$E^2 = p^2c^2 + (mc^2)^2$	$E = pc + mc^2$
Momentum from energy	$p = \frac{\sqrt{E^2 - m^2c^4}}{c} = \frac{\sqrt{\left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 - m^2c^4}}{c}$	$p = \frac{E - mc^2}{c} = \frac{\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}{c}$
Momentum from energy	$p = \frac{\sqrt{E^2 - m^2c^4}}{c} = \frac{E}{c} = \frac{h\frac{c}{\lambda}}{c}$ Partly "trickery" derivation, but correct	$p = \frac{E - mc^2}{c} = \frac{\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}{c}$ Consistent and correct
Negative: energy, momentum, and mass	Cannot be excluded	Totally excluded
Negative probability	Suggested as solution	Absurd and not needed

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