Beal Conjecture Convincing Proof

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

The author proves directly the original Beal conjecture that if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor. In the numerical examples, two approaches have been used to change the sum, $A^x + B^y$, of two powers to a single power, C^z. In one approach, the application of factorization is the main principle, while in the other approach, a derived formula from $A^x + B^y$ was applied. The two approaches changed the sum $A^x + B^y$ to the single power, C^z, perfectly. The derived formula confirmed the validity of the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived. It was shown that if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor.

Beal Conjecture Convincing Proof
Preliminaries

Process and Requirements Involved in Changing the Sum of Two Powers to a Single Power

The process will be guided as follows. There is a sum of two powers $A^x + B^y$. One will write this sum as a single power C^z such that $A^x + B^y = C^z$,.noting that A, B, C, x, y, z are positive integers and $x, y, z > 2$.
The necessary condition is that the two powers must have a common power as exemplified below. If this requirement is not satisfied, the sum of the two powers cannot be changed to a single power, because of the intermediate product step.

Two main steps are involved in changing the sum $A^x + B^y$ to a single power C^z.

Step 1
In step 1, the **sum** of the two powers is changed to a **produc**t by factorization and also by a derived formula. The factorization is a common "monomial" factoring which involves a division process. It is the quotient involved which requires the common prime factor requirement so that the resulting product satisfies the requirements, A, B, x, y are positive integers and $x, y, > 2$. Thus, A and B must have common prime factor (as illustrated below) for the division result to satisfy the conditions where A, B, x, y are positive integers and $x, y, z > 2$. Any product obtained also has the same common prime factor as the sum of the powers. Note below as in Approach A that it is the second term (a quotient) of the critical sum, where some of the common factors are needed for cancellation and simplification.

Approach A Without factoring the powers first		**Approach B**
Change $34^5 + 51^4$ to a single term		Factoring each power first (more efficient)
$= 34^5(\underbrace{1 + \frac{51^4}{34^5}}_{\text{critical sum}})$ (A)	(factoring out the 34^5	Change $34^5 + 51^4$ to a single term
		$= (17 \bullet 2)^5 + (17 \bullet 3)^4$
		$= 17^5 \bullet 2^5 + 17^4 \bullet 3^4$
$= 34^5(1 + \frac{17^4 \bullet 3^4}{17^5 \bullet 2^5})$ (B)	(some common factors divided out	$= 17^4\underbrace{(17 \bullet 2^5 + 3^4)}_{\text{critical sum}}$
$= 17^5 \bullet 2^5(1 + \frac{3^4}{17 \bullet 2^5})$ (C)		$= 17^4(17 \bullet 32 + 81)$
$= 17^5 \bullet 2^5(\frac{17 \bullet 2^5 + 3^4}{17 \bullet 2^5})$ (D		$= 17^4(625)$
$= 17^4(\underbrace{17 \bullet 2^5 + 3^4}_{\text{critical sum}})$ <--------(E)	Critical sum satisfies A, B, x, y being, positive integers; $x, y, z > 2$.	$= 17^4(5^4)$
$= 17^4(625)$		$= (17 \bullet 5)^4$
$= 17^4(5^4)$		$= 85^4$
$= (17 \bullet 5)^4$		
$= 85^4$		

Note above that Steps (A) to (C) constitute Step 1.
Note also that Approach B takes less steps.

Step 2: In Step 2, the product from step 1 is changed to a single power.

Example 1A: $\boxed{2^3+2^3=2^4}$ $A=2,\ B=2,\ C=2,\ x=3,\ y=3,\ z=4$; $A^x+B^y=C^z$.

Change the sum 2^3+2^3 to a single power of 2.

Factor out the greatest common factor.	
2^3+2^3	
$=2^3(\underbrace{1+1}_{\text{critical sum}})$ (G) <.-------------------------------	This step requires that 2^3 and 2^3 have a common prime factor
$=2^3(2)$	It is interesting how the "(1+1)" provided the much needed 2.
$=2^4$	

The 2^4 must have a common factor as 2^3 and 2^3, from which it was obtained..
From above, the common prime factor is 2,

Example 1B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x+E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A=Dr$, $B=Es$, and $C=Ft$, $\boxed{(Dr)^x+(Es)^y=(Ft)^z}$ (from $A^x+B^y=C^z$) . Also $r^x(D^x+E^y r^{y-x})\ =F^z t^z$, where $s=r$.

Assuming that A and B have a common prime factor r, the above equation becomes $(Dr)^x+(Er)^y=(Ft)^z$. From the left-hand side of this equation, one obtains the conversion formula, $r^x(D^x+E^y r^{y-x})$. This formula will be applied to numerical equations to test the validity of the assumption that A, and B have a common prime factor before being converted to C.

Conversion Formula: $r^x(D^x+E^y r^{y-x})$, where $r=s$ (i.e., A and B have a common prime factor)

The conversion formula will convert the two-term sum A^x+B^y to a single term, C^z.

For the left-hand-side	For the right-hand side
Change 2^3+2^3 to a single term	$F=1,\ t=r=2,\ z=4$
$(1\bullet 2)^3+(1\bullet 2)^3=(1\bullet 2)^4$	$\boxed{F^z t^z=1\bullet 2^4 \\ =2^4}$
$\boxed{\text{Formula}: r^x(D^x+E^y r^{y-x})}$	
$r=2, D=1, x=3, E=1, y=3$	Observe above that it has been shown that
$=2^3(1^3+1^3\bullet 2^{3-3})$	$r^x(D^x+E^y r^{y-x})\ =F^z t^z$
$=2^3(1+2^0)$	
$=2^3(1+1)$	
$=2^3(2)$	
$=2^4$	

Example 1B confirmed the assumption that it is necessary that the sum A^x+B^y has a common prime factor before C^z can be derived. (from the formula $r^x(D^x+E^y r^{y-x})\ =F^z t^z$**).**

Example 2A $\boxed{7^6 + 7^7 = 98^3}$ $A = 7,\ B = 7,\ C = 98,\ x = 6,\ y = 7,\ z = 3,\ A^x + B^y = C^z$

Change the sum $7^6 + 7^7$ to a single power of 98.

Factor out the greatest common factor.	
$7^6 + 7^7$	
$= 7^6(\underbrace{1+7}_{\text{critical sum}})$ (G)<-------------------	This step requires that 7^6 and 7^7 have a common prime factor
$= 7^6(8)$	
$= 7^6(2^3)$	It is interesting how the "(1+ 7)" provided the much needed 2^3.
$= (7^2)^3(2^3)$	
$= (7^2 \bullet 2)^3$	
$= (49 \bullet 2)^3$	
$= (98)^3$	
$= 98^3$	
Note that if $7^6 + 7^7$ did not have any common factor, one could not factor, and one will not be able write the sum as a product and subsequently change the product to power form.	

Since 98^3 was obtained from the sum $7^6 + 7^7$, which has a common prime factor. 7, 98^3 has the same common prime factor, 7. Therefore 7^6, 7^7 and 98^3 have the common prime factor of 7.

Example 2B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$, $(Dr)^x + (Es)^y = (Ft)^z$

$$7^6 + 7^7 = 98^3$$

$$(1 \bullet 7)^6 + (1 \bullet 7)^7 = (14 \bullet 7)^3$$

Conversion Formula: $r^x(D^x + E^y r^{y-x}) = F^z t^z$, where $r = s$ (i.e., A and B have a common prime factor)

For the left-hand-side	For the right-hand side
$\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$	$F = 14,\ t = r = 7,\ z = 3$
$r = 7, D = 1, x = 6, E = 1, y = 7$	$(Ft)^z = (14 \bullet 7)^3$
$= 7^6(1^6 + 1^7 \bullet 7^{7-6})$	$\boxed{= 98^3}$
$= 7^6(1 + 1 \bullet 7)$	
$= 7^6(1+7)$	Observe above that it has been shown that $r^x(D^x + E^y r^{y-x}) = F^z t^z$.
$= 7^6(8)$	
$= 7^6 \bullet 2^3$	
$= (7^2)^3 \bullet 2^3$	
$= (7^2 \bullet 2)^3$	
$= (49 \bullet 2)^3$	
$\boxed{= 98^3}$	

Example 2B confirmed the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived.(from the formula $r^x(D^x + E^y r^{y-x}) = F^z t^z$**).**

Example 3A: $\boxed{3^3 + 6^3 = 3^5}$ $A = 3,\ B = 6,\ C = 3,\ x = 3,\ y = 3,\ z = 5$, $A^x + B^y = C^z$

Change the sum $3^3 + 6^3$ to a single power of 3..

Factor out the greatest common factor.	
$3^3 + 6^3$	
$= 3^3 + (3 \bullet 2)^3$	
$= 3^3 + 3^3 \bullet 2^3$	
$= 3^3(\underbrace{1 + 2^3}_{\text{critical sum}})$ (G)<--------------------	This step requires that 3^3 and 6^3 have a common prime factor
$= 3^3(1 + 8)$	It is interesting how the "(1+ 8)" provided the much needed 3^2.
$= 3^3(9)$	
$= 3^3 \bullet 3^2$	
$= 3^5$	

Since 3^5 was obtained from the sum $3^3 + 6^3$, which has a common prime factor. 3, 3^5 has the same common prime factor, 3,

Example 3B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$, $\boxed{(Dr)^x + (Es)^y = (Ft)^z}$

$$3^3 + 6^3 = 3^5$$

$$(1 \bullet 3)^3 + (2 \bullet 3)^3 = (1 \bullet 3)^5$$

Conversion Formula: $r^x(D^x + E^y r^{y-x}) = F^z t^z$, where $r = s$ (i.e., A and B have a common prime factor)

For the left-hand-side	For the right-hand side
$\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$	$F = 1,\ t = r = 3,\ z = 5$
$(1 \bullet 3)^3 + (2 \bullet 3)^3 = (1 \bullet 3)^5$	$(Ft)^z = (1 \bullet 3)^5$
$r = 3, D = 1, x = 3, E = 2, y = 3$	$\boxed{= 3^5}$
$= 3^3(1^3 + 2^3 \bullet 3^{3-3})$	
$= 3^3(1 + 2^3 \bullet 1)$	
$= 3^3(1 + 2^3)$	Observe above that it has been shown that
$= 3^3(1 + 8)$	$r^x(D^x + E^y r^{y-x}) = F^z t^z$.
$= 3^3(9)$	
$= 3^3(3^2)$	
$\boxed{= 3^5}$	

Example 3B confirmed the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived (from the formula $r^x(D^x + E^y r^{y-x}) = F^z t^z$**).**

Example 4A $\boxed{2^9 + 8^3 = 4^5}$ $A = 2,\ B = 8,\ C = 4,\ x = 9,\ y = 3,\ z = 5$, $A^x + B^y = C^z$

Change the sum $2^9 + 8^3$ to a single power of 4.

Factor out the greatest common factor.	
$2^9 + 8^3$	
$= 2^9 + (2^3)^3$	
$= 2^9 + 2^9$	
$= 2^9 \underbrace{(1+1)}_{\text{critical sum}}$ (G)<----------------------	This step requires that 2^9 and 8^3 have a common prime factor
$= 2^9 \bullet 2$	It is interesting how the "(1+ 1)" provided the much needed 2.
$= 2^{10}$	
$= (2^2)^5$	
$= (4)^5$	
$= 4^5$	

Since 4^5 was obtained from the sum $\boxed{2^9 + 8^3}$, which has a common prime factor. 2, 4^5 has the same common prime factor, 2,

Example 4B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$, $\boxed{(Dr)^x + (Es)^y = (Ft)^z}$

$\boxed{2^9 + 8^3 = 4^5}$

$(1 \bullet 2)^9 + (4 \bullet 2)^3 = (2 \bullet 2)^5$

Conversion Formula: $r^x(D^x + E^y r^{y-x}) = F^z t^z$, where $r = s$ (i.e., A and B have a common prime factor)

For the left-hand-side	For the right-hand side
$\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$	$F = 2,\ r = 2,\ z = 5$
$(1 \bullet 2)^9 + (4 \bullet 2)^3 = (2 \bullet 2)^5$	$(Ft)^z = (2 \bullet 2)^5$
$r = 2, D = 1, x = 9, E = 4, y = 3$	$\boxed{= 4^5}$
$= 2^9(1^9 + 4^3 \bullet 2^{3-9})$	
$= 2^9(1 + (2^2)^3 \bullet 2^{-6})$	
$= 2^9(1 + 2^6 \bullet 2^{-6}$	
$= 2^9(1 + 2^0)$	Observe above that it has been shown that
$= 2^9(1+1)$	$r^x(D^x + E^y r^{y-x}) = F^z t^z$.
$= 2^9(2)$	
$= 2^{10}$	
$= (2^2)^5$	
$\boxed{= 4^5}$	

Example 4B confirmed the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived. (from the formula $r^x(D^x + E^y r^{y-x}) = F^z t^z$**).**

Example 5A $\boxed{34^5 + 51^4 = 85^4}$ $A = 34,\ B = 51,\ C = 85,\ x = 5,\ y = 4,\ z = 4$, $A^x + B^y = C^z$

Change the sum $34^5 + 51^4$ to a single power of 85.

Factor out the greatest common factor.	
$34^5 + 51^4$	
$= (17 \bullet 2)^5 + (17 \bullet 3)^4$	
$= 17^5 \bullet 2^5 + 17^4 \bullet 3^4$	
$= 17^4 \underbrace{(17 \bullet 2^5 + 3^4)}_{\text{critical sum}}$ (G) <--------------	This step requires that 34^5 and 51^4 have a common prime factor
$= 17^4(17 \bullet 32 + 81)$	It is interesting how the $\underbrace{17 \bullet 2^5 + 3^4}_{\text{magic}}$ provided the much needed
$= 17^4(625)$	$625 = 5^4$
$= 17^4(5^4)$	
$= (17 \bullet 5)^4$	
$= 85^4$	

Since 85^4 was obtained from 34^5 and 51^4 which have the common prime factor , 17, 85^4 has the same common factor, 17.

Example 5B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$, $\boxed{(Dr)^x + (Es)^y = (Ft)^z}$

$34^5 + 51^4 = 85^4$

$(2 \bullet 17)^5 + (3 \bullet 17)^4 = (5 \bullet 17)^4$

Conversion Formula: $r^x(D^x + E^y r^{y-x}) = F^z t^z$, where $r = s$ (i.e., A and B have a common prime factor)

For the left-hand-side	For the right-hand side
$(2 \bullet 17)^5 + (3 \bullet 17)^4 = (5 \bullet 17)^4$	$F = 5,\ t = r = 17,\ z = 4$
$\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$	$\boxed{F^z t^z = 5^4 17^4}$
$r = 17, D = 2, x = 5, E = 3, y = 4$	$= (5 \bullet 17)^4$
$= 17^5(2^5 + 3^4 \bullet 17^{4-5})$	$\boxed{= 85^4}$
$= 17^5(2^5 + 3^4 \bullet 17^{-1})$	
$= 17^5(2^5 + \frac{3^4}{17})$	
$= 17^5(\frac{17 \bullet 2^5 + 3^4}{17})$	Observe above that it has been shown that $r^x(D^x + E^y r^{y-x}) = F^z t^z$.
$= 17^4(17 \bullet 2^5 + 3^4)$	
$= 17^4(17 \bullet 32 + 81)$	
$= 17^4(625)$	
$= 17^4(5^4)$	
$= (17 \bullet 5)^4$	
$\boxed{= 85^4}$	

Example 5B confirmed the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived. (from the formula $r^x(D^x + E^y r^{y-x}) = F^z t^z$)..

Example 6A: $\boxed{3^9 + 54^3 = 3^{11}}$ $A = 3,\ B = 54,\ C = 3,\ x = 9,\ y = 3,\ z = 11,\ A^x + B^y = C^z$

Change the sum $3^9 + 54^3$ to a single power of 3.

Factor out the greatest common factor. $3^9 + 54^3$ $= 3^9 + (9 \bullet 6)^3$ $= 3^9 + (3 \bullet 3 \bullet 3 \bullet 2)^3$ $= 3^9 + (3^3 \bullet 2)^3$ $= 3^9 + 3^9 \bullet 2^3$ $= 3^9 \underbrace{(1 + 2^3)}_{\text{critical sum}}$ (G) <---------------------- $= 3^9(1 + 8)$ $= 3^9(9)$ $= 3^9 \bullet 3^2$ $= 3^{11}$	This step requires that 3^9 and 54^3 have a common prime factor It is interesting how the $1 + 2^3$ provided the much needed 9. .

Since 3^{11} was obtained from 3^9 and 54^3 which have the common prime factor , 3.
3^{11} has the common factor 3.

Example 6B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$, $\boxed{(Dr)^x + (Es)^y = (Ft)^z}$

$3^9 + 54^3 = 3^{11}$

$(1 \bullet 3)^9 + (3 \bullet 18)^3 = (1 \bullet 3)^{11}$

Conversion Formula: $r^x(D^x + E^y r^{y-x}) = F^z t^z$, where $r = s$ (i.e., A and B have a common prime factor)

For the left-hand-side	For the right-hand side
$(1 \bullet 3)^9 + (3 \bullet 18)^3 = (1 \bullet 3)^{11}$ $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$ $(1 \bullet 3)^9 + (18 \bullet 3)^3 = (1 \bullet 3)^3$ $r = 3,\ D = 1,\ x = 9,\ E = 18,\ y = 3$ $= 3^9(1^9 + 18^3 \bullet 3^{3-9})$ $= 3^9(1 + 3^6 \bullet 2^3 \bullet 3^{3-9})$ $= 3^9(1 + 3^6 \bullet 3^{3-9} \bullet 2^3)$ $= 3^9(1 + 2^3)$ $= 3^9(1 + 8)$ $= 3^9(9)$ $= 3^9(3^2)$ $\boxed{= 3^{11}}$	$F = 1,\ r = 3,\ z = 11$ $(Ft)^z = (1 \bullet 3)^{11}$ $\boxed{= 3^{11}}$ Observe above that it has been shown that $r^x(D^x + E^y r^{y-x}) = F^z t^z$.

Example 6B confirmed the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived. (from the formula $r^x(D^x + E^y r^{y-x}) = F^z t^z$).

Example 7A: $\boxed{33^5 + 66^5 = 33^6}$ $A = 33,\ B = 66,\ C = 33,\ x = 5,\ y = 5,\ z = 6$, $A^x + B^y = C^z$

Change the sum $33^5 + 66^5$ to a single power of 33..

Factor out the greatest common factor.	
$33^5 + 66^5$	
$= (11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5$	
$= 11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5$	
$= 11^5 \bullet 3^5 \underbrace{(1 + 2^5)}_{\text{critical sum}}$ (G) <---------------------	This step requires that 33^5 and 66^5 have a common prime factor
$= (11 \bullet 3)^5 (1 + 2^5)$	It is interesting how the $1 + 2^5$ provided the much needed 33
$= 33^5 (33)$	
$= 33^6$	

Similarly, as from above, 33^6 has the common prime factors 3 and 11.

Example 7B Using the derived formula: $\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers. Then $A = Dr$, $B = Es$, and $C = Ft$, $\boxed{(Dr)^x + (Es)^y = (Ft)^z}$

$\boxed{33^5 + 66^5 = 33^6}$

$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5 = (11 \bullet 3)^6$

Conversion Formula: $r^x(D^x + E^y r^{y-x}) = F^z t^z$, where $r = s$ (i.e., A and B have a common prime factor)

For the left-hand-side		For the right-hand side
There are two prime factors 3 and 11. Here one uses the prime factor 3.	Here one uses the prime factor ,11	**For Prime factor, 3**
$\boxed{\text{Formula}: r^x(D^x + E^y r^{y-x})}$	$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5$	$F = 11,\ t = 3,\ z = 6$
$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5$	$r = 11, D = 3, x = 5, E = 6, y = 5$	$(Ft)^z = (11 \bullet 3)^6$
$r = 3, D = 11, x = 5, E = 22, y = 5$	$= 11^5(3^5 + 6^5 \bullet 11^{5-5})$	$\boxed{= 33^6}$
$= 3^5(11^5 + 22^5 \bullet 3^{5-5})$	$= 11^5(3^5 + 6^5 \bullet 1)$	
$= 3^5(11^5 + 22^5 \bullet 1)$	$= 11^5(3^5 + 6^5)$	
$= 3^5(11^5 + 22^5)$	$= 11^5(3^5 + 2^5 \bullet 3^5)$	**For Prime factor, 11**
$= 3^5(11^5 + (2^5 \bullet 11^5)$	$= 11^5 \bullet 3^5(1 + 2^5)$	$F = 3,\ t = 11,\ z = 6$
$= 3^5 \bullet 11^5(1 + 2^5)$	$= (11 \bullet 3)^5(1 + 2^5)$	$(Ft)^z = (3 \bullet 11)^6$
$= (3 \bullet 11)^5(1 + 2^5)$	$= 33^5(1 + 2^5)$	$\boxed{= 33^6}$
$= 33^5(1 + 2^5)$	$= 33^5(33)$	Observe above that it has been shown that
$= 33^5(33)$	$\boxed{= 33^6}$	$r^x(D^x + E^y r^{y-x}) = F^z t^z$
$\boxed{= 33^6}$	**Note:** One gets the same result as using 3 as the prime factor.	

Example 7B confirmed the assumption that it is necessary that the sum $A^x + B^y$ has a common prime factor before C^z can be derived. (from the formula $r^x(D^x + E^y r^{y-x}) = F^z t^z$.)

Example 8 is to show that the application of the substitution axiom is valid

$\boxed{33^5 + 66^5 = 33^6}$

$33^5 + 66^5$

$33^5(1 + \frac{66^5}{33^5})$

$33^5(\frac{33^6}{33^6} + \frac{66^5}{33^5})$ <-------------------------- $(\frac{33^6}{33^6} = 1,$ applying the substitution axiom.

$= 33^5(\frac{33^6 \bullet 33^5 + 33^6 \bullet 66^5}{33^6 \bullet 33^5}$

$= \frac{33^6 \bullet 33^5 + 33^6 \bullet 66^5}{33^6}$

$= \frac{33^6(33^5 + 66^5)}{33^6}$

$= \frac{33^6(33^5 + 33^5 \bullet 2^5)}{33^6}$

$= \frac{33^6 \bullet 33^5(1 + 2^5)}{33^6}$

$= 33^5(1 + 2^5)$

$= 33^5(33)$

$= 33^6$

Note above in Example 1, 2, 3 and 5 , 6 and 7, that the derivation of C^z from the sum $A^x + B^y$ is more efficient by factoring than by applying the formula, $r^x(D^x + E^y r^{y-x})$ **.**

Generalized Conversion of $A^x + B^y$ to C^z and Common Prime Factor Conclusion

Given: $A^x + B^y = C^z$, A,B,C,x,y,z are positive integers and $x,y,z > 2$.
Required: To prove that A, B and C have a common prime factor.
Plan: A necessary condition for A, B and C to have a common prime factor is that A and B must have a common prime factor. The proof would be complete after showing that If A and B have a common prime factor, C^z can be produced from the sum $A^x + B^y$.

Proof: Let r be a common prime factor of A and B. Then $A = Dr$, and $B = Er$., where D and E are positive integers. Also let t be a prime factor of C. Then $C = Ft$, where F is a positive integer. Beginning with $(Dr)^x + (Er)^y$ one will change this sum to the single power, $C^z = (Ft)^z$ as was done in the preliminaries.

$(Dr)^x + (Er)^y$

$= (Dr)^x[1 + \frac{(Er)^y}{(Dr)^x}]$ (Factoring out the $(Dr)^x$)

$= (Dr)^x[\frac{(Ft)^z}{(Ft)^z} + \frac{(Er)^y}{(Dr)^x}]$ $(\frac{(Ft)^z}{(Ft)^z} = 1$, applying the substitution axiom

$= (Dr)^x[\frac{(Ft)^z(Dr)^x + (Ft)^z(Er)^y}{(Ft)^z Dr)^x}]$ (Adding the terms within the brackets)

$= \frac{(Ft)^z(Dr)^x + (Ft)^z(Er)^y}{(Ft)^z}$ (canceling out the $(Dr)^x$)

$= \frac{(Ft)^z[(Dr)^x + (Er)^y]}{(Ft)^z}$ (Factoring out $(Ft)^z$)

$= \frac{(Ft)^z \bullet (Ft)^z}{(Ft)^z}$ $(Dr)^x + (Er)^y = (Fr)^z$

$= (Ft)^z$

Since $C^z = (Ft)^z$ was obtained from $A^x = (Dr)^x$ and $B^y = (Er)^y$ which have the common prime factor r, C^z also has the common prime factor, r. and one can write $(Dr)^x + (Er)^y = (Fr)^z$, where $t = r$. Therefore, A, B and C have a common prime factor.

Conclusion

The main principle in this paper is that the two powers, A^x and B^y of a common prime factor are being added to form a single third power, C^z. A factorization process and a conversion formula derived from $A^x + B^y$ tested perfectly in converting $A^x + B^y$ to C^z on the numerical sample equations. Thus the conversion formula confirmed the necessity that A and B must have a common prime factor, otherwise, the sum $A^x + B^y$ cannot be converted to a single power of C. Step (G) in each numerical equation requires that A and B have a common power. Since C is derived from $A^x + B^y$, C will have the same common factor as $A^x + B^y$. Therefore, without $A^x + B^y$ with a common factor, there would be no C. Note in the examples that C is derived solely from the sum $A^x + B^y$. Thus, to derive C, A and B must have a common prime factor, and if C is derived from $A^x + B^y$ with a common prime factor, C will also have the same common prime factor. Therefore if $A^x + B^y = C^z$, where A,B,C,x,y,z are positive integers and $x,y,z > 2$, then A, B and C have a common prime factor.

PS: Other proofs of Beal Conjecture by the author are at viXra:1702.0331; viXra:1609.0383, viXra:1609.0157.
Adonten